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Standard course – Modelling
Lesson SM3.2 – Ecosystem modelling - Exercises

Fasma Diele and Carmela Marangi - CNR

Growth functions

EXERCISE 2

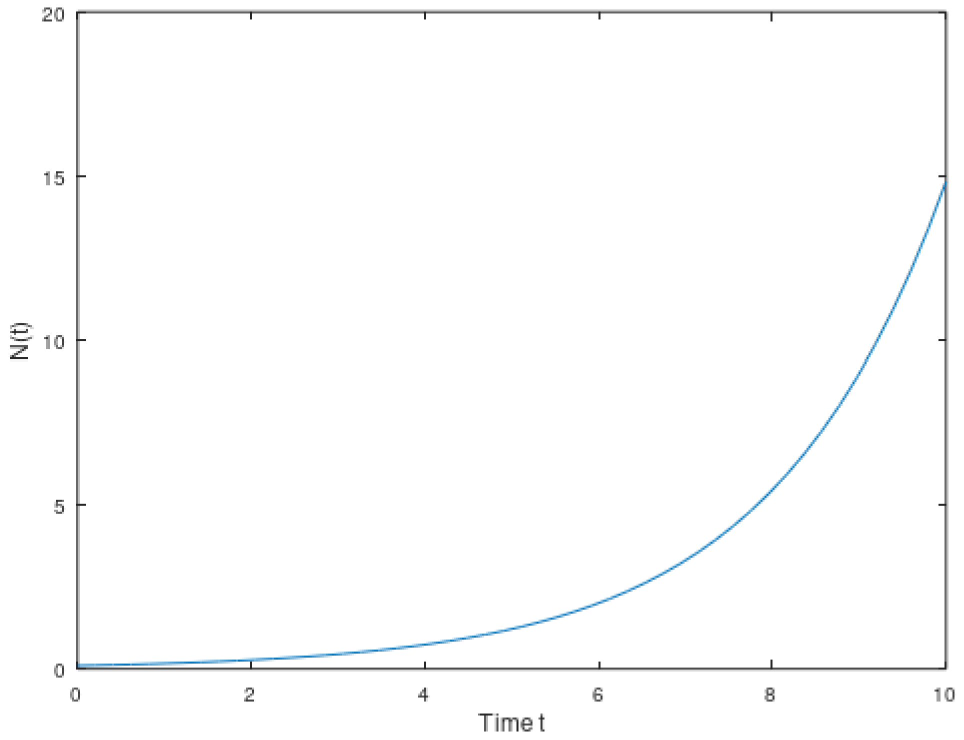
- Consider the continuous-time population balance equation described by the

$$\frac{dN}{dt} = r\,N$$

- Suppose $N(0) = 0.1$
- Integrate the equation in the time interval [0,10] with 50 steps.

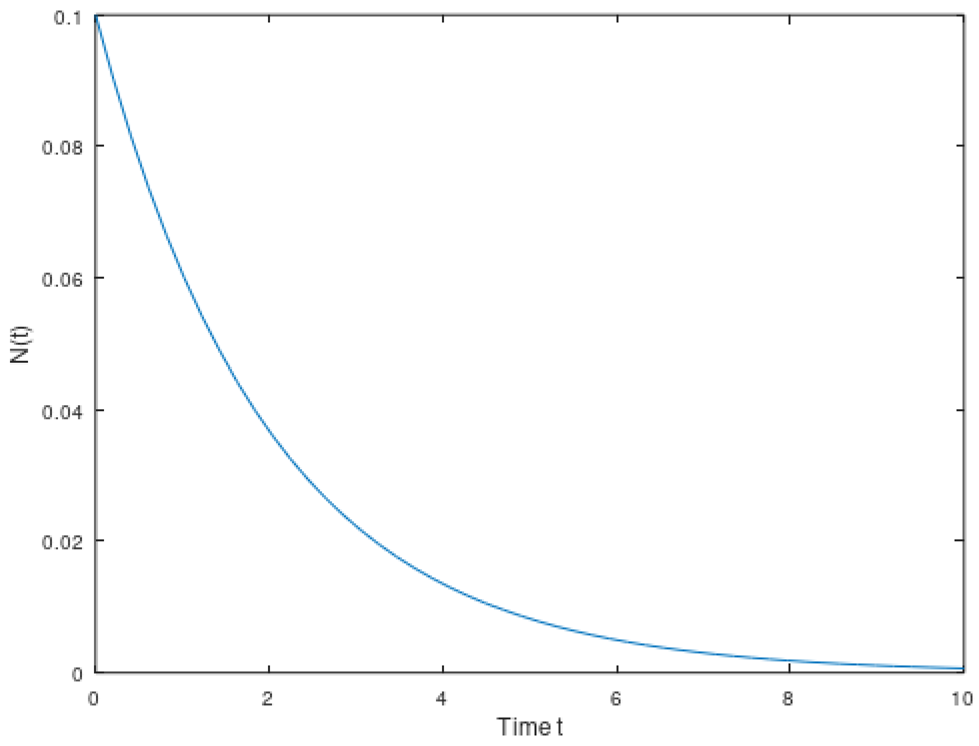
Exponential growth

- r=0.5



Exponential growth

- r=-0.5 A negative growth provides a decrease



Logistic growth

EXERCISE 3

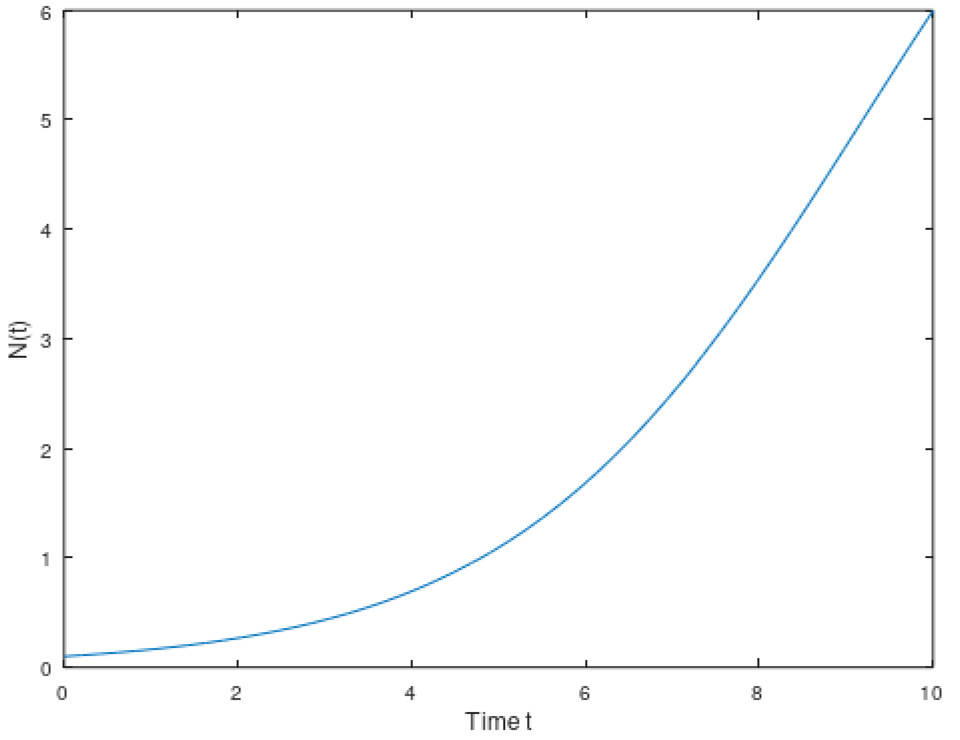
- Consider the continuous-time population balance equation described by the

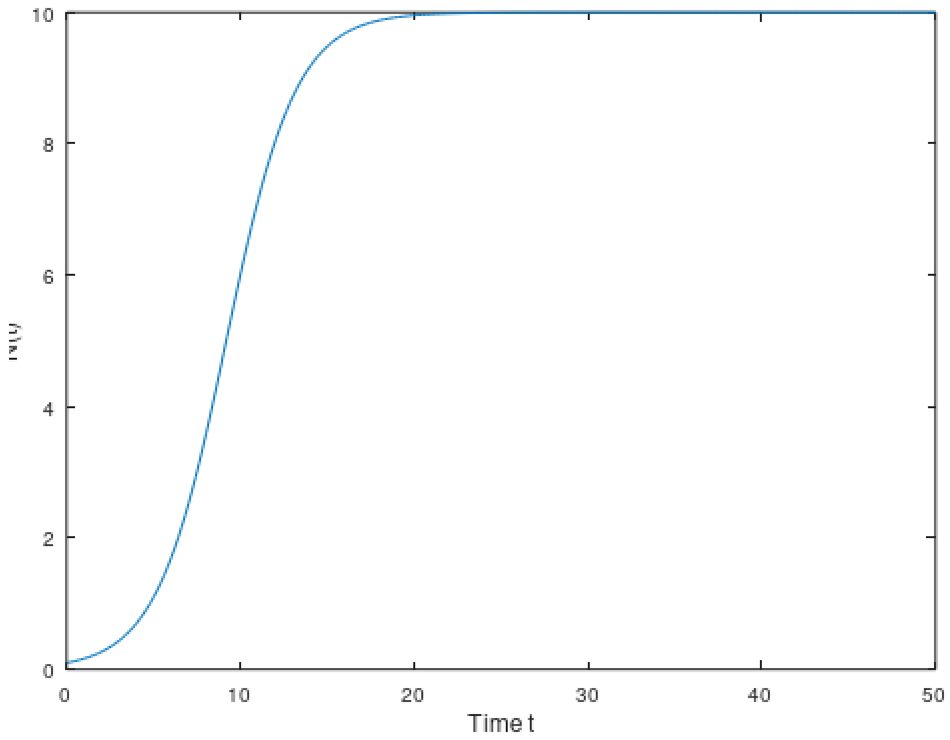
$$\frac{dN}{dt} = r\,N\left(1 - \frac{N}{K}\right)$$

- Suppose $r = 0.5, K = 10.$
- Suppose $N(0) = 0.1$
- Integrate the equation in the time interval [0,10] with 50 steps.
- Integrate the equation in the time interval [0,50] with 250 steps.

Logistic growth

Integrate the equation in the time interval [0,10] with 50 steps.





Integrate the equation in the time interval [0,50] with 250 steps

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Allee effect

EXERCISE 4

Consider the continuous-time population balance equation described by the

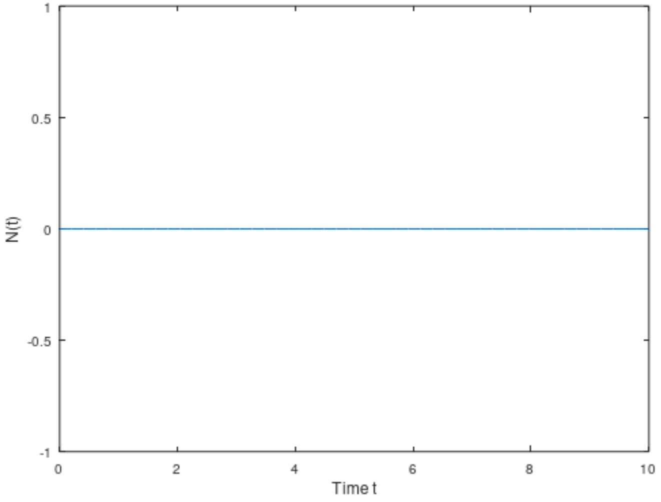
$$\frac{dN}{dt} = r\,N\left(1 - \frac{N}{k}\right)\left(\frac{N}{A} - 1\right)$$

- Suppose $r = 0.5, k = 10, A = 2.$

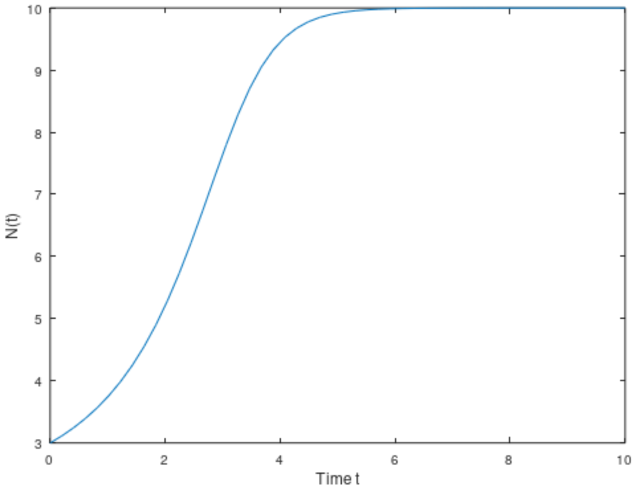
- Integrate the equation in the time interval [0,10] with 50 steps.

Allee effect

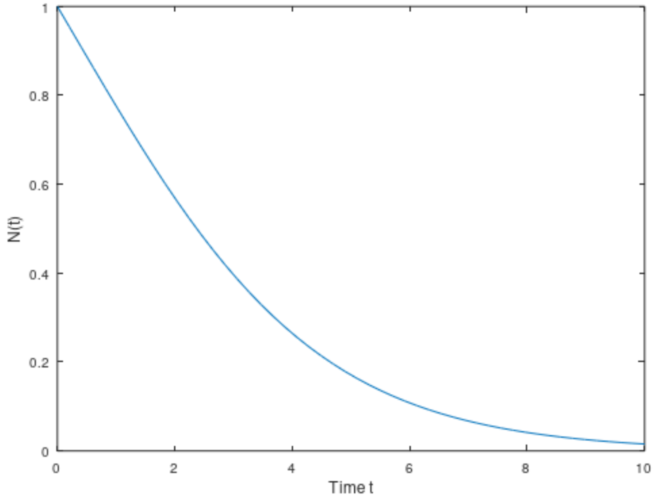
- $N(0) = 0$



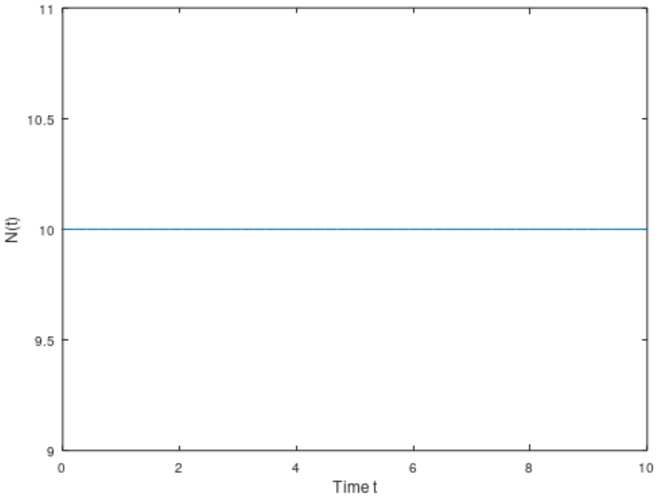
$N(0) = 3$



$N(0) = 1$



$N(0) = 10$



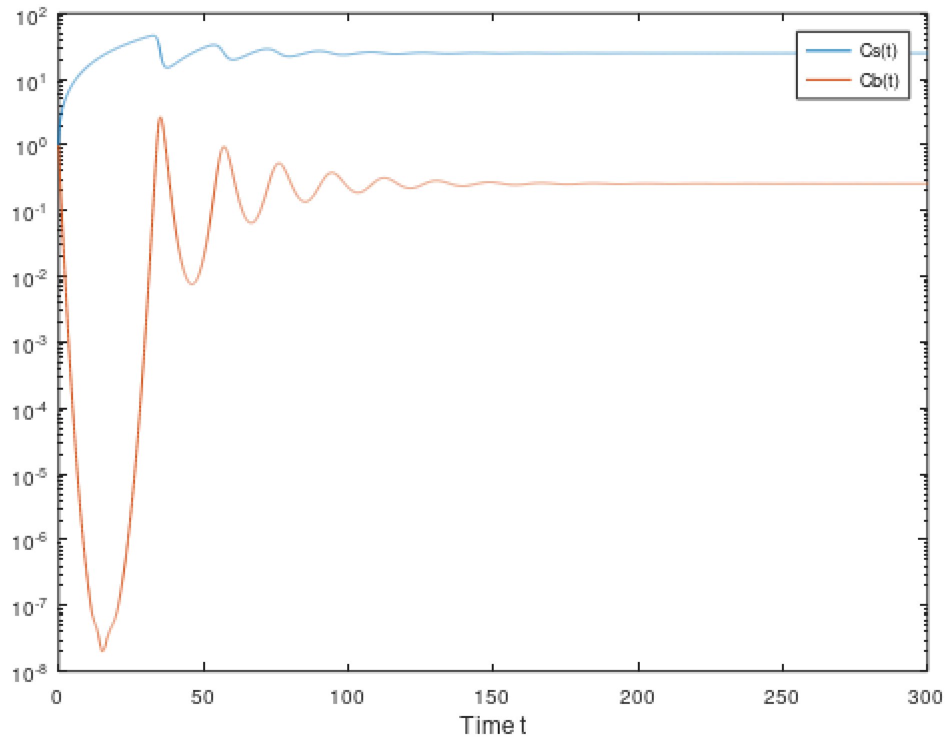
SOC dynamics: a two pools model

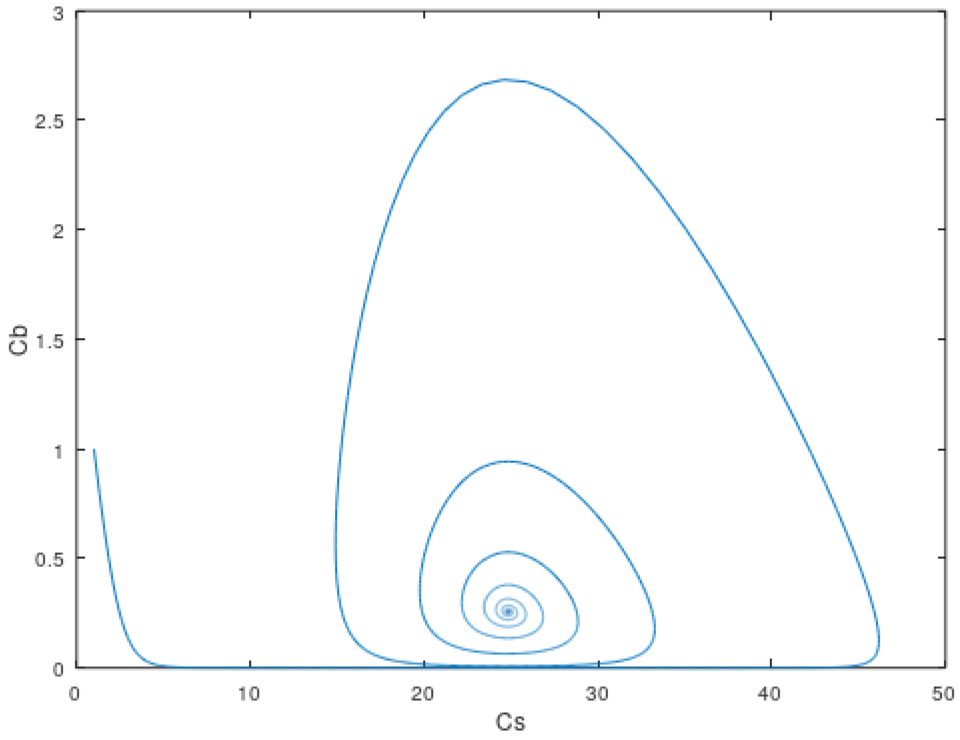
Consider the two pools soil organic carbon model

$$\frac{dC_s}{dt} = I - \frac{V_{max,U}\, C_b\, C_s}{K_{M,U} + C_s} + k_b C_b$$

$$\frac{dC_b}{dt} = \varepsilon \frac{V_{max,U}\, C_b\, C_s}{K_{M,U} + C_s} - k_b C_b$$

Temporal evolution and phase plane





Lotka-Volterra competition model

- Two populations N_1 and N_2 competing for a resource : for each species the presence of a competitor reduce its growth.

$$\frac{dN_1}{dt} = r_1 \; N_1\left(1 - \frac{N_1}{K_1}\right) - \frac{r_1\,\beta_{12}}{K_1} N_1 N_2$$

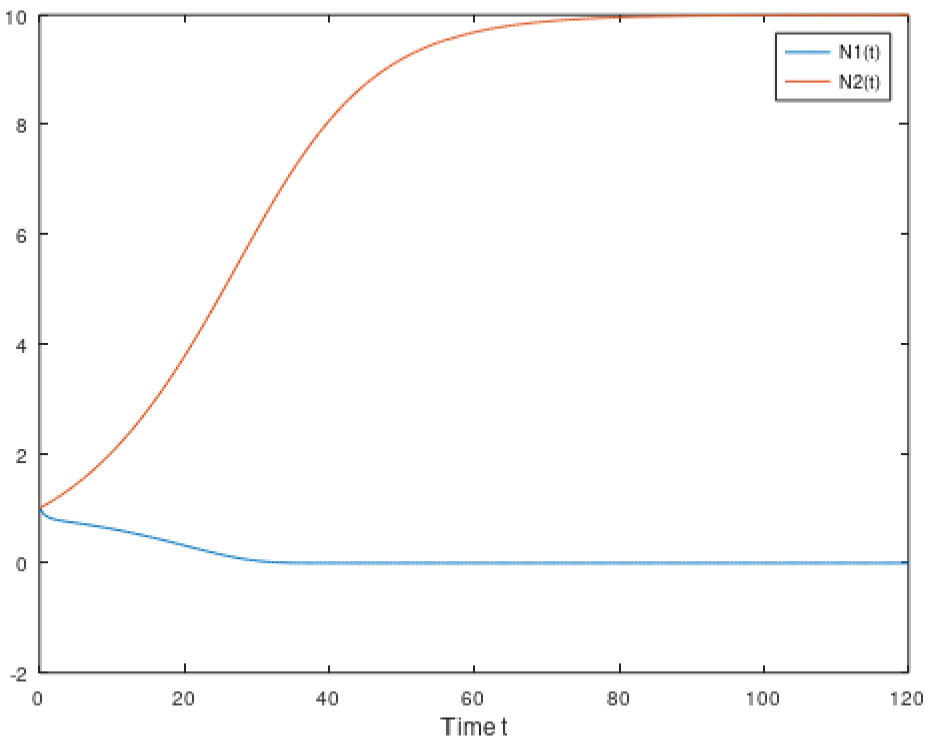
$$\frac{dN_2}{dt} = r_2 \; N_2\left(1 - \frac{N_2}{K_2}\right) - \frac{r_2\,\beta_{21}}{K_2} N_1 N_2$$

LV competition model : scenario 1

Parameters

r_1 = 1.3824; β_{12} = 0.2; K_1 = 1;

r_2 = 0.1; β_{21} = 2; K_2 = 10;

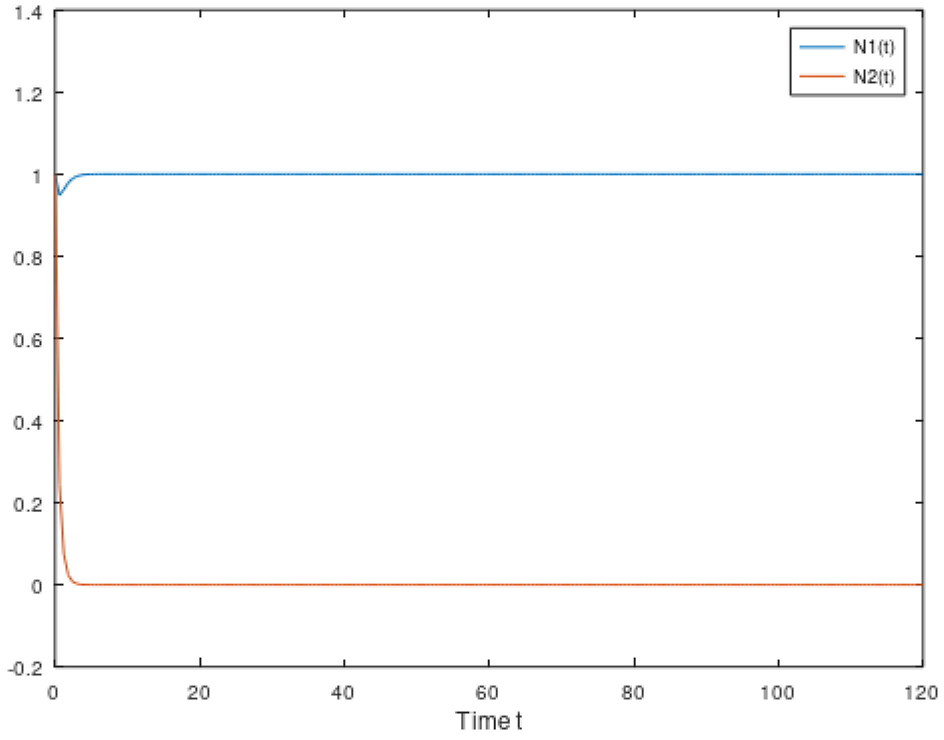


LV competition model : scenario 2

Parameters

r_1 = 1.3824; β_{12} = 0.2; K_1 = 1;

r_2 = 0.1; β_{21} = 2; K_2 = 0.1;

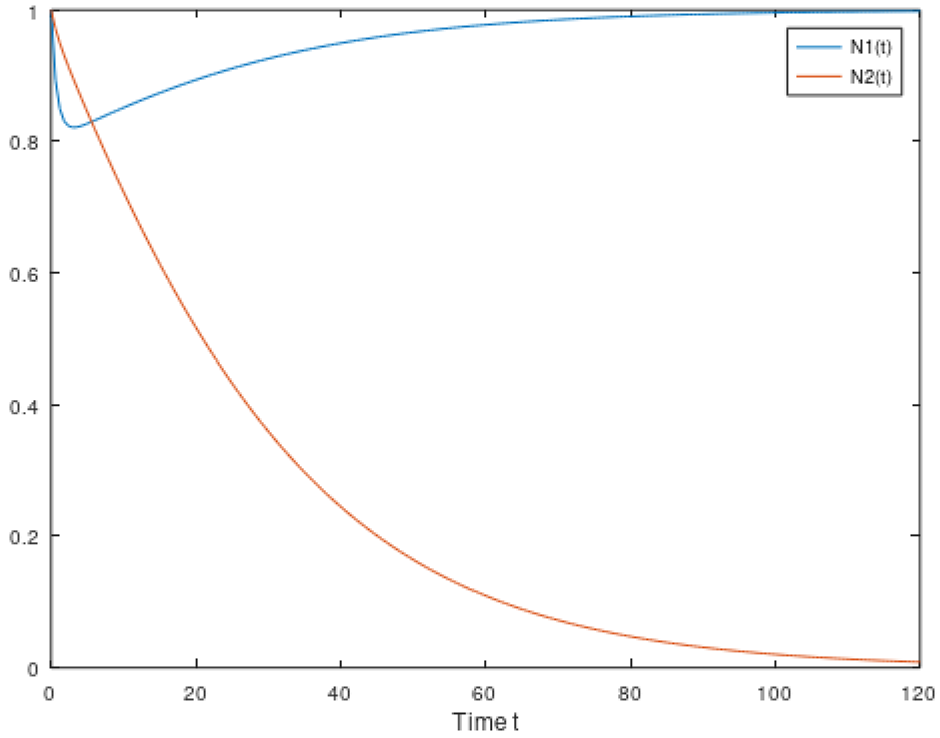


LV competition model : scenario 3

Parameters

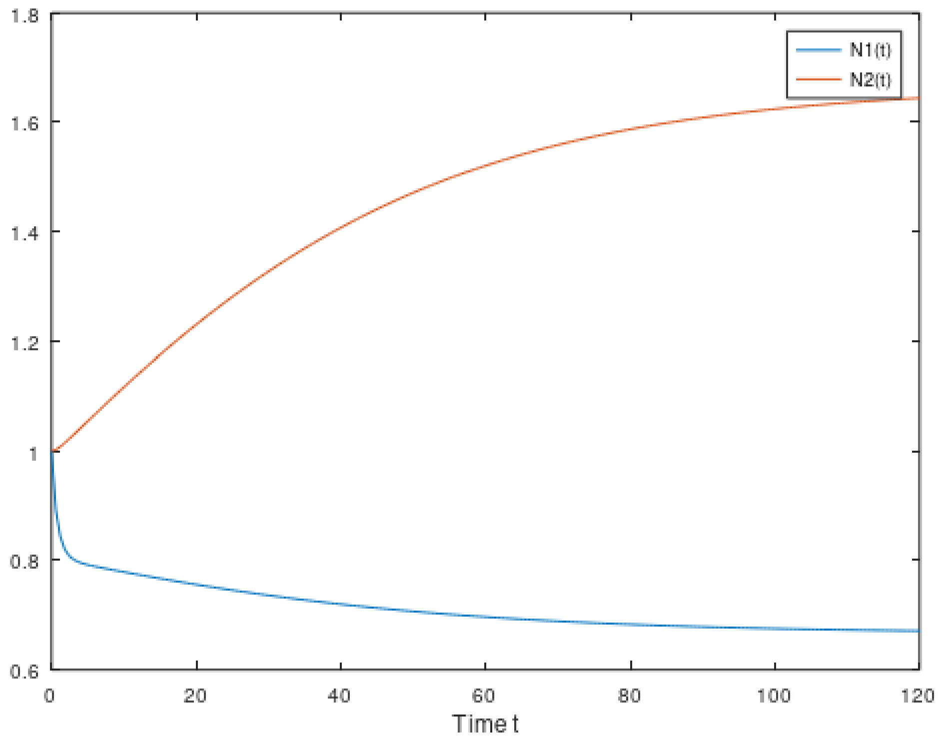
r_1 = 1.3824; β_{12} = 0.2; K_1 = 1;

r_2 = 0.1; β_{21} = 10; K_2 = 7;



LV competition model : scenario 4

r1 = 1.3824; beta12 = 0.2; K1 = 1;r2 = 0.1; beta21 = 2; K2 = 3;



Parameters

r_1 = 1.3824; β_{12} = 0.2; K_1 = 1;

r_2 = 0.1; β_{21} = 2; K_2 = 3;

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Lotka - Volterra predator-prey model

- Two populations: P predator and N prey

$$\frac{dN}{dt} = r\,N - a\,N\,P$$

$$\frac{dP}{dt} = \mathcal{E}\,a\,N\,P - \mu\,P$$

Temporal evolution and phase-plane