Course title: EOTIST Standard course

Course subject: Modelling

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LESSON SM3.2

ECOSYSTEM MODELLING

EXERCISES

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1. THE SOFTWARE

GNU-OCTAVE

A FREE SOFTWARE COMPATIBLE WITH MATLAB

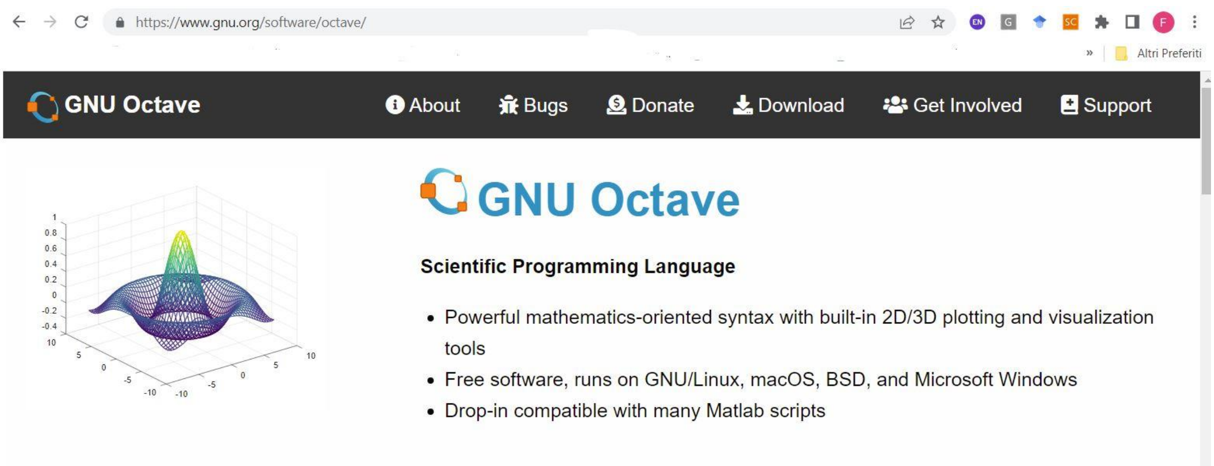
Octave is free software under the GNU General Public License.

GNU Octave is a high-level language, primarily intended for numerical computations. It provides a convenient command line interface for solving linear and nonlinear problems numerically using a language that is mostly compatible with Matlab.

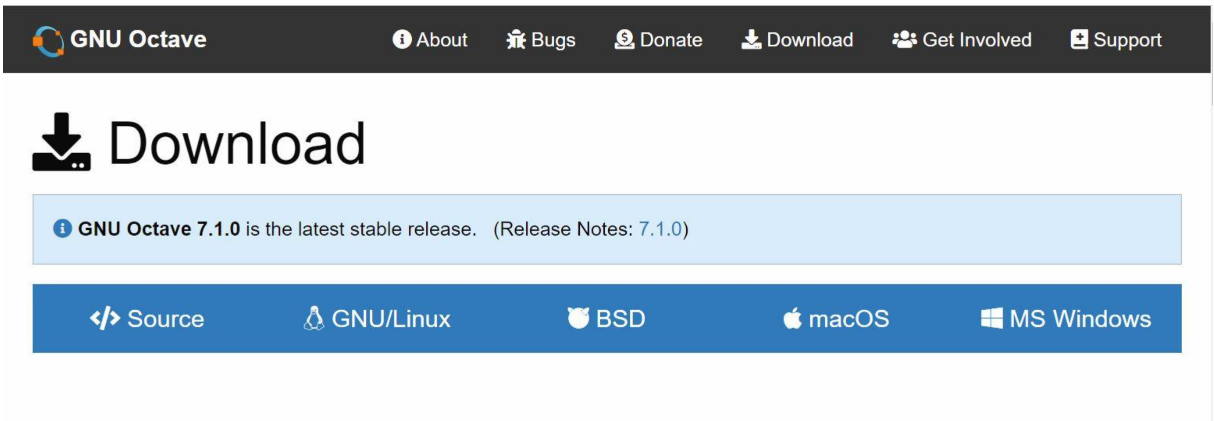
Octave has extensive tools for solving integrating ordinary differential equations.

HOW TO INSTALL OCTAVE

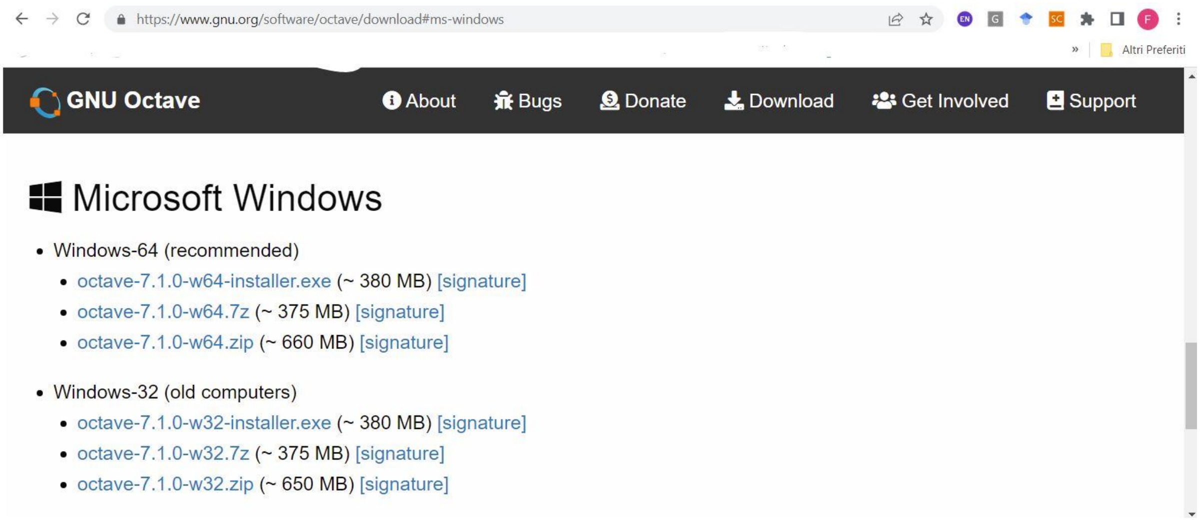
Visit the page https://www.gnu.org/software/octave/



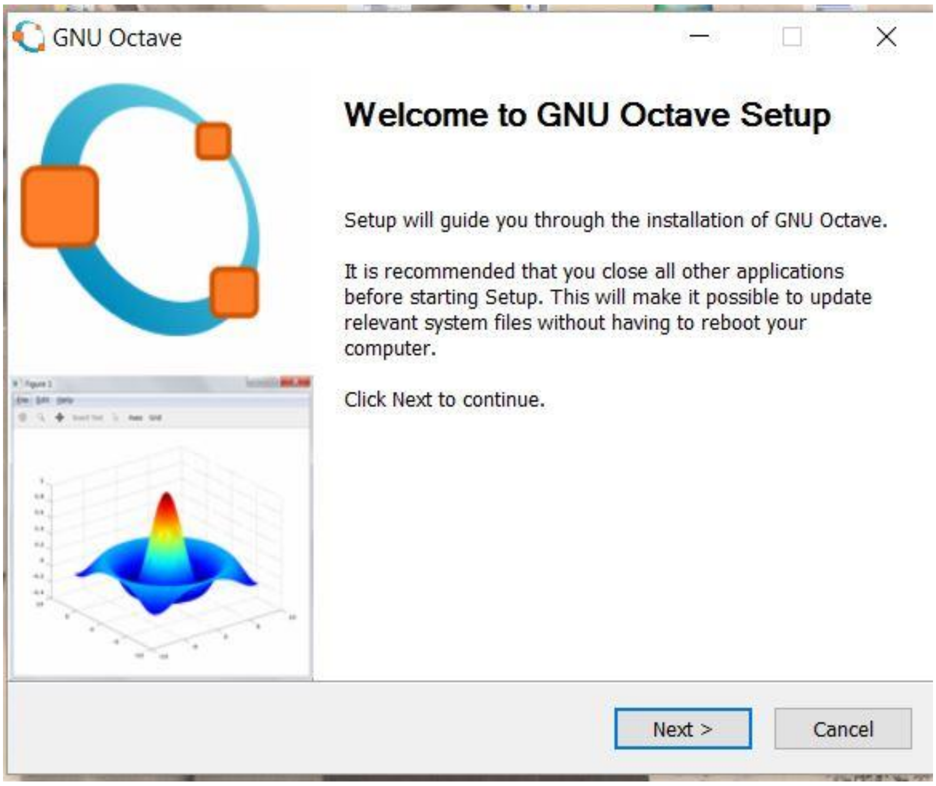
and select 'Download'



Choose and select the correct framework. For example, by selecting **MS Windows** you will reach the page

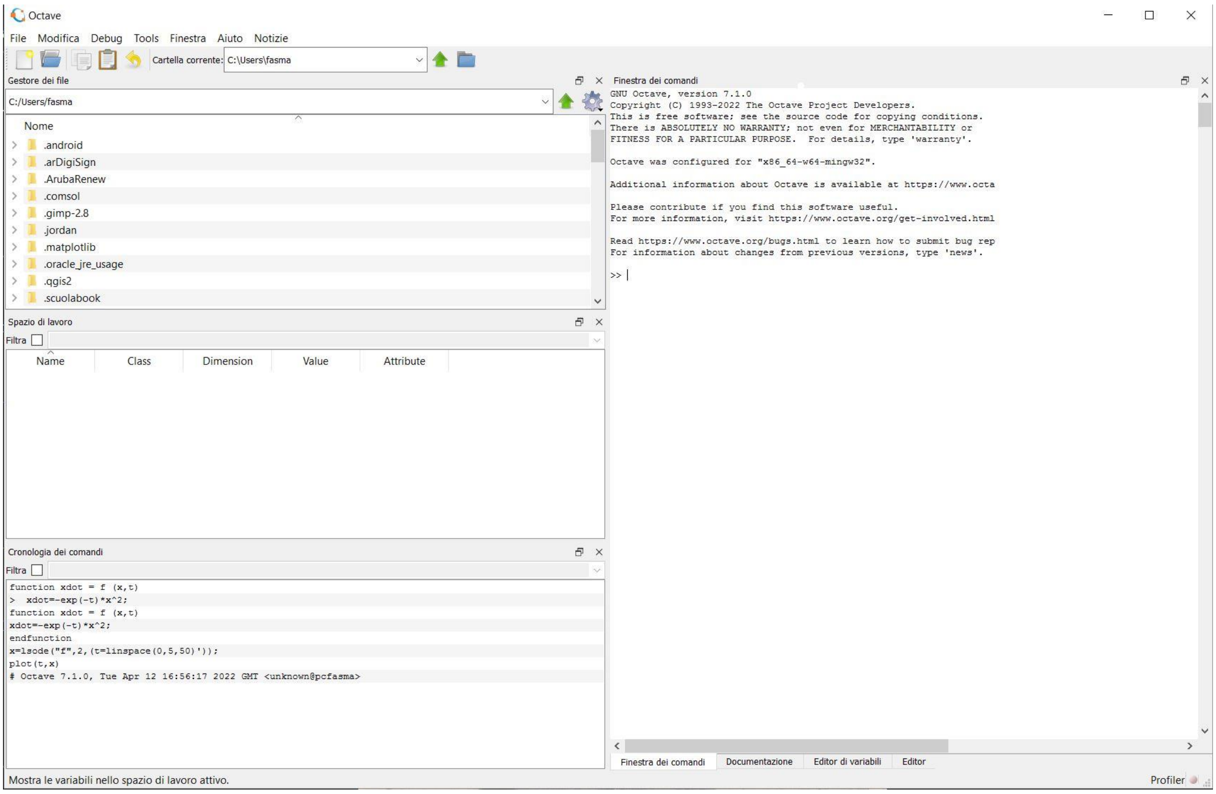


Download and execute the _installer.exe



2. A TUTORIAL FOR THE BASICS OF USING THE SOFTWARE

THE WINDOWS



THE MANUALS

A tutorial for the basics of using GNU Octave is available at

https://wiki.octave.org/Using_Octave.

Before you begin with the ODEs exercises, first read how a system of ODEs has to be implemented in Octave

The online manual at

https://octave.org/doc/v7.1.0/Ordinary-Differential-Equations.html

describes how to implement the ODEs and how to use the functions.

3. DISCRETE POPULATION MODELS

EXERCISE 1

Only 80% of the bamboo seeds actually generate a plant that becomes an adult, and ripening takes place in a year. The plantation will then increase thanks to the autonomous reproduction of adult bamboos (roughly, every two bamboos present, a new seed is obtained every year). Suppose to sow 100 bamboo seeds annually and that 50% of plants are cut for the need to thin out the plants.

Give a graphical representation of the dynamics of seeds and adult pools.

Write the discrete model for the annual growth of adult bamboo abundance. Assuming that one plant is initially present in the land, evaluate the abundances in the first 10 years and plot the graph of the growth (abundances versus years).

4. ORDINARY DIFFERENTIAL EQUATIONS

GROWTH FUNCTIONS

EXERCISE 2

Consider the continuous-time population balance equation described by the

$$\frac{dN}{dt} = r\,N$$

Suppose $r = 0.5$. What kind of growth does it describe?

Suppose that $N(0) = 0.1$ and that we want to integrate the equation in the time interval [0,10] with 50 steps.

Implement in Octave the following code. In the command window define the function

```
function Ndot = f(N,t)

r=0.5;

Ndot = r*N;

endfunction

t = linspace (0, 10, 50);

N = lsode("f",0.1,t);

plot(t,N)

xlabel('Time t', 'FontSize', 12);

ylabel('N(t)', 'FontSize', 12);
```

Compare the plot with the one obtained setting $r = -0.5$.

What kind of dynamics is described ?

Now, suppose $N(0) \; = \; 1$ and change the code accordingly.

Set $N(0) = 0$ and comment about the outcome.

EXERCISE 3

Consider the continuous-time population balance equation described by the

$$\frac{dN}{dt} = r\,N\left(1 - \frac{N}{K}\right)$$

Suppose $r = 0.5, K = 10$. What kind of growth does it describe?

Suppose that $N(0) = 0.1$ and that we want to integrate the equation in the time interval [0,10] with 50 steps.

Implement in Octave the following code. In the command window define the function

```
function Ndot = f(N,t)

r=0.5; K=10;

Ndot = r*N*(1-N/K);

endfunction

t = linspace (0, 10, 50);

N = lsode("f",0.1,t);

plot(t,N)

xlabel('Time t', 'FontSize', 12);

ylabel('N(t)', 'FontSize', 12);
```

Compare the plot with the one obtained by integrating in the time interval [0,50] with 250 steps.

EXERCISE 4

Consider the continuous-time population balance equation described by the

$$\frac{dN}{dt} = r\,N\left(1 - \frac{N}{10}\right)\left(\frac{N}{2} - 1\right)$$

Suppose $r = 0.5$. What kind of growth does it describe?

Suppose that $N(0) = 3$ and that we want to integrate the equation in the time interval [0,10] with 50 steps.

Run in Octave the following code. In the command window cut and paste the following lines

```
function Ndot = f(N,t)

r=0.5; K=10; A=2;

Ndot=r*N*(1-N/K)*(N/A-1);

endfunction

t = linspace (0, 10, 50);

N = lsode("f",10,t);

plot(t,N)

xlabel('Time t', 'FontSize', 12);

ylabel('N(t)', 'FontSize', 12);
```

Compare the resulting plot with the ones obtained by integrating the same problem starting with $N(0) = 0$, $N(0) = 1$, $N(0) = 2$, $N(0) = 5$, $N(0) = 10$.

Comment about.

5. ODE SYSTEMS

SOIL ORGANIC CARBON DYNAMICS: A TWO POOLS MODEL

Consider the two pools soil organic carbon model

$$\begin{aligned}\frac{dC_s}{dt} &= I - \frac{V_{max,U}\, C_b\, C_s}{K_{M,U} + C_s} + k_b C_b \\ \frac{dC_b}{dt} &= \varepsilon \frac{V_{max,U}\, C_b\, C_s}{K_{M,U} + C_s} - k_b C_b\end{aligned}$$

with parameters (expressed in yearly units) given in *Georgiou, K., Abramoff, R. Z., Harte, J., Riley, W. J., & Torn, M. S. (2017). Microbial community-level regulation explains soil carbon responses to long-term litter manipulations. Nature Communications, 8(1), 1-10.*

EXERCISE 5

Cut and paste in the command window the following lines. 'f_soilmodel ' defines the model

```
function Cdot = f_soilmodel(C, t)
```

```
##Parameters
I = 1.3824; Vmaxu = 86.4; Kmu = 250; kb = 2.4192; epsilon = 0.31;

## Auxiliaries
g_Cs = C(1)/( Kmu + C(1));

## State equations
# Soil carbon pool
Cdot(1, 1) = I- Vmaxu *C(2)*g_Cs + kb*C(2);

# Microbial pool
Cdot(1, 2) = epsilon* Vmaxu *C(2)*g_Cs - kb*C(2);

endfunction



t = linspace (0, 300, 3000);
C=lsode("f_soilmodel",[1,1],t);
figure(1)
semilogy(t,C)
Cs = C(:,1);
Cb = C(:,2);
legend('Cs(t)', 'Cb(t)');
xlabel('Time t', 'FontSize', 12);


figure(2)
plot(Cs,Cb)
xlabel('Cs', 'FontSize', 12);
ylabel('Cb', 'FontSize', 12);
```

Comment about the resulting plots.

LOTKA-VOLTERRA COMPETITION MODEL

In the Lotka-Volterra competition model we have two populations N_1 and N_2 competing for a resource (which does not appear explicitely in the equations): for each species the presence of a competitor reduce its growth.

$$\frac{dN_1}{dt} = r_1 \, N_1 \left(1 - \frac{N_1}{K_1}\right) - \frac{r_1 \, \beta_{12}}{K_1} N_1 N_2$$

$$\frac{dN_2}{dt} = r_2 \, N_2 \left(1 - \frac{N_2}{K_2}\right) - \frac{r_2 \, \beta_{21}}{K_2} N_1 N_2$$

EXERCISE 6

Cut and paste the following lines in the command window

```
function Ndot = f_LV_competition(N, t)
##Parameters
r1 = 1.3824; beta12 = 0.2; K1 = 1;
r2 = 0.1; beta21 = 2; K2 = 10;

## Auxiliaries
f1=r1*N(1)*(1-N(1)/K1); g1 = r1*beta12*N(1)*N(2)/K1;
f2=r2*N(2)*(1-N(2)/K2); g2 = r2*beta21*N(1)*N(2)/K2;

## State equations
Ndot(1, 1) = f1-g1;
Ndot(1, 2) = f2-g2;

endfunction
t = linspace (0, 120, 200);
N=lsode("f_LV_competition",[1,1],t);
figure(1)
plot(t,N)
legend('N1(t)', 'N2(t)');
xlabel('Time t', 'FontSize', 12);
```

From the obtained results identify if one of the two species outcompetes the other or if there is coexistence.

Now, change the values of parameters as follows and identify in what case the dynamics falls

```
r1 = 1.3824; beta12 = 0.2; K1 = 1;
r2 = 0.1; beta21 = 2; K2 = 0.1;

r1 = 1.3824; beta12 = 0.2; K1 = 1;
r2 = 0.1; beta21 = 10; K2 = 7;

r1 = 1.3824; beta12 = 0.2; K1 = 1;
r2 = 0.1; beta21 = 2; K2 = 3;
```

LOTKA-VOLTERRA PREDATOR-PREY MODEL

Under the assumption of uniform space distribution of populations, the Lotka Volterra predator-prey model is given by

$$\frac{dN}{dt} = r\,N - a\,N\,P$$

$$\frac{dP}{dt} = \varepsilon\,a\,N\;P - \mu\,P$$

EXERCISE 7

Cut and paste the following lines in the command window

```
function Cdot = f_LV_predator_prey(C, t)
##Parameters
r = 1.3824; a = 0.2;
epsilon = 0.1; mu = 2;

## State equations
Cdot(1, 1) = r*C(1)-a*C(1)*C(2);
Cdot(1, 2) = epsilon*a*C(1)*C(2)-mu*C(2);

endfunction
t = linspace (0, 50, 1000);
C=lsode("f_LV_predator_prey",[10,10],t);
figure(1)
plot(t,C)
xlabel('Time t', 'FontSize', 12);
legend('N(t)', 'P(t)');
```

```
N = C(:,1);
P = C(:,2);
figure(2)
plot(N,P)
xlabel('N', 'FontSize', 12);
ylabel('P', 'FontSize', 12);
```

Comment about the resulting plots.