

Lightcones

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Abstract

A lightcone is necessarily an orbifold. It is a Penrose space (or product of penrose spaces), alongside a designated singularity (base point) which is denoted \mathfrak{b} . This singularity has an effective action, denoted by $\hat{\mathfrak{b}}$, which is akin to the four-momentum.¹ In this paper, I shall summarize the facts about lightcones, and shed some light on the dynamics of massless antiparticles.

Preamble

I would like to thank my mother Dawn, my sisters Megan, Kiya, and Ayla, Derik Monroe, Thunder Wishteyah, my brother-in-law Robert and my nephew Zaayden. I would like to thank O. Hancock, P. Emmerson, as well my closest friend and coworker Kyle Flynn. Without the support of all of the named individuals, the publication of this work would not be possible, and so I am indebted to them in a very literal way.

When I last saw Esther Jepsen, we were at Stewart Park and I explained to her Einstein's work on lightcones. She said she would see me on TV one day. Probably not, but if you are reading this Esther, thank you for the support.

However, we have come a long way since the early days of relativity. Specifically, many of the benefits we have gained since that time have come from switching from general Riemannian geometry to more categorical and topological approaches. It is clear through Lurie's work on cobordisms that there is a singularity at the center of a Minkowski lightcone, and in order to treat this properly one needs to shift their view from smooth manifolds to the more general orbifolds, which are the quotient of a smooth manifold with a group.

With one time dimension and three of space, we have the orbifold:

$$\mathbb{L}^{1,3} = \mathbb{C}^2/U(1) \tag{0.1}$$

where $U(1)$ is the electromagnetic circle group, generating the classical "lines of force" (field lines) of a charged particle. When gauge theory enters the picture, these lines of force become co-linear with the *Wilson loops* of a configuration space, which determines the gauge transformations that must be performed to maintain a constant value while a particle is transported around the lightcone.

The lightcone, for all intents and purposes, is a completely real space, but that doesn't stop us from treating functions on the lightcone (and, by extension, operators of those functions) as being

¹Of, presumedly, a force-carrying quantum.

holomorphic in nature. Usually, however, we will want to perform a Wick rotation, which allows us to describe actions on the lightcone as the vector sum

$$\mathbb{R}^3 \oplus \mathbb{R}^1 \tag{0.2}$$

and when we discretize, we have a functor $\delta : \mathbb{R}^1 \mapsto \mathbb{Z}$.

We should think of discretization as overlaying a meshgrid onto the space we are interested in, but there is actually a deeper idea than that: we can treat this meshgrid as a graph, and take Laplacians of small subgraphs and stitch them together to weave the bigger picture. Vertices of the graph represent incredibly dense clusters of gravity, which Penrose interprets as residue from a previous manifestation of the universe. So, in a way, it would be congenial to think of the universe as inheriting its discrete properties from certain anomalies in a previous global lifespan, which manifest as solitons.

I think also that the ideas presented here would work well with a Wolfram-style physics project; however, we cannot divorce the metaphysical from the physics. Luckily, the metaphysics are elegantly described by a graph of complex dimension 2. We would have to embed a hypergraph in this space before modding by the electromagnetic circle; actually, it makes sense to define a hypergraph in just this way. The kinematic structure of the observable universe (as dictated by the infinite tower of lightcones) is governed by the topology of this graph. All loops of the hypergraph are necessarily preserved by modding by $U(1)$, but in addition, some new ones are created close to the singularity of the lightcone. These manifest as instantaneous data transfer, which we denote by $\hat{\mathbf{b}}$.

Once we have identified $\hat{\mathbf{b}}$, we can describe torsion from the ground-up as a rotation of the principle fiber linking the present moment \square to the past and future. Then we have

$$\int_{\mathbb{R}} \frac{\partial \hat{\mathbf{b}}}{\partial t} \cdot \mathfrak{C}_{Prin} = \square \pm \varepsilon \square = \square^{\pm} \tag{0.3}$$

which allows us to talk about “edge states” of the current universe, which we define as the collection of all simultaneously occurring Dowker events. Notice that simultaneity is a gauge dependent quality, and so, in principle, the definition of universe used thus far is little better than a “ghost number.” In order to improve this definition, we induce a preorder:

$$\mathcal{U} = \mathbb{L}^- \prec \square^- \prec \square \prec \square^+ \prec \mathbb{L}^+ \tag{0.4}$$

and we define the universe to be \mathcal{U} .

Emmerson has convinced me that the universe, when written as a process, is totally non-ergodic. At the very least, when we map out of his energy numbers and into a more digestible system like the integers, we obtain a difference between the time average and ensemble average at the local scale almost everywhere. In the phenomenal cases where this does happen, we obtain a so-called *resonance window* which is described by crossing the gauge group with the average energy of a phonon. As Emmerson notes, however, defining (let alone calculating) the energy of a phonon is almost entirely inaccessible for real world purposes.

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1 Necessity

Necessity is opposed to probability; i.e., these are radically conformed to the “God doesn’t play dice” line of Einstein. I argue that necessity is only found in the immediate present moment; however, the past may be sufficient. The past, then, is capable of unlocking the present moment through a transaction of photons and any potential superpartners. We denote the necessary number:

$$\square \tag{1.1}$$

which decomposes as

$$\square^- \oplus \square^+ \tag{1.2}$$

where the left-hand-side is connected to the immediate past and the right-hand-side is connected to the immediate future.

The totality of the present, is essentially a spinor, which we can write as a composition of two vector bundles. Actually, this combination is known as the “Whitney sum” or “product” of those vector bundles.

The necessary is L^2 -normed; in other words, we can impose the Manhattan metric on it. This is essentially a meshgrid. This allows for different *splines* to be constructed, which are all very regular in character. A spline is an excellent analogue to a *sheaf*; actually, it is a *sheaf of frame bundles*. The definition of a “frame bundle” involves attaching principal fibers from each frame to a holonomic line.

A holonomic line is actually a marvelous simplification of the real world, but it is one where we can agree with the real world in many ways by investigating. It defines transportation of local variables along pre-specified lines; for instant, lines of force which act transversely to become phonons. If we are working with symplectic forms, we impose a non-degenerate, closed two-form; i.e,

$$\partial^2 = 0 \tag{1.3}$$

We can apply composition:

$$\partial^2 \square^\pm \tag{1.4}$$

to form a zero manifold. This is essentially a Ricci flatness condition. This keeps things safe (asymtotically at least), as there are no transcendentals.

Since we can contract every ball to points, we should look for negative dimensional sphere spectra.

Let’s take a moment to define the fiber spectrum.

Definition 1.1. *The fiber spectrum, $x = \int_B x_b$, is the minimal ergodic process contained within a ball centered at base-point x of radius r homeomorphic to the unit 2-sphere.*

If it is a higher-order fiber spectrum, we call it a *gerbe*. When Reidemeister moves are performed on these gerbes, we get torsion. Torsions are corruptions of the cross-product of two matrices.

$$\mathfrak{Tors} = \partial^n(A \times B) \quad \forall n \neq 2 \quad (1.5)$$

These entries of these matrices are called “*orbits*” of GL_n . We just have to make sure A does not have 2 columns, or B two rows. As long as they have the same number of columns in A for rows in B , we can perform the cross product and obtain this torsion. The best space to do so is on the four-dimensional spacetime.

This spacetime is encoded by its minimal structure: the smallest possible lightcone. This can be appropriately called the n -torsion, and to the great dismay of the mathematicians.

Since these processes are ergodic, we have:

$$\sum_i \frac{\sum_j K_j}{(i+j)K_i} = \sum_{ij} K_{ij} = H \quad (1.6)$$

This describes the gluon field, which is a huge component of quantum chromodynamics (QCD).

The K_{ij} , I like to call a “diquark.” The K_i and K_j are individual monoquarks, or just *quarks*. The different isotopes, if you will, are made by varying a single quark, and leaving the diquark configuration fixed.

A pentaquark can be explained as a simple hadron, along with a diquark:

$$q^{\otimes 5} = H + K_{ij} \quad (1.7)$$

2 Anti-Photons

Each of the quarks has a specific *color charge*; we denote the positive charges as follows: red (r), green (g), and blue (b); we denote the anticharges as \bar{r}, \bar{g} and \bar{b} . When the corresponding charges meet with their anticharge, they annihilate and they become a vacuum color charge.

The colors and anticolors are not dependent upon the specific *kind of quark*, i.e., top, bottom, strange, charm, up, down. Nor do the kinds of quark depend on color. These are completely separate issues.²

The photon, since it does not consist in quarks, does not possess a color charge. Therefore, a hypothetical superpartner would also have no color charge. This is a property of the field being massless. What my colleagues and I propose, is a massless antiphoton, which can access areas in the fabric of spacetime that the massive particles cannot, alongside the photon.

When these two meet, they should hypothetically annihilate one another, but no exchange of mass, i.e., no gravity, should take place.

We denote these anti-photons by γ^- . The transaction is associative, so:

$$\gamma^- \circ \gamma^+ = \gamma^+ \circ \gamma^- \quad (2.1)$$

Right off the bat, assume we know nothing about this equation. Firstly, we know that it describes a cross-section of intersecting configuration spaces. These are basically simplicial complexes, which

²We will discuss this later.

only really work with model categories such as the ∞ -category of quasi-categories, $\mathcal{Q}ats$. These are basically Riehl categories in my notation, of course named after the amazing Emily Riehl, who collaborated with Dominick Verity to bring us an excellent account of the quasi-categories of complete Segal spaces. Rezk would later describe these as Θ -spaces.

Now, what do I add to the story? Don't hold your breath, but I think these Θ -spaces can be thought of as classes of Mochizuki's θ -links. These are essentially antiholomorphic transitions which allow one to translate between "mutually alien copies," so they are like Baez's "Rosetta Stones."

First off, note that photons (or other massless particles) do not satisfy Newton's equations of motions. They must be thought of entirely relativistically, as they always represent the relativistic limit. They have a momentum, despite the fact that the Newtonian equation for momentum is:

$$\dot{p} = \text{Mass} \times \text{Velocity} \tag{2.2}$$

However, the complicated picture is that mass varies depending upon which relativistic frame one selects. This is because the *resting frame* is defined in reference to an infinitesimal deviation from a null trajectory. This causes stochastic gravitational fluctuations, which locally distort the truth values of particles/waves in a neighborhood of the semiclassical Einstein-Minkowski-Lorentz spacetime. The revised equation is:

$$p_v = \frac{m_0 V}{\sqrt{(1 - \frac{V^2}{c^2})}} \tag{2.3}$$

where c is the speed of light in a vacuum and m_0 is the rest mass of the particle, which may be affected by relativistic forces. This revised equation causes a lot of very fascinating (and observable!) effects such as time dilation (length contraction), and quantum tunneling. Quantum tunneling happens in ideal systems, or close-to-ideal superconductors like Josephson junctions. In these systems, a great approximation for the relativistic speed of an electron is infinity, close to the classical speed of light.

We can rewrite this as:

$$p_v = \frac{V}{\sqrt{1 - V^2}} \tag{2.4}$$

when we use natural units, and set the resting mass equal to the momentum of a graviton.³

The easiest way to think about this is by imagining that carrier particles like photons and their antiparticles gain a form of mass, or appear to have a greater mass in an accelerated reference frame. I like to think of this as a "finely tuned" reference frame. However, this is somewhat of a fiction; we aren't actually gaining mass, but a ghost number describing the eigenvalues of the torsion.

2.1 Speculation

We should be able to detect differences in the color spectra of photons emitted in a Bose-Einstein condensate (BEC). This is because cold photons age faster than warm ones as they "detect" each reference frame slower, and therefore expend more time to reach the same space travelling frame-by-frame. Mathematically, this frame traversing mechanism is defined by a *timestep operator* or *Ricci iteration*, which is a map between two distinct states of Ricci flow. We could measure the relativistic "speed of time," as Carroll puts it.

³This is by its nature a recursive function.

We may find a value for ε in the equation:

$$Ric_{g_{i+\varepsilon}}(x_b) = x_b \rightarrow x_{b'} \quad (2.5)$$

by measuring the relativistic momentum of apparent photons or gravity waves at the time of creation or annihilation.

In the future, perhaps a lattice QCD experiment will allow us to modify the thermodynamic environment through modulations in the freedom to do work.

$$g_i = g_{i-\varepsilon+\Delta(\varepsilon+i)} \quad (2.6)$$

$$\Delta(\varepsilon+i) = \partial^n F \quad (2.7)$$

and we have:

$$\partial^n F \notin \square \quad (2.8)$$

which means this value does not contribute to \mathfrak{Sors} , so that when we contract to the relativistic limit near the singularity of the lightcone, we do not encode this information.

So:

$$\lim_{\mathfrak{b} \rightarrow *}\mathbb{L}^{1+3} = \mathcal{E}_{Shannon} \rightarrow 0 \quad (2.9)$$

where $\mathcal{E}_{Shannon}$ is the Shannon entropy, and is equal to

$$\rho \log \rho \quad (2.10)$$

where

$$\rho = p_v M_H \quad (2.11)$$

and M_H is the mass of the nearest hadron. We call ρ the *charge* of the field. This says that in order to recreate the vacuum, we would have to map every singularity to a single point. In the strong field, this point is the product:

$$GL_{2n} \times SU(2) \quad (2.12)$$

which is a quaternionic group.

The key idea to remember here is, as time progresses, due to gravitational forces, the distance to the nearest hadron should change in one unit time, and so ρ is a relativistic quality. The simplest way to describe ρ would be to use Weinstein's version of Ehresmannian geometry, with the prescription that connections describe differences between simultaneous frames, which are embedded in the necessary moment. Specifically, there is a principle connection on \square that turns it into \square^\pm .

Attached to this principle connection, we have regions of spacetime where the observables of the Higgs field may change. These are moments on a particle's worldline that may be punctured. By doing some bookkeeping on the puncture locations, we can count these points as potential singularities for minimal lightcones.

Actually, I am not sure how to formalize this yet, but I think if one applies Penrose's CCC, we can actually identify some (if not all) of these points as Hawking points. These points are evidence of the activity of a black hole in a previous iteration of the universe, and their location may be conformally mapped to points in the new universe. This theory assumes that the universe essentially ends in 10^{100} years, when the very last black hole evaporates.

However, black holes, I believe, are not the only things which beget Hawking points. Any singularity, such as those formed by the annihilation of a particle-antiparticle pair, will give rise to a Hawking point, through a process known as “quantum scarring.” The scars are of the form of quantum dots, which emit photons of certain wavelengths if excited to a sufficient degree.

If the information of two entangled quantum dots transitions to a new location on a single tangent frame of a worldsheet, then we say that it is “quantum teleportation.”

3 Transmission Categories

Feynman described the photoelectric effect, which supervened upon emission and absorption. We can describe these as outward and inward projections, respectively.

Emission is described as:

$$f : e \rightarrow b \in B \quad e \subset A \tag{3.1}$$

where the subset is proper. Absorption is defined as:

$$g : e \hookrightarrow B : B \oplus e \tag{3.2}$$

Definition 3.1. *A transmission category is a collection of emission morphisms composed with absorption morphisms, and vice versa. The objects of this category are energy potentials, and the morphisms are Ricci iterations of the holonomy.*

We abbreviate these by TC.

Theorem 3.1.

$$TC \cong E$$

where the r.h.s. is the category of Dowker events.

We are allowed to make this statement, since the generalnonsense at this level is very arbitrary, and for the equivalence, all we need to do is smash TC with some observable (call it \mathcal{O}):

$$E = TC \otimes \mathcal{O} \tag{3.3}$$

and, since we are topologists, the notation \mathcal{O} means a sheaf of model categories. Since this observable is (classically, at least) a uniquely identifiable node on a graph of spacetime, perfect knowledge of the Laplacian of the node transfers to perfect knowledge of a large subgraph of spacetime.

We can translate between causal set theory and bootstrap theory by calling each event category a “*spacetime pixel*.” This is a purely spatial configuration which is installed into the bedrock of reality as neurotypical humans perceive it. When we map the spacetime bulk to a point, this weight of this pixel cancels out, and we get:

$$\omega \mathcal{P}^{\otimes n} \rightarrow 0 \tag{3.4}$$

which means either ω goes to 0, or $\mathcal{P}^{\otimes n}$, or both. Since the symplectic form isn’t an observable, if this were at all a physical condition, it means the n -fold Penrose space in question would have to vanish in order to realize this “law.”

For the whole of spacetime, this law is untestable. However, for small, relatively isolated systems, the dream of obeying this rule can be detected. When a force carrier is annihilated, the speed of

time should become infinite. This means that the quantum Zeno effect has been maximized, and we get:

$$[Z \rightarrow \max(Z, 1)](\hbar e^{i\theta\pi k} \mathcal{O}) = |\hbar e^{i\theta\pi k}| \cong \hbar \cdot p_{Emmerson} = E_{\alpha, B_b, r} \quad (3.5)$$

where $E_{\alpha, B_b, r}$ is the energy of a matter wave α throughout an isotropic ball of radius r centered at a point b . This is a dimensionless quantity which describes the failure of the region to remain perfectly sedentary. This is the Emmersonian of the phonon in an elastic medium.

This is a vibration of a Morse string, which acts as the creation operator for a single phonon. Recall the Morse string is written:

$$s_{Morse} = Diff([\gamma]) = \gamma / \sim \quad (3.6)$$

so it is the diffeomorphism of any force carrier. The equivalence relation is given up to a smooth perturbation of the principal curvature:

$$\mathfrak{C}_{Prin} = \alpha_\mu + \beta_\nu - (\alpha_\mu - \beta_\nu) \quad (3.7)$$

$$= [Sum - Difference](\alpha, \beta)_{\mu, \nu} \quad (3.8)$$

So, basically, the equivalence is bounded by minimum Emmersonian of the regime in question.

We have:

$$SO(3) \times SU(2) \times U(1) \times \mathfrak{C}_{Prin}[\theta] = \mathfrak{M}_\mathbb{L} \quad (3.9)$$

generating the moduli space of lightcones, which is the standard model crossed with the rotations of the principle fiber.

We can also write:

$$\mathfrak{M}_\mathbb{L} \simeq \theta \curvearrowright [\mathfrak{C}_{Prin} \times \mathfrak{b}] \quad (3.10)$$

and when we replace \mathfrak{b} with $\hat{\mathfrak{b}}$, we get \mathfrak{M}_\square , the moduli space of necessary moments.

4 Towers of Lightcones

Let us take a moment to formally define the discretization functor. It is a category $\mathbf{Man} \rightarrow \mathbf{TV}_{Disc}$ from the category of smooth manifolds to the category of discrete topological vector spaces. Suppose we have a wavefunction $\Psi(\alpha)$ on \mathbb{R}^3 . We can write $\delta(\Psi(\alpha))$ to give us a formal distribution over \mathbb{Z} . Now, keep in mind, a Hawking point can form at any point the codomain of this map. If we accept that a Hawking point can be rewritten as a lightcone singularity, or in other words, that \mathfrak{b} admits a Hawking point, we can form a lightcone anywhere in the intersection of $\Psi(\alpha)$ with the integers.

That is to say, for any integer valued location on a lightcone, we can perform a base change that will transform that location into the center of a lightcone. Formally:

$$\forall y = \Psi(\alpha) \cap \mathbb{Z} \quad \exists \mathfrak{b} \wedge \exists \mathbb{L}^{1,3} \text{ such that } \mathfrak{b} = Sing(\mathbb{L}^{1,3}) \quad (4.1)$$

which gives us

$$\forall y \quad \exists \iota : y \hookrightarrow \mathcal{P}^1 \setminus \mathbb{A}^1 \quad (4.2)$$

What this really means, is that we have an “infinite spin tower,” or an infinite tower of bosons. Thus, for every force carrier φ , we can set $\lim_{i \rightarrow \infty} \varphi_i = \mathcal{U}$. This works, at least when he have supersymmetry, as a very simple “master equation” for every particle in the universe. This should be allowed in any theory that involves quantization. We write the Pauli exclusion principle as:

$$\varphi_{i \equiv_2 0} \wedge \neg \varphi'_{i \equiv_2 0} \quad (4.3)$$

where \equiv_2 is equivalence mod 2. For $\mathcal{N} = N$ supersymmetry, we have:

$$\varphi_{i \equiv_m 0} \wedge \neg \varphi'_{i \equiv_m 0} \quad m = (\log_2 N) + 1 \quad (4.4)$$

We have $x \propto \alpha$ for every fiber spectrum x and for every particle α ; this means that the probability density of the fiber spectrum (analogous to the wavefunction) *predicts* the action of each particle; however, this is not an ordinary equality. This is actually given by:

$$(Diff(\square) / \sim) = \hat{\square} \quad (4.5)$$

where the similarity is given by $\square \pm k$ where $k \leq 2\varepsilon$, so the moment lies on either the immediate past or future of the singularity, or its outbound or incoming temporal section. In the presence of Hawking points, k is forced to lie exactly on either $\square \pm \varepsilon$ or $\square \pm 2\varepsilon$, but in the absence it is free to lie in the intermediate points. It is the stochastic deviations from these Hawking points in smooth spacetime that causes the profile of the universe to differ from iteration to iteration. These small but appreciable changes are tantamount to what in causal set theory is known as a “*Poisson sprinkling*.”

4.1 Critical Points

Let \mathcal{S}_b be a spline; choose the vacuum points of the spline to be critical, so that for any function $\phi : \mathcal{S}_b \rightarrow [0, 1]$, we obtain either zero or one. Recall from MacLane’s definition of the skyscraper sheaf that it is the sheaf $Sky_x(\alpha)(U)$ which returns a value of α if $x \in U$ and a value of 1 otherwise. We can pick certain “good neighborhoods,” (i.e., second countable and contractible) which contain a particle x , and assign them to a local excitation of a field α . Everything outside this neighborhood will be mapped to 1, though for bookkeeping purposes, it may be easier to choose zero for us.

Define:

$$\exists A = \int_{\mathbb{1}, 1, 3} \frac{\sum_{i=0}^{\infty} Sky_{x_i}(\alpha)(U)}{\#U} \equiv 1 \quad (4.6)$$

Thus, $\exists A$ entails the existence of a maximally dense region of spacetime. Suppose each of the α is a hadron. Then, A should theoretically be maximally work-locked. However, it will emit Hawking radiation which is entangled with its internal contents. By “entangled,” we mean the entropy of the emissions is the same as the entropy of the infalling particles. If we are talking about Shannon entropy, then the black hole must find some distant, but dense, partner to couple to and exchange information with.

Let \mathfrak{h} be the black hole and \mathfrak{p} its partner. Then, obviously, we must have some string $\mathfrak{s} = s_{Morse}^{\mathfrak{h}, \mathfrak{p}}$ which links the two. The critical points of this string are actually small transaction categories, containing a lone creation or annihilation operator. Write

$$\phi_{Goldstone} = e^{\pi i \theta k} \frac{\mathfrak{s}}{2^N \pi} \quad (4.7)$$

for the Goldstone boson exchanging the force transferred between the black hole and its partner.

Definition 4.1. *An ideal black hole is a region of spacetime in which every point is a critical point of a Morse string.*

We could actually argue that ideal black holes are more likely to arise through dark-light matter interactions (i.e., photon-antiphoton) than through any other process in the universe. This is because, of course, the volume of the black hole is incredibly small, and so it is more likely to be filled entirely by hadrons than if it were distributed over a larger region of the bulk.