ROBUST ADAPTIVE CONTROLLER FOR UNCERTAIN NONLINEAR SYSTEMS

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ABSTRACT

Control robustness is study for uncertain nonlinear systems transformed in semi strict feedback systems. An adaptive backstepping sliding mode control and an adaptive backstepping control robustness is considered. Robustness is studied when the unknown parameter are constants and varied. The simulation results prove that the sliding mode control is more robust than the backstepping control to tracking trajectory and to estimate the unknown parameter in two cases..

KEYWORDS

adaptive backstepping control, adaptive backstepping sliding mode control, uncertain nonlinear system

1. INTRODUCTION

The systems are made more complicated due to the technology development. Due to modeling nonlinear systems, an uncertainty term appears. Many conditions caused the uncertainty like the parameter variation, neglected term………………

To solve the tracking trajectory problem, many controllers are designed to uncertain nonlinear systems like sliding mode controller, backstepping controller, PID controller………..
The sliding mode technique is mostly applied to build a controller. The sliding mode controller is known as one of robust controller to reject perturbation and uncertainty. The sliding mode control problem is the chattering. Many solutions are proposed to eliminate its.

In Literature ([1], [2], [3]), it was shown that the chattering phenomenon disappears with the higher order sliding mode control algorithms.

The construction of the lyapunov function is complicated. The backstepping is a recursive method to build a lyapunov function.

The backstepping technique is recently applied to construct an adaptive controller for some systems form like uncertain nonlinear systems, electrical machine.

The backstepping control solves both the problem of stabilization and tracking trajectory of nonlinear systems [4]

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In [5], the author proves that the backstepping control does not guarantee the tracking trajectory asymptotic convergence in the presence of measurement noise on the state and the output and input constraints.

In ([6], [7], [8]) the authors studied the backstepping technique for uncertain systems which is proved that the backstepping is a simple method to construct the control law and the simulation results show a good performance to stabilize and ensure the output tracking trajectory for class of systems.

The combined backstepping and sliding mode technique is one of proposed solution which the chattering is eliminated and the control law still robust to reject perturbation and uncertainty. Besides the backstepping sliding mode ensure simplicity to construct the lyapunov function.

In [9], a comparison between backstepping sliding mode control and PID control is made which the author shown that the backstepping sliding mode control applied to an electrical machine improve the tracking trajectory error of 50% than that obtained by the PID control.

In ([10], [11], [12]), the authors compared two technique, the backstepping and the sliding mode which prove that the backstepping is robust to eliminate the chattering and ensure the tracking trajectory.

The theoretical backstepping sliding mode control studies are limited for uncertain nonlinear systems transformed in a semi strict feedback system. This control was extended to an adaptive form which is showed that the backstepping sliding mode control is robust to tracking trajectory ([6], [7], [9], [13]).

In this work, a comparative study between an adaptive backstepping control and an adaptive backstepping sliding mode control for an uncertain nonlinear system are presented. A parameter variation case is considered to test the robustness of the two control technique .A simulation results are presented.

2. SYSTEM CLASS

The backstepping controller is designed practically for an uncertain nonlinear system
Considering the uncertain nonlinear system:

$$\dot{X} = F(X) + G(X)\theta + Q(X)u + D(X, w, t)$$

With $X \in R^n$ is the state vector and u is a scalar control law. The functions $F(X) \in R^n, G(X) \in R^{nxp}$ *and* $Q(X) \in R^n$ are known, $D(X, w, t) \in R^n$ is an unknown functions, w is the uncertain parameter vector and time variation and θ the constant and known parameter vector.

If the diffeomorphisme $x = x(X)$ and an input output linearization conditions are satisfied then the systems is transformed in a semi strict feedback form

$$\begin{cases} \dot{x}_1 = x_2 + \rho_1^T(x_1)\theta + \eta_1(x,w,t) \\ \dot{x}_2 = x_3 + \rho_2^T(x_1,x_2)\theta + \eta_2(x,w,t) \\ \vdots \\ \dot{x}_n = f_n(x) + g_n(x)u + \rho_n^T(x_1,x_2\ldots x_n)\theta + \eta_n(x,w,t) \\ y = x_1 \end{cases}$$

With $x \in R^n$ a state vector, $y \in R^p$ the system output, $f_n(x), g_n(x) \in R$ and $\rho_i(x_1, x_2, \ldots x_n) \in R^p, i = 1 \ldots n$ the known nonlinear function, which are smooth and Lipchitz, u the scalar control

$\eta_i(x,w,t), i = 1 \ldots n$ is a known scalar nonlinear incorporate all perturbation. It is bounded by a positive known function $h_i(x_1 \ldots x_n) \in R$

$$|\eta_i(x,w,t)| \leq h_i(x_1 \ldots x_n), i = 1 \ldots n$$

The output reference is bounded and n order derivative

3. ADAPTIVE BACKSTEPPING CONTROL

The backstepping aims is to use the state as a virtual drive. However the system is then divided into a set of subsystems united descending order. The control law appears at the last step of the backstepping algorithm. At intermediate stages, the instability of the nonlinear system is processed and the order of the system increased from stage to another. Global stability is guaranteed, it ensures the continuation and regulation of nonlinear system. The backstepping algorithm is organized as follow:

The adaptive backstepping algorithm consists by the following steps:

Step 1: Let the variable error $z_1 = x_1 - y_r$

Or $\dot{z}_1 = \dot{x}_1 = x_2 + \rho_1^T(x_1)\theta + \eta_1(x,w,t) - \dot{y}_r$ (1)

$\tilde{\theta} = \theta - \hat{\theta}$ is the parameter error with $\hat{\theta}$ is the estimate of θ

We have: $\dot{z}_1 = \dot{x}_1 = x_2 + \rho_1^T(x_1)\hat{\theta} + \eta_1(x,w,t) + \rho_1^T(x_1)\tilde{\theta} - \dot{y}_r$

Considering the subsystems (1) is stable and the lyapunov function

$$V_1(z_1, \hat{\theta}) = \frac{1}{2} z_1^2$$

The lyapunov function V_1 derivative is:

$$\dot{V}_1(z_1, \hat{\theta}) \leq -cz_1^2 + z_1 z_2 + z_1 \rho_1^T \tilde{\theta}$$

With c_1 is a positive constant

$$z_2 = x_2 - \dot{y}_r - \alpha_1$$

$$\alpha_1 = -c_1 z_1 - h_1 - \rho_1^T(x_1)\hat{\theta}$$

the z_2 derivative is:

$$\dot{z}_2 = \dot{x}_2 - \ddot{y}_r - \frac{\partial \alpha_1}{\partial x_1}\dot{x}_1 - \frac{\partial \alpha_1}{\partial y_r}\dot{y}_r - \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}}$$

$$V_2\left(z_2, \hat{\theta}\right) = V_1\left(z_1, \hat{\theta}\right) + \frac{1}{2}z_2^2$$

The V_2 derivative is:

$$\dot{V}_2\left(z_2, \hat{\theta}\right) = \dot{V}_1\left(z_1, \hat{\theta}\right) + z_2\dot{z}_2$$

$$\dot{V}_2\left(z_2, \hat{\theta}\right) \le -c_1 z_1^2 - c_2 z_2^2 + z_2 z_3 + z_1\rho_1^T\tilde{\theta} + \tilde{\theta} z_2(\rho_2^T - \frac{\partial \alpha_1}{\partial x_1}\rho_1^T)$$

With $z_3 = x_3 - \ddot{y}_r - \alpha_2$

$$\alpha_2 = -z_1 - c_2 z_2 - \rho_2^T\hat{\theta} + \frac{\partial \alpha_1}{\partial x_1}x_2 + \frac{\partial \alpha_1}{\partial x_1}\rho_1^T\hat{\theta} + \frac{\partial \alpha_1}{\partial x_1}h_1 + \frac{\partial \alpha_1}{\partial y_r}\dot{y}_r + \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}} - h_2$$

Step i *(*$1 < i \le n-1$*):* Define $z_i = x_i - \alpha_{i-1} - y_r^{(i-1)}$

$$\alpha_i\left(x_1, \dots x_i, \hat{\theta}\right) = -z_{i-1} - c_i z_i - \rho_i^T\hat{\theta} + \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}(x_{j+1} + h_j) + \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}$$
$$+ \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}\rho_j^T\hat{\theta} + \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial y_r^{(j-1)}}y_r^{(j)} - h_i \tag{2}$$

Where $c_i > 0$ and the z_i derivative is:

$$\dot{z}_i = z_{i+1} + \alpha_i + \rho_i^T\theta - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}(x_{j+1} + \eta_j) - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}\rho_j^T\theta -$$
$$\sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial y_r^{(j-1)}}y_r^{(j)} + \eta_i$$

$$V_i = V_{i-1} + \frac{1}{2}z_i^2$$

$$\dot{V}_i = \dot{V}_{i-1} + z_i\dot{z}_i$$

$$\dot{V}_i \le -\sum_{j=1}^{i}c_j z_j^2 + z_i z_{i+1} + \tilde{\theta}^T(\sum_{j=1}^{i}z_j(\rho_j - \sum_{k=1}^{j}\frac{\partial \alpha_{k-1}}{\partial x_k}\rho_k))$$

Step n:

Define

$$z_n = x_n - \alpha_{n-1} - y_r^{(n-1)}$$

Where α_{n-1} is the virtual control, it is obtained by the equation (2) in the case $n = i$.

The z_n derivative is:

$$\dot{z}_n = f_n(x) + g_n(x)u + \rho_n^T(x,t)\theta + \eta_n - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_i}(x_{i+1} + \eta_i) - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} -$$

$$\sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(i-1)}} y_r^{(i)} - y_r^{(n)} - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_i} \rho_i^T \theta$$

Considering the lyapunov function

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 + \frac{1}{2} \tilde{\theta}^T \Gamma \tilde{\theta}$$

The V_n derivative is :

$$\dot{V}_n \le -\sum_{i=1}^{n} c_i z_i^2 + \tilde{\theta}^T (\sum_{i=1}^{n} z_i (\rho_i - \sum_{j=1}^{i} \frac{\partial \alpha_{j-1}}{\partial x_j} \rho_j)) - \tilde{\theta}^T \Gamma \dot{\hat{\theta}} + z_n (z_{n-1} + g_n(x)u +$$

$$\rho_n^T (x_1, \ldots, x_n)\hat{\theta} + h_n + c_n z_n - y_r^{(n)} - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_i} (x_{i+1} + h_i) - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_i} \rho_i^T (x_1, \ldots, x_i)\hat{\theta} -$$

$$\sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(i-1)}} y_r^{(i)} - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} + f_n(x))$$

The adaptive backstepping control is stable and robust if :

$$\dot{V}_n \le -\sum_{i=1}^{n} c_i z_i^2$$

$$\dot{V}_n \le -Z^T C Z$$

With $c_i > 0$, $C = [c_1 \ c_2 \ldots\ldots c_n]^T$, $Z = [z_1 \ z_2 \ldots\ldots z_n]^T$

- The adaptation parameter law is :

$$\dot{\hat{\theta}} = \Gamma \sum_{i=1}^{n} z_i (\rho_i - \sum_{j=1}^{i} \frac{\partial \alpha_{j-1}}{\partial x_j} \rho_j)$$

- The backstepping control is:

$$u=\frac{1}{g_n(x)}\begin{bmatrix} -z_{n-1}-c_n z_n - f_n(x)-\rho_n^T(x,t)\hat{\theta}+\sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}(x_{i+1}+h_i) \\ +\sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}\rho_i^T(x_1,\dots,x_i)\hat{\theta}+\sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_r^{(i-1)}}y_r^{(i)}+\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}- \\ h_n+y_r^{(n)} \\ \end{bmatrix}$$

4. ADAPTIVE BACKSTEPPING SLIDING MODE CONTROLLER

The construction of the backstepping control law didn't affected by the nonlinearity. For that it is widely studied and used as a synthesis tools to build several control laws like the sliding mode control. Combining backstepping and sliding mode technique eliminates the chattering and ensures robustness to uncertainties and disturbances. This command is extended to the adaptive form and showed robustness in regulation and tracking trajectory.
The backstepping sliding mode control is expressed by:

$$u_{bsm}=u+u_{disc}$$

The backstepping sliding mode control has the same form as a sliding mode control. Which the backstepping algorithm is used to determine the control u as an equivalent control

And the control u_{disc} is chosen such that the trajectories of the states reach the sliding surface and stay there.

In this study, on choice the discontinuous control u_{disc} as follow:

$$u_{disc}=sign(z_1)(kz_1+\mu)$$

Which:

z_1 : The backstepping variable error

k and μ : Are positive constant

5. ROBUSTNESS STUDY

Consider the uncertain nonlinear system is transformed to a semi strict parameter:

$$\begin{cases} \dot{x}_1=x_2+\theta x_1+2x_1^2\cos(3x_1x_2) \\ \dot{x}_2=u \\ y=x_1 \end{cases}$$

With : $\rho_1^T(x_1)=[x_1 \quad 0]$; $\rho_2^T(x_1,x_2)=[0 \quad 0]$; ; $f_n(x)=0$; $\eta_1=2x_1^2\cos(3x_1x_2)$,

$\eta_2=0$; $h_1=2x_1^2$, $h_2=0$, $g_n(x)=1$, $y_r=0.2$

Adaptation law :

$$\dot{\hat{\theta}} = \Gamma z_2 \left(x_1 \hat{\theta} + 4x_1^2 - c_1 x_1 \right)$$

Backstepping control :

$$u = \ddot{y}_r - z_1 + c_1(c_1 z_1 - z_2) - x_1 \dot{\hat{\theta}} - \hat{\theta} x_2 - \hat{\theta}^2 x_1 - 6x_1^2 \hat{\theta} - 8x_1^3 - 4x_1 x_2 - c_2 z_2)$$

Backstepping sliding mode control:

$$u = \ddot{y}_r - z_1 + c_1(c_1 z_1 - z_2) - x_1 \dot{\hat{\theta}} - \hat{\theta} x_2 - \hat{\theta}^2 x_1 - 6x_1^2 \hat{\theta} - 8x_1^3 - 4x_1 x_2 - c_2 z_2 + sign(z_1)(kz_1 + \mu)$$

Where z_1 et z_2 are an algorithm backstepping intermediate variables such as:

$$z_1 = x_1 - y_r$$

$$z_2 = x_2 - \dot{y}_r + c_1 z_1 + \hat{\theta}_1 x_1 + 2x_1^2$$

The simulation parameters are:

- Backstepping control :

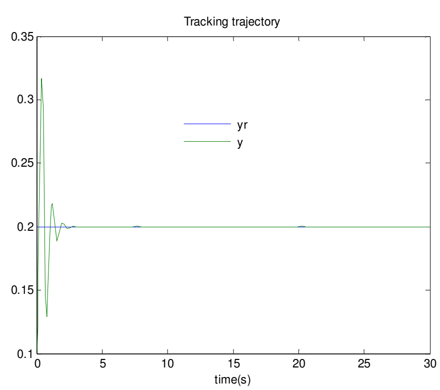
$$\hat{\theta}(0) = 0.1, x_1(0) = 1, x_2(0) = 0;\ \theta = 1, c_1 = 1, c_2 = 1, \Gamma = 20$$

- Backstepping sliding mode control:

$$\hat{\theta}(0) = 0.1, x_1(0) = 1, x_2(0) = 0;\ \theta = 1, c_1 = 5, c_2 = 1; k = 0.01, \mu = -80, \Gamma = 20$$

Simulation results show that the output converges to the reference yr in 4s adaptive backstepping sliding mode control (Figure.1.a, Figure.2.a). To estimate the unknown parameter, the adaptive backstepping control takes 12s to reach the desired value whereas the adaptive control backstepping sliding mode make 5s to converges (Figure.1.b, Figure.2.b).

From these results, we conclude that the backstepping adaptive control made more time to converge, tracking trajectory and estimate unknown parameter, than the adaptive sliding mode control.



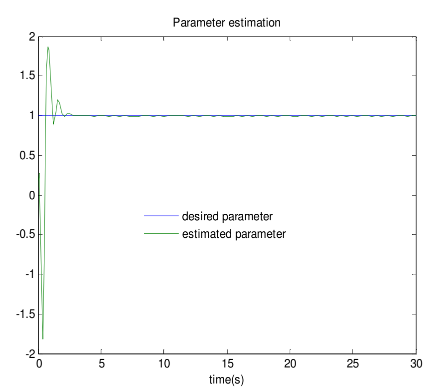


Fig.1.a. Tracking trajectory

Fig. 1.b parameter estimation θ

Fig.1. Trajectory tracking, parameter estimation by backstepping sliding mode control with constant parameter

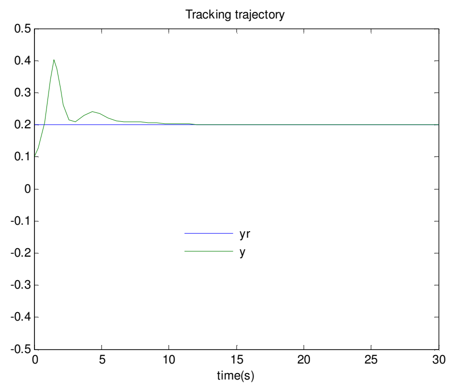


Fig. 2.a. Tracking trajectory

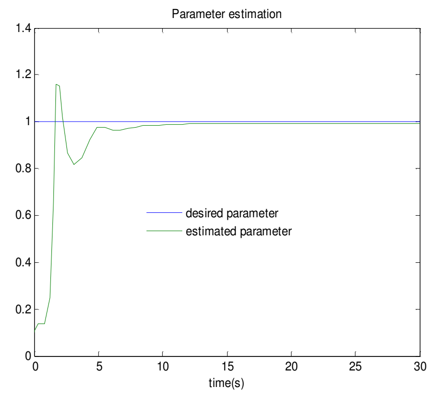


Fig. 2.b parameter estimation θ

Fig.2. Trajectory tracking and parameter estimation by backstepping control with constant parameter

Varying the unknown parameters such as:

$$\theta = \begin{cases} 1 & si\, 0 \leq t \leq 15s \\ 2 & si\, 15s \leq t \leq 30s \end{cases}$$

We show that the adaptive backstepping sliding mode control is more robust to tracking trajectory and estimate the unknown parameters (Fig.3.a, Fig.3.b, Fig.4.a, and Fig.4.b). At parameters variation instants, the adaptive backstepping sliding mode control guarantees the convergence of the output to the reference in a time less than the adaptive backstepping control (Fig.3.a)

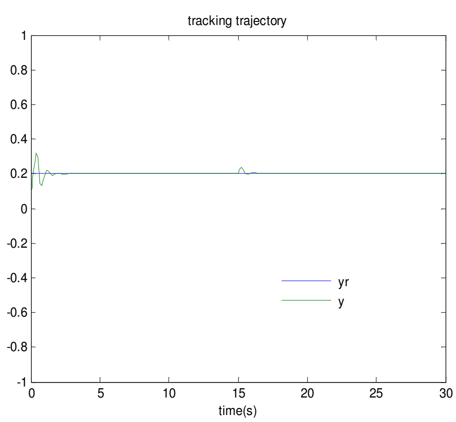


Fig. 3.a. Tracking trajectory

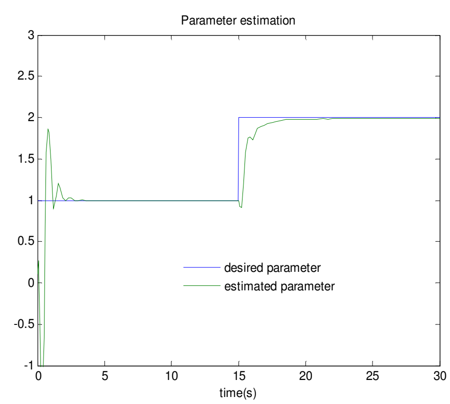
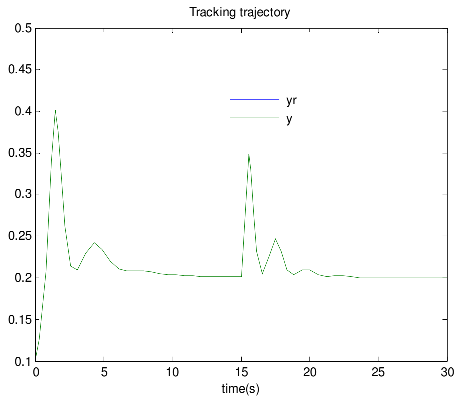


Fig. 3.b parameter estimation θ

Fig.3. Trajectory tracking, parameter estimation by backstepping sliding mode control with parameter variation



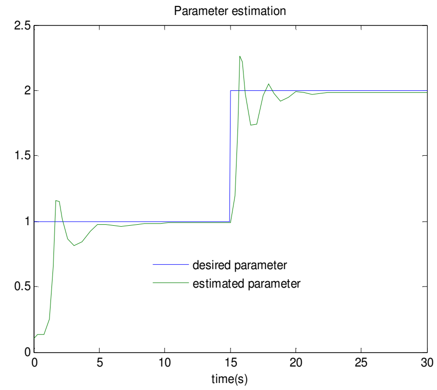


Fig. 4.a. Tracking trajectory

Fig. 4.b parameter estimation θ

Fig.4. Trajectory tracking, parameter estimation by backstepping control with parameter variation

Simulation results show that the adaptive backstepping sliding mode control is more robust to tracking trajectory and to estimate the unknown parameters and to ensure stability of the system which the unknown parameters are constant or variants.

6. CONCLUSION

In this paper, a comparative study is made between a backstepping sliding mode control and a backstepping control. The studied system is an uncertain semi strict feedback systems which is most applied to a backstepping sliding mode control. The robustness study show that the adaptive backstepping sliding mode control ensures the asymptotic convergence and eliminate the chattering. To conclude the backstepping sliding mode control is robust than the adaptive backstepping control to tracking trajectory and to estimate the unknown parameter when the parameter are constant or varied.

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