

1 PyCVI: A Python package for internal Cluster Validity 2 Indices, compatible with time-series data

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5 Summary

6 PyCVI is a Python package specialized in internal Clustering Validity Indices (CVIs) compatible
7 with both time-series and non time-series data.

8 Clustering is a task that aims at finding groups within a given dataset. CVIs are used to select
9 the best clustering among a pre-computed set of clusterings. In other words, CVIs select the
10 division of the dataset into groups that best ensures that similar datapoints belong to the
11 same group and non-related datapoints are in different groups.

12 PyCVI implements 12 state-of-the-art *internal* CVIs to improve clustering pipelines as well as
13 the Variation of Information (VI) ([Meilă, 2003](#)), a distance measure between clusterings. VI
14 can have many purposes, among which being used as an *external* CVI and to evaluate internal
15 CVIs or clustering methods when true labels are known. The *internal* qualifier here refers to
16 the general case in practice where no *external* information is available about the dataset such
17 as the true association of the datapoints with groups, as opposed to *classification* tasks.

18 Statement of need

19 There exist many mature libraries in python for machine learning and in particular clustering:
20 [scikit-learn](#) ([Pedregosa et al., 2011](#)), [TensorFlow](#) ([Abadi et al., 2015](#)), [PyTorch](#) ([Paszke et al.,](#)
21 [2019](#)), [scikit-learn-extra](#) ([Scikit-learn Extra, n.d.](#)), and even several specifically for time series
22 data: [aeon](#) ([Aeon, n.d.](#)), [sktime](#) ([Löning et al., 2019](#)), [tslearn](#) ([Tavenard et al., 2020](#)).

23 However, although being fundamental to clustering tasks and being an active research topic,
24 very few internal CVIs are implemented in standard python libraries (only 3 in [scikit-learn](#), more
25 were available in R but few were maintained and kept in CRAN ([Charrad et al., 2014](#))). Thus
26 for a given CVI, there is currently no corresponding maintained and public implementation. This
27 is despite the well-known limitations of all existing CVIs ([Arbelaitz et al., 2013](#)), ([Gagolewski](#)
28 [et al., 2021](#)), ([Gurrutxaga et al., 2011](#)), ([Theodoridis & Koutroumbas, 2009](#)) and the need
29 to use the right one(s) according to the specific dataset at hand, similarly to matching the
30 right clustering method with the given problem. A crucial step towards developing better CVIs
31 would be an easy access to an implementation of existing CVIs in order to facilitate larger
32 comparative studies.

33 In addition, all CVIs rely on the definition of a distance between datapoints and most of them
34 on the notion of cluster center.

35 For static data, the distance between datapoints is usually the euclidean distance and the
36 cluster center is defined as the usual average. Libraries such as [scipy](#), [numpy](#), [scikit-learn](#), etc.
37 offer a large selection of distance measures that are compatible with their main functions.

38 For time-series data however, the common distance used is Dynamic Time Warping (DTW)
39 ([Berndt & Clifford, 1994](#)) and the barycenter of a group of time series is then not defined

40 as the usual mean, but as the DTW Barycentric Average (DBA) (Petitjean et al., 2011).
 41 Unfortunately, DTW and DBA are not compatible with the libraries mentioned above. This,
 42 among other reasons, made additional machine learning libraries specialized in time series data
 43 such as `aeon`, `sktime` and `tslearn` necessary.

44 PyCVI fills that gap by implementing 12 state-of-the-art internal CVIs: Hartigan (Strauss
 45 & Hartigan, 1975), Calinski-Harabasz (Calinski & Harabasz, 1974), GapStatistic (Tibshirani
 46 et al., 2001), Silhouette (Rousseeuw, 1987), ScoreFunction (Saitta et al., 2007), Maulik-
 47 Bandyopadhyay (Maulik & Bandyopadhyay, 2002), SD (Halkidi et al., 2000), SDbw (Halkidi
 48 & Vazirgiannis, 2001), Dunn (Dunn, 1974), Xie-Beni (Xie & Beni, 1991), XB* (Kim &
 49 Ramakrishna, 2005) and Davies-Bouldin (Davies & Bouldin, 1979). Furthermore, in PyCVI
 50 their definition is extended in order to make them compatible with DTW and DBA in addition
 51 to static data. Finally, PyCVI is entirely compatible with `scikit-learn`, `scikit-learn-extra`, `aeon`
 52 and `sktime`, in order to be easily integrated into any clustering pipeline in python. To ensure a
 53 fast a reliable computation of DTW and DBA, PyCVI relies on the `aeon` library.

54 Example

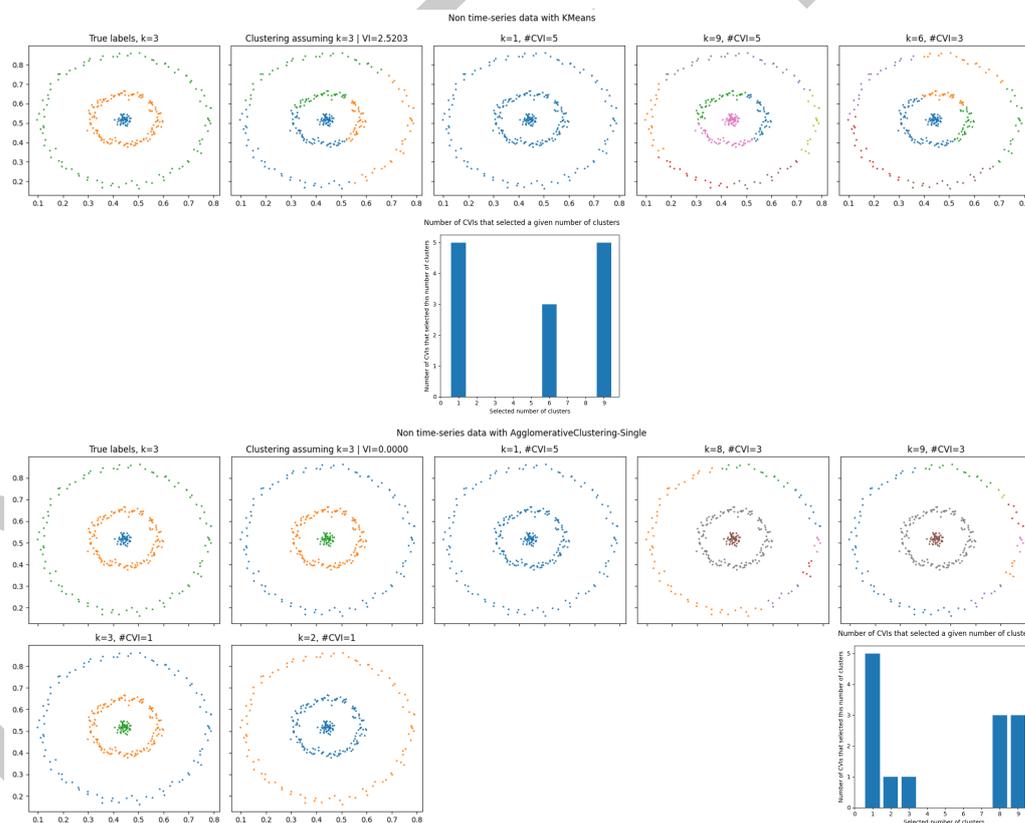


Figure 1: KMeans and AgglomerativeClustering on static data. Selected clusterings according to each implemented CVI.

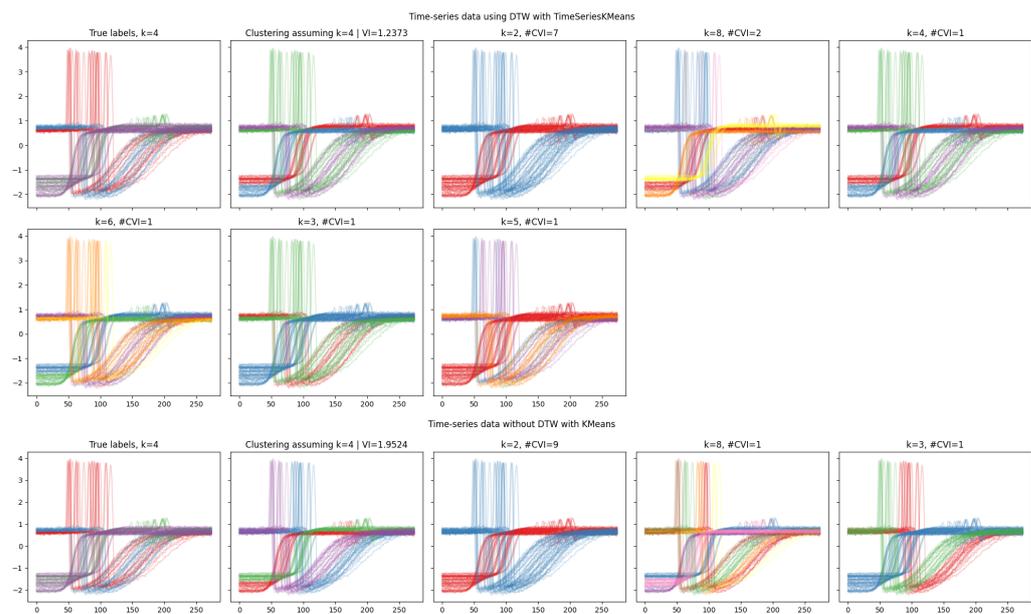


Figure 2: KMeans on time-series data, with and without DTW.

55 We experimented 3 cases: static data (Barton, 2015), time-series data (Dau et al., 2018) with
 56 euclidean distance and then with DTW as distance measure and DBA as center of clusters.
 57 In addition, we used different clustering methods from different libraries: KMeans (Lloyd, 1982)
 58 and AgglomerativeClustering (Ward, 1963) from scikit-learn, TimeSeriesKMeans from aeon
 59 and KMedoids (“Partitioning Around Medoids (Program PAM),” 1990) from scikit-learn-extra
 60 to showcase PyCVI integration with other clustering libraries.

61 As a first example, we individually ran all CVIs implemented in PyCVI, selected the best
 62 clustering according to each CVI and plotted the selected clustering. In addition, we computed
 63 the variation of information (VI) between each selected clustering and the true clustering.
 64 High VI values mean large distances between the true clustering and the computed clusterings,
 65 meaning computed clusterings of poor quality. In Figure 1, we can see the difference of quality
 66 when assuming the correct number of clusters between the AgglomerativeClustering and the
 67 KMeans clustering method on static data. This is independent of the CVI used, meaning that
 68 a poor clustering quality will be due to the clustering method.

69 In Figure 1, since the quality of clusterings generated by KMeans is bad due to the clustering
 70 method, the poor selection results gives us no information about the correct clustering, nor
 71 about the quality of the CVIs used. This motivates further research on clustering methods.
 72 However, using AgglomerativeClustering, the quality of the clustering is excellent, as indicated
 73 by a null VI. The corresponding selection results shown in the corresponding histogram tells
 74 us that the CVIs used here are not adapted to this dataset. This was expected since most
 75 CVIs rely on the cluster center to compute a good separation between clusters. The dataset
 76 here consisting of concentric circles, most CVIs fail to measure how well separated the clusters
 77 actually are. This illustrates the need of further research on CVIs, which is facilitated by PyCVI,
 78 notably in the case of concentric subgroups.

79 Similarly, with time-series data in Figure 2, the quality of the clustering assuming the correct
 80 number of clusters varies although the same clustering method is used on the same dataset.
 81 This illustrates the difference between using DTW as a distance measure compared to using
 82 the euclidean distance, and between using DBA to compute the average of a group of time
 83 series and using the usual average.

84 In a second example, we demonstrate cases of successful clustering and clustering selection,

85 while showcasing an additional feature of PyCVI: CVIAggregator. CVIAggregator selects the
86 best clustering by combining several CVIs and by using the majority vote among the clusterings
87 individually selected by the combined CVI.

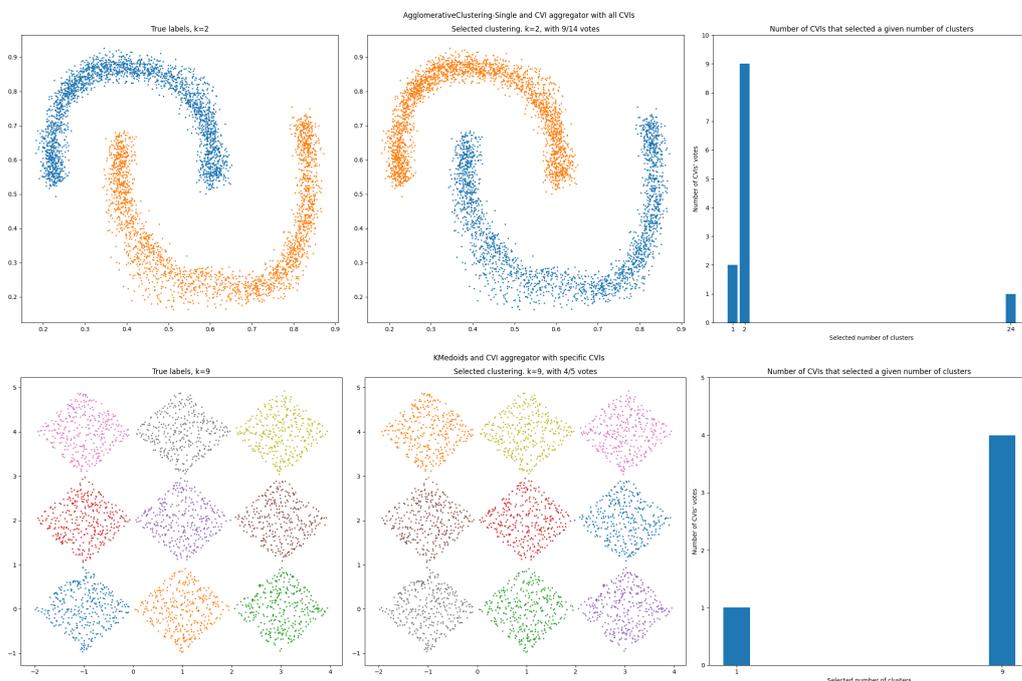


Figure 3: Selection done by a CVIAggregator using all implemented CVIs first and then with specific CVIs (GapStatistic, Silhouette, Dunn, CalinskiHarabasz and XieBeni).

88 In [Figure 3](#), we used CVIAggregator with first all CVIs implemented in PyCVI and then only
89 with some of the implemented CVIs, as it could be done in practice when known characteristics
90 of the dataset can help identify unadapted CVIs. We see that in both cases, the data was
91 correctly clustered by the clustering method and the best clustering correctly selected. This is
92 in spite of clusters of non-convex shapes in the first case and clusters “touching” each other
93 in the second

94 The code of these examples is available on the [GitHub repository](#) of the package, and its
95 [documentation](#).

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