Interpolation of Missing Frequencies

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Summary

The ability of marine vibrators to accurately control the frequency and phase of the emitted signal offers new and interesting possibilities. In terms of deblending, one could, for example, imagine having simultaneously operating vibrators in narrow non-overlapping frequency bands. Deblending, could then be done by applying a simple bandpass filter.
In a sensitive survey area, one could imagine that vibrators omit the frequencies used by the local mammal population to communicate, thus reducing the overall environmental impact.

In such cases, there is a need to interpolate or fill in the missing frequencies. In seismic processing, interpolating missing frequencies is a new problem, not directly related to the more well studied problem of interpolating missing spatial data.
In this work, we present both classical signal processing methodology as well as CNN-based approaches for interpolation of missing frequency bands in seismic reflection data.

Interpolation of Missing Frequencies

Introduction

Geophysically, MVs offer precise control over signal spectrum and phase. This enables highly effective deblending, removal of residual sweep noise, and enhanced resolution or improved efficiency by an alternating sequence of monopole and dipole source points (Laws et al., 2019).
This work looks at another set of novel applications that could be enabled by accurate control of the emitted signal. These applications are probably best explained through examples, where the common theme is that each vibrator source omits specific frequencies from the emitted signal. The 'missing' frequencies will then have to be interpolated, which is the topic of this paper.

In the first example, we can envision two vibrators that operate simultaneously. Vibrator #1 emits 5-10, 15-20 and 25-30Hz while vibrator #2 emits 10-15, 20-25 and 30-35Hz. In the recorded reflection data, the contribution from each source can easily be recovered (deblended) by applying bandpass filters. However, for the individual source data, the missing frequencies will have to be interpolated or reconstructed.

In a second example, we imagine a vibrator survey that takes place in an area where a certain species is present. This species uses a known frequency band to communicate, for example, 30-40Hz. We could then design sweeps that omit this frequency range, thereby reducing the amount of disturbance to the species. Seismic reflection data in the missing frequency band could then be interpolated or reconstructed.

Other scenarios relate to strong narrowband noise that could be muted and then interpolated, or situations where, for example, mechanical resonances in the vibrator system made it difficult to emit certain frequencies. One could then omit a predetermined frequency range, and then instead just interpolate the missing frequency band(s) (data). Finally, we mention that if one only emitted half the frequencies in a band, the SNR in the emitted frequencies could be doubled.

The problem of interpolating a missing frequency band is very different from the commonly studied seismic problem of interpolating missing traces. To solve this new problem, we have investigated both classical signal processing techniques as well as more modern approaches utilizing machine learning.

Classical Signal Processing

In image processing, super-resolution algorithms are an active field of research, and have been used to extend the spectrum of images (Park et al., 2003). In seismic processing Wang et al., (2016) use the so-called TV norm minimization method to extrapolate low frequencies using the convex optimization method and knowledge of the sparsity of seismic data. The idea is to estimate the lacking low frequencies to ensure convergence when performing Full Waveform Inversion (FWI). In its simplest single trace form, a time series is generated with its L1 norm being minimized, subject to preserving the spectral content of the trace within a particular frequency region. The L1 norm minimization, based on the knowledge of the sparsity of seismic data, intends to put the extrapolated energy where the reflections already are, not creating new events. Extrapolation can, in theory, be at lower or higher frequencies. Wang et al., (2016) then use the spatial similarity between traces, in an extension using multiple traces, in addition to the above optimization scheme to reduce the estimation error.

We adapted this algorithm for interpolating a gap in the frequency spectra. Figure 1 illustrates how TV norm minimization can be used to interpolate a gap in the spectrum of the data between 30Hz and 40Hz. The phase is also preserved by putting the interpolated energy where it belongs in time, as seen from the time-series plot. In a second example, we took inspiration from the work of Bekara et al., (2022) and Djebbi et al., (2022).

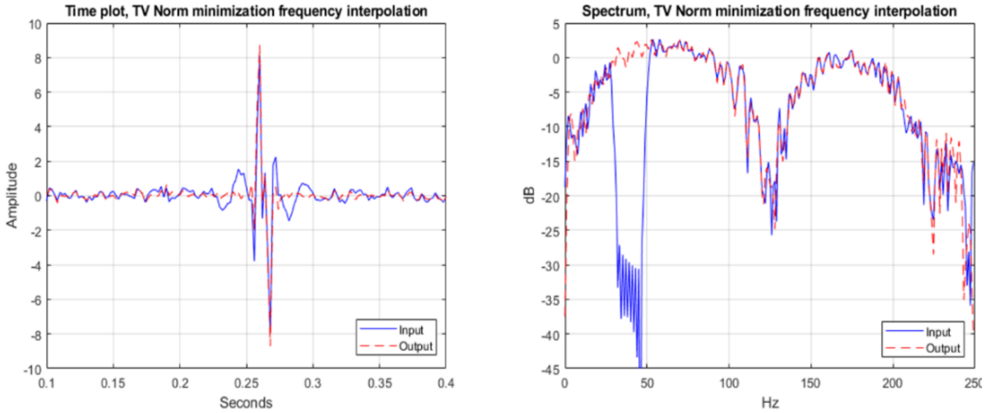


Figure 1. *Left: Time series plot of a synthetic trace before and after interpolation. Right: The corresponding spectra showing frequency band reconstruction.*

They described an autoregressive method used to extrapolate the low frequencies in the slowness domain to be used in FWI. In our case, we want to interpolate a gap in the spectra rather than extrapolating to lower frequencies. We transform the data from the f-k domain to the frequency versus slowness domain, f-p, where an event with a specific dip appears as a vertical line and a frequency gap appears as horizontal lines, as shown in Figure 2.

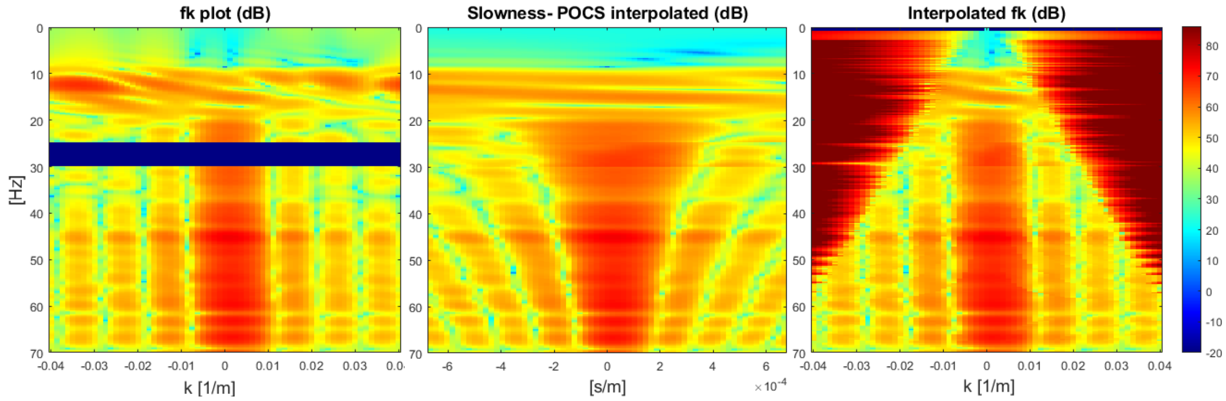


Figure 2. *Filling in a band of missing frequencies. From left to right: f-k data (in dB) before interpolation of missing data in the 25-30Hz range. f-p data after POCS 2D interpolation for velocities greater than 1500m/s and f-k data after interpolation. Note that only amplitudes are illustrated though all operations are performed on complex values. The underlying seismic data consisted of a block of data with 512 samples by 8 traces.*

The frequency gap can then be treated the same way as traces when interpolating dead traces, though this time these are complex valued so that both amplitude and phase are reconstructed. We experimented with several different methods (AR, ARMA, ARIMA and POCS) for reconstructing the gap. Generally, the methods were sensitive to parameter settings. The best results were obtained using a 2D POCS interpolator (Abma and Kabir, 2006). In Figure 2, the operations are performed on a limited number of traces and samples to illustrate the approach. In production software, a 2D overlapping sliding window approach will likely need to be applied. The interpolation should be performed within each window, and the results should be merged to recreate the entire gather in the t-x domain.

A Machine Learning Approach

The other approach we tried for interpolating missing frequency band(s) is based on deep learning, using a convolutional neural network (CNN) of U-Net architecture (Ronneberg, et al., 2015). From previous work by Larsen Geiner et al., (2020), we know that such networks have a good ability to perform trace interpolation. Training, on 2D shot gathers was performed by band-stop filtering existing seismic data gathers to simulate missing frequencies from the acquisition. To mimic the real-life situation, an appropriate amount of recorded noise was added back onto the frequency band(s) that had been filtered out. Conversely, the same unfiltered shot gathers were used as the learning targets for the network. In the course of our work, we experimented with training the CNN on data in both the t-x and f-k domains. Figure 3 shows an example from a t-x interpolation, where we look at how well a single trace taken from the 2D gather was reconstructed by the CNN in the frequency domain. The results are encouraging, but there is clear room for improvement.

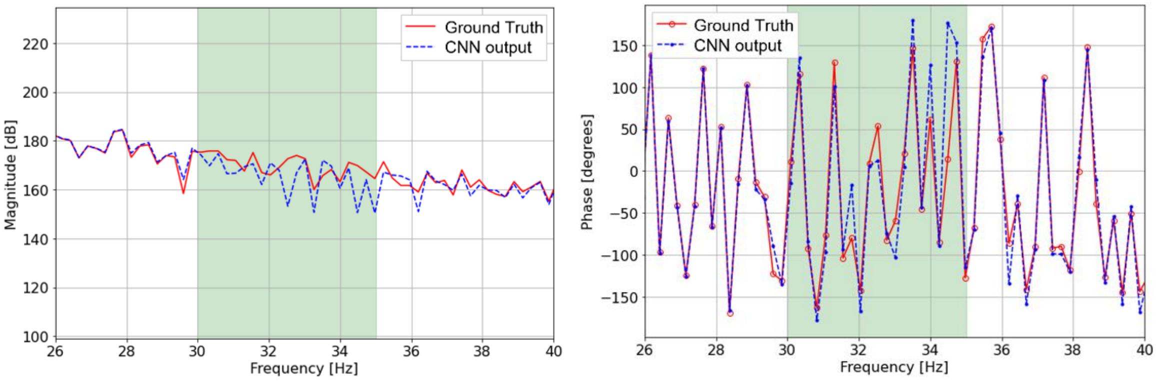
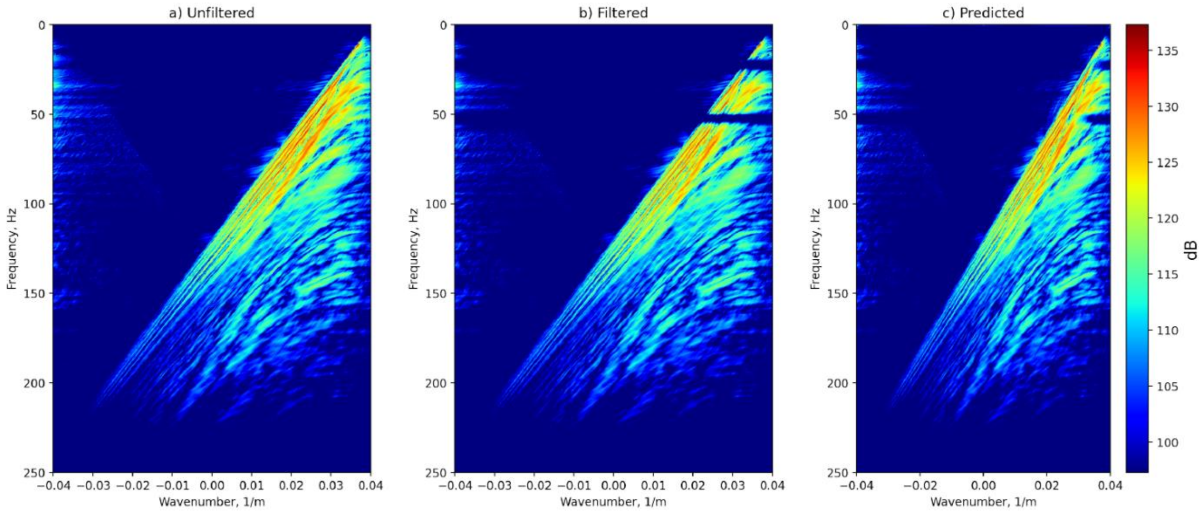


Figure 3. *The CNN interpolation results on a seismic trace with the 30-35Hz frequency band zeroed out. Both the amplitude (left) and the phase spectrum (right) show a reasonable fit between the interpolated and the true frequency band.*



Applying CNN-based frequency interpolation in the f-k domain proved challenging. The main problem was that in the f-k domain the values are complex. It is not a straightforward task to adapt a CNN to work with complex numbers directly. An alternative was to put in the real and imaginary parts as two different channels, similarly to how colour images normally are processed, where red, green, and blue

Figure 4. A seismic shot gather transformed into the f-k domain. *From left to right: Unfiltered – truth (a), filtered (b) and reconstructed (c) f-k gathers. Notice the two horizontal slices in the middle image, showing where frequencies were missing.*

(RGB) goes into three different channels. However, with such an approach, the phase relationship was difficult to maintain. Figure 4 shows one example of interpolating two missing frequency bands in the f-k domain. Again, the results are encouraging, but with clear room for improvement.

Conclusion and future work

In this paper, we have looked at several different methods for interpolating missing frequency bands of marine seismic data.

Such novel functionality could be an enabler for new acquisition setups with marine vibrators. The results are promising, but they are still not at a level where we would be comfortable deploying this in a commercial setting. By transforming the data into the f-p domain, the missing frequency bands were represented by gaps that could be filled with classical trace interpolation algorithms. We also tried to fill in the missing frequencies directly using CNN-based deep learning. In both cases, a challenge was that we ended up with complex numbers, meaning that existing algorithms needed to be adapted or newly developed to better handle this issue. Going forward, we see several possible ways to improve results further:

1) With f-p based frequency interpolation, we could refine the algorithm, and possibly look for methods to take further advantage of the structure of the data. By structure, we mean the way data in one frequency range could best be used to estimate data in another frequency range.
2) With CNN-based interpolation, we likely need to work on adapting such networks to handle complex numbers. This is not a trivial task. Furthermore, we believe that by developing new network structures and cost functions that better adapt to the nature of seismic data and this task, better results can be achieved.
3) We could utilize harmonics to help fill in missing frequencies (Wang et al., 2023).
4) Finally, we mention the possible use of other transform domains as a pre-processing step before doing the actual interpolation. We would like to utilize domains where missing frequencies are represented as compactly as possible.

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