



# COMPUTATIONAL STUDY OF HIGH-PRESSURE LIQUID INJECTION PROCESS BY MEANS OF LES AND PANS APPROACHES

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For internal combustion engine, the determination of combustion characteristics and subsequent emissions formation relies heavily on the fuel injection process. With the increasing demand for enhanced fuel efficiency and reduced emissions, it becomes vital to develop fundamental understanding of physical process involved in the fuel injection process. In this study, an optimal numerical approach to predict high pressure liquid injection process in the context of industrial computations has been investigated. In particular, this study focuses on the respective performance of the Partially-Averaged Navier-Stokes and Large Eddy Simulation models to predict turbulent igniting sprays. Both approaches are coupled with widely used Lagrangian Discrete Droplet Method for spray modelling. The results are validated against well established ECN Spray A case in reactive and non reactive conditions. For reacting conditions, Flamelet Generated Manifold (FGM) combustion model is employed in the present work. Comparative study and validation against experimental data showed that PANS turbulence model allows for coarser grids while still maintaining accurate results.

**KEY WORDS:** Engine Combustion Network (ECN) Spray A, Large Eddy Simulation, Partially Averaged Navier Stokes, Tabulated chemistry

1

## 2 **1. INTRODUCTION**

3 Internal combustion (IC) engine remains a significant source of harmful pollution despite the  
4 effort made to improve its efficiency and reduce emissions to some extent. To mitigate harm-  
5 ful effects linked to the pollutants emission, transition to cleaner and more sustainable forms  
6 of transportation is crucial. Recent European (Hooftman et al. (2018)) and US (Gerard and  
7 Lave (2005)) emission legislation dictate stricter emission standards, development of hybrid  
8 and electric vehicles, and adoption of alternative fuels. Nevertheless, electrification strat-  
9 egy for medium and large heavy-duty engines, such as engines for cargo ships, heavy-duty  
10 trucks and marine engines is still not foreseen. One of the promising concepts for this kind  
11 of applications, that has emerged in recent years, is dual fuel engine. The concept of dual  
12 fuel combustion has garnered increased interest due to its potential to reduce engine noise,  
13 soot and NO<sub>x</sub> emissions (Xu et al. (2020)). Additional benefits of dual fuel engines are high  
14 fuel flexibility, reliable ignition and combustion as well as robustness. Hence establishing an  
15 effective methodology for design of dual fuel engines and its injection equipment represents  
16 a key priority.

17 Multiphase flow mixing and evaporation phenomena are essential in numerous indus-  
18 trial applications, including IC engine. In IC engine, fuel injection is crucial process used to  
19 disperse liquid fuel over a wider area of the combustion chamber, thereby increasing the  
20 surface needed for more intensive evaporation. Proper fuel injection process is essential  
21 for ensuring that the fuel is efficiently and effectively burned, which results in more efficient  
22 and stable combustion process leading to better engine performance. Moreover, amount of  
23 pollutant emissions released from IC engine strongly depends on the spray process, in par-  
24 ticular on fuel atomization and fuel-air mixing process (Petranovic (2016)). As the demand  
25 for improved fuel efficiency and reduced emissions continues to grow, detailed understand-  
26 ing of spray processes is essential for the design and optimization of IC engines and fuel  
27 injection equipment (FIE). In dual fuel operational mode injector is operated in so called bal-  
28 listic mode, characterized by short injection duration and high pressure. This leads to more

1 complex and challenging design process compared to the design of injectors for pure diesel  
2 engines. Therefore, effective design of the FIE is considered as crucial step in industrial de-  
3 velopment of dual fuel IC engines (Gaballa (2023)). To address this challenge, fuel injector  
4 manufacturers require advance simulation tools to achieve optimized design for fuel injec-  
5 tors specifically tailored to meet demands of dual fuel engines. Consequently, this leads to  
6 improved performance and efficiency of dual fuel IC engines.

7 Computational fluid dynamics (CFD) tools have been widely used for simulating fuel in-  
8 jection under various operating conditions. However, spray process is highly turbulent and  
9 transient process which involves wide range of time and length scales. Moreover, it involves  
10 many closely related physical processes such as in-nozzle cavitation, liquid atomization,  
11 phase change, mixing and chemical reactions. Consequently, numerical modelling of fuel  
12 injection is a challenging task. Several numerical approaches with different level of complex-  
13 ity have been developed to address this problem. For turbulent flow simulations of industrial  
14 interest, common practice is to adapt Reynolds-Averaged Navier-Stokes (RANS) turbulence  
15 modelling approach due to its low computational cost. However, the range of flow physics  
16 that can be accurately represented by RANS models is severely limited. This limitation arises  
17 from the fact that RANS models are single-point closures relying on the assumption of self-  
18 similarity of turbulence spectrum (Jakirlic et al. (2012)). This assumption implies that entire  
19 turbulence spectrum is characterized by only one characteristic turbulent length scale. To  
20 overcome these limitations, Large Eddy Simulation (LES) could be employed. LES resolves  
21 a wide range of turbulent physics, capturing large coherent structures and significant portion  
22 of the inertial scales. Hence, it provides more detailed and accurate description of turbulent  
23 flow phenomena, especially in applications where unsteady and complex turbulent struc-  
24 tures are essential. However, the increased range of resolved flow physics and high fidelity  
25 flow details come at the expense of much greater computational effort (Girimaji and Abdol-  
26 Hamid (2005)). Indeed, LES is still prohibitively expensive for engineering applications if  
27 only the resolution of the largest scales is needed. With the desire to combine the benefits  
28 of RANS and LES methods for complex turbulent flows in practical engineering applications,



1 the hybrid LES/RANS modelling approaches have been developed. The objective of these  
2 models is to extract important large-scale unsteady coherent structures, which dominate the  
3 flow, without a burden of resolving inertial scales. Thus, providing accurate predictions in a  
4 computationally efficient manner for industrial every day use. One of the hybrid LES/RANS  
5 models that has shown capability to accurately and cost-efficiently predict turbulent flow in  
6 various canonical flows (see Reyes et al. (2014); Tazraei and Girimaji (2019)) as well as  
7 in real-life industrial applications (see Basara et al. (2017); Jakirlic et al. (2012); Krajnović  
8 et al. (2012)) is Partially-Averaged Navier-Stokes (PANS) turbulence model. The PANS tur-  
9 bulence model is a scale resolving method that can smoothly vary from RANS to direct nu-  
10 merical simulation based on the model resolution parameters. Compared to RANS models,  
11 PANS simulations exhibit improved accuracy due to resolving a portion of turbulent flow. Fur-  
12 thermore, PANS simulations have demonstrated to produce comparable results to LES on  
13 coarser computational meshes. Therefore, PANS method is practical and efficient modelling  
14 approach for many engineering applications. In this paper, we propose to couple Lagrangian  
15 Discrete Droplet Method (DDM) for spray modelling with PANS  $k - \zeta - f$  turbulence model  
16 for an accurate and computationally affordable 3D CFD simulation of fuel injection process.  
17 The Engine Combustion Network (ECN) Spray A is used as a reference case to validate pro-  
18 posed numerical methodology, in both, non-reactive and reactive conditions. Although, this  
19 generic geometry differs significantly from a standard production nozzle, it has been widely  
20 used for collaborative research and openly accessible data sets with numerical and exper-  
21 imental results are available. For the reactive simulation the Flamelet Generated Manifold  
22 (FGM) tabulated chemistry combustion model is employed.

23 The work presented here is structured as follows. Firstly, the LES Coherent Structure  
24 Model (CSM) coupled with DDM is applied to simulate non-reactive and reactive ECN Spray  
25 A conditions. The original LES CSM, as developed by Kobayashi (2005) is designed for sin-  
26 gle phase flow application. Therefore, the model is slightly adapted to obtain better results  
27 for two phase flow application. Development of the two phase flow modelling framework is  
28 far beyond the scope of the present work. Here a modification to the original formulation of

1 the LES CSM model is introduced only to obtain better results in cases involving high pres-  
2 sure liquid injection, such as fuel injection in IC engines. This modification of the model is  
3 explained in detailed in Section 4.1. Secondly, PANS  $k - \zeta - f$  turbulence model is employed  
4 to simulate ECN Spray A test case. Numerical study on dependence of the PANS model on  
5 the discretization schemes is performed. Additionally, the same test case is simulated uti-  
6 lizing the RANS  $k - \zeta - f$  turbulence model. Finally, the proposed DDM PANS simulation  
7 methodology is employed to simulate ECN Spray A case in the dual fuel configuration, where  
8 methanol is utilized as a primary fuel. The methanol/oxidizer mixture is assumed to be ho-  
9 mogeneously mixed in the combustion chamber and the same amount of n-dodecane fuel,  
10 as in single fuel configuration, is used to provided required ignition energy. Due to the lack  
11 of experimental data for dual fuel conditions this investigation is purely numerical. Obtained  
12 results are qualitatively compared to the results of ECN Spray baseline single fuel case and  
13 the results obtained from numerical simulation of the same case. The primary objectives of  
14 this study are (i) to demonstrate potential of PANS turbulence model to predict high pressure  
15 fuel injection process (ii) to propose an effective numerical simulation workflow for develop-  
16 ment and design of DFIC engines and its fuel injection equipment suitable for every day  
17 industrial use.

## 18 2. MATHEMATICAL MODEL

19 The following section will describe, in some detail, numerical models relevant to the present  
20 work. Initially, Euler Lagrangian approach for spray modelling along with its sub-models is  
21 introduced. Afterwards, the modification of the LES coherent structure model is elaborated.  
22 Additionally, a brief explanation of the PANS  $k - \zeta - f$  model is provided. Finally, the FGM  
23 combustion modelling approach is briefly described.

## 1 **2.1 Spray Modelling**

2 Spray simulations require simultaneous numerical solution of conservation equations for  
3 both, the gas and the liquid phases. With the respect to the liquid phase, for spray calcula-  
4 tions in the engineering environment, a commonly used method is a statistical method re-  
5 ferred to as the Discrete Droplet Method (DDM). In this method, the dispersed liquid droplets  
6 are tracked as Lagrangian parcels within the continuous gas phase, which is represented us-  
7 ing Eulerian framework. The parcels represent group of droplets with similar properties such  
8 as diameter and velocity. This approach suffers from several limitations which are described  
9 in Petranovic (2016); Xue et al. (2014). Nevertheless, it proved to be efficient and accurate  
10 in predicting the spray dynamics under turbulent conditions in the industrial development  
11 process. To accurately capture spray phenomena with Lagrangian particle tracking method,  
12 a set of spray sub-models has to be employed to account for different aspects of spray be-  
13 havior. These models include primary and secondary breakup, evaporation, wall interaction,  
14 atomization, collision and turbulent dispersion. Selection and implementation of these sub-  
15 models depends on the specific system and its characteristics. Moreover, the computational  
16 cost and accuracy trade-off must be considered to ensure practical and efficient numerical  
17 simulation. The present study employs the standard WAVE breakup model of Reitz (1987).  
18 Droplet evaporation is modelled with multi-component evaporation approach capable of han-  
19 dling evaporation of droplets composed of arbitrary number of components according to Fink  
20 (2005). As the spray droplets pass through the continues gas phase they interact with in-  
21 dividual turbulent eddies. This interaction can't be directly resolved by the flow. Hence, the  
22 turbulent dispersion model is used. Modification of the spray sub-models for PANS and LES  
23 applications is not necessary. The effects of the unresolved scales are assumed to be neg-  
24 ligible compared to the droplet interaction with the resolved scales. Hence, for the present  
25 PANS and LES simulations, the turbulent dispersion effects are fully covered by interaction  
26 of the spray droplets with the scales resolved by the LES or PANS simulation.

## 1 2.2 Large Eddy Simulation

2 The RANS turbulence models are still most commonly used approach in design and opti-  
 3 mization process of IC engine due to their efficiency and reliability. However, this approach  
 4 is limited when it comes to investigating important aspects of combustion process such as  
 5 cycle to cycle variations and related phenomena. On the other hand, the LES approach  
 6 enables capturing of cycle-to-cycle variations by directly simulating the large turbulent struc-  
 7 tures and modeling the influence of subgrid scales on the resolved ones. In the LES frame-  
 8 work, set of equations governing the fluid flow are based on the filtered Navier-Stokes equa-  
 9 tions. Filtering operation is performed to separate the resolved scales of the flow from the  
 10 unresolved scales. In this work, explicit filtering is not performed, meaning the computational  
 11 mesh serves as an implicit filter that removes the subgrid scales. Hence, the resolution of  
 12 the computational grid requires special attention to ensure accurate simulation results. Due  
 13 to limitations of computational resources, it might not be possible to have fine resolution in  
 14 the entire domain, but locally where specific phenomena of interest might occur.

15 The governing equations within the LES framework are given as:

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \nu \frac{\partial \bar{u}_i}{\partial x_j} - \underbrace{(u_i u_j - \bar{u}_i \bar{u}_j)}_{\tau_{ij}} \right] \quad (1)$$

16 where  $\tau_{ij}$  represents represents the subgrid scale tensor which is modelled according to  
 17 Boussinesq assumption:

$$\overline{u_i u_j} - \bar{u}_i \bar{u}_j = -2\nu_t \bar{S}_{ij} \quad (2)$$

18 where  $\nu_t$  is the turbulent viscosity which is a property of the flow and has to be modelled.  
 19 Turbulent viscosity is an artificial viscosity, representing equivalent dissipation of unresolved  
 20 scales of motion Perkovic (2014). In this work, turbulent viscosity is modelled according to  
 21 Kobayashi (2005). This model is based on coherent structure function which enables avoid-  
 22 ing expensive averaging in the homogeneous direction. Another benefit of subgrid scale  
 23 model based on the coherent structure function is that it doesn't require a wall damping

- 1 function of Van Driest type to vanish the eddy viscosity on the wall. The formulation of tur-  
 2 bulent viscosity is given in Eq. 3:

$$\nu_{SGS} = C_{CSM} \Delta^2 |\bar{S}| \quad (3)$$

- 3 where the  $|\bar{S}|$  is resolved rate of strain tensor.  $C_{CSM}$  is model constant which is locally  
 4 defined according to following expression:

$$C_{CSM} = C_2 |F_{CS}|^{3/2} F_\Omega \quad (4)$$

- 5 Where  $F_\Omega$  is energy-decay suppression function given by Eq.5:

$$F_\Omega = (1 - F_{CS}), \quad (5)$$

- 6 and  $F_{CS}$  is coherent structure function (see Eq. 6) defined as a ratio of second invariant of  
 7 a velocity gradient tensor  $Q$  and the magnitude of a velocity gradient tensor  $E$  as defined in  
 8 Eq.7:

$$F_{CS} = \frac{Q}{E} \quad (6)$$

$$Q = \frac{1}{2} (\tilde{W}_{ij} \tilde{W}_{ij} - \tilde{S}_{ij} \tilde{S}_{ij}); \quad E = \frac{1}{2} (\tilde{W}_{ij} \tilde{W}_{ij} + \tilde{S}_{ij} \tilde{S}_{ij}) \quad (7)$$

- 10 The resolved velocity strain tensor  $\tilde{S}_{ij}$  and resolved vorticity  $\tilde{W}_{ij}$  tensor are defined as  
 11 follows:

$$\tilde{S}_{ij} = \frac{1}{2} \left( \frac{\partial \tilde{u}_j}{\partial x_i} + \frac{\partial \tilde{u}_i}{\partial x_j} \right); \quad \tilde{W}_{ij} = \frac{1}{2} \left( \frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j} \right) \quad (8)$$

- 12 While the  $C_{CSM}$  is locally updated parameter based on the local coherence in the velocity  
 13 field, the  $C_2$  in Eq.4 is fixed model constant with value of 1/22. This value is obtained from  
 14 DNS data in non-rotating homogeneous turbulence at the center of turbulent channel flow  
 15 ( Kobayashi (2005)). The performance of CSM model has been well exhibited in variety of  
 16 idealized and canonical flows. Kobayashi et al. (2008) applied CSM model for simulation of  
 17 a flow over a backward-facing step and for a flow in an asymmetric plane diffuser, as well as

1 for staggered jets in crossflow.

2 In this study, it has been observed that when employing this model in cases involving  
 3 high-pressure liquid injection, such as in IC engines, the model effectively damps both, eddy  
 4 viscosity on the wall and in the spray region, where dissipation has its maximum. According  
 5 to Klein et al. (2019), there is not standard set of governing equations for two-phase flow  
 6 LES simulation, but rather a variety of different formulations, all with advantages and disad-  
 7 vantages. This paper discusses and analyses suitable closure for eddy viscosity of coherent  
 8 structure model in the context of fuel injection process. A three dimensional rectangular  
 9 computational domain with average cell size of 0.25 mm is used to illustrate behaviour of  
 10 the original coherent structure model. For this purpose n-dodecane is injected into gaseous  
 11 nitrogen under conditions of 60 bar and 900 K, similarly as in Spray A case. In the Fig. 1  
 12 the upper figure shows total eddy dissipation field where it is notable that maximum dissipa-  
 13 tion occurs in the spray region. The middle figure shows unresolved turbulence representing  
 14 small scale turbulent motions that are not explicitly resolved by the numerical grid and again  
 15 the maximum of unresolved energy lies in the spray region. Lower figure shows turbulent  
 16 eddy viscosity as obtained by coherent structure subgrid model. It can be observed that  
 17 resulting eddy viscosity is damped in the region of highest dissipation of unresolved scales  
 18 of motion. Consequently, influence of the unresolved scales on the resolved ones is not  
 19 captured satisfactorily.

20 It has been observed, that this leads to overprediction of liquid length and vapor pene-  
 21 tration as shown in Sec. 4.1, which in turn results with a wrong prediction of the combustion  
 22 process. To overcome this issue, a well-established Smagorinsky model with a value of  
 23 model constant  $C_S = 0.1$ , is used to obtain eddy viscosity in this case, according to Eq. 9:

$$\nu_{SGS} = (C_s f \Delta)^2 |S| \quad (9)$$

24 Where  $f$  is a wall damping function of Van Driest type employed to vanish the eddy viscosity

1 on the wall, thus:

$$f = 1 - \exp\left(-\frac{y^+}{25}\right) \quad (10)$$

2 The obtained eddy viscosity field is then utilized to determine a new value of the coherent  
 3 structure model constant  $C_2$  from the Eq. 4 and Eq. 3. As it can be seen from the Fig. 2 it  
 4 appears that a suitable value for  $C_2$  constant in the spray region is unity.

5 According to Popovac and Hanjalic (2007), damping functions introduce additional non-  
 6 linearity and often numerical stiffness, which together with dense clustering of the com-  
 7 putational grid in the wall-normal direction, may lead to excessive computational cost. On  
 8 the other had, the coherent structure model is suitable for engineering applications since it  
 9 accounts for local coherence in the flow without expensive averaging in a homogeneous di-  
 10 rection. Additionally, wall-damping function of Van Direst type is not required to vanish eddy  
 11 viscosity on the wall. Thus to optimize the coherent structure model for wall vicinity and vis-  
 12 cous effects in the spray region, in this study it has been proposed to employ a blending  
 13 function as described in Popovac and Hanjalic (2007):

$$\Gamma = \frac{0.01y^{+4}}{1 + 5y^+} \quad (11)$$

14 By means of this blending function one can combine two values of  $C_2$  model constant.  
 15 Namely, in the viscous near wall region where eddy viscosity should vanish, an original  
 16 value of  $C_2 = 1/22$  is kept, and further away from the wall, in the full turbulent field, the  $C_2$   
 17 is changed to suitable value which is unity. We now apply the blending principle to the Eq. 3  
 18 and obtain a formulation for optimized eddy viscosity within coherent structure model which  
 19 considers wall effects and local flow properties:

$$\nu_{opt} = \nu_{org}e^{-\Gamma} + \nu_{new}e^{-1/\Gamma} \quad (12)$$

20 Expression 12 defines optimized, or tuned, turbulent eddy viscosity  $\nu_{opt}$ , which smoothly  
 21 varies between original viscosity  $\nu_{org}$  with  $C_2 = 1/22$  near the wall, and  $\nu_{new}$  defined

1 with the same equation 3 and 4 but with  $C_2 = 1$ . This formulation enables blending of  
 2 the turbulent eddy viscosity between the viscous and fully turbulent definitions utilizing the  
 3 blending function  $\Gamma$ . Eddy viscosity obtained by this formulation is shown in Fig. 3. It can  
 4 be seen that the new eddy viscosity has its maximum in the region of highest dissipation of  
 5 unresolved scales, at the same time the viscosity on the wall is vanished. This formulation  
 6 is employed to calculate the Spray A case and the results are shown in the Section 4.1.

### 7 **2.3 Partially Averaged Navier Stokes**

8 The PANS model is a scale resolving turbulence model developed to overcome some of the  
 9 limitations of traditional turbulence modelling approaches, such as RANS and LES models.  
 10 Model is envisioned to offer a balance between computational efficiency of RANS models  
 11 and the accuracy of LES models. The PANS model as proposed by Girimaji et al. (2003)  
 12 seamlessly vary from RANS to the direct numerical solution of Navier-Stokes equations. The  
 13 specific closure model used in PANS can vary depending on implementation and application.  
 14 There are several variants of PANS model derived up to now, throughout this work PANS  
 15 version based on  $k - \zeta - f$  model formulation is used (see Basara et al. (2011, 2018)).

16 In PANS models, the instantaneous velocity and pressure fields are decomposed into  
 17 two component, partially filtered component ( $U_i$  and  $P$ ) and unresolved component ( $u_i$  and  
 18  $p'$ ) as:

$$V_i = U_i + u_i; \quad p = P + p' \quad (13)$$

19 After applying filtering operator, which commutes with spatial and temporal differentiation,  
 20 Navier Stokes equations for partially filtered pressure and velocity fields are written as:

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} + \frac{\partial \tau(V_i, V_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j} \quad (14)$$

21 The above equation needs closure for the sub-filter stress  $\tau(V_i, V_j)$ , which takes into account  
 22 influence of unresolved motion on the resolved flow field. A detailed description on the choice  
 23 of the closure model can be found in Girimaji (2005). Here we proceed the closure obtained



1 by using the Boussinesq approximation as:

$$\tau(V_i, V_j) = -\nu_u \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) + \frac{2}{3} k_u \delta_{ij} \quad (15)$$

2 where  $\nu_u$  is eddy viscosity of unresolved scales defined as:

$$\nu_u = C_\mu \frac{k_u^2}{\epsilon_u} \quad (16)$$

3 Additionally, to fully close the system of equations given above, models for unresolved tur-  
 4 bulent kinetic energy  $k_u$  and unresolved eddy dissipation  $\epsilon_u$  are needed. A starting point  
 5 for this development is the RANS  $k - \zeta - f$  turbulence model, see reference of Hanjalic  
 6 et al. (2004). Basara et al. (2011) developed the PANS variant from the  $k - \zeta - f$  model  
 7 due to its excellent near-wall wall behaviour in complex flows. It is important to note that in  
 8 many industrial applications, the computational cost of wall-resolved LES can be prohibitive  
 9 due to limited computational resources and time constraints. As described in Basara et al.  
 10 (2011), the  $k - \zeta - f$  model can be effectively combined with universal wall approach, which  
 11 combines the integration up to the wall with wall functions. Therefore the model is well-suited  
 12 for enhancing the PANS model. Detailed derivation of the model equations can be found in  
 13 Basara et al. (2011), here only final form is presented. The equation for unresolved turbulent  
 14 kinetic energy is defined as follows:

$$\frac{\partial k_u}{\partial t} + U_j \frac{\partial k_u}{\partial x_j} = (P_u - \epsilon_u) + \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k_u}{\partial x_j} \right] + (U_j - \bar{U}_j) \frac{\partial k_u}{\partial x_j} \quad (17)$$

15 Eq.17 introduces an additional unclosed term  $(U_j - \bar{U}_j)$  known as the turbulent transport  
 16 term, which accounts for the convection of unresolved energy by resolved fluctuations. We  
 17 use maximum transport model which assumes that transport is directly proportional to eddy  
 18 viscosity of resolved fluctuations  $\nu_r$ , for detailed derivation of the possible transport models  
 19 and its performance see Girimaji (2005) and Murthi et al. (2010). This leads to following

1 closure and final form of equation for unresolved turbulent kinetic energy:

$$(U_j - \bar{U}_j) \frac{\partial k_u}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{\nu_r}{\sigma_k} \frac{\partial k_u}{\partial x_j} \right); \quad \sigma_{k_u} = \sigma_k \quad (18)$$

2

$$\frac{\partial k_u}{\partial t} + U_j \frac{\partial k_u}{\partial x_j} = (P_u - \epsilon_u) + \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\nu_r}{\sigma_k} \right) \frac{\partial k_u}{\partial x_j} \right] \quad (19)$$

3 Equations for unresolved eddy dissipation and wall-normal unresolved velocity scale ratio in

4 their final form are defined as follows:

$$\frac{\partial \epsilon_u}{\partial t} + U_j \frac{\partial \epsilon_u}{\partial x_j} = C_{\epsilon 1} P_u \frac{\epsilon_u}{k_u} - C_{\epsilon 2}^* \frac{\epsilon_u^2}{k_u} + \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\nu_r}{\sigma_\epsilon} \right) \frac{\partial \epsilon_u}{\partial x_j} \right] \quad (20)$$

5

$$\frac{\partial \zeta_u}{\partial t} + U_j \frac{\partial \zeta_u}{\partial x_j} = f_u - \frac{\zeta_u}{k_u} P_u + \frac{\zeta_u}{k_u} \epsilon_u (1 - f_k) + \frac{\partial}{\partial x_j} \left( \frac{\nu_r}{\sigma_\zeta} \frac{\partial \zeta_u}{\partial x_j} \right) \quad (21)$$

6 where  $C_{\epsilon 2}^* = C_{\epsilon 1} + \frac{f_k}{f_\epsilon} (C_{\epsilon 2} - C_{\epsilon 1})$ . The level of physical resolution depends entirely upon  
 7 model resolution parameters, unresolved to total ratio of kinetic energy  $f_k$  and unresolved  
 8 to total ratio of eddy dissipation  $f_\epsilon$ . Resolution parameters have values between zero and  
 9 unity. When the resolution parameters equal unity the PANS model transitions back to its  
 10 parent RANS model. As the values of resolution parameters decrease more of the turbulent  
 11 flow structures is resolved. Lastly, in the limit when resolution parameters equal zero, a  
 12 direct numerical simulation is performed. In this study  $f_\epsilon$  is set to unity while  $f_k$  is specified  
 13 according to Basara et al. (2018) where a dynamic update of the  $f_k$  parameter as a function  
 14 of cell size is proposed:

$$f_k \geq \frac{1}{\sqrt{C_\mu}} \left( \frac{\Delta}{\Lambda} \right)^{\frac{2}{3}} > \frac{k_u}{k_{tot}} \quad (22)$$

15 with  $\Delta = (\Delta_x + \Delta_y + \Delta_z)^{\frac{1}{3}}$  being cell dimension and  $\Lambda$  an integral length scale. Note that  
 16 total turbulent kinetic energy required to determine integral length scale is  $k_{tot} = k_u + k_r$   
 17 and it can be calculated only after the resolved turbulent kinetic energy is obtained. Resolved  
 18 turbulent kinetic energy is obtained from Eq. 23:

$$k_r = \frac{1}{2} (U_i - \bar{U}_i)^2 \quad (23)$$

1 This step involves expensive averaging of the resolved field, as a result this approach is im-  
2 practical for complex unsteady flow with moving boundaries. Therefore, Basara et al. (2018)  
3 further improved this approach by adding transport equation to determine resolved turbulent  
4 kinetic, thus supplying the information for the correct cut-off scale. This equation is simply  
5 called scale supplying variable (SSV) and it is given by Eq. 24:

$$\frac{\partial k_{ssv}}{\partial t} + U_j \frac{\partial k_{ssv}}{\partial x_j} = (1 - f_k)(P - \epsilon) + \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\nu_u}{\sigma_{ku}} \right) \frac{\partial k_{ssv}}{\partial x_j} \right] \quad (24)$$

6 Finally, by continuous calculation of both, unresolved and resolved turbulent kinetic energy,  
7 assuming that  $k_r = k_{ssv}$ ,  $f_k$  parameter is efficiently specified in every cell at the end of every  
8 time step depending on the flow conditions and mesh resolution.

9 With a desire to accurately depict high-pressure fuel injection with conditions relevant  
10 to Diesel engines under minimal computational effort the PANS  $k - \zeta - f$  SSV model is  
11 employed in this work. The PANS simulation results of ECN Spray A test case are compared  
12 to experimental data and LES simulation in Section 4.2.

## 13 2.4 Flamelet Generated Manifold

14 As already stated in Section 1 of this paper, the primary goal of this study is to establish  
15 accurate, and for today's industry, computationally affordable numerical methodology to pre-  
16 dict fuel injection process relevant for IC engines. Due to the inherent complexity of diesel  
17 combustion which includes thousands of species and tens of thousands of chemical reac-  
18 tions between them, computational power that is necessary quickly grows beyond practical  
19 limits, especially in the industrial environment (Tvrdojevic et al. (2019)). Hence, to minimize  
20 computational effort, the Flamelet Generated Manifold (FGM) tabulated chemistry model is  
21 employed to describe chemical kinetics. The model is presented in van Oijen et al. (2016)  
22 as a practical and efficient method for accurate combustion modelling under reduced com-  
23 putational cost compared to detailed chemistry simulation. This model allows for separate  
24 computation of flow and flame structure by assuming that a turbulent multi-dimensional flame

1 can be represented as a collection of locally one-dimensional flamelets, and that the chem-  
 2 ical scales are significantly smaller than the turbulent scales. This involves pre-computing  
 3 combustion chemistry using detailed chemical mechanism of any desired level of complex-  
 4 ity. The resulting thermochemical data of the flamelets are then stored in a lookup table,  
 5 which is subsequently utilized to interpolate the data during the CFD simulation. In this  
 6 study lookup table is generated using AVL TABKIN™ table generation tool based on per-  
 7 fectly stirred reactor (PSR) simulations carried out at constant pressure conditions. More  
 8 details about TABKIN™ generation tool can be found in FIRE™ (2022b). All thermochemi-  
 9 cal data obtained from PSR simulations are stored in the lookup table as a function of two  
 10 independent variables, progress variable and mixture fraction, for which transport equations  
 11 are solved during CFD simulation. The mean general transport equation for control variables  
 12 reads:

$$\frac{\partial \bar{\rho} \tilde{\phi}}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_i \tilde{\phi}}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \bar{\rho} (D + D_T) \frac{\partial \tilde{\phi}}{\partial x_i} \right) + \bar{\omega}_{\phi} \quad (25)$$

13 where  $\phi$  represents either progress variable or mixture fraction. Here,  $\tilde{u}_i$  denotes Favre av-  
 14 eraged or filtered velocity component. For more detailed description of software implementa-  
 15 tion see FIRE™ (2022a). The progress variable represents the progress of the combustion  
 16 reaction and it is defined as scalar that varies between zero (unburnt) and unity (burnt). It  
 17 quantifies the degree of fuel consumption and provides a measure of the local flame posi-  
 18 tion. The mixture fraction is a scalar quantity that also varies between zero and unity, where  
 19 zero represents pure oxidizer and unity represents pure fuel. It characterizes mixing process  
 20 in turbulent combustion. In this study, turbulent flow field is resolved by adopting LES, PANS  
 21 and RANS turbulence models, which depending on the modelling approach results in unre-  
 22 solved subfilter scales. To account for influence of turbulence on combustion a  $\beta$ -Presumed  
 23 Probability Density Function (PPDF) averaging approach is performed over control variables  
 24 and its variances. General transport equation for control variable variances is defined as:

$$\frac{\partial \bar{\rho} \tilde{\phi}_{var}}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_i \tilde{\phi}_{var}}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \bar{\rho} (D + D_T) \frac{\partial \tilde{\phi}_{var}}{\partial x_i} \right) + 2\bar{\rho} D_T \left( \frac{\partial \tilde{\phi}_{var}}{\partial x_i} \right)^2 - \bar{\rho} \tilde{\chi}_{\phi} \quad (26)$$

1 where  $\tilde{\chi}_\phi$  is scalar dissipation rate defined as follows:

$$\tilde{\chi}_\phi = 2 \frac{\epsilon}{k} \tilde{\phi}_{var} \quad (27)$$

2 In case of premixed combustion regime, averaging is performed over progress variable,  
3 while for non-premixed cases over mixture fraction. ECN Spray A is application featuring  
4 non-premixed combustion conditions. Hence, in this study  $\beta$ -PPDF averaging over mixture  
5 fraction is adopted. Nevertheless, dual fuel conditions require simultaneous consideration  
6 of premixed and non-premixed combustion regimes. Therefore, for dual fuel simulation tur-  
7 bulence chemistry interactions were considered over both, progress variable and mixture  
8 fraction. Additionally, the lookup table generated for dual fuel simulation has one dimension  
9 more compared to table used for single fuel simulation, specifically the fuel composition  
10 parameter.

### 11 3. DESCRIPTION OF THE SIMULATED TEST CASE

12 The test case used for validation of proposed methodology is the Engine Combustion Net-  
13 work (ECN) Spray A which features typical operating conditions of diesel engines. Detailed  
14 specifications on this experiment are available at ECN (2023). Main operating conditions are  
15 summarized in Table 1. The ECN experimental database of the nonreactive and reactive  
16 conditions is used to validate numerical results.

17 Additionally in the present work, the ECN Spray A is configured for dual fuel conditions.  
18 Due to the lack of experimental data for dual fuel configuration, results were qualitatively  
19 compared to single fuel case and other numerical studies. Similar study can be found in  
20 Gaballa (2023), where two-phase Real Fluid Model (RFM) has been applied to study evap-  
21 oration and mixing in dual fuel configuration. In the work of Xu et al. (2020) a numerical  
22 study on effects of ambient methanol on pollutants formation in dual-fuel spray combustion  
23 can be found. The ECN Spray A condition was selected because it closely represents the  
24 typical operating parameters of modern dual fuel internal combustion engines. Although, it

1 is not a direct representation of the dual fuel configuration, it serves as a reasonable start-  
2 ing point for studying and understanding dual fuel combustion process. For that purpose  
3 non-reactive and reactive ECN Spray A baseline condition is investigated in dual fuel con-  
4 figuration, where methanol is utilized as a primary fuel. The methanol/oxidizer mixture is  
5 considered to be homogeneously mixed and has equivalence ratio of 0.3, corresponding to  
6 medium-load conditions in dual fuel engine (Xu et al. (2020)). The initial pressure is slightly  
7 modified in such way that the initial ambient density of  $22.8 \text{ kg/m}^3$  is maintained.

### 8 **3.1 Computational domain and numerical setup**

9 The computational domain, shown in Fig. 4, is a cubic hexahedral mesh, with the same  
10 characteristic dimensions as experimental device. The maximum cell size is 1 mm, located  
11 mainly outside of the spray area, 3 refinement levels are employed in the spray region while  
12 for the liquid core 4 refinement levels are performed. The resulting mesh has a minimum cell  
13 size of  $62.5 \mu\text{m}$  in the liquid core and total number of cells in the computational domain is 14  
14 594 828.

15 The AVL FIRE™ 3D-CFD solver has been adopted to perform spray simulations under  
16 realistic engine conditions. The solution method is based on the fully conservative finite vol-  
17 ume approach with all the dependent variables evaluated at the center of the control volume.  
18 The method allows any type of the computational meshes. The overall solution procedure  
19 is iterative and based on the combination of SIMPLE algorithm and PISO corrections. The  
20 temporal discretization method is based on the first-order accurate Euler implicit scheme.  
21 To obtain a value at the cell-face center, a second order linear approximation is used. A  
22 second order midpoint rule is used for integral approximation. For solving convection a va-  
23 riety of differencing schemes is employed. For turbulence, energy and species transport  
24 equations, first order upwind differencing scheme was employed. The continuity equation is  
25 discretized utilizing central differencing scheme. In case of LES and RANS simulations for  
26 the momentum equation MINMOD (see Sweby (1984)) differencing scheme is used. It has  
27 been reported in Basara et al. (2011) that when the unresolved turbulence as modeled by

1 PANS model is small, applied differencing scheme on the momentum equation is getting to  
2 be of higher importance. Therefore, in this study we apply upwind, central, MINMOD and  
3 AVL SMART (see Przulj and Basara (2001)) schemes for the momentum equation to inves-  
4 tigated influence of numerical schemes on the PANS simulation results. See the comparison  
5 in the Section 4.2.

6 The FGM lookup table is generated utilizing a hybrid reduced n-dodecane mechanism  
7 (Lapointe et al. (2019)) which considers 65 chemical species and 363 elementary reactions,  
8 the mechanism is available at (LLNL (2022)). The table discretization is summarized in the  
9 Table 2. Table is discretized in such way that the higher refinement is obtained at the most  
10 reactive conditions, considering n-dodecane auto-ignition. Fuel ratio and progress variable  
11 variances are dimensions which are employed in the table only for the dual fuel configuration.

12 In Fig. 5, the 3D CFD simulation workflow used in this study to calculated high-pressure  
13 liquid injection process is summarised.

#### 14 4. RESULTS AND DISCUSSION

15 The obtained numerical results are validated against experimental data such as vapor pen-  
16 etration, liquid length, gas velocity and mixture fraction in radial and axial direction, as well  
17 as ignition delay time. All of the values are calculated as recommend by (ECN). Following  
18 ECN recommendations are considered: the vapor penetration is defined as the maximum  
19 distance from the nozzle outlet to the point where the fuel mass fraction is 0.1%. The liquid  
20 length is defined as maximum distance from the nozzle outlet to the farthest axial position  
21 where projected liquid volume in the cross-stream direction decreases to  $0.2e - 3mm^3$  liquid  
22 per  $mm^2$ . The projected liquid volume is  $PLV = \int LVF dy$ , where LVF is the liquid vol-  
23 ume fraction. Finally, high temperature ignition delay time is defined as the time of maximum  
24 gradient  $\frac{dT}{dt}$  in temperature.

#### 1 4.1 Model validation: LES

2 Firstly, numerical results obtained from LES simulation utilizing the blending function as de-  
3 scribed in Section 2.2 are compared to the results obtained from original LES coherent  
4 structure model as well as to the experimental data. The Fig. 6 shows comparison for liquid  
5 length and vapor penetration. As visible from the Fig. 6 results obtained utilizing blending  
6 function for turbulent eddy viscosity resulted with improved results in terms of both, liquid  
7 and vapor penetrations. The liquid length reported by the original LES CSM model with the  
8 constant value of  $C_2 = 1/22$  is significantly overestimated. In addition, vapor penetration  
9 predicted by non modified LES CSM model is over-predicted. Modifying turbulent eddy vis-  
10 cosity within LES CSM model resulted with liquid and vapor penetration that fairly agree  
11 with experimental data. This can be attributed to the fact that we increased the turbulent  
12 eddy viscosity of the gaseous phase in the spray region and thus enable the liquid to exhibit  
13 longer penetration length, while the original formulation of the LES CSM model is somewhat  
14 over-diffusive. This behavior can be seen in Fig. 7 and Fig. 8 where eddy viscosity of the  
15 original model and that obtained from the new formulation are shown, for the time instances  
16 of 0.1 and 1 ms, respectively. It is notable that keeping the  $C_2$  at the constant value of  $1/22$  in  
17 the entire numerical domain results with damped eddy viscosity in the spray region. The low  
18 eddy viscosity of the gaseous phase implies minimal to no resistance to liquid penetration.  
19 As a result, the original model formulation tends to overestimate both, the liquid and vapor  
20 penetrations. Introducing the blending function to combine two values of  $C_2$ ,  $1/22$  in the  
21 outer edge of the domain and 1 in the spray region resulted with increased values of eddy  
22 viscosity in the spray region where the dissipation of the unresolved scales is the highest.  
23 However, even with modified eddy viscosity an over-prediction of the liquid tip in the initial  
24 stage of the spray is notable in the Fig. 6.

25 Additionally, comparison of the radial distribution of n-dodecane mass fraction obtained  
26 from two LES simulations and experimental measurements is shown in Fig. 9. Comparisons  
27 are made at 18, 30 and 40 mm from the nozzle outlet. Numerical results are time averaged  
28 in the interval between 1.5 and 3 ms. Once again, the simulation results obtained from LES



1 simulation with modified eddy viscosity are in better agreement with the experimental profile  
2 of the mixture fraction. The simulation results agree reasonably with the experimental data  
3 in terms of transversal distribution of the mixture fraction along the radial direction. However,  
4 the peak values of the mixture fraction on the jet axis are not captured satisfactorily. Based  
5 on this results, it can be deduced that, employed numerical mesh resolution is inadequate to  
6 accurately capture involved physical phenomena and the main features of the spray. Possible  
7 reason for this is higher diffusion caused by the big cell surface, which is directly reflected in  
8 the accuracy of the results obtained by the LES approach (Žilić (2021)). Instead of refining  
9 the computational mesh to improve accuracy of the results in this study, a PANS turbulence  
10 model is applied as an efficient alternative to the LES models.

#### 11 **4.2 Model validation: PANS**

12 As reported by Basara et al. (2011), turbulence models with variable resolution, such as  
13 PANS model, pose a challenge to numerical schemes. In regions with the coarse resolution,  
14 the computational behavior of the PANS model resembles that of RANS, and the choice  
15 of the numerical scheme prioritizes computational robustness. Typically, RANS models uti-  
16 lize second-order upwind schemes, for example MINMOD. Conversely, in regions with fine  
17 resolution, PANS model requires computational capabilities similar to LES. In the present  
18 work, we perform PANS computations employing first-order upwind scheme, second-order  
19 MINMOD, AVL SMART and central differencing schemes for the momentum conservation  
20 equation, and compare their results. Fig. 10 and Fig. 11 show comparison of the liquid length  
21 and vapor penetration predicted with PANS simulations using different numerical schemes.  
22 The best agreement with experimental data is obtained with MINMOD and AVL Smart differ-  
23 encing schemes. Employing upwind scheme produced nonphysical peak in the initial stage  
24 of the spray, while after 0.25 ms predicted liquid length converged to the experimental curve.  
25 Additionally, a small overestimation of the liquid length predicted by simulation with central  
26 differencing scheme is visible.

27 The reason for the differences in predicted liquid length and vapor penetration can be

1 attributed to different numerical dissipation and diffusion properties of the employed differ-  
2 encing schemes. These properties can affect the ability of the numerical method to accu-  
3 rately capture dynamics of turbulent flow, and how the small modeled scales fluctuations  
4 interact with the resolved scales. The choice of numerical scheme can effect the accuracy  
5 and fidelity of the results. Fig. 12 and Fig. 13 show temperature and equivalence ratio distri-  
6 bution as predicted by PANS simulations employing different numerical schemes. As visible  
7 from the presented results upwind scheme introduced excessive numerical diffusion, which  
8 damped the small scale turbulent fluctuations and led to an over smoothing of the turbulent  
9 flow. On the contrary, pure central scheme doesn't introduce smoothing. Additionally, due  
10 to insufficient dissipation it can result with numerical instability and nonphysical oscillations  
11 (Ferziger and Peric (2002)). Thus as visible from the temperature and equivalence ratio  
12 distribution in Fig. 12 and Fig. 13 the predicted two phase interface is artificially over sharp-  
13 ened. Both the MINMOD and SMART methods exhibit second-order accuracy on refined  
14 grids and show robust convergence properties. However, it is worth noting that compared  
15 to the MINMOD method, the SMART method is characterized by lower numerical diffusivity  
16 and therefore higher accuracy especially on coarser grids. Our results clearly show that the  
17 SMART scheme reproduces the expected fine-scale flow structures much more distinctly  
18 for a given grid resolution. As a result, all PANS simulation results presented in this paper  
19 use the AVL SMART differentiation scheme, while the LES results were derived using the  
20 MINMOD scheme.

21 In Fig. 14 PANS model resolution parameter, total, resolved and unresolved turbulent  
22 kinetic energy are shown. It is visible that maximum value of the resolution parameter lies in  
23 the region close to the nozzle exit, indicating that this part of the flow can't be resolved by  
24 the employed mesh resolution. Hence, in the cells where resolution parameter equals unity,  
25 PANS model will reduce to its parent RANS model and turbulent flow will be completely  
26 modeled. As a result, adequate description of the dense spray region is preserved, despite  
27 insufficient mesh resolution for resolved simulation in this region. Correspondingly, maximum  
28 of unresolved turbulent kinetic energy lies in the region where values of resolution parameter

1 are high. Contrary, maximum of the resolved energy is where resolution parameter has lower  
2 values. Total turbulent kinetic energy is obtained by summing up resolved and unresolved  
3 turbulent kinetic energy.

4 Fig. 15 shows the liquid length and vapor penetration obtained with PANS simulation and  
5 compared to the results obtained from LES simulation with the blending function. It can be  
6 seen that PANS simulation exhibited better agreement with experiment compared to LES.  
7 This can be attributed to the fact that PANS model is designed to be adaptive, meaning it  
8 can transition smoothly from completely modeled to partially resolved regions, based on the  
9 the grid resolution and the local flow conditions. As already seen in the Fig. 14, grid reso-  
10 lution in the region close to the nozzle exit is insufficient to resolve turbulent flow structures  
11 of this flow. Therefore, PANS model showed its benefit of retrieving its parent RANS model  
12 equations and completely modelling this region of the flow. Consequently, PANS model out-  
13 performed LES on this particular mesh for given flow conditions. Comparison of the gas  
14 velocity and the mixture fraction axial distribution as predicted by LES simulation with blend-  
15 ing function and PANS simulation to experimental data is shown in Fig. 16. Results obtained  
16 from PANS simulation appear to be in better agreement with experimental data compared  
17 to the LES results. It can be seen that gas velocity close to the nozzle exit is overestimated  
18 by LES, while PANS result shows a good agreement with experimental profile. In addition,  
19 axial distribution of the mixture fraction is well captured by PANS simulation.

20 Additionally, radial distribution of the mixture fraction at 18, 30 and 40 mm from the nozzle  
21 exit as obtained from LES and PANS simulations is compared to the experiment in Fig. 17.  
22 For all three distances from the nozzle exit mixture fraction distribution as predicted by PANS  
23 simulation is in better agreement compared to the mixture fraction distribution predicted by  
24 LES simulation. The predicted peak values and radial distribution of the mixture fraction  
25 obtained by PANS simulation agrees reasonably well with experimental profile.

26 In Fig. 18 temperature distribution as obtained by LES simulation with blending function,  
27 RANS  $k - \zeta - f$  simulation and PANS  $k - \zeta - f$  simulation is shown. All three simulations  
28 reported similar temperature range and peak temperatures values. In addition predicted

1 distance from the nozzle exit where the mixture ignites is similar in all three simulations.  
2 However, RANS simulation predicted considerably longer flame penetration, while PANS  
3 simulation is in agreement with LES result. The same comparison is made for equivalence  
4 ratio distribution in Fig. 19. All the results appear similar, in terms of equivalence ratio values.  
5 It should be noted from the presented results that predicted temperature and equivalence  
6 ratio field obtained from the RANS simulation is very smooth, and no fluctuations or small  
7 scale structures are visible. On the other hand, PANS results show much more fine scale  
8 structures due to resolving portion of the fluctuating scales. Comparing the PANS results  
9 with LES results on the same mesh, it can be seen that PANS delivered the same level of  
10 detail in terms of resolving portion of the turbulent flow. However, PANS simulation exhibited  
11 more accurate results than LES when compared to experimental data.

### 12 **4.3 Dual fuel configuration**

13 Current section presents the results of ECN Spray A baseline case in dual fuel configura-  
14 tion. This case is calculated employing PANS turbulence model. In the Fig. 20, liquid length  
15 and vapor penetration between the single fuel and dual fuel configuration is shown. The  
16 obtained results are in agreement from the findings of Gaballa (2023), where it has been  
17 shown that the gaseous phase doesn't have high influence on the penetration length of the  
18 liquid phase. As can be seen from the obtained results, both liquid and vapor penetrations  
19 of dual fuel case are very similar to the single fuel case. However, presence of the methanol  
20 is expected to have significant influence on the combustion process characteristics, such as  
21 ignition delay time and pollutant formation rates. Indeed, different fuel mixtures have signif-  
22 icant influence on the ignition delay time. It has already been shown in many experimental  
23 (see Schlatter et al. (2012); Xu et al. (2022)) and numerical (see Eder et al. (2018); Xu  
24 et al. (2020)) studies that the presence of the lean premixed high-octane fuel retards the  
25 ignition delay time of the directly injected high-cetane fuel. In addition, the delay of ignition  
26 becomes more apparent when the ambient temperature decrease (Xu et al. (2020)). The be-  
27 haviour of ignition delay time for different different ambient temperatures in single and dual

1 fuel configuration of ECN Spray A case is shown in Fig. 21. The from the PANS simulation  
2 obtained ignition delay time for ECN Spray A baseline case at the temperature of 900 K is  
3 accurately predicted comparing to the ignition delay time measured in experiment. It can  
4 be seen that ignition delay time of the dual fuel case for the same ambient temperature is  
5 significantly longer. In addition, as the ambient temperature increases ignition delay time of  
6 both single and dual fuel case decreases, which is in line with other relevant findings avail-  
7 able in the literature. Longer ignition delay time in dual fuel case results with longer time for  
8 mixing before high-temperature combustion starts (Xu et al. (2020)). Consequently, as it can  
9 be seen from the Fig. 22, dual fuel case exhibits locally lower temperatures compared to  
10 the single fuel case. In addition, equivalence ratio of the dual fuel case is lower in the high  
11 fuel region compared to the single fuel case, shown in Fig. 23. Lower equivalence ratio is  
12 beneficial for soot reduction (Xu et al. (2020)), while lower local temperatures are beneficial  
13 for  $NO_x$  reduction (Alla et al. (2000)). The spacial distribution of OH mass fraction plays  
14 important role in indicating start of the high temperature combustion, while the HCHO mass  
15 fraction distribution is indicator of low-temperature ignition delay. Fig. 24 and Fig. 25 show  
16 the comparison of OH and HCHO mass fraction spatial distribution for single and dual fuel  
17 case, respectively. The highest distribution of OH in the single fuel case is found along the  
18 stoichiometry line, while for the dual fuel case OH appear to be distributed over wider range  
19 of the stoichiometry line. The mass fraction of HCHO is relatively higher and wider spread  
20 out in terms of spatial distribution for single fuel case. For the dual fuel case HCHO is mostly  
21 found close to the nozzle exit. All of the presented results are consistent with the above  
22 stated fact that methanol retards ignition of the n-dodecane spray. Additionally, scatter plots  
23 with HCHO mass fraction as color map, and the temperature and equivalence ratio as axes  
24 are prepared. The Fig. 26 shows scatter plots prepared from the results obtained from the  
25 PANS and LES simulation for the single fuel case, as well as PANS simulation for dual fuel  
26 case. Results from LES and PANS simulation for single fuel case are very similar, both of  
27 them yield the highest mass fractions of HCHO in the subspace of temperature between  
28 750 K and 1250 K, and equivalence ratio between 3 and 8. Comparing the scatter plots

1 of dual fuel case to the single fuel case it can be observed that dual fuel strategy reduced  
2 equivalence ratio of the pilot fuel spray in the high temperature region.

### 3 **5. CONCLUSION**

4 The main objective of this work was to propose a reliable and efficient CFD simulation work-  
5 flow for two-phase simulation in the context of dual fuel internal combustion engine. To this  
6 goal, present study investigated performance of different numerical approaches to simu-  
7 late high-pressure fuel injection with the respect of industrial requirements. LES turbulence  
8 modelling approach with a subgrid model based on the coherent structure function and the  
9 scale resolving PANS turbulence model have been coupled with Euler Lagrangian Discrete  
10 Droplet Method for spray modelling. In comparison to the well-established and viable turbu-  
11 lence modelling technique of LES for single-phase flow, the application of LES to two phase  
12 flows is still at the early stage of the development. Therefore, a modification to the original  
13 LES coherent structure model has been introduced to improve a prediction of high-pressure  
14 liquid fuel injection process. The modification is based on a blending function used to tune  
15 the turbulent eddy viscosity for two-phase application. The new formulation of eddy viscosity  
16 enables smooth transition from viscosity defined for near wall region to viscosity appropriate  
17 for turbulent region. Viscosity formulation in the near wall region is kept the same as in the  
18 original model with the same model constant. For the fully turbulent region the value of the  
19 model constant is changed.

20 In this study ECN Spray A benchmark case, in both reactive and non-reactive conditions,  
21 was utilized to validated proposed modification of LES coherent structure model and to in-  
22 vestigate the performance of the PANS turbulence model. The modified LES model showed  
23 improved results of predicted liquid and vapor penetrations, and mixture fraction distribution,  
24 compared to the original model formulation. However, the PANS turbulence model outper-  
25 formed the results of LES on the same computational mesh. The generated computational  
26 mesh is designed to be computationally affordable for everyday industrial use. However, this  
27 compromises the mesh resolution, rendering it insufficient for proper LES calculation. On

1 the other hand, PANS model can provide good compromise by gradually transitioning from  
2 RANS in regions where grid resolution is coarser to partially resolving the flow in regions  
3 the grid is sufficiently fine. This feature allowed PANS model to maintain accurate predictions  
4 for coarser grids compared to the ones required for proper LES calculation. Some potential  
5 improvements of the results from both, LES and PANS, involve investigation on sensitivity  
6 of the results on the chemical mechanism. Nevertheless, the PANS model exhibited highly  
7 remarkable overall performance. Hence, it can be concluded that PANS turbulence model  
8 has potential do deliver accurate results under computational expense affordable for today's  
9 industrial standards. Following this conclusion, the PANS model can be employed to inves-  
10 tigate high pressure fuel injection and combustion process in dual fuel configuration.

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1     **DECLARATION**

2     The authors declare that the article has not been published elsewhere and that it has not  
3     been simultaneously submitted for publication elsewhere.



1 **Nomenclature**

- 2  $\chi_\phi$  Scalar dissipation rate, (-)
- 3  $\Delta$  Mesh size, (m)
- 4  $\delta_{ij}$  Kronecker delta, (-)
- 5  $\epsilon_u$  Unresolved eddy dissipation, ( $\text{m}^2/\text{s}^3$ )
- 6  $\Lambda$  Integral length scale, (m)
- 7  $\nu$  Kinematic viscosity, ( $\text{m}^2/\text{s}$ )
- 8  $\nu_t$  Turbulent viscosity, ( $\text{m}^2/\text{s}$ )
- 9  $\nu_{opt}$  Optimized turbulent viscosity, ( $\text{m}^2/\text{s}$ )
- 10  $\omega$  Source term, (-)
- 11  $\phi$  Progress variable or mixture fraction scalar, (-)
- 12  $\rho$  Density, ( $\text{kg}/\text{m}^3$ )
- 13  $\tau_{ij}$  Tangential stress tensor, ( $\text{N}/\text{m}^2$ )
- 14  $\zeta$  Velocity scale ratio, (-)
- 15  $C_{CSM}$  Coherent structure model constant, (-)
- 16  $D$  Laminar diffusion coefficient of the fuel species, (-)
- 17  $D_T$  Turbulent diffusion coefficient of the fuel species, (-)
- 18  $E$  Velocity gradient tensor, (1/s)
- 19  $f$  Elliptic relaxation function, (-)
- 20  $f_k$  PANS Resolution parameter, (-)
- 21  $F_\Omega$  Energy decay function, (-)

- 1  $F_{CS}$  Coherent structure function, (-)
- 2  $k$  Turbulent kinetic energy, ( $\text{m}^2/\text{s}^2$ )
- 3  $k_u$  Unresolved turbulent kinetic energy, ( $\text{m}^2$ )
- 4  $k_{ssv}$  Scale supplying variable, ( $\text{m}^2/\text{s}^2$ )
- 5  $k_{tot}$  Total turbulent kinetic energy, ( $\text{m}^2/\text{s}^2$ )
- 6  $p$  Pressure, (Pa)
- 7  $P_k$  Production of turbulent kinetic energy, ( $\text{m}^2/\text{s}^2$ )
- 8  $PV$  Mixture fraction, (-)
- 9  $PV$  Progress variable, (-)
- 10  $Q$  Second invariant of the velocity flow field, (-)
- 11  $S_{ij}$  Velocity strain tensor, (1/s)
- 12  $T$  Temperature, (K)
- 13  $t$  Time, (s)
- 14  $u_i$  Velocity component in direction i, (m/s)
- 15  $W_{ij}$  Vorticity tensor, (1/s)
- 16  $x_i$  Position vector, (-)

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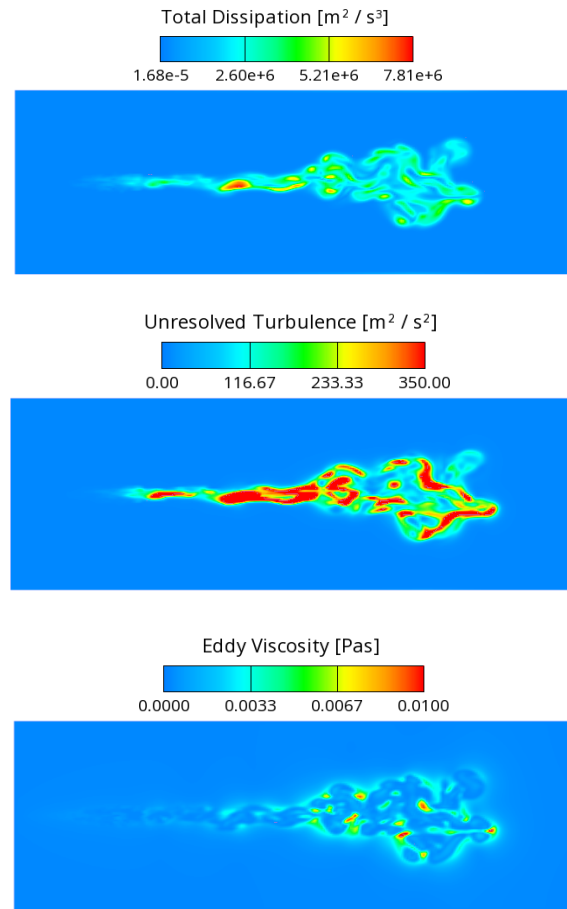
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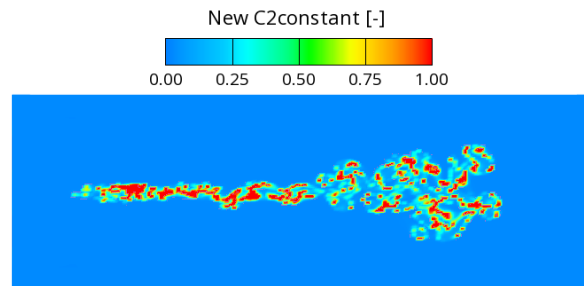
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**FIG. 1:** Total eddy dissipation, unresolved turbulence and turbulent eddy viscosity as obtained by coherent structure model in case of high-pressure liquid injection process.



**FIG. 2:** Distribution of calculated values for the  $C_2$  constant of the coherent structure subgrid model.

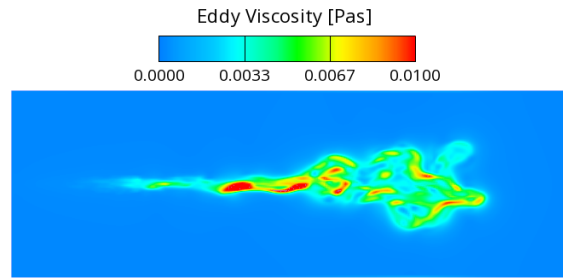


FIG. 3: Turbulent eddy viscosity as obtained by Eq. 12.

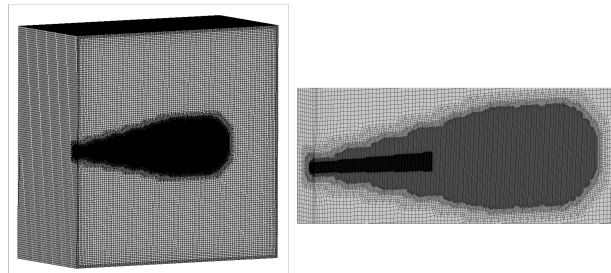


FIG. 4: Computational domain used for the CFD simulation with a zoom on spray area

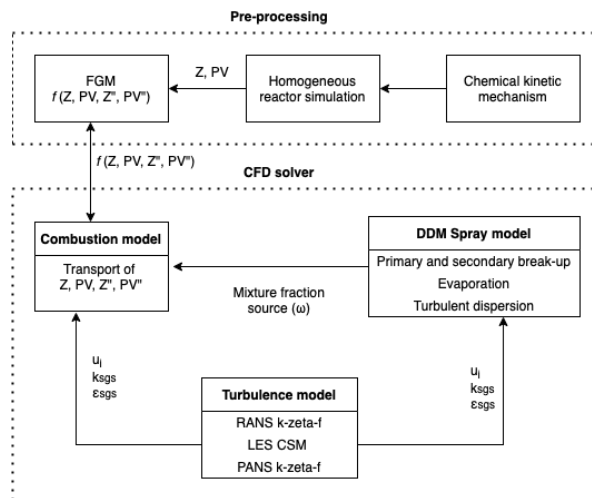
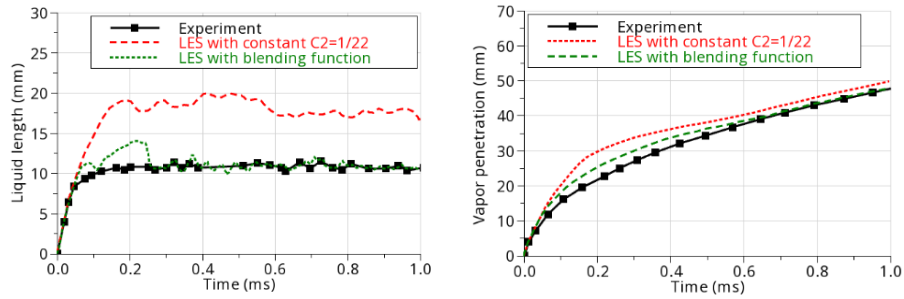
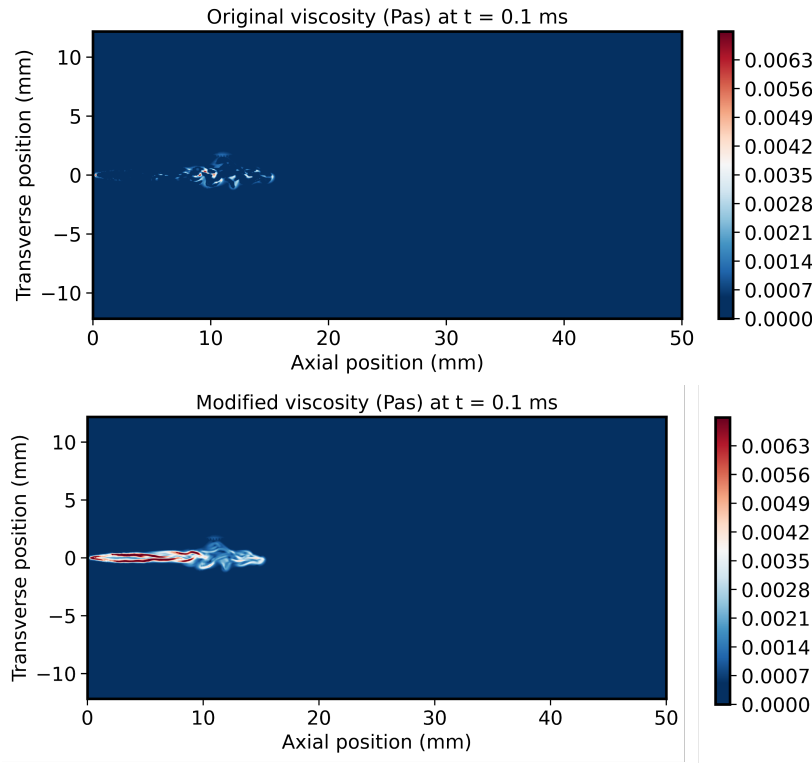


FIG. 5: Employed simulation workflow scheme.

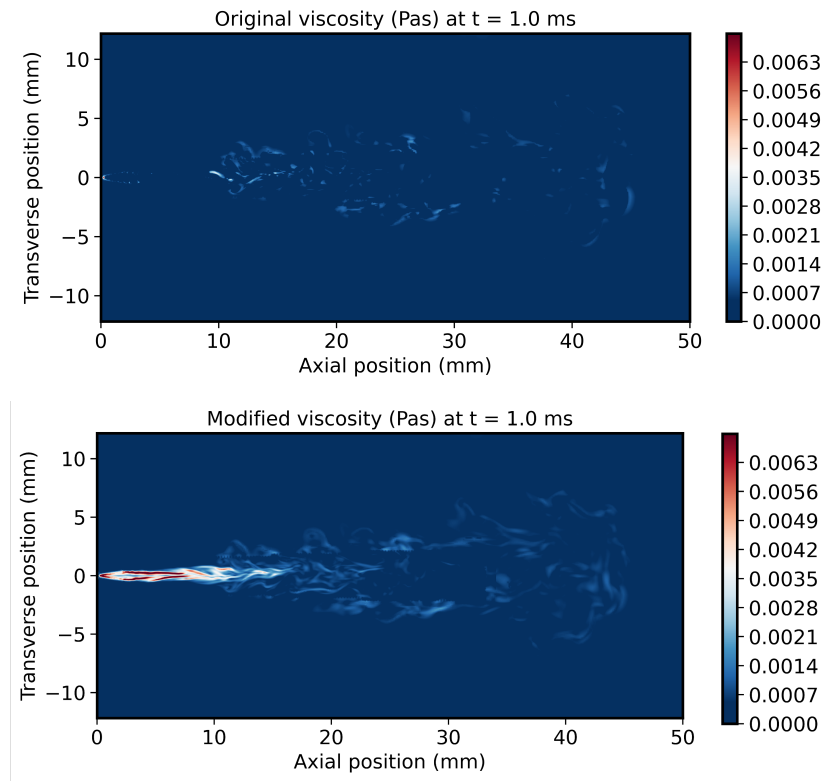




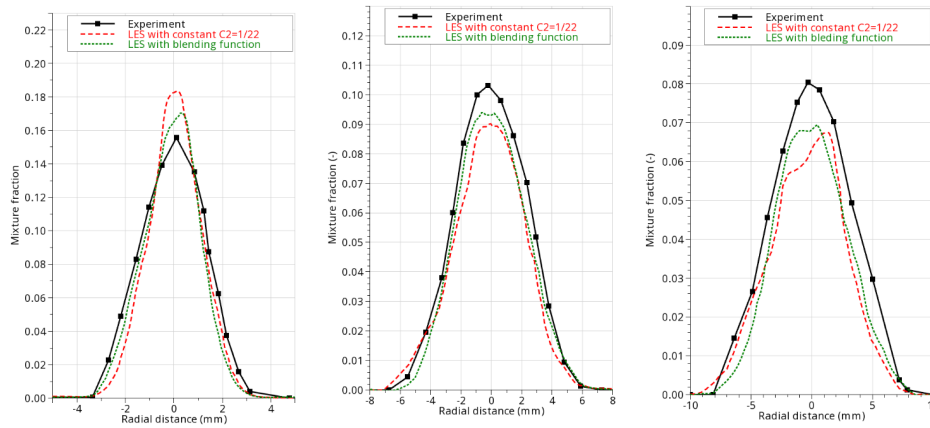
**FIG. 6:** Comparison of liquid length (left) and vapor penetration (right) obtained from LES simulation utilizing the blending function to experimental data and numerical results obtained utilizing original LES coherent structure model.



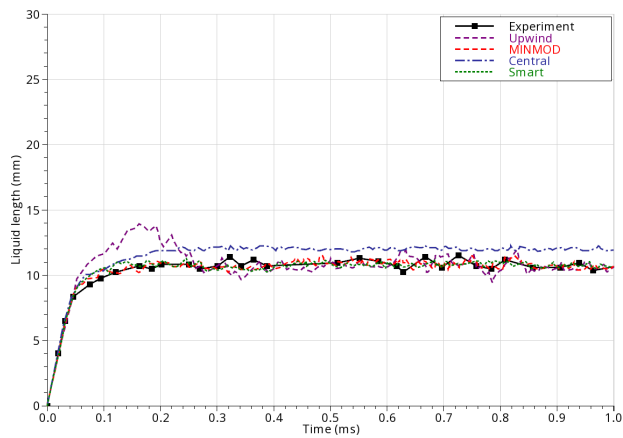
**FIG. 7:** Turbulent eddy viscosity as predicted by the original formulation of the LES CSM model with the constant value of  $C_2 = 1/22$  (upper figure), compared to the eddy viscosity obtained from LES CSM model when the viscosity is blended between viscosity near wall, with  $C_2 = 1/22$  and viscosity in the turbulent region, with  $C_2 = 1$  (lower figure). Time instance is 0.1 ms.



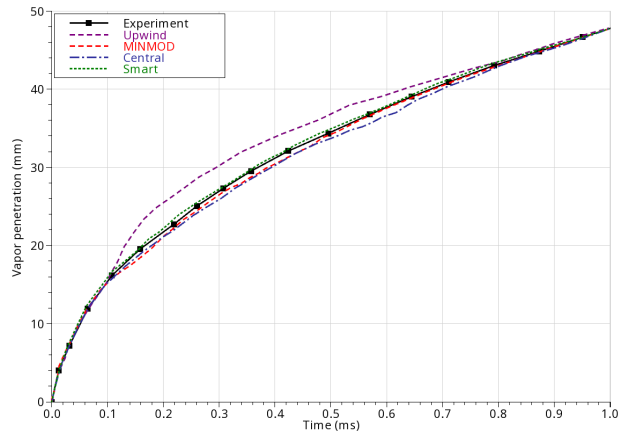
**FIG. 8:** Turbulent eddy viscosity as predicted by the original formulation of the LES CSM model with the constant value of  $C_2 = 1/22$  (upper figure), compared to the eddy viscosity obtained from LES CSM model when the viscosity is blended between viscosity near wall, with  $C_2 = 1/22$  and viscosity in the turbulent region, with  $C_2 = 1$  (lower figure). Time instance is 1 ms.



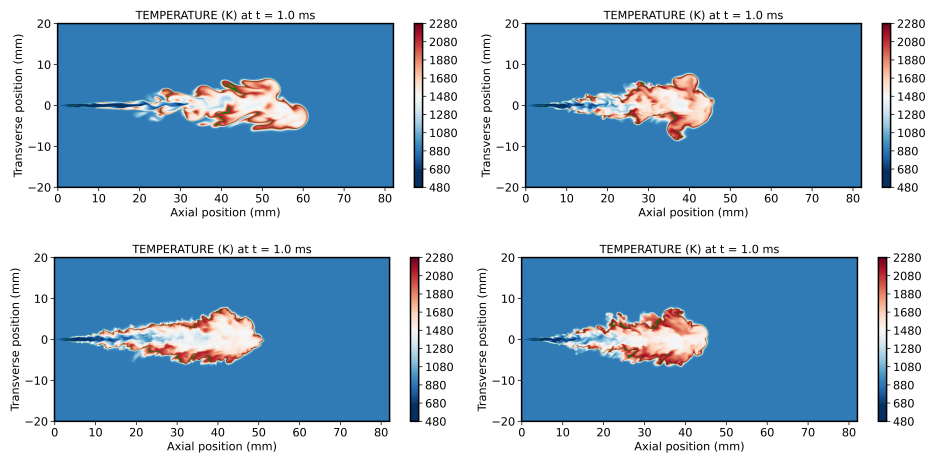
**FIG. 9:** Radial distribution of n-dodecane mass fraction at 18 mm from the nozzle exit (left), 30 mm from the nozzle exit (middle) and 40 mm from the nozzle exit (right). Comparison between experimental data, CFD results from LES simulation utilizing the blending function and original coherent structure model.



**FIG. 10:** Liquid length as obtained by PANS simulation employing upwind, MINMOD, AVL Smart and central differencing schemes for momentum compared to experimental data.



**FIG. 11:** Vapor penetration as obtained by PANS simulation employing upwind, MINMOD, AVL Smart and central differencing schemes for momentum compared to experimental data.



**FIG. 12:** Temperature distribution as obtained by PANS simulations utilizing Upwind (upper left), MINMOD (upper right), Central (lower left) and AVL Smart (lower right) differencing schemes.

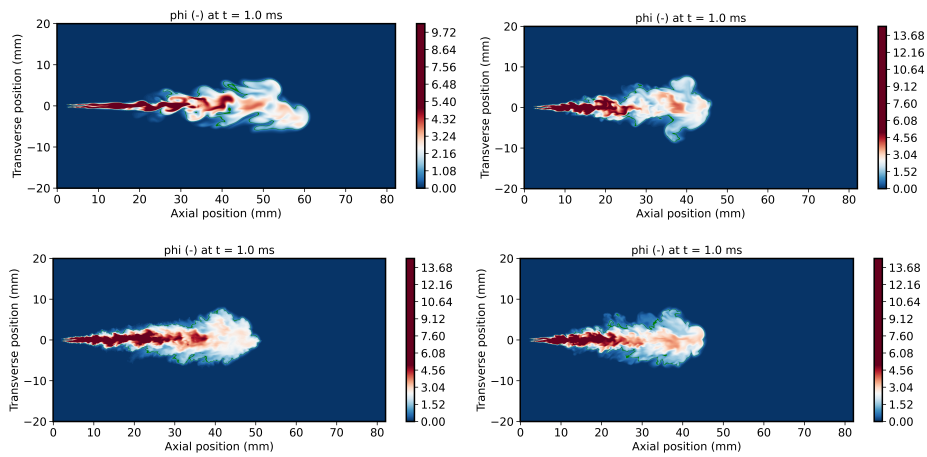


FIG. 13: Equivalence ratio distribution as obtained by PANS simulations utilizing Upwind (upper left), MINMOD (upper right), Central (lower left) and AVL Smart (lower right) differencing schemes.

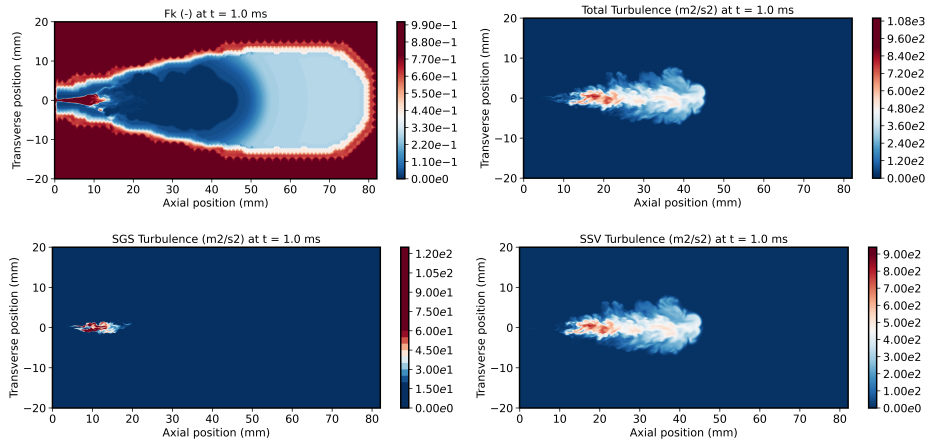


FIG. 14: Resolution parameter (upper left), total (upper right), unresolved (lower left) and resolved (lower right) turbulent kinetic energy as obtained by PANS turbulence model.

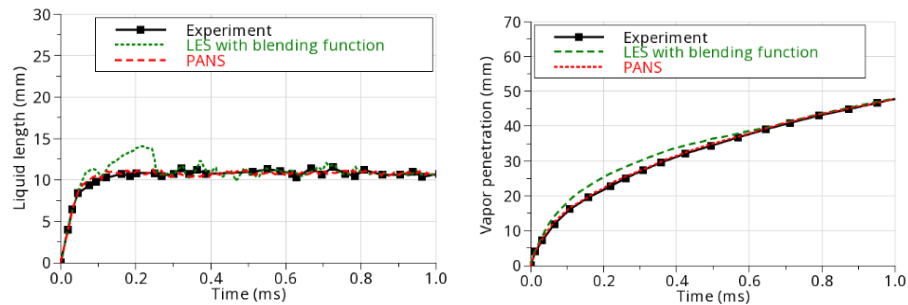
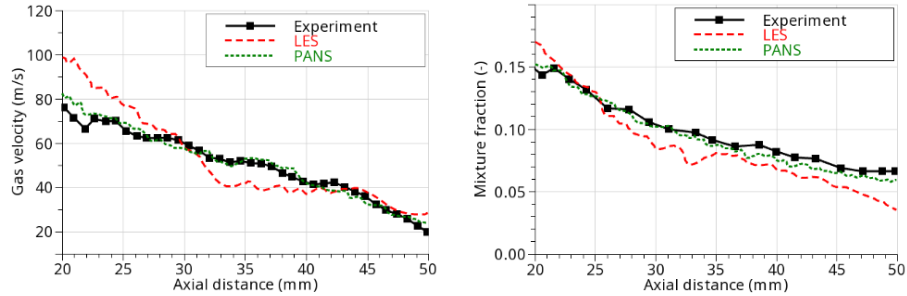
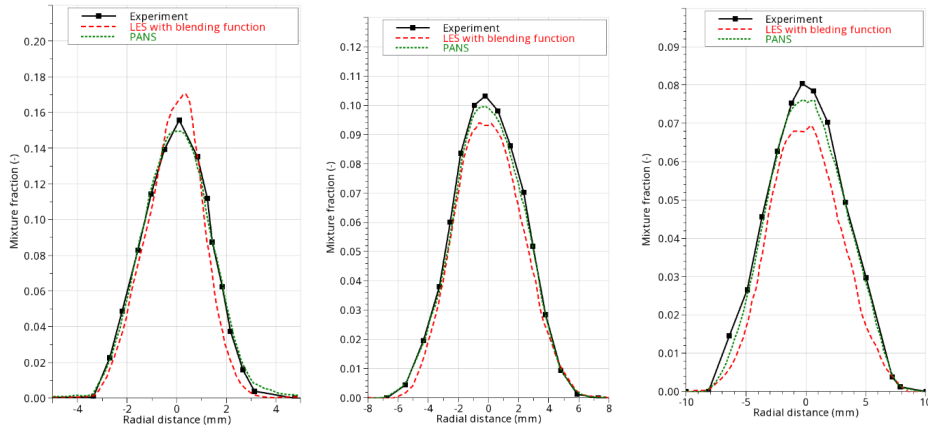


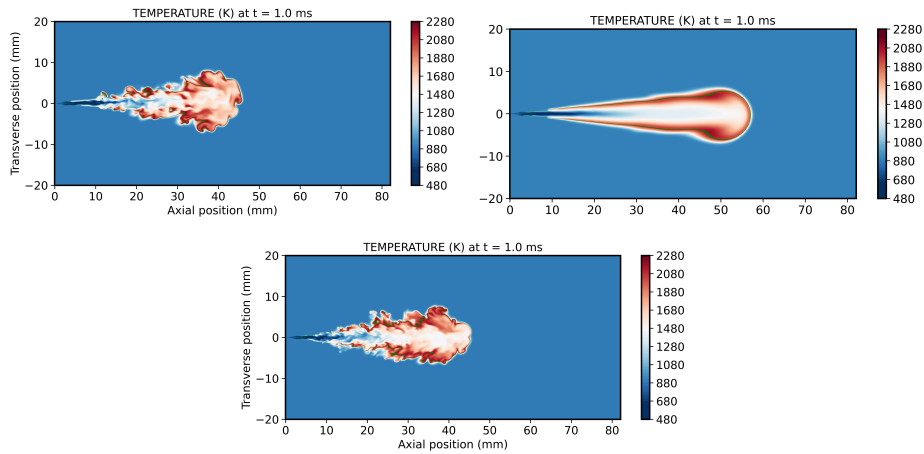
FIG. 15: Comparison of liquid length (left) and vapor penetration (right) predicted by LES coherent structure model with blending function and PANS  $k - \zeta - f$  turbulence model.



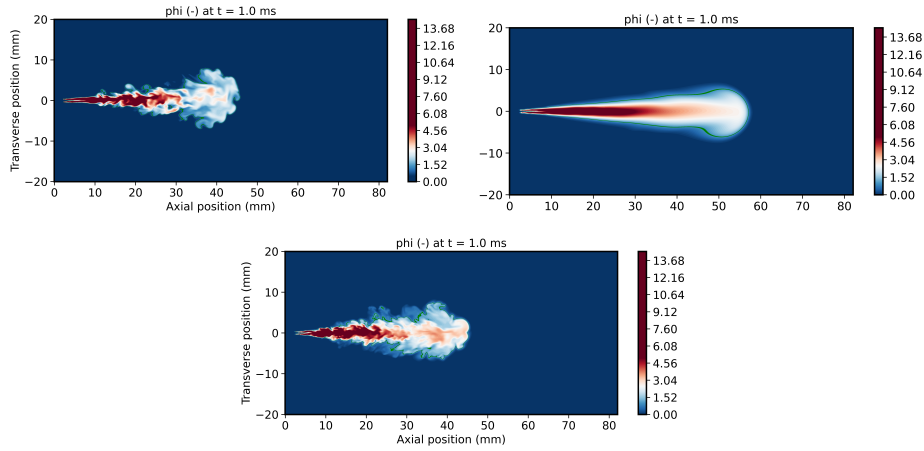
**FIG. 16:** Comparison of the results obtained with LES coherent structure model with blending function and PANS  $k - \zeta - f$  turbulence model for gas velocity (left) and mixture fraction axial distribution (right).



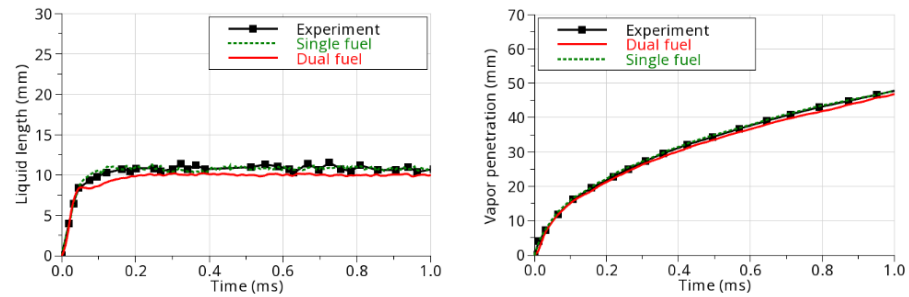
**FIG. 17:** Radial distribution of n-dodecane mass fraction at 18 mm from the nozzle exit (left) 30 mm from the nozzle exit (middle) and 40 mm from the nozzle exit (right). Comparison between experimental data, CFD results from LES simulation utilizing the blending function and PANS simulation.



**FIG. 18:** Comparison of the temperature distribution as predicted by LES CSM with blended eddy viscosity (upper left), RANS  $k - \zeta - f$  model (upper right) and PANS  $k - \zeta - f$  SSV model (lower).



**FIG. 19:** Comparison of the equivalence ratio distribution as predicted by LES CSM with blended eddy viscosity (upper left), RANS  $k - \zeta - f$  model (upper right) and PANS  $k - \zeta - f$  SSV model (lower).



**FIG. 20:** Comparison of liquid length (left) and vapor penetration (right) between single fuel and dual fuel Spray A configuration.

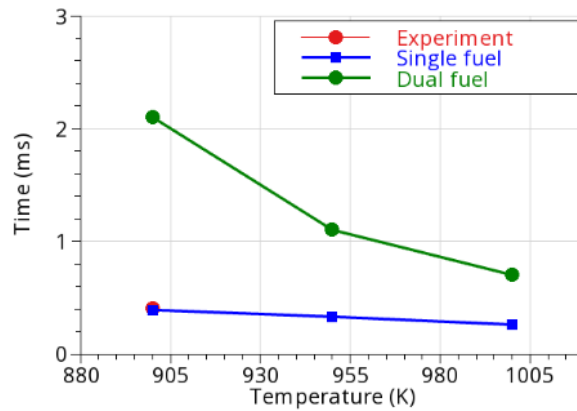


FIG. 21: Comparison of ignition delay time between single fuel and dual fuel configuration for three different temperatures.

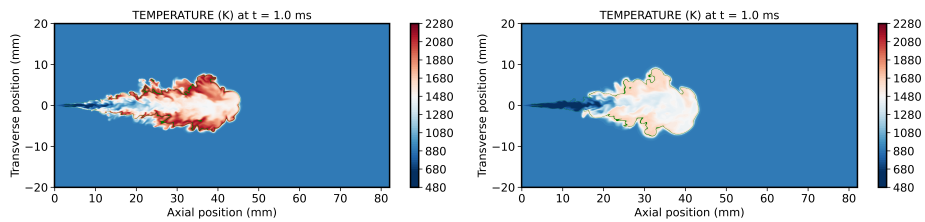


FIG. 22: Temperature distribution as obtained by PANS simulation for single fuel configuration (left) and dual fuel configuration (right).

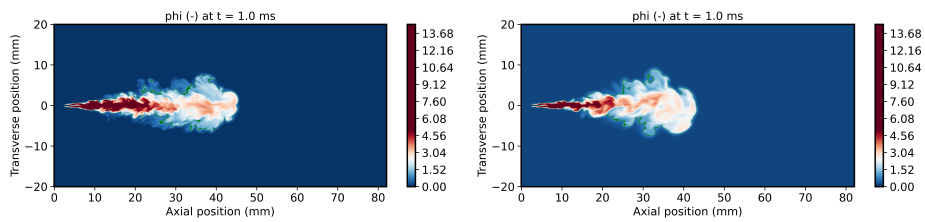


FIG. 23: Equivalence ratio distribution as obtained by PANS simulation for single fuel configuration (left) and dual fuel configuration (right).

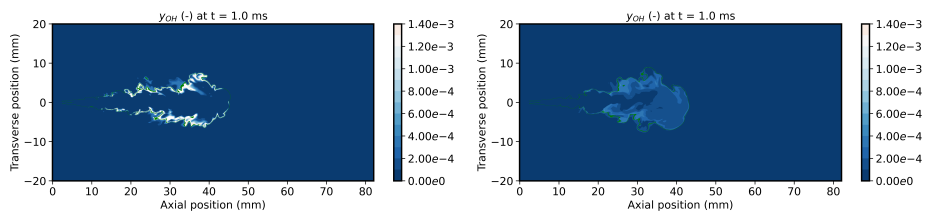


FIG. 24: OH mass fraction as obtained by PANS simulation for single fuel configuration (left) and dual fuel configuration (right).



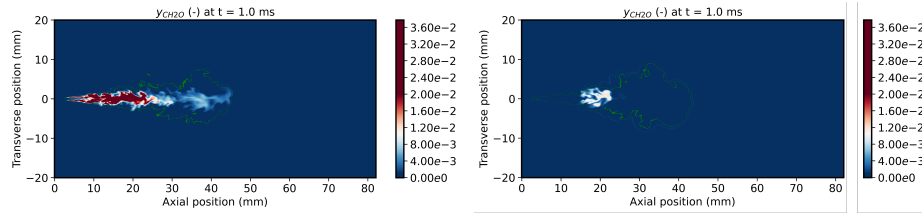


FIG. 25: HCOH mass fraction as obtained by PANS simulation for single fuel configuration (left) and dual fuel configuration (right).

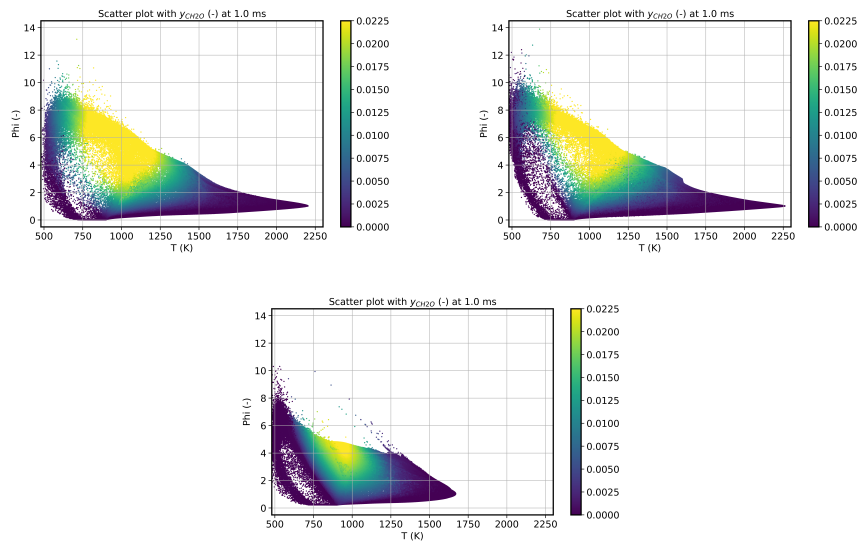


FIG. 26: HCHO scatter plots as predicted by PANS simulation for single fuel configuration (upper left), LES simulation for single fuel configuration (upper right) and PANS simulation for dual fuel configuration (lower).

Fuel	n-dodecane
Injection pressure	150 MPa
Injection temperature	363 K
Injection duration	1.5 ms
Number of nozzle holes	1
Ambient gas temperature	900 K
Ambient gas pressure	6 bar
Ambient gas density	22.8 kg/m <sup>3</sup>

**TABLE 1:** ECN Spray A operating conditions as prescribed by ECN (2023)

Chemical mechanism	Hybrid reduced n-dodecane
Pressure	9 points from 1 to 80 bar
Temperature	22 points from 300 to 2000 K
Mixture fraction	55 points from 0 to 1
Progress variable	110 points from 0 to 1
Mixture fraction segregation	10 points from 0 to 1
Progress variable segregation	12 points from 0 to 1
Fuel ratio	6 points from 0 to 1

**TABLE 2:** FGM table discretization.