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MEASUREMENTS OF THE MODULATION OF SHORT WAVES FROM IMAGE SEQUENCES

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Abstract. Three image processing techniques are described to measure the modulation of the amplitude, phase speed, frequency, and wavenumber of short waves as a function of the phase of the long wave. First, long and short waves are separated by appropriate bandpass filtering. Secondly, the phase speed is computed by determining the local orientation in the space-time domain. Thirdly, Hilbert filters are used to compute the phase and amplitude of the long and short waves. Local wavenumbers and frequencies can then be estimated from spatial and temporal phase gradients, respectively. These techniques are successfully used to analyze long wave/short wave interaction of mechanically generated waves in the glass-wall wave channel at the Scripps Hydraulic Facility and the modulation of short wind waves by larger waves in the large wind-wave flume of Delft Hydraulics (The Netherlands).

1 Introduction

Image sequences from short wind waves are the key experimental technique to gain further experimental insight into the dynamics and kinematics of short wind-waves. So far, almost exclusively only wavenumber spectra or wave number frequency spectra have been computed from wave image sequences. In power spectra, however, all phase relations are lost and it is hardly possible to distinguish different models about the energy fluxes in short wind waves (Jähne, 1989). Here a first attempt is made to extend wave image processing techniques beyond wavenumber spectra. Image processing techniques are described to determine the local phase speed of waves (section 3) and the local wavenumber and frequency (section 4). These techniques are applied in sections 5 and 6 to study the modulation of short waves by long waves.

2 Experimental Techniques

Optical imaging techniques used here are based on light refraction. This technology was pioneered by Keller and Gotwols (1983). The first systematic laboratory investigations of 2-D wavenumber spectra derived from wave slope images were published by Jähne and Riemer (1990) and Klinke and Jähne (1992). Detailed theoretical studies of refraction-based wave imaging can be found in Jähne *et al.* (1992) and Jähne and Schultz (1992). In the context of this paper, it is only important to realize that the brightness of all images discussed here is proportional to the along-wind slope component of the water surface waves.

3 Local Phase Speed Determination in Space-Time Images

Motion in an image sequence results in oriented gray value structures, as can be seen from Fig. 1. A sector in an image that moves with the velocity $\boldsymbol{u}$, can be

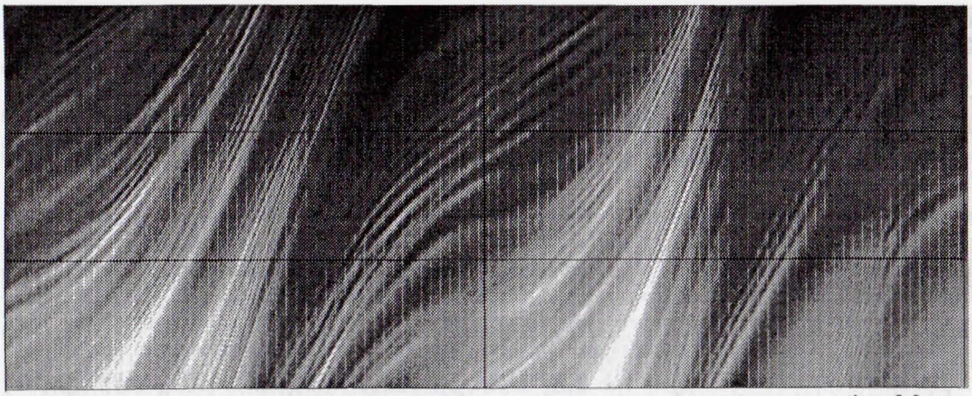

Fig. 1. Space-time slice in along-wind direction of a wind wave image sequence taken in the Delft wind/wave flume at conditions as indicated (fetch and wind speed).

described by $g(\boldsymbol{x}, t) = g(\boldsymbol{x} - \boldsymbol{u}t)$. The corresponding inclination of the direction of constant gray values with respect to the t axis in x_i direction is given by $\varphi_i = \arctan u_i$.

The analysis of local orientation in images has been pioneered by Granlund (1978). Bigün and Granlund (1987) and Jähne (1993) showed that local orientation and thus $\boldsymbol{u}$ can be estimated using the structure tensor

$$J(\boldsymbol{x})_{kl} = \int_{-\infty}^{\infty} \mathrm{d}^n x' \; w(\boldsymbol{x} - \boldsymbol{x}') \left(\frac{\partial g(\boldsymbol{x}')}{\partial x'_k} \frac{\partial g(\boldsymbol{x}')}{\partial x'_l} \right) \quad \text{with} \quad \int_{-\infty}^{\infty} \mathrm{d}^n x \; w(\boldsymbol{x}) = 1, \tag{1}$$

where kl denotes the klth component of the tensor; the t coordinate has been written as the nth spatial coordinate with an appropriate velocity scaling $x_n = u_0 t$. The structure tensor is symmetrical and is computed for every point in the space-time image. The width of the window function $w(\boldsymbol{x}, t)$ determines the spatial and temporal resolution. An eigenvalue analysis of this tensor reveals the orientation of gray value structures. In 2-D space, the solution of the eigenvalue problem is given by simple analytical expressions, summarized in the *orientation vector* $\boldsymbol{o}$ using the components of the structure tensor (Jähne, 1993):

$$\boldsymbol{o} = (J_{xx} - J_{tt}, 2J_{xt}). \tag{2}$$

The orientation vector gives both the direction of the oriented gray value structures and a confidence measure:

$$\tan 2\varphi = \frac{2J_{xt}}{J_{xx} - J_{tt}} \quad \text{and} \quad c = \frac{\sqrt{4J_{xt}^2 + (J_{xx} - J_{tt})^2}}{J_{xx} + J_{tt}}. \tag{3}$$

The confidence measure is one for an ideally oriented gray value structure (constant motion, $g(\boldsymbol{x}, t) = g(\boldsymbol{x} - \boldsymbol{u}t)$) and zero for isotropically orientated patterns.

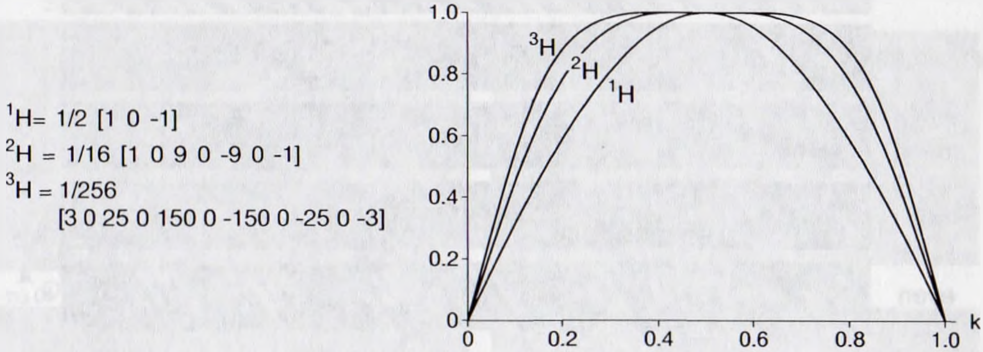

Fig. 2. Convolution kernels and transfer functions (imaginary part) for simple Hilbert operators suitable for bandpass filtered images; the wavenumber k is scaled with the Nyquist wavenumber.

Jähne (1991) devised a fast algorithm to compute local orientation by using a combination of derivative filters, binomial smoothing filters, and point operations:

$$\mathcal{J}_{kl} = \mathcal{B}(\mathcal{D}_k \cdot \mathcal{D}_l), \tag{4}$$

where $\mathcal{B}$ represents the operator for a binomial smoothing filter, $\mathcal{D}_k$ the operator for a spatial derivative filter in k direction, and $\cdot$ pointwise multiplication of images.

4 Local Wavenumber and Frequency Determination

The key to a direct estimation of the local wavenumber and frequency is the determination of the *phase*, since both quantities are given as the spatial and temporal derivatives of the phase function, respectively:

$$\boldsymbol{k} = \nabla_x \Phi(\boldsymbol{x}, t) \quad \text{and} \quad \omega = \frac{\partial \Phi(\boldsymbol{x}, t)}{\partial t}. \tag{5}$$

The phase and also the amplitude (envelope) of a periodic signal can be computed by using Hilbert filters. A Hibert filter shifts the phase by $\pi/2$ but does not change the amplitude. Thus the phase and amplitude of the signal at each point in an image can be derived from

$$\Phi(\boldsymbol{x}, t) = \arctan \frac{\mathcal{H}g(\boldsymbol{x}, t)}{g(\boldsymbol{x}, t)} \quad a(\boldsymbol{x}, t) = \sqrt{(\mathcal{H}g(\boldsymbol{x}, t))^2 + g^2(\boldsymbol{x}, t)}, \tag{6}$$

where $\mathcal{H}$ represents the Hilbert operator. Fleet and Jepson (1990) showed that phase gradients can be computed efficiently without using trigonometric func-

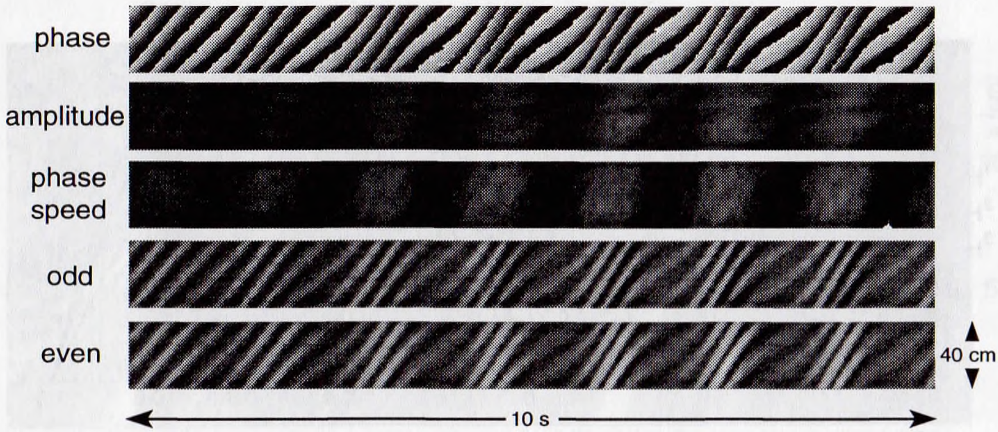

Fig. 3. Modulation of a 3 Hz mechanical wave by a 0.7 Hz mechanical wave as measured in the glass wave channel of the Scripps Hydraulic Facility. Shown are from bottom to top: original sequence (even), Hilbert filtered sequence (odd), phase speed, amplitude, and phase of the 3 Hz wave.

tions directly from the signal g and its Hilbert transform $\mathcal{H}g$:

$$\frac{\partial \Phi(x,t)}{\partial x_k} = \frac{g \dfrac{\partial \mathcal{H}g}{\partial x_k} - \mathcal{H}g \dfrac{\partial g}{\partial x_k}}{g^2 + (\mathcal{H}g)^2} \tag{7}$$

Jähne (1992) demonstrated that simple Hilbert filters can be applied for bandpass-filtered images (Fig. 2).

5 Results I: Long-Wave/Short-Wave Interaction of Mechanically Generated Waves

The Hilbert filter technique has been applied to wave image sequences taken in the Scripps glass-wall wave facility. Using the same wave-maker first a train of short waves and, after a certain delay, a second train of long waves was generated in such a way that the two wave trains superimpose each other in the measuring section.

Fig. 3 shows an example of the interaction of a 3 Hz wave with a 0.7 Hz wave. From the bottom, the first and second row contain the original and Hilbert transformed image sequences, respectively. The other rows nicely show the modulation of the phase speed, the amplitude, and local frequency and wave number of the short wave. The latter two quantities can be inferred directly from the distances of the contour lines — corresponding to a phase shift of 2π — in the phase diagram.

6 Results II: Phase Speed Determination of Short Wind Waves

For wind-generated waves, it is crucial to separate waves of different scales before the determination of the phase speed by an orientation analysis. This can be

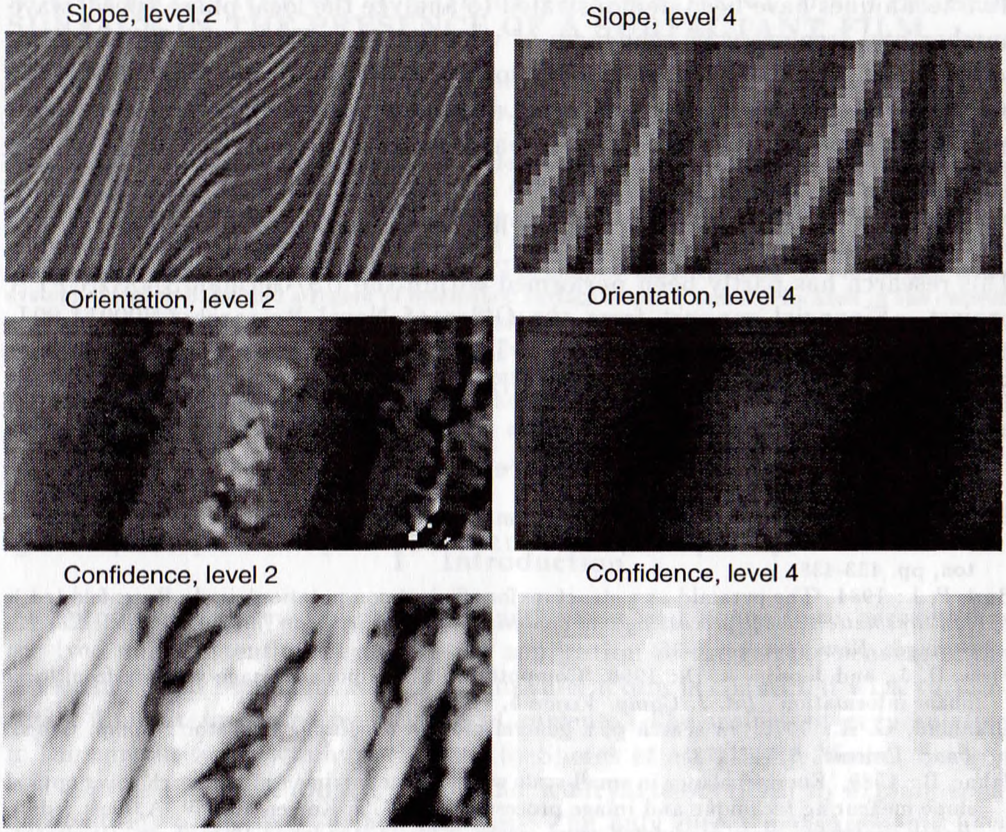

Fig. 4. Multiscale orientation analysis of wind wave image sequences. The left and right column show levels two and four of the Laplacian pyramid computed from the image in Fig. 1, respectively. The first row contains the wave slope, the second the orientation (darker gray values mean higher phase velocities), and the third the confidence measure for the phase speed according to equation (3).

done efficiently by a Laplacian pyramid of the spatiotemporal image shown in Fig. 1. The Laplacian pyramid is an isotropic bandpass decomposition of an image into coarse wavenumber intervals spaced by a factor of two (octaves) (Burt, 1984).

Fig. 4 demonstrates the results of the orientation analysis. The second row shows two images in which the gray values are inversely proportional to the phase speed. It can immediately be seen that the phase speed of the small waves is modulated by the orbital motion of a long wave with a period of about half the temporal extension of the image. The third row in Fig. 4 shows the confidence measure. It is only low (dark) in those areas where no short waves are present. In this case, of course, the determination of phase speed becomes meaningless.

7 Conclusions

Two techniques have been demonstrated to analyze the local phase speed, wave number and frequency in sequences of wave images. While the Hilbert filter technique is most useful to determine local wavenumbers and frequencies, the orientation analysis of image sequences, bandpass filtered by a Laplacian pyramid, is a robust method to determine phase speeds.

8 Acknowledgements

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