

**BIR TO'G'RI CHIZIQDA JOYLASHGAN ZARYADLAR SISTEMASINI  
MUNTAZAM KO'PBURCHAK SHAKLGA KELITIRISHDA BAJARGAN  
ISHINI HISOBLASH METODIKASI**

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**Annotatsiya**

*Ushbu maqolada bir to'g'ri chiziqda joylashgan zaryadlar sistemasini muntazam ko'pburchak shaklga keltirishda bajargan ishini topish masalasi ko'rib otilgan.*

**Kalit so'zlar**

*Nuqtaviy zaryad, zaryadlar sistemasining potensial energiyasi, elektr maydon potentsiali, zaryadni ko'chirishda bajarilgan ish.*

**Аннотация**

*В данной работе рассматривается задача нахождения работы по приведению нескольких точечных зарядов, находящихся на одной прямой в правильный многоугольник.*

**Ключевые слова**

*Точечный заряд, потенциальная энергия системы зарядов, потенциал электрического поля, работа по перемещению заряда.*

**Abstract**

*This paper considers the problem of finding the work of bringing several point charges lying on the same straight line into a regular polygon.*

**Key words**

*electric charge, potential energy of a system of charges, electric field potential, work to move a charge.*

Oliy ta'lim muassasalarida talabalar elektr va magnetizm bo'yicha masalalar yechishda ko'pgina qiyinchiliklarga duch kelishadi. Sababi o'rta maxsus ta'limda yetarlicha bilim olishmaydi. Natijada oily ta'limda o'qiyotgan vaqtlarida murakkabroq masalalarni yecha olishmaydi. Shuning uchun yuqori malakali fizik o'qituvchi kadrlar yetishtirib chiqarishda ba'zi muammolar tug'iladi. Ularni

bartaraf etish uchun talabalar turli xil qiyinlikdagi masalalarni yechishga o'rganishlari kerak.

Ushbu maqolada fizikaning eletr qismiga tegishli bo'lgan bir to'g'ri chiziqda joylashgan zaryadlar sistemasini muntazam ko'pburchak shaklga keltirishda bajargan ishini hisoblashga oid misollar keltirilgan. Umumiy holda zaryadlar sistemasini turli xil qiymatli zaryadga ega bo'lsa va ular orasidagi masofa turlicha bo'lsa bu masalani yechish ancha murakkab bo'ladi. Shuning uchun biz bir-biridan bir xil masofada joylashgan va bir to'g'ri chiziqda yotgan, biroq turlicha zaryadlarga bo'lgan zaryadlar sistemasini muntazam ko'pburchaklar shakliga olib kelish uchun bajariladigan ishni hisoblash masalasini ko'rib chiqamiz.

Elektrostatik maydon bajaradigan ishi [1] quyidagicha aniqlanadi:

$$A=W_{p2}-W_{p1}, \tag{1}$$

bunda,  $W_{p1}$  va  $W_{p2}$  - zaryadlar sistemasining boshlang'ich va ko'chirilgandan keyingi potensial energiyalari. Soddalik uchun  $p$  indesni tushirib qoldiramiz.

Elektrostatik maydon bajarargan ishi topish uchun nuqtaviy zaryadlar sistemasining potensial energiyalarini [2] aniqlash lozim.  $q_1, q_2, \dots, q_N$  nuqtaviy zaryadlar sistemasidan iborat sistemaning potensial energiyasi:

$$W_p = \frac{1}{2} \sum_{k=1}^N q_k \varphi_k, \tag{2}$$

bunda,  $q_k$  -  $k$ -chi nuqtaviy zaryad,  $\varphi_k$  - ushbu zaryad joylashgan nuqtada boshqa zaryadlar hosil qilinayotgan maydon potentsiali. Endi masalalar ko'rishga o'tamiz.

**1-masala.** Zaryadlari  $q_1, q_2, q_3, q_4, q_5, q_6$  bo'lgan va 1-rasmda ko'rsatilgandek joylashgan nuqtaviy zaryadlar bir-biridan  $a$  masofada va bir to'g'ri chiziqda joylashishgan. Zaryadlar shunday ko'chirilganki,  $q_1$  va  $q_2$  zaryadlarning joylashishi o'zgartirilmay,  $q_1$  zaryad atrofiga  $q_3$ , va  $q_5$  zaryadlar soat strelkasiga teskari yo'nalishda,  $q_2$  zaryad atrofiga  $q_4$  va  $q_6$  zaryadlar soat strelkasi yo'nalishi bo'ylab burib muntazam oltiburchak shakliga keltirilgan. Zaryadlarni bunday ko'chirishda bajarilgan ishni toping.

**Berilgan:**

$q_1; q_2; q_3;$

$q_4; q_5; q_6;$

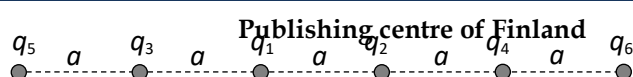
$a, \alpha=120^\circ$

**Topish**

**kerak**

$A=?$

(2) ifodani 1-rasmdagi holatga qo'llaymiz:



1-rasm

$$W_1 = q_1\varphi_2 + q_1\varphi_3 + q_1\varphi_4 + q_1\varphi_5 + q_1\varphi_6 + q_2\varphi_3 + q_2\varphi_4 + q_2\varphi_5 + q_2\varphi_6 + q_3\varphi_4 + q_3\varphi_5 + q_3\varphi_6 + q_4\varphi_5 + q_4\varphi_6 + q_5\varphi_6, \quad (3)$$

bu yerda,  $\varphi_i$  -  $i$ -indeksli zaryadning boshqa zaryadlar turgan nuqtalarda hosil qiluvchi potensial. Ushbu potensialning ifodasi quyidagicha [2]:

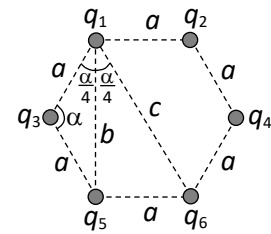
$$\varphi = \frac{1}{4\pi\epsilon_0} \frac{q}{r}, \quad (4)$$

bu yerda,  $\epsilon_0$  - elektr doimiysi,  $r$  - zaryadlar orasidagi masofa.

(4) ifodani (3) ifodaga qo'yib quyidagini olamiz:

$$W_1 = \frac{k}{a} (q_1q_2 + q_1q_3 + \frac{1}{2} q_1q_4 + \frac{1}{2} q_1q_5 + \frac{1}{3} q_1q_6 + \frac{1}{2} q_2q_3 + q_2q_4 + \frac{1}{3} q_2q_5 + \frac{1}{2} q_2q_6 + \frac{1}{3} q_3q_4 + q_3q_5 + \frac{1}{4} q_3q_6 + \frac{1}{4} q_4q_5 + q_4q_6 + \frac{1}{5} q_5q_6). \quad (5)$$

Endi 1-rasmda keltirilgan zaryadlarni masala shartiga ko'ra muntazam oltiburchak shakliga keltiramiz (2-rasm). Muntazam oltiburchak uchun potensial energiya ifodasini yozamiz:



2-rasm  
(6)

$$W_2 = q_1\varphi_2' + q_1\varphi_3' + q_1\varphi_4' + q_1\varphi_5' + q_1\varphi_6' + q_2\varphi_3' + q_2\varphi_4' + q_2\varphi_5' + q_2\varphi_6' + q_3\varphi_4' + q_3\varphi_5' + q_3\varphi_6' + q_4\varphi_5' + q_4\varphi_6' + q_5\varphi_6'.$$

(4) ifodani (6) ifodaga qo'ysak quyidagini olamiz:

$$W_2 = \frac{k}{a} (q_1q_2 + q_1q_3 + q_2q_4 + q_3q_5 + q_4q_6 + q_5q_6) + \frac{k}{b} (q_1q_4 + q_1q_5 + q_2q_3 + q_2q_6 + q_3q_6 + q_4q_5) + \frac{k}{c} (q_1q_6 + q_2q_5 + q_3q_4), \quad (7)$$

bu yerda,  $\alpha$  - muntazam oltiburchakning ichki burchagi,  $a$  - qo'shni zaryadlar orasidagi masofa  $b$  va  $c$  masofalar esa sinuslar teoremasi yordamida topiladi:

$$b = a \frac{\sin \alpha}{\sin \alpha / 4}, \quad c = a \frac{1}{\sin \alpha / 4}. \quad (8)$$

(8) ifodani (7) ifodaga qo'yamiz:

$$W_2 = \frac{k}{a} [q_1q_2 + q_1q_3 + q_2q_4 + q_3q_5 + q_4q_6 + q_5q_6 + \frac{\sin \alpha / 4}{\sin \alpha} \cdot (q_1q_4 + q_1q_5 + q_2q_3 + q_2q_6 + q_3q_6 + q_4q_5) + \sin \alpha / 4 \cdot (q_1q_6 + q_2q_5 + q_3q_4)]. \quad (9)$$

Hosil bo'lgan (9) va (5) ifodalarni (1) ifodaga qo'yib zaryadlarni ko'chirishda bajarilgan ishni topamiz:

$$A = \frac{k}{a} \left[ \left( \frac{\sin \alpha / 4}{\sin \alpha} - \frac{1}{2} \right) \cdot (q_1q_4 + q_1q_5 + q_2q_3 + q_2q_6) + \left( \sin \alpha / 4 - \frac{1}{3} \right) \cdot (q_1q_6 + \right.$$

$$+q_2q_5+q_3q_4)+\left(\frac{\sin \alpha / 4}{\sin \alpha}-\frac{1}{4}\right) \cdot\left(q_3q_6+q_4q_5\right)+\frac{4}{5}q_5q_6\left]. \quad (10)$$

$\alpha=120^\circ$  ekanligini hisobga olsak (10) ifoda quyidagiga keladi:

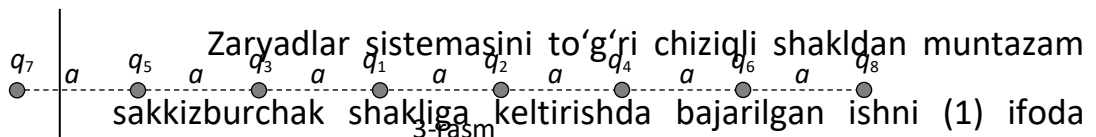
$$A=\frac{k}{a}\left[\frac{2\sqrt{3}-3}{6} \cdot\left(q_1q_4+q_1q_5+q_2q_3+q_2q_6\right)+\frac{1}{6} \cdot\left(q_1q_6+\right.\right.$$

$$\left.\left.+q_2q_5+q_3q_4\right)+\frac{4\sqrt{3}-3}{12} \cdot\left(q_3q_6+q_4q_5\right)+\frac{4}{5}q_5q_6\right].$$

**Javob:**  $A=\frac{k}{a}\left[\frac{2\sqrt{3}-3}{6} \cdot\left(q_1q_4+q_1q_5+q_2q_3+q_2q_6\right)+\frac{1}{6} \cdot\left(q_1q_6+\right.\right.$

$$\left.\left.+q_2q_5+q_3q_4\right)+\frac{4\sqrt{3}-3}{12} \cdot\left(q_3q_6+q_4q_5\right)+\frac{4}{5}q_5q_6\right].$$

**2-masala.**  $q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8$  zaryadlarga ega bo'lgan va 3-rasmda ko'rsatilgandek joylashgan nuqtaviy zaryadlar bir-biridan  $a$  masofada va bir to'g'ri chiziqda joylashishgan. Zaryadlar shunday ko'chirilganki,  $q_1$  va  $q_2$  zaryadlarning joylashishi o'zgartirilmay,  $q_1$  zaryad atrofiga  $q_3, q_5$  va  $q_7$  zaryadlar soat strelkasiga teskari yo'nalishda,  $q_2$  zaryad atrofiga  $q_4, q_6$  va  $q_8$  zaryadlar soat strelkasi yo'nalishi bo'ylab burib muntazam sakkizburchak shakliga keltirilgan. Zaryadlarni ko'chirishda bajarilgan ishni toping.



**Berilgan:**

$q_1; q_2; q_3; q_4;$

$q_5; q_6; q_7; q_8;$

$a; \alpha=135^\circ$

**Topish kerak:**

$A=?$

Buning uchun (2) ifodadan foydalanamiz:

$$W_1=q_1\varphi_2+q_1\varphi_3+q_1\varphi_4+q_1\varphi_5+q_1\varphi_6+q_1\varphi_7+q_1\varphi_8+q_2\varphi_3+q_2\varphi_4+q_2\varphi_5+$$

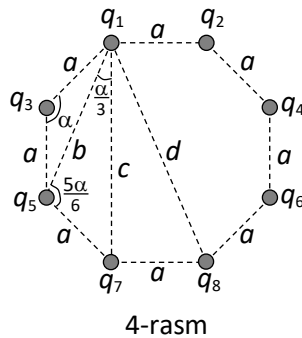
$$+q_2\varphi_6+q_2\varphi_7+q_2\varphi_8+q_3\varphi_4+q_3\varphi_5+q_3\varphi_6+q_3\varphi_7+q_3\varphi_8+q_4\varphi_5+q_4\varphi_6+$$

$$+q_4\varphi_7+q_4\varphi_8+q_5\varphi_6+q_5\varphi_7+q_5\varphi_8+q_6\varphi_7+q_6\varphi_8+q_7\varphi_8. \quad (11)$$

Potensialning o'rniga (4) ni qo'yib potensial energiya uchun quyidagini olamiz:

$$W_1=\frac{k}{a}\left(q_1q_2+q_1q_3+\frac{1}{2}q_1q_4+\frac{1}{2}q_1q_5+\frac{1}{3}q_1q_6+\frac{1}{3}q_1q_7+\frac{1}{4}q_1q_8+\frac{1}{2}q_2q_3+\right.$$

$$\begin{aligned}
 &+q_2q_4+\frac{1}{3}q_2q_5+\frac{1}{2}q_2q_6+\frac{1}{4}q_2q_7+\frac{1}{3}q_2q_8+\frac{1}{3}q_3q_4+q_3q_5+\frac{1}{4}q_3q_6+ \\
 &+\frac{1}{2}q_3q_7+\frac{1}{5}q_3q_8+\frac{1}{4}q_4q_5+q_4q_6+\frac{1}{5}q_4q_7+\frac{1}{2}q_4q_8+\frac{1}{5}q_5q_6+ \\
 &+q_5q_7+\frac{1}{6}q_5q_8+\frac{1}{6}q_6q_7+q_6q_8+\frac{1}{7}q_7q_8). \tag{12}
 \end{aligned}$$



Endi birinchi holatda turgan zaryadlar sistemasini muntazam sakkizburchak shakliga keltiramiz (4-rasm). Hosil bo'lgan sistema uchun potensial energiyaning ifodasini yozamiz:

$$\begin{aligned}
 W_1 = & q_1\varphi_2' + q_1\varphi_3' + q_1\varphi_4' + q_1\varphi_5' + q_1\varphi_6' + q_1\varphi_7' + q_1\varphi_8' + q_2\varphi_3' + q_2\varphi_4' + q_2\varphi_5' + \\
 & + q_2\varphi_6' + q_2\varphi_7' + q_2\varphi_8' + q_3\varphi_4' + q_3\varphi_5' + q_3\varphi_6' + q_3\varphi_7' + q_3\varphi_8' + q_4\varphi_5' + q_4\varphi_6' + \\
 & + q_4\varphi_7' + q_4\varphi_8' + q_5\varphi_6' + q_5\varphi_7' + q_5\varphi_8' + q_6\varphi_7' + q_6\varphi_8' + q_7\varphi_8' \tag{13}
 \end{aligned}$$

(4) ifodani (13) ifodaga qo'ysak quyidagini olamiz:

$$\begin{aligned}
 W_2 = & \frac{k}{a} (q_1q_2 + q_1q_3 + q_2q_4 + q_3q_5 + q_4q_6 + q_5q_7 + q_6q_8 + q_7q_8) + \frac{k}{b} (q_1q_4 + q_1q_5 + \\
 & + q_2q_3 + q_2q_6 + q_3q_7 + q_4q_8 + q_5q_8 + q_6q_7) + \frac{k}{c} (q_1q_6 + q_1q_7 + q_2q_8 + q_3q_4 + \\
 & + q_3q_8 + q_4q_7 + q_5q_6) + \frac{k}{d} (q_1q_8 + q_2q_7 + q_3q_6 + q_4q_5), \tag{14}
 \end{aligned}$$

bu yerda,  $\alpha$  – muntazam sakkizburchakning ichki burchagi,  $a$  – qo'shni zaryadlar orasidagi masofa  $b$ ,  $c$  va  $d$  masofalar esa sinuslar teoremasi yordamida topiladi:

$$b = a \frac{\sin \alpha}{\sin \alpha / 6}, \quad c = a \frac{\sin 5\alpha / 6}{\sin \alpha / 6}, \quad d = a \frac{1}{\sin \alpha / 6}. \tag{15}$$

(15) ifodani (14) ifodaga qo'yamiz:

$$\begin{aligned}
 W_2 = & \frac{k}{a} [q_1q_2 + q_1q_3 + q_2q_4 + q_3q_5 + q_4q_6 + q_5q_7 + q_6q_8 + q_7q_8 + \frac{\sin \alpha / 6}{\sin \alpha} \cdot (q_1q_4 + q_1q_5 + \\
 & + q_2q_3 + q_2q_6 + q_3q_7 + q_4q_8 + q_5q_8 + q_6q_7) + \frac{\sin \alpha / 6}{\sin 5\alpha / 6} \cdot (q_1q_6 + q_1q_7 + q_2q_5 +
 \end{aligned}$$

$$+q_2q_8+q_3q_4+q_3q_8+q_4q_7+q_5q_6)+\sin\alpha/6\cdot(q_1q_8+q_2q_7+q_3q_6+q_4q_5)]. \quad (16)$$

Hosil bo'lgan (16) va (12) ifodalarni (1) ifodaga qo'yib zaryadlarni ko'chirishda bajarilgan ishni topamiz:

$$A = \frac{k}{a} \left[ \left(1 - \frac{1}{7}\right) q_7 q_8 + \left(\frac{\sin \alpha / 6}{\sin \alpha} - \frac{1}{2}\right) \cdot (q_1 q_4 + q_1 q_5 + q_2 q_3 + q_2 q_6 + q_3 q_7 + q_4 q_8) + \left(\frac{\sin \alpha / 6}{\sin \alpha} - \frac{1}{6}\right) \cdot (q_5 q_4 + q_6 q_7) + \left(\frac{\sin \alpha / 6}{\sin 5\alpha / 6} - \frac{1}{3}\right) \cdot (q_1 q_6 + q_1 q_7 + q_2 q_5 + q_2 q_8 + q_3 q_4) + \left(\frac{\sin \alpha / 6}{\sin 5\alpha / 6} - \frac{1}{5}\right) \cdot (q_3 q_8 + q_4 q_7 + q_5 q_6) + \left(\sin \alpha / 6 - \frac{1}{4}\right) \cdot (q_1 q_8 + q_2 q_7 + q_3 q_6 + q_4 q_5) \right]. \quad (17)$$

$\alpha = 135^\circ$  ekanligini hisobga olsak (17) ifoda quyidagiga keladi:

$$A = \frac{k}{a} \left[ \frac{6}{7} q_7 q_8 + \frac{\sqrt{4 - 2\sqrt{2}} - 1}{2} \cdot (q_1 q_4 + q_1 q_5 + q_2 q_3 + q_2 q_6 + q_3 q_7 + q_4 q_8) + \frac{3\sqrt{4 - 2\sqrt{2}} - 1}{6} \cdot (q_5 q_4 + q_6 q_7) + \frac{3\sqrt{2} - 4}{3} \cdot (q_1 q_6 + q_1 q_7 + q_2 q_5 + q_2 q_8 + q_3 q_4) + \frac{5\sqrt{2} - 6}{5} \cdot (q_3 q_8 + q_4 q_7 + q_5 q_6) + \frac{2\sqrt{2 - \sqrt{2}} - 1}{4} \cdot (q_1 q_8 + q_2 q_7 + q_3 q_6 + q_4 q_5) \right].$$

**Javob:** 
$$A = \frac{k}{a} \left[ \frac{6}{7} q_7 q_8 + \frac{\sqrt{4 - 2\sqrt{2}} - 1}{2} \cdot (q_1 q_4 + q_1 q_5 + q_2 q_3 + q_2 q_6 + q_3 q_7 + q_4 q_8) + \frac{3\sqrt{4 - 2\sqrt{2}} - 1}{6} \cdot (q_5 q_4 + q_6 q_7) + \frac{3\sqrt{2} - 4}{3} \cdot (q_1 q_6 + q_1 q_7 + q_2 q_5 + q_2 q_8 + q_3 q_4) + \frac{5\sqrt{2} - 6}{5} \cdot (q_3 q_8 + q_4 q_7 + q_5 q_6) + \frac{2\sqrt{2 - \sqrt{2}} - 1}{4} \cdot (q_1 q_8 + q_2 q_7 + q_3 q_6 + q_4 q_5) \right].$$

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