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RESEARCH ARTICLE

PREDICTING PRESSURE FILTER NET SOLIDS YIELD USING BUCKINGHAM-π METHOD OF LMT DIMENSIONAL ANALYSIS.

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Abstract

A model for predicting Pressure filter Net Solids Yield has been derived using Buckingham-π method of LMT Dimensional Analysis. It was observed in previous studies that addition of large amounts of chemical conditioners could alter the initial solids content of the original slurry to be filtered. Increment in the initial solids content also occurs when skeleton builders are mixed with chemical conditioners. The filter cake yield of such slurry would definitely be higher while the resulting specific resistance would be lower. To account for these additions, it is important that a valid filter cake yield model with a correction factor, F be derived. In furtherance of the above, a new equation expressing the filter cake yield as net sludge solids was derived with further incorporation of the cake compressibility coefficient. It was established that filter net solids yield is directly proportional with the operating pressures, initial sludge solids content, and conditioner dosages while being inversely proportional to specific resistance to filtration, pressing time, sludge viscosity and sludge compressibility . The difference between the actual cake yields and the net solids yield accounts for the quantity of the ferric chloride coagulant used in this study. The new equation enables performance of a pressure filter (Filter Press) to be predicted from a simple laboratory determination of cake yields. Moreover, curves derived from the model show how increasing ferric chloride dosage from 11.87% to 22.61% increased net solids yield from 2.4254g/Cm^2.S to 2.7052g/Cm^2.S while reducing specific resistance from 1.7372 x 10^{10}Cm/g to 1.5940 x 10^{10}Cm/g. The optimum dosage from the graph to attain acceptable filtrate quality was 19.61% at an operating pressure of 5098.58g/Cm^2. Also, considering the differences in the parameters tested, it is significant that they all responded similarly when compared with previous research works.

Introduction:-

Sludge management from both domestic and industrial process operations occasioned by modern advancement in technology is highly complex and cost effective, hence, if poorly accomplished, may jeopardize the environmental and sanitary advantages expected from the treatment. The importance attached to sludge management was widely

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acknowledged by Agenda 21, which included in the theme of environmentally wholesome management of solid wastes; sludge issues and as well defined the following modalities towards its administration; reduction in production, maximum increase in reuse and recycling and the adoption of environmentally wholesome treatment and disposal, Andreoli C.V et al. (2006).

Moreover, due to the low indices of wastewater and other industrial processes resulting from high population increase and industrialization especially in developing countries, there is need to explore and adopt more scientific options to meet the demand. As a consequence, the amount of sludge produced is expected to increase posing serious environmental and health concerns to the developing nations (Afangideh et al., 2014; Ghazy et al., 2009).
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Brix (2017) had stated that Wastewater treatment processes such as activated sludge treatment systems produce surplus sludge which has to be disposed of. The surplus sludge contains valuable nutrients and organic matter that can be used to improve soil quality and as a fertilizer for agricultural crops. Sludge treatment processes generally have two main purposes: (i) thickening and dewatering whereby the sludge volume and hence the costs of subsequent handling, transportation, and disposal are reduced Uggetti et al. (2010).

Recent studies conducted by (Randal,2001; revealed that despite the fact that the volume of sludge tends to be less than 1% of the total plant influent, sludge handling costs ranges between 21-50% of total plant operation and maintenance costs,. Dewatering of sewage sludge is not only found in removal of the excess moisture but to render the sludge odourless and nonputrescible, Garg (2009). Ademiluyi et al. (2012), Olukanni et al. (2013) maintained that dewatering of sewage sludge prior to drying or disposal is an important step because the lower the water content of the sludge, the lesser the transport costs. Proper waste management system should be established and enhanced in view of menace imposed in our community due to improper handling and disposal of wastes to our environment.

Dimensional Analyses is a conceptual tool often applied in physics, chemistry and engineering to understand physical solution involving a mixture of different kinds of physical quantities. Dimensional formulae provide a useful catalogue system for physical quantities according to Rajput (2004). The principle of dimensional homogeneity states that in a physical equation consisting of an algebraic sum of two or more terms, the exponent of the dimension of Length, Mass and Time in any term of the equation must be the same as that in any other term. The system of fundamental units commonly used in Newtonian mechanics is the LMT System.

The Concept of Filter Cake Yield Modeling:-

A Panoramic view of Previous Works on Cake Yield Models:-

The essence of the research was aimed at formulating a new cake yield equation with compressibility attribute as a measure of filterability. The incorporation of the compressibility coefficient 'S' is against the traditional filtration equations already suggested by (Carman,1934 & 1938; Gale et al., 1970, Rebhun et al., 1989 and Jones, 1956) where the sludge compressibility effects on filterability was obviously unaccounted for. It has been discovered in literature that the traditional equations were embedded with uncertainties in the areas of formulating them. Peter and Donald (1975) stressed that since the literature is replete of dewatering operations which have unsatisfactory performance predictions and formulations and considering the controversies among prominent researcher to the present knowledge of filtration equations, it is justified that an acceptable equation which characterize the filtration process has to be derived. The equation to be derived must contain the compressibility coefficient 'S' as an attribute. The incorporation of 'S' will make such equation acceptable to the previous researchers.

Carman derived his equation based on non-compressible sludge cakes. In his equation, which was a modification of Darcy's equation, he stressed that the specific resistance is constant throughout the filtration process. Hence, Carman (1934) proposed the equation

$$\frac{dV}{d\theta} = \frac{P.A^2}{\mu(RCV + R_m A)} \qquad (1)$$

Where:
A = filter area, m^2
R_m = septum resistance, kg/m
V = volume of filtrate. m^3
P = pressure drop, Kg/m^2

C = concentration of solids in the feed, Kg/m^3
$R =$ specific cake resistance, kg/m
μ = liquid viscosity, poise
θ = filtration time, s
Integrating the above and neglecting septum resistance, a modified form of the equation by relating specific resistance, R with cake yield is obtained as:

$$Y = (\frac{2PC}{\mu Rt})^{\frac{1}{2}} \quad (2)$$

Where Y is the cake yield (Kg/m^2S)

Jones (1956) while modifying the previous work of Coackely et.al (1956) for sludge undergoing rotary filtration process assumed that for a yield equation to be fully described as parabolic, the initial specific resistance must be assumed as zero. Sludge compressibility coefficient was not accounted for in his derivation.

Gale et al. (1970) questioned Dahlstron et al. (1958) for oversimplifying their consideration of the effects of the resistance of the filter medium when dealing with highly compressible cake and thereafter considered mathematically only those equations where the filter medium is negligible. Based on that, they developed a yield model for a rotary vacuum filter from basic filtration equations. It was suggested that in order to express yield in place of the true specific resistance, without approximations of the variable as evidenced in Buchner funnel tests and in agreement with Anazodo's (1974) latter observations, then a true value reflecting both specific resistance and Initial solids contents should be used as suggested in their model.

A study on the effects of skeleton builders on oily sludge using filter press apparatus was carried out by Zall et al. (1987). They found out that filter press raw sludge yield increased continuously with added conditioner dose. They also maintained that even though the yield curves represent the rate of raw sludge solids capture during the filtration process, they cannot be used directly to determine the practical optimum conditioner dose. They thereafter developed a filter press yield model at 90% completion of filtration.

Rebhum et al. (1989) in their model maintained that Carman's modified cake yield equation can only be applied to determine a sludge filtration process if sludge conditioning does not appreciably alter the solids content of the original slurry, as is usually with chemical coagulants. He stated that if the main objective of sludge conditioning is to improve filter yield, it is better to express filterability as yield. He thereafter modified Dick (1972) equation to account for both the original sludge solids and conditioner solids.

However, It is important to note that the traditional Carman's equations, Rebhun, Jones, Gale and co-workers did not account for the compressibility coefficient in the formulation of their equations as highlighted earlier. The mere absence of the compressibility attribute of the sludge in their models rendered them unfit for compressible sludge filtation process. The introduction of compressibility coefficient 'S' as an attribute of the new equation cannot be over emphasized and makes it a novelty. It is in the light of the uncertainties associated with previous cake yield models that this new equation has judiciously accounted for compressibility coefficient 'S' bringing the total variables to nine in it's formulation.

Materials and Method:-

Preparation of Sludge Slurry:-

A photograph of the pressure filter apparatus is seen in Figure 1.

Brewery sludge slurry was prepared by mixing given quantities of the desired sludge in 1 liter of distilled water and agitating it for few minutes. 60 ml was drawn from the slurry using a pre-weighed cylinder. The weight of the cylinder and slurry was measured and recorded as w_1. The slurry was then oven dried at 103^0C for 24hours. The new weight was thereafter noted and recorded as w_2. The concentration of the slurry was then calculated as fully described. The filtration process was started by connecting the compressed air line to the top of the sample holder using an easy push through arrangement. Thus, the suspension was forced to flow through the filter cell producing a filter cake at the surface of the filter paper. As the filter cake deposited, the flow rate kept declining. The filtrate was collected in a graduated cylinder placed in a tilted position so that the filtrate traveled along the walls of the cylinder without causing a splash, enabling accurate determinations of equal increments of the filtrate volume.

After about 60 ml of the suspension had filtered through and had formed a cake on the surface of the filter paper, time increments required to produce successive equal increments of filtrate volume were observed. The data were plotted in the appropriate manner and both the cake yield and the specific resistance of the filter cake were then calculated using equation 27.

Constant pressure filtration experiments were carried out respectively 0n $0.0128g/Cm^3$, $0.0194g/Cm^3$.$0.0220g/Cm^3$,$0.02465g/Cm^3$ and $0.0393g/Cm^3$ sludge samples using different doses of Ferric Chloride suspensions. The filtration pressure ranges investigated was between 2039.43g/Cm^2-6628.155g/Cm^2. Filtration was allowed to proceed and stopped once deliquoring, which was determined when the *t/V* against *V* plot experienced a sudden change in accordance to traditional filtration behavior was deemed to be beginning. On the conclusion of filtration the formed cake in the filter cell was removed and put in a pre-weighed beaker. It was then dried over a period of twenty four hours and reweighed. The raw data from the laboratory, pilot filter runs made to evaluate the effect of pressure drop, initial solids contents, conditioner dosages, specific resistance and compressibility on cake yields are as shown in table 3.

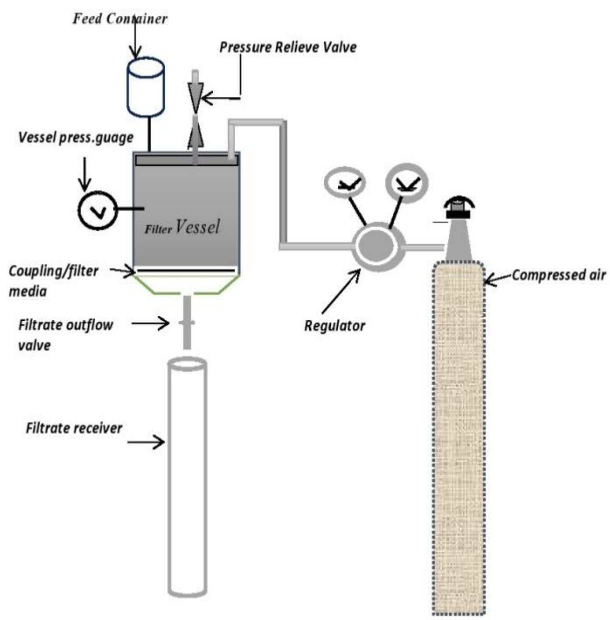

Fig. 1:-The Constant Volume Filter Press Assembly

Developing the New Filtration Equation:-

In order to derive the new Filter Cake Yield equation, the Buckingham's π-method of dimensional analysis was employed. The sludge cake yield (Y) is a function of volume of the sludge (V), filter paper area (A), time of filtration (θ), mass of solids per unit volume of filtrate (C), net filtration pressure (P), viscosity of filtrate (μ), the average specific resistance of filter cake (R) and Sludge Compressibility (S). This is mathematically expressed as Equation (4). Table 1 is a summary of the relevant variables and their dimensions as applied in this derivation.

The relationship between the parameters can be established by assigning any arbitrary value to the exponent of the variables of interest and expressing the variable as a product of others hence,

$$Y = P^a A^b \mu^c C^d V^e R^f \theta^g S^h \qquad (3)$$

$$Y = f(P, A, C, V, \mu, \theta, Y, R, S) \qquad (4)$$

Or

$$f(P, A, C, V, \mu, \theta, Y, R, S) = 0 \qquad (5)$$

From the Buckingham's π-method theories, the total number of variables (n) is nine while the number of fundamental dimensions (m) is three.

Hence, the number of π- terms is n – m, 9 – 3 = 6. Therefore, number of π-terms in the equation can be written as:

$$f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6) = 0 \quad (6)$$
$$\pi_1 = P^a A^b \mu^c y \quad (7)$$
$$\pi_2 = P^a A^b \mu^c R \quad (8)$$
$$\pi_3 = P^a A^b \mu^c C \quad (9)$$
$$\pi_4 = P^a A^b \mu^c \theta \quad (10)$$
$$\pi_5 = P^a A^b \mu^c V \quad (11)$$
$$\pi_6 = P^a A^b \mu^c S \quad (12)$$

Where π_1 to π_6 are dimensionless terms while a, b, and c are exponents to be determined by dimensional Analysis.

Table 1:-Summary of LMT Dimensional formula

Physical Variables	Symbols	Dimensions
Yield	Y	$ML^{-2}T^{-1}$
Volume	V	L^3
Filtration Area	A	L^2
Time for Filtration	θ	T
Mass of cake dry solids per unit volume volume of filtrate	C	ML^{-3}
Net filtration Pressure	P	$ML^{-1}T^{-2}$
Viscosity of filtrate	μ	$ML^{-1}T^{-1}$
Average specific resistance of filter cake	R	LM^{-1}
Compressibility coefficient	S	$M^{-1}LT^2$

Considering π1– Term:-

By replacing the right hand side of equation (7) with the corresponding dimensions of the variables and the dimensionless term on the left hand side with $M^0 L^0 T^0$, equation obtained is given as:

$$M^0 L^0 T^0 = (ML^{-1}T^{-2})^a (L^2)^b (ML^{-1}T^{-1})^c (ML^{-2}T^{-1}) \quad (13)$$

Where a, b, c, are unknowns to be determined using dimensional homogeneity between variables.

Equating the exponents of M, L and T on the left hand side to the corresponding exponents on the right hand side, we get,

M :0 = a + c + 1 (14)

L : 0 = -a + 2b –c -2 (15)

T : 0 = -2a –c -1 .(16)

From equation (16), c = -2a-1 (17),

Combining equation (14) and (16), => a-2a-1 +1 =0

=➔ a = 0, c= -1

Solving equation (15) for the values of a=0 and c=-1 yields, b= ½

Substituting the values of a, b and c in Equation (7), we obtain:

$\Pi_1 = \frac{A^{1/2}Y}{\mu}$ (18)

Similarly, analyzing $\pi_2, \pi_3, \pi_4, \pi_5$ and π_6 terms respectively gives:

$\pi_2 = \frac{R\mu^2}{P}, \pi_3 = \frac{PAC}{\mu^2}, \pi_4 = \frac{P\theta}{\mu}, \pi_5 = \frac{V}{A^{3/2}}, \pi_6 = PS$

Substituting the specific expressions for the dimensionless terms $\pi_1, \pi_2, \pi_3, \pi_4, \pi_5$ and π_6 into equation (6), yields:

$$f\left(\frac{A^{1/2}Y}{\mu} . \frac{\mu^2 R}{P} . \frac{PAC}{\mu^2} . \frac{P\theta}{\mu} . \frac{V}{A^{3/2}} .PS\right) = 0 \quad (19)$$

Since equation (19) does not give the exact relationship between the parameters being investigated; there is need to generate experimental data. Following Buckingham's π-method, any of the dimensionless terms of equation (19) can be written as a function of the others hence, it transforms to:

$$\frac{A^{1/2}Y}{\mu} = K\left(\frac{\mu^2 R}{P}\right)^a \left(\frac{PAC}{\mu^2}\right)^b \left(\frac{P\theta}{\mu}\right)^c \left(\frac{V}{A^{3/2}}\right)^d (PS) \quad (20)$$

The exponents in equation (20) can be obtained by regression analysis using experimental data.
For easy determination of the exponents, the above equation can be transformed as;

$$\text{Ln}\frac{A^{1/2}Y}{\mu} = \text{Ln K} + a\,\text{Ln}\frac{\mu^2 R}{P} + b\,\text{Ln}\frac{PAC}{\mu^2} + c\,\text{Ln}\frac{P\theta}{\mu} + d\,\text{Ln}\frac{V}{A^{3/2}}\ e\,\text{Ln PS} \quad (21)$$

Assumptions:-

Let $M = Ln\frac{A^{1/2}Y}{\mu}$, $X_1 = Ln\frac{\mu^2 R}{P}$, $X_2 = Ln\frac{PAC}{\mu^2}$, $X_3 = Ln\frac{P\theta}{\mu}$, $X_4 = Ln\frac{V}{A^{3/2}}$, and $X_5 = PS$

Hence, equation (21) becomes,

$$M = \text{Ln K} + aX_1 + bX_2 + cX_3 + dX_4 + eX_5 \quad (22)$$

From the experimental data obtained (Data too large to reproduce), values of the constants a, b c, d and e were evaluated by Regression using SPSS (Table 2).

Table 2:-LMT Model Coeffiecents

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-0.1571	1.0451	-0.1503	0.8811	-2.2516	1.9374
2.1147	-0.8266	0.0787	-10.5044	0.0000	-0.9843	-0.6689
21.0151	0.1485	0.0620	2.3958	0.0200	0.0243	0.2728
19.0380	-0.1565	0.0441	-3.5456	0.0008	-0.2450	-0.0681
-1.8157	-1.1885	0.4252	-2.7951	0.0071	-2.0407	-0.3364
7.0733	-0.0494	0.0301	-1.6424	0.1062	-0.1096	0.0109

From Table 2 above, LnK = -0.1571, ➔K = 0.8546
a = -0.8270, b = 0.1485, c = -0.1565
d = -1.1885, e = -0.0494

$$\text{But} \quad \frac{A^{1/2}Y}{\mu} = K\left(\frac{\mu^2 R}{P}\right)^a \left(\frac{PAC}{\mu^2}\right)^b \left(\frac{P\theta}{\mu}\right)^c \left(\frac{V}{A^{3/2}}\right)^d (PS)^e,$$

Hence, substituting the values of K, a, b, c, d and e in equation (20) yields,

$$\frac{A^{1/2}Y}{\mu} = K\left(\frac{\mu^2 R}{P}\right)^{-0.827} \left(\frac{PAC}{\mu^2}\right)^{0.1485} \left(\frac{P\theta}{\mu}\right)^{-0.1565} \left(\frac{V}{A^{\frac{3}{2}}}\right)^{-1.1885} (PS)^{-0.0494} \quad (23)$$

$$\boldsymbol{Y = 0.8546\ x\ \frac{P^{0.7696}\,C^{0.1485}A^{1.4313}}{R^{0.827}\,\mu^{0.7945}\,\theta^{0.1565}V^{1.1885}S^{0.0494}}} \quad (\mathbf{24})$$

Equation (24) can be transformed as follows:
By multiplying both sides of equation (24) by V^8 and rearranging,

$$V^{9.189} = \frac{K\ x\ P^{0.7696}\ C^{0.1485} A^{1.4313}}{Y\ R^{0.827}\ \mu^{0.7945}\ \theta^{0.1495}\ S^{0.0494}}\ x\ \frac{V^8}{\theta^{0.007}} \qquad (25)$$

A plot of $\mathbf{V^{9.189}}$ and $\frac{V^8}{\theta^{0.007}}$ gives a straight line with slope, b_2 = 120.68

From the graph (Figure 2), slope, b_2 = 120.68

Hence, equation (25) becomes,

$$Y = \frac{0.8546\ P^{0.7696}\ C^{0.1485} A^{1.4313}}{R^{0.827}\ \mu^{0.7945}\ \theta^{0.1495}\ S^{0.0494}\ x\ b_2} \qquad (26)$$

Equation (26) is the desired cake equation for total pressure filter solids yield.
Substituting the values of b_2 and K in the model yields,

$$Y_{LMT} = 0.007082 \frac{P^{0.770}\ C^{0.149} A^{1.431}}{R^{0.827}\ \mu^{0.795}\ \theta^{0.1495}\ S^{0.049}} \qquad (27)$$

The above model predicts the total filter solids yield comprising of sludge and conditioner solids

However, in order to predict the net sludge solids yield, especially at higher conditioner dosages, a correction factor, F is introduced in equation 27 as follows;

$$Y_N = 0.007082 \frac{F\ P^{0.770}\ C^{0.149} A^{1.431}}{R^{0.827}\ \mu^{0.795}\ \theta^{0.1495}\ S^{0.049}} \qquad (28)$$

Where;

Y_N = Net Sludge Solids Yield, $Kg/M^2.S$,

$$F = \frac{\text{Original sludge solids, } W1}{\text{Original sludge solids, } W1 + \text{Conditioner solids, } W2}$$

Actual cake yield was calculated using the relation, $Y_A = \frac{x_2 - x_1}{Filter\ area\ x\ Cycle\ time}$, ($Kg/M^2.S$), (29)

Where; x_2 is the weight of dry mass deposited on filter paper and beaker (kg)
x_1 is the weight of empty beaker and filter paper (kg)

Results and Discussion:-

Evaluation of the Process Parameters:-

The formulation of the model involved nine (9) parameters listed in equation (5) above. The first six (6) were experimentally determined while the last three were evaluated. Actual cake yields were determined by measuring the weight of the dry mass deposited on the filter medium for each filtration cycle. For the purpose of the parameters mentioned above, a detailed analyses of the raw data obtained from the pressure filtration of the Brewery Sludge Samples from Nigerian Brewery, 9th Mile, Enugu, Nigeria (Data too large to reproduce here) were made. The slopes of t/V against V for the evaluation of specific resistance were done and values of specific resistance evaluated based on Ademiluyi (1981) equation. The actual cake yield from the laboratory analyses were evaluated using equation (29) while the compressibility coefficient was evaluated as slope from the logarithmic plot of the proportional relation between specific resistance and pressure. Moreover, Table 3 values were used for the evaluation of slope, b_2 in equation (26).

Table 3:-Data for the plot of $V^{9.189}$ against $\frac{V^8}{\theta^{0.007}}$

T(s)	V(Cm^3)	$T^{0.007}$ ($S^{0.007}$)	V^8 x 10^{13} (Cm^{24})	$V^8/T^{0.007}$ x 10^{13} ($Cm^{24}/S^{0.007}$)	$V^{9.189}$ x 10^{13} ($Cm^{27.57}$)
10	18	1.016249	0.001102	0.001084	0.034253
20	26	1.021192	0.020883	0.020449	1.005056
30	34	1.024094	0.178579	0.174378	11.82387
40	36	1.026158	0.282111	0.27492	19.99234
50	38	1.027763	0.434779	0.423035	32.85727
60	40	1.029075	0.65536	0.636844	52.64162
70	42	1.030186	0.968265	0.939893	82.42097
80	44	1.031149	1.404822	1.362385	126.3824
90	46	1.032	2.004761	1.942598	190.1436
100	48	1.032761	2.817928	2.728537	281.142
110	50	1.033451	3.90625	3.779813	409.1055
120	52	1.03408	5.345973	5.169785	586.6171
130	53	1.03466	6.225969	6.017407	698.8291
140	54	1.035197	7.230196	6.984369	829.7863
150	55	1.035697	8.373394	8.084792	982.1838

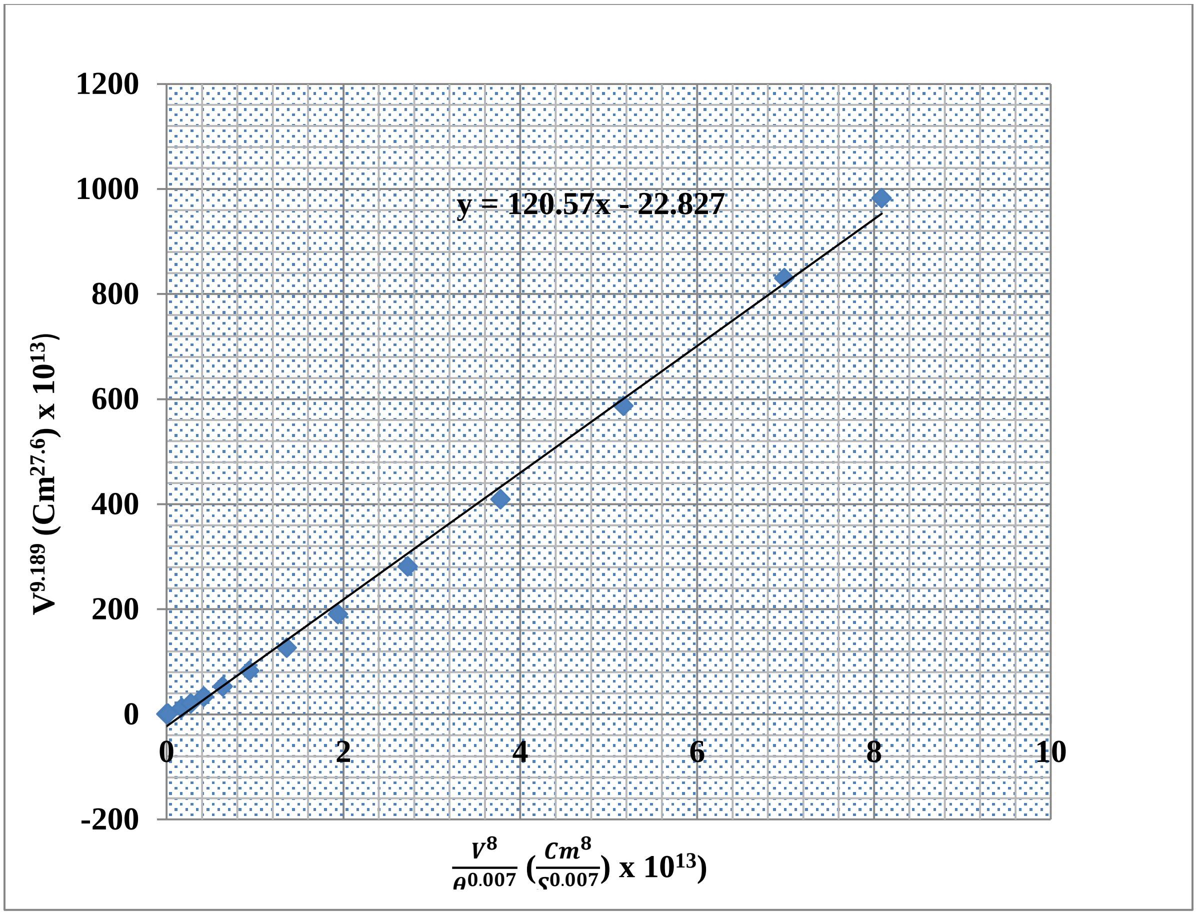

Figure 2:-A plot of $V^{9.189}$ and $\frac{V^8}{\theta^{0.007}}$ for the determination of slope, b_2

Variation of Net Solids Yield with Pressing Time

From the model, net solids yield is inversely proportional with the pressing time. Hence, from equation (27), $\mathbf{Y_N} \propto \frac{1}{\theta^{0.1495}}$.

Hence, $Y_N = \frac{K}{\theta^{0.1495}}$

Where K is proportionality constant given as $[0.007082 \frac{P^{0.770} C^{0.149} A^{1.431}}{R^{0.827} \mu^{0.795} S^{0.049}}]^{0.69}$

The variation of net solids yield with time is however affected by chemical dosages which tend to decrease the amount of period needed for the filtration process to complete.

For instance, if the pressing time is increased from θ to θ^i, net solids yield would decrease from Y_N to Y_N^i. It is important to note that in pressure filters, time of filtration varies as cake thickness increases Jones, (1956). However, the dependence of pressing time on cake yield is shown on figure 2.

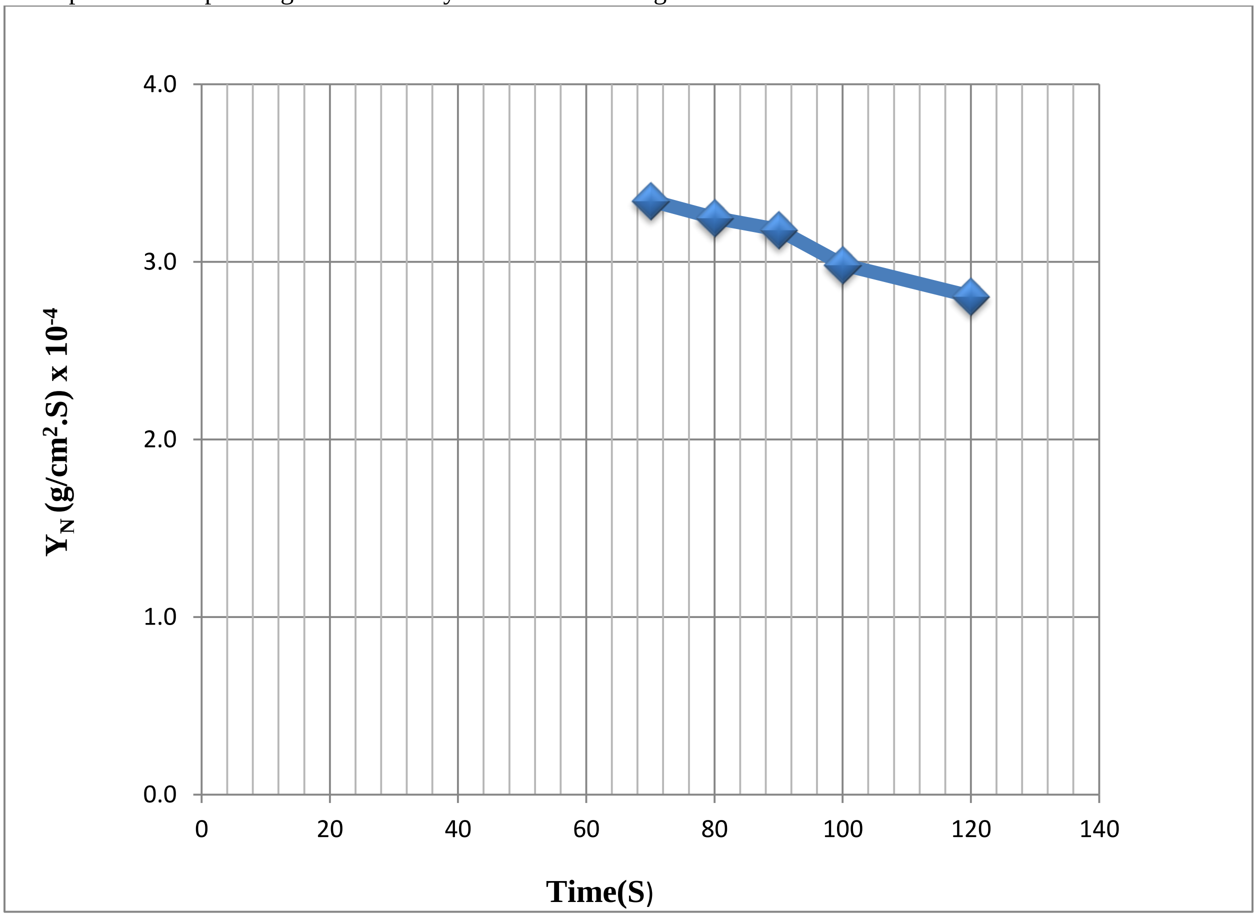

Figure 3:-Variation of Net Solids Yield with Pressing time at various dosages.

Table 4:-Variation of Net Solids Yield with Pressure at different Values of Compressibility, S for 0.02465g/Cm^3 Sludge Sample.

S_1		**S_2**		**S_3**		**S_4**	
Y_N(g/cm^2.S) x 10^{-4}	P(g/Cm^2)	Y_N(g/cm^2.S) x 10^{-4}	P(g/Cm^2)	Y_N(g/cm^2.S) x 10^{-4}	P(g/Cm^2)	Y_N(g/cm^2.S) x 10^{-4}	P(g/Cm^2)
2.7986	2039.43	2.9733	2039.43	2.8875	2039.43	3.2506	2039.43
2.8436	3059.15	2.8239	3059.15	3.2914	3059.15	3.3004	3059.15
3.1623	4078.87	3.2262	4078.87	3.1486	4078.87	3.3191	4078.87
3.0658	5098.58	3.1038	5098.58	3.9852	5098.58	4.0058	5098.58
3.6108	6628.16	3.9979	6628.16	3.2327	6628.16	3.3246	6628.16

Effects of Pressures on Filter Net Solids Yield:-

According to the relationship derived from Darcy's law which relates pressure drop to dry solids yield (equation 3), an increase in pressure drop should result in an increase in dry cake production. This is the case if the filter cake is not highly compressible such that the specific cake resistance increases with pressure drop Svarovsky (2000). It is also beneficial to gradually increase the pressure until a constant pressure is reached. This is because the solids are non-homogeneous and a high initial pressure drop can result in particles plugging the interstices of the cloth Reynolds, et al.(2003).

The graph in figure 4 shows the effects of pressures on filter net solids yields. Yield increases as the operating pressure increases, which is in agreement with both Carman and Jones's findings earlier cited above. As filtration continues, more and more solid settles reducing the porosity of particles so that the pressure of water increases and also the cake yield.

Jones (1956) found that the proportional increase in cake yield with pressure was also a function of sludge compressibility. The effects of operating pressures on the cake yield as studied were partially is in agreement with the findings of Half (1952) where he stated that the yield per hour on a rotary filter was practically constant for pressures greater than 1054.60g/Cm^2. It is important to note that while Half's theory does not hold for less compressible sludge, He could not demonstrate that in his approach.

Mathematically from the model,

$$Y_N \propto P^{0.77}$$

Furthermore, the deterioration of filtrate quality as the pressures were increased cannot be ignored as was the case with previous Researchers. However, physically, it is quite easy to explain. As the operating pressures were increased, sludge flocs were ruptured accounting for the poor filtrate quality.

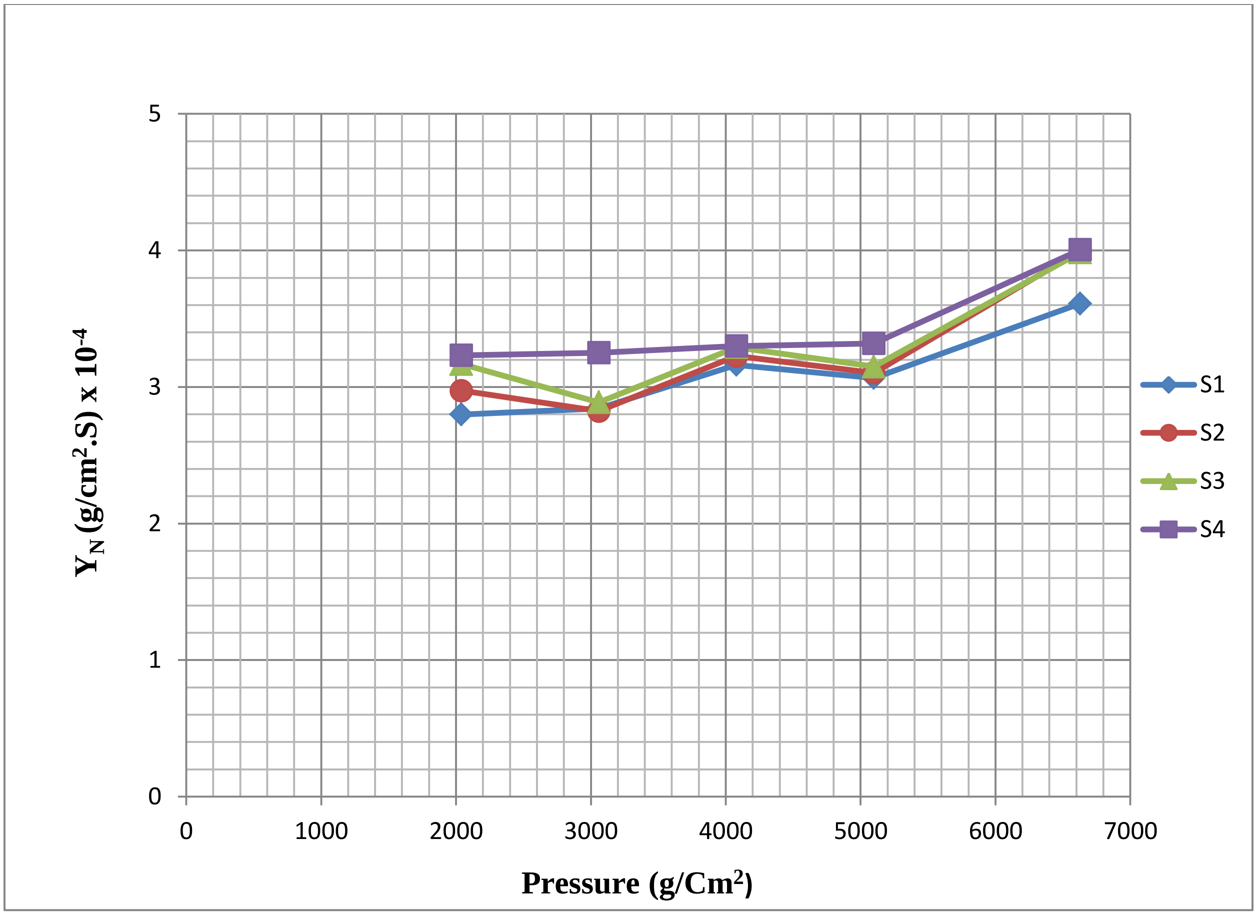

Figure 4:-Effects of Operating Pressures on Filter Net Solids Yield

Effects of Conditioner Dosages on Net Solids Yield:-

From Figures 5, net solids yield increases with increased conditioner dosage until an optimum dosage is reached, all other conditions being equal. For instance, increasing ferric chloride dosage from 11.87% to 22.61% increased filter net solids yield from 3.785g/Cm^2.S to 4.3859g/Cm^2.S while reducing specific resistance from 1.7372 x 10^{10}Cm/g to 1.5940 x 10^{10}Cm/g. The optimum dosage from the graph to attain acceptable filtrate quality is 19.61% for P_5 = 5098.58g/Cm^2. It is important to note that overdosing the unconditioned sludge beyond the optimum requirement mares the solids yield while the possibility of increasing the specific resistance cannot be ruled out. This is due to de-flocculation as a result of excessive surface coverage and charge reversal. Overall, the most important benefit of polymer conditioning of brewery sludge is the improvement in sludge dewaterability. This benefit is based on the proper use of polymers and their integrated effects on sludge characteristics.

In summary, the new model conforms favourably with previous works of (Jones,1956; Zall et al 1989; Gale et al.1970) where the cake yield increased as the rate of conditioning increased while the specific resistance of the sludge decreases.

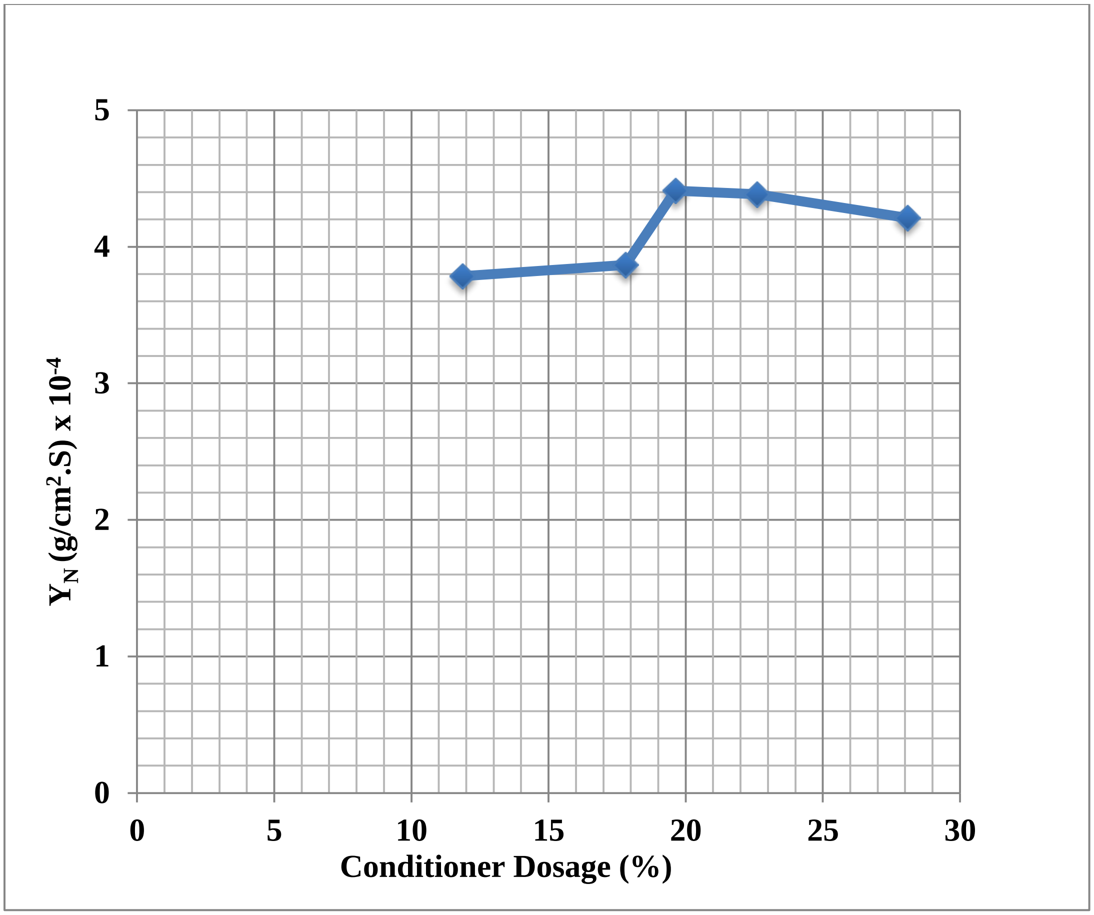

Figure 5:-Effects of Conditioner Dosages on Net Solids yield at operating Pressure of 5098.58 g/Cm^2

Variation of Net Solids Yield with Initial Solids Content at Various Pressures:-

Many Researchers including (Ademiluyi et al.,2018; Gale et al.,1970; Onosakponome, et al., 2014; Rebhun, 1989) have indicated this filterability dependence on initial solids content especially when considering the effects on specific resistance on filtration, but the effect was never reported to be as great as observed in this study.

The variation of net solids yield for different values of initial solids content at different operating pressures is shown in Figure 6. It is important to note here that the effect of initial solids moisture on performance is much more pronounced in Filter Presses than in Vacuum filtration, Jones (1956). Net solids yield increases with initial solids content at higher pressures. From the developed model, net yield is mathematically related to operating initial solids content, C as shown below;

$$Y_N = \; KC^{0.149}$$

Where K is proportionality constant.

Similarly, results show that an increase in the concentration of solids in the feed results in an increase in dry cake production and that an increase in the specific cake resistance can result in a decrease in the dry cake production. This was depicted in equation (2) where the derivation of Darcy's law led to a relationship between the solids concentration and the dry cake production capacity (in kg/m^2/s) (Svarovsky, 2000):

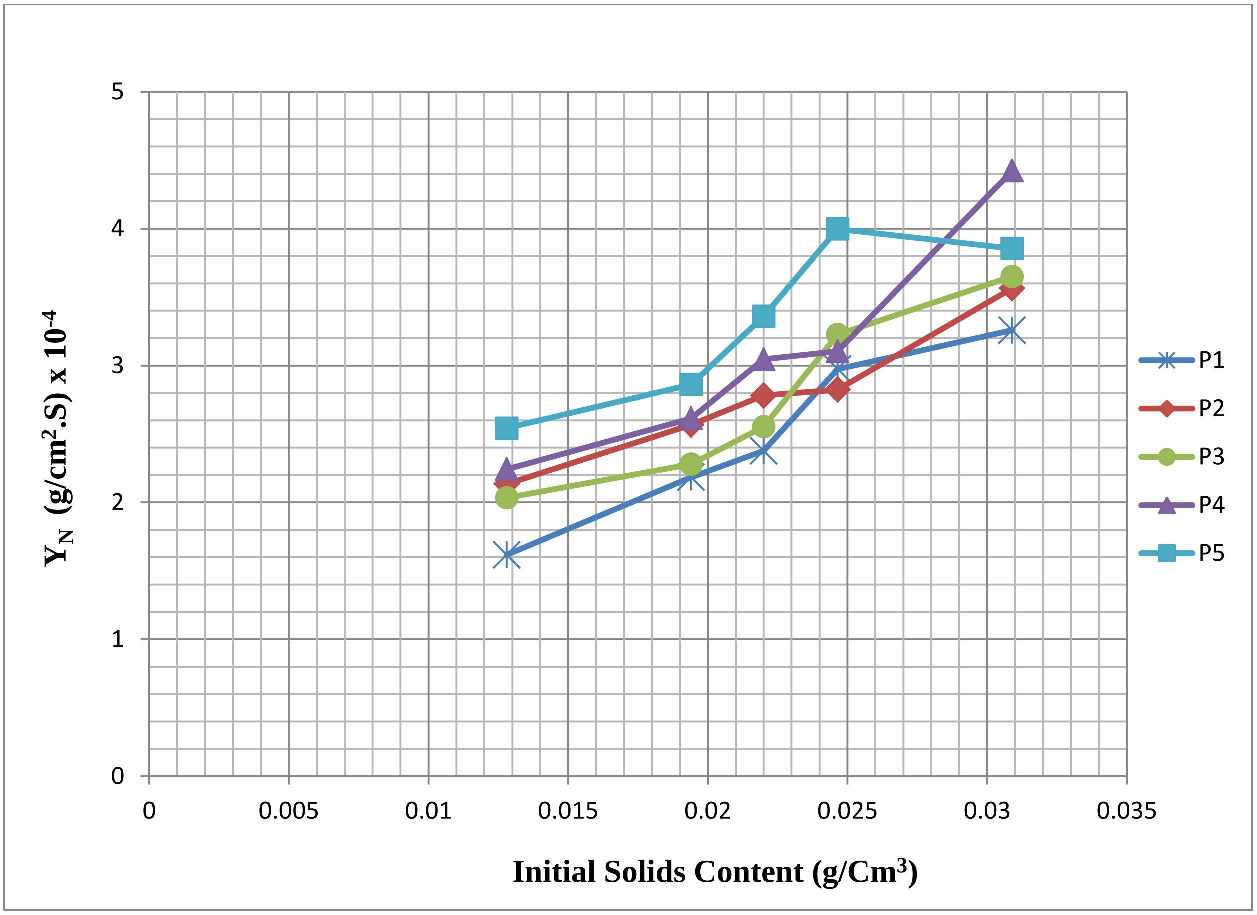

Figure 6:-Variation of Net Solids Yield (Y_N) with Initial Solids Content for 17.81% Conditioned Sludge at various Pressures.

Deviation from linearity can be due to the effect of the filter medium resistance, which is neglected in the derivation of the relationship.

Variation of Net Solids Yield with Specific Resistance:-

The developed cake yield model predicts that more solids are captured on the filter as specific resistance decreases. The effect of specific resistance on yield is shown on Figure 7. However, the reason for this is that more cake are deposited when there is less restriction to filtration, taking into account other conditions such filtration pressures, time and conditioner dosages. This can be mathematically represented as follows;

$Y_N = k/R^{0.827}$

Where k is proportionality constant given as $0\ .007082\ \boldsymbol{x}\ \frac{P^{0.770}\ C^{0.149} A^{1.431}}{\mu^{0.795}\ S^{0.049}\ \theta^{0.1495}}$

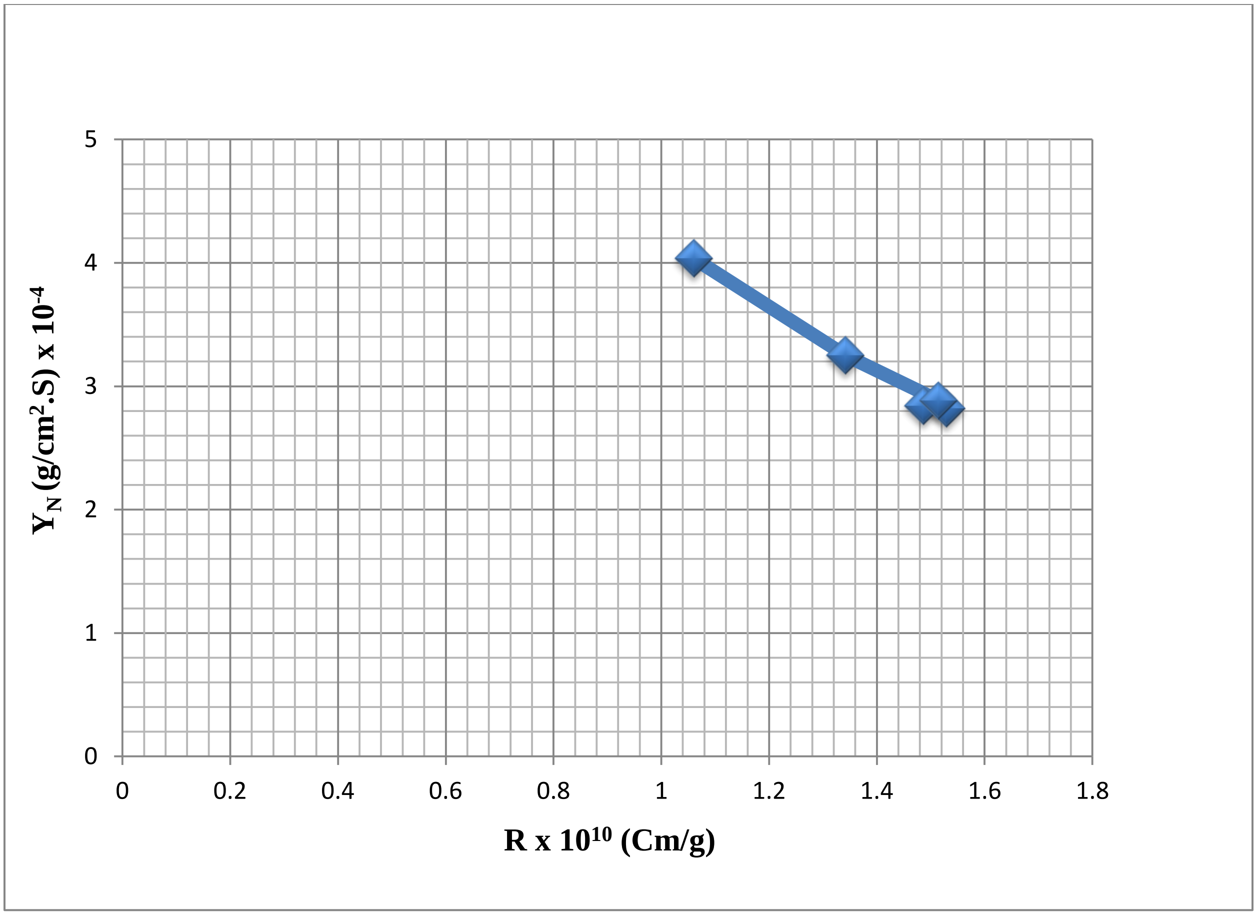

Figure 7:-Effects of Specific Resistance on Net Solids Yield at an operating Pressure of 4078.87 g/Cm^2.

Variation of Net Solids Yield with Sludge Compressibility

From the developed yield equation, net solids yield increases and decreased correspondingly with compressibility for 0.0194g/Cm^3 tested sludge sample. Figure 8 shows cake yield increasing with compressibility but falls steeply when compressibility value increased above 0.7981Cm.S^2/g. The initial short rise in yield value with compressibility may be attributable to other operating conditions such as pressure and chemical dosage. In summary, the graph agrees with the model theoretical prediction given as;

$Y_N = k/S^{0.049}$

Where: k is proportionality constant.

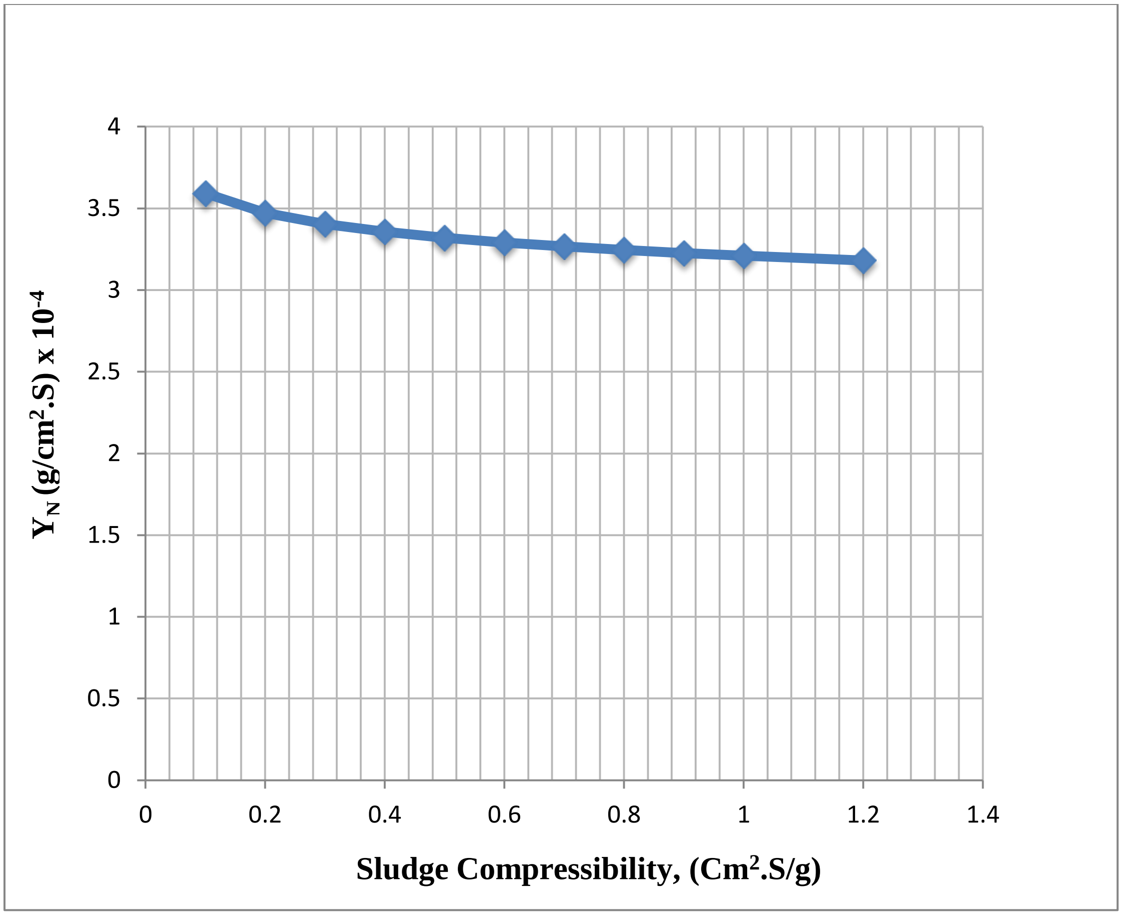

Figure 8:-Effects of Compressibility on Net Solids Yield.

Conclusion:-

The LMT Model equation developed (Equation 28) shows that the net solids yield from a filter press is directly proportional to the filter area of the pressure vessel, applied pressure and initial solids content of the sludge while being inversely proportional to specific resistance of the filtrate, viscosity of filtrate, compressibility coefficient of the sludge and pressing time.

This is in agreement with scientific reasoning and experimental observation. Equations 27 and 28 enable performance of a pressure filter (Filter Press) to be predicted from a simple laboratory determination of cake yields. Moreover, experimental verification of the equation 28 and the derived curves has been described and it may be concluded that for practical purposes, the predicted performance agrees with measured values and conclusions from previous works as cited. What makes the model a novelty is the incorporation of the compressibility attribute of the sludge. Moreover, Ademiluyi (2014) had stated that the only way through which a filtration can validly predict a filtration process is when the equation's plot of t/V and V gives a straight line, the developed model agreed with the above assertion as the plot of $V^{9.189}$ against $\frac{V^8}{\theta^{0.007}}$ gave s straight line with a correlation coefficient of 0.998.

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