Theodor Kaluza's Theory of Everything: revisited

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Abstract

Using a Kaluza-type of model, describing the laws of electromagnetism within the formalism of differential geometry, provides a coherent, comprehensive and quantitative description of phenomena related to particles, including a convergent series of quantized particle energies, with limits given by the energy values of the electron and the Higgs vacuum expectation value, and the values for electroweak coupling constants. The geometry of the solutions for spin 1/2 defines 6 lepton-like and 6 quark-like objects and allows to calculate the fractional electric charges as well as magnetic moments of baryons.

Electromagnetic and gravitational terms will be linked by a series expansion, the corresponding relation suggests the existence of a cosmological constant in the correct order of magnitude.

The model can be expressed *ab initio*, necessary input parameters are the electromagnetic constants.

1 Introduction

Theodor Kaluza in 1919 developed a unified field theory of gravitation and electromagnetism that produced the formalism for the field equations of the general theory of relativity (GR) and Maxwell's equations of electromagnetism (EM) thus unifying the major forces known at his time. His 5-dimensional model [1] is not suited to give properties related to particles, a problem addressed by Oskar Klein [2] who introduced the idea of compactification and attempted to join the model with the emerging principles of quantum mechanics. Therefore the theory is mainly known as Kaluza-Klein theory today. This version became a progenitor of string theory. The original Kaluza model was developed further as well [3], Wesson and coworkers elaborated a general non-compactified version to describe phenomena extending from particles to cosmological problems. The equations of 5D space-time may be separated in a 4D Einstein tensor and metric terms representing mass and the cosmological constant, Λ. Particles may be described as photon-like in 5D, traveling on time-like paths in 4D. This version is known as space-time-matter theory [4]. Both successor theories give general relationships rather than providing quantitative results for specific phenomena such as particle energy.

Kaluza realized that Maxwell's equations may be described within the formalism of GR. To get both these *and* the Einstein field equations (EFE) he needed an additional dimension and had to insert the constant of gravitation, G, in his metric. However, usually one effect dominates, gravitation on the scale of astrophysics, EM on the scale of particles. Since the latter is the main subject of this article the minor - gravitational - terms will be neglected in 1st approximation, eliminating the need for introducing the extra coefficient G. Gravitation may be recovered by a series expansion and G may be expressed as an EM-term.

Curvature of space-time based on an electromagnetic version of the field equations of GR will be strong enough to localize a photon in a self-trapping kind of mechanism, yielding energy states in the range of the particle zoo. Circular polarized light is part of conventional electromagnetic theory, in the following this feature will be treated equivalently with the terms angular momentum or spin as intrinsic property of a photon. In particular, unless noted otherwise, it is assumed that particles posses spin 1/2 or are composed of spin 1/2 components. Spin will be a necessary boundary condition to determine an integration limit for the equations used. Since at this point there is no obvious ansatz for integrating spin into the metric of this model, any metric discussed in the following should be considered as an approximation only.

The basic proceeding will be as follows:

Kaluza's equations for flat 5D-space-time may be arranged to give [4, chapter 6.6]

1) Einstein-like equations for space-time curved by electromagnetic and scalar fields (equ. (2)),
2) Maxwell equations where the source depends on the scalar field,
3) a wave-like equation connecting the scalar Φ with the electromagnetic tensor (equ. (3)).

Solutions of (3) for Φ in a flat 5D-metric will be used in a general ansatz for a metric. Due to 3) Φ has to be a function of the EM-potentials, in the static approximation of this work the electric potential, A_{el}. The only other parameter included in Φ will be a function of the fine-structure constant [1], α, which enters the equations

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[1] The relation of the masses e, μ, π with α was noted first in 1952 by Nambu [5]. MacGregor calculated particle mass and constituent quark mass as *multiples* of α and related parameters [6].

through the boundary condition spin 1/2[ħ], see chpt. 2.4 [2]. Since a geometric interpretation allows to give α in terms of Γ-functions, the results of this model can be calculated *ab initio,* using electromagnetic and mathematical constants only.

The model yields absolute particle energies in the range expected for a neutrino and as a set of converging series with limits given by the energy of the electron and the Higgs vacuum expectation value (VEV). Assuming that a 2nd order term in a series expansion of EM-terms represents gravitation and should not exceed the EM-term, some of the α-terms included in Φ can be identified with the ratio of electron and Planck energy, see chpt. 2.6, 4. With this ansatz additional minor terms in the field equations will be in the correct order of magnitude for the cosmological constant, Λ.

Focusing on the angular momentum aspects of the model, in chpt. 3 the rotation of a set of orthogonal E, B, C-vectors, attributed to the electromagnetic fields and the propagation with the speed of light, C, will be modeled via quaternions. This gives 3 possible solutions for spin 1/2 defining 6 distinct geometric objects with partial charges of 1/3 and 2/3. Combining 2 solutions gives 6 lepton like entities as the simplest, node-free case, combining 3 appropriate solutions allows to calculate magnetic moments of baryons.

Typical accuracy of the calculations is in the order of 0.0001 [3]. The deviation of calculated results from the experimental values is typically in the range 0.01 - 0.001, consistent with a variation of input parameters related to elementary charge in an order of magnitude of QED corrections, which are not included in this model.

To focus on the more fundamental relationships some minor aspects of the model are exiled to an appendix, related topics will be marked as [A].

2 Calculation of energies and coupling constants

2.1 Kaluza theory

Kaluza theory is an extension of general relativity to 5D-space-time with a metric given as [cf. 4, equ. 2.2]:

$$g_{AB} = \begin{bmatrix} (g_{\alpha\beta} - \delta^2 \Phi^2 A_\alpha A_\beta) & -\delta \Phi^2 A_\alpha \\ -\delta \Phi^2 A_\beta & -\Phi^2 \end{bmatrix} \tag{1}$$

In (1) roman letters correspond to 5D, Greek letters to 4D. δ corresponds to a general constant appropriate for an EM unit system that turns δA_α into a dimensionless quantity. To get the field equations of GR Kaluza assigns δ to a gravitational term, κ. A is the electromagnetic potential. In the context of the electrostatic approximation of this model A will be assumed to be represented by the electric potential, $A_{el} = e_c/(4\pi\varepsilon_c r) = \rho_0/r$ [-]. In the following ρ_0 will refer to the A_{el}-term, while dropping subscript 0 will indicate a general solution where ρ may contain additional terms.

Assuming 5D space-time to be flat, i.e. $G_{AB} = 0$, gives for the 4D-part of the field equations [cf. 4, equ. 2.3]:

$$G_{\alpha\beta} = \frac{\delta^2 \Phi^2}{2} T^{EM}_{\alpha\beta} - \frac{1}{\Phi}\left(\nabla_\alpha(\partial_\alpha \Phi) - g_{\alpha\beta} \Box \Phi\right) \tag{2}$$

From $R_{44} = 0$ follows:

$$\Box \Phi = -\frac{\delta^2 \Phi^3}{4} F_{\alpha\beta} F^{\alpha\beta} \tag{3}$$

In the following only derivatives with respect to r of a spherical symmetric coordinate system will be considered. A function Φ_N

$$\Phi_N \approx \left(\frac{\rho}{r}\right)^{N-1} \exp\left(-\left(\frac{\rho}{r}\right)^N /2\right) \tag{4}$$

yields solutions for an equation of general type of (3), where the term of highest order of exponential N, given by $\Phi'' \sim \rho^{3N-1}/r^{3N+1}$, may be interpreted to provide the terms for $A_{el}(r)' \sim e_c/(4\pi\varepsilon_c r^2) \sim \rho_0/r^2$, see [A1].The significance of (4) lies in providing the relationship of exponential and pre-exponential terms and first of all in the requirement to contain powers of (ρ_0/r) in the exponent of Φ_N.

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[2] The coefficient of angular momentum may be interpreted as either σ, which will in general indicate the integration limit, $(r/\rho)^3$ for calculating the incomplete gamma functions, or its main component $\alpha_{lim} \approx 1.5/\alpha$, see chpt. 2.4, 2.5.

[3] Including e.g. errors due to the numerical approximation of incomplete Γ-functions.

2.2 Modification of Kaluzas metric

In the following the focus will be on EM-terms only, in particular A_{el}, minor terms and off-diagonal terms will be omitted. Equation (1) will turn into:

$$g_{AB} = \begin{bmatrix} -\delta^2 \Phi^2 A_\alpha A_\beta & -1 \\ -1 & -1 \end{bmatrix} \qquad (5)$$

A term for Φ according to 3f with N = 3 will be inserted in what is essentially a 4D-metric. According to Campbells theorem [4] a flat N-D metric is mathematically equivalent to a curved (N-1)-D metric so both approaches are compatible in principle.

Concerning dimensions and unit systems, the 4D electrostatic term for energy density in the stress energy tensor requires an expansion with some appropriate EM-coefficient, ε, turning the square of the electric field into energy density: $w = T_{00} = 1/\varepsilon\ (\varepsilon\delta^2 E^2)$, with E being some general electric field. Since E is the derivative of A_{el}, δ^2 will be needed for consistent units.

When equating G_{00} with T_{00} coefficient δ will cancel, 1/ε will replace the G-term in the field equation, giving

$$(8\pi) G/c_0^4 \quad => \quad \approx \ 1/\varepsilon \qquad (6)$$ [4]

$$\text{in} \quad G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\ \frac{1}{\varepsilon} T_{\alpha\beta} \qquad (7)$$

2.2.1 System of natural units

A unit system that is particularly suited for such an approach, featuring a dimensionless A_{el}, will have charge in units of energy. In the following a unit system based on SI [5] for units of mechanics but with modified EM-units, indicated by subscript c, will be used.

$$c_0^2 = (\varepsilon_c\ \mu_c)^{-1} \qquad (8)$$

with $\varepsilon_c = (2.998\text{E+8}\ [\text{m}^2/\text{Jm}])^{-1} = (2.998\text{E+8})^{-1}\ [\text{J/m}]$

$\mu_c = (2.998\text{E+8}\ [\text{Jm/s}^2])^{-1} = (2.998\text{E+8})^{-1}\ [\text{s}^2/\text{Jm}]$.

From the Coulomb term $b_0 = e^2/(4\pi\varepsilon_0) = e_c^2/(4\pi\varepsilon_c) = 2.307\text{E-28}$ [Jm] follows for the square of the elementary charge: $e_c^2 = 9.671\text{E-36}\ [\text{J}^2]$. In the following $e_c = 3.110\text{E-18}$ [J] and $e_c/(4\pi\varepsilon_c) = 7.419\text{E-11}$ [m] may be used as natural unit of energy and length.

2.3 Point charge energy

The approach of chpt. 2.1, 2.2 is an approximation not only in neglecting contributions of the magnetic potentials but also in not considering spin. It is therefore not possible to give a specific metric for the problem considered here [6], [A2] introduces 2 versions for illustrative purposes. However, due to the difference in order of magnitude between ρ_0 and ρ, to be elaborated on below, the leading term for particle energy in a large class of approaches for a metric will be due to the Christoffel symbols of the angular coordinates, giving a "-1" component in the Ricci-tensor [7] and a result for G_{00} as:

$$G_{00} = -\Phi^2 E^2 = -\rho_0^2/r^4 \exp(-(\rho/r)^3) = w/\varepsilon_c \qquad (9)$$

The exponential of function Φ allows to integrate the electric field of the point charge. The volume integral over the energy density of (9) gives the energy of particle n according to:

$$W_n = \varepsilon_c \rho_0^2 \int_0^{r_n} \exp(-(\rho_n/r)^3) r^{-4}\ d^3 r = 4\pi\varepsilon_c \rho_0^2 \int_0^{r_n} \exp(-(\rho_n/r)^3) r^{-2}\ dr \qquad (10)$$

Solutions for integrals over exp(-(ρ/r)³) times some function of r can be given by:

$$\int_0^{r_n} \exp(-(\rho_n/r)^N) r^{-(m+1)} dr = \Gamma(m/N, (\rho_n/r_n)^3)\ \frac{\rho_n^{-m}}{N} = \int_{(\rho_n/r_n)^3}^{\infty} t^{\frac{m}{N}-1} e^{-t} dt\ \frac{\rho_n^{-m}}{N} \qquad (11)$$

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[4] In deriving the EFE an additional factor of 2 is needed to equate a perturbation term relative to a Minkowski metric to a gravitational potential term. The ansatz given here does not require this factor, however, it might be related to equ. (13) and factor 8 of (14)ff.

[5] Note: In SI proper ρ_0, ρ have units of [Vm]; coefficient δ [1/V] is part of the argument of the exponential.

[6] On which the details of a solution for Φ will depend as well.

[7] Consistent with curvature being due to the lateral extension of the E-vector in the quaternion model of chpt. 3;

in this work used for N = {3; 4}, m = {-2; -1; 0; +1;+2}. The term $\Gamma(m/N, (\rho_n/r_n)^3)$ denotes the upper incomplete gamma function, given by the Euler integral of the second kind [8]. In the range of values relevant in this work, for m/N ≥ 1 the complete gamma function $\Gamma_{m/N}$ is a sufficient approximation, for m/N ≤ 0 the integrals have to be calculated numerically, requiring an integration limit, see 2.4.

Equation (10)f will give as energy for a particle n:

$$W_{n,elstat} = 4\pi\varepsilon_c\rho_0^2 \int_0^{r_n} \exp\left(-\left(\rho_n/r\right)^3\right) r^{-2}\ dr \ = \ b_0\,\Gamma(+1/3,\,(\rho_n/r_n)^3)\,\rho_n^{-1}/3 \approx \ b_0\,\Gamma_{+1/3}\,\rho_n^{-1}/3 \qquad (12)$$

including the integral for the energy of a point charge term modified by $\exp(-(\rho/r)^3)$. Particles are supposed to be electromagnetic objects possessing photon-like properties, thus it will be assumed that particle energy has equal contributions of electric and magnetic energy, i.e.:

$$W_n = W_{n,elstat} + W_{n,mag} = 2W_{n,elstat} \ \approx \ 2\ b_0\,\Gamma_{+1/3}\,\rho_n^{-1}/3 \qquad (13)$$

2.4 Angular momentum, coefficient σ

The integral limits required for integrals of (11) with m/N ≤ 0 are r_n („particle radius" of state n; with respect to J_Z; $r_n \neq \lambda_C$, see (64)) for integrals over $\exp(-(\rho/r)^3)$ and $(\rho_n/r_n)^3$ for the Euler integrals. The latter will be expressed via a constant defined as 8/σ [9]:

$$(\rho_n/r_n)^3 = 8/\sigma \qquad (14)$$

whose value may be derived from the condition for angular momentum J_z = 1/2 [ħ]. A simple relation with angular momentum J_z for spherical symmetric states will be given by applying a semi-classical approach [10]:

$$J_z = r_2\,x\,p(r_1) = r_2\,W_n(r_1)/c_0 \quad \equiv 1/2\ [\hbar] \qquad (15)$$

Using term $2b_0$ of equ. (13) as constant factor and integrating over a circular path of radius $|r_2| = |r_1|$, equation (11) will give for m = 0:

$$J_z = \int_0^{r_n}\int_0^{2\pi} J_z(r,\varphi)\,d\varphi\,dr = \ 4\pi\frac{b_0}{c_0}\int_0^{r_n} e^{-\left(\frac{\rho}{r}\right)^3} r^{-1}dr = 4\pi\alpha\hbar\int_0^{r_n} e^{-\left(\frac{\rho}{r}\right)^3} = \frac{4\pi}{3}\alpha\hbar\int_{8/\sigma}^{\infty} t^{-1}e^{-t}dt \ \equiv \ \frac{[\hbar]}{2} \qquad (16)$$

To obtain J_z = 1/2 [ħ] the integral over $\exp(-(\rho/r)^3)/r$ dr of (16), *has to* yield $\alpha^{-1}/8\pi$.

$$\int_0^{r_n} \exp\left(-\left(\rho_n/r\right)^3\right) r^{-1} dr \ = \ 1/3\int_{8/\sigma}^{\infty} t^{-1}e^{-t}\,dt \ \equiv \ \frac{\alpha^{-1}}{8\pi} \approx 5.45 \qquad (17)$$

Relation (17) may be used for a numerical calculation of the integration limit, 8/σ, giving a value of σ_0 (assumed to represent spherical symmetry), σ_0 = 1.810E+8 [-]. Assuming the coefficient $\Gamma_{-1/3}/3$ according to (11) has to be part of the expression for σ_0 [11] this results in $\sigma_0 \approx 8(1.5\alpha^{-1}\Gamma_{-1/3}/3)^3$. This value may be interpreted as a coefficient representing geometry, giving a value close to the numerical one:

$$\sigma_0 \approx 8\ (1.5\ \alpha^{-1}\ \Gamma_{-1/3}\ /3)^3 \ \approx 8 \ \left(\frac{4\pi\,\Gamma_{-1/3}^{\ \ 3}}{3}\right)^3 \ = 1.772\text{E}+8\ [-] \qquad (18)$$

As a consequence a dimensionless volume-like term appears in the denominator of the energy expression (13) for spherical symmetry. Expression (18) is closely related to the value of α and will be used in this context in chpt. 2.10.

In [A3] some additional aspects of the terms supposed to constitute σ will be discussed, giving an alternate expression for (18) and demonstrating that coefficient σ has to be part of the exponent of Φ: $\rho_n^3 \approx \sigma\rho_0^3$,

$$\Phi_\sigma \approx (\rho_0/r)^2\,\exp(-\sigma(\rho_0/r)^3) \qquad (19)$$

Calculating energy according to Φ_σ and (13) will give W ≈ 0.1 eV, a value in the estimated energy range for a neutrino.

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[8] Euler integrals yield positive values, the sign convention of Γ-functions gives negative values for negative arguments. Abbreviations such as $\Gamma_{-1/3}$ for $|\Gamma(-1/3)|$ will be used;

[9] Chosen to give coefficient σ as component in the argument of the exponential of Φ, see [A3.1].

[10] In 0th approximation: using the term for energy (13) and length (29) requires $\sigma^{1/3}$ to be of order of the inverse fine-structure constant α^{-1}: $1/c_0\int w(r)\,dr * \int dr \approx b_0/\rho_n * \sigma^{1/3}\rho_n/c_0 \equiv \hbar/2 \ \Rightarrow \ \sigma^{1/3} \approx \alpha^{-1}$.

[11] Since according to (14) $\sigma^{1/3}$ is proportional to a length parameter, r_n, which according to (11) includes $\Gamma_{-1/3}/3$.

2.5 Lower limit of σ

The minimal possible value for σ is defined by the Γ-term in the integral expression for length, $\Gamma_{-1/3}/3$, (11), and the integers in (56):

$$\sigma_{min} = 8(\Gamma_{-1/3}/3)^3 \quad {}^{12} \tag{20}$$

leaving a term

$$\alpha_{\lim} \approx 1.5\alpha^{-1} \approx 4\pi\Gamma_{-1/3}^{2} \tag{21}$$

as variable part in σ to consider non-spherical symmetric states (see 2.7, [A3]) [13]).

2.6 Quantization with powers of $1/3^n$ over α

Most relations given here are valid for any particle energy which should be expected as there is a continuous spectrum of energies according to special relativity. However, a particular set of energies may be identified by relaxing the condition of orthogonality of different states according to quantum mechanics to requiring different states to be expressible in simple terms of a ground state coefficient, α_0, in the exponent of Φ and not to exhibit any dependence on intermediary states [14].

There are 2 lines of thought for an estimation of α_0.

2.6.1 Condition a)

The condition that energy/length of a charged particle has to be higher/smaller than the value given by a pure electrostatic term.

Since r_l according to (11), (14), (18)f will be proportional to α^{-2} the term in the exponential has to be: $\alpha_0 < \alpha^6$.

The relationship between a photon-like object and a point charge object of elementary charge is based on the coefficient α, suggesting a photon-like state to differ by a factor of α from a pure point charge state and to use a ground state coefficient $\alpha_0 \approx \alpha^9$. This fits the relationship of a set of fundamental particle energies with the charged particle of lowest energy, the electron, as a ground state quite well, however, requiring an ad hoc factor ≈ 3/2 for the electron itself. With W_e as ground state, W_n would be given by (22)ff *relative* to the electron state as:

$$W_n/W_e \approx 3/2\frac{\alpha\wedge(1.5/3^n)}{\alpha^{1.5}} \approx 3/2\,\Pi_{k=1}^{n}\alpha\wedge(-3/3^k) \qquad n = \{1;2;..\} \tag{22}$$

see table 1.

However, to calculate absolute values of energy requires another factor in addition to α_0.

2.6.2 Condition b)

In a series expansion of the exponential in terms of force, potential, etc., such as given below, particles beyond the electron enter the terms according to their coefficients from (22)ff. In order for the 2nd order term not to exceed the 1st order term the energy of spherical symmetric particles - including relativistic effects - should not exceed $\alpha_0^{-1} = \alpha^{-9}$. However, this restriction should apply for non-spherical symmetric particles as well, requiring $\alpha_{\lim} \approx 1.5/\alpha$ as additional factor. Including the additional factor of the electron, ≈ 3/2, and restricting to electrostatic contributions (cf. chpt. 4) gives:

$$\frac{1.5^3\alpha_0}{2\alpha_{\lim}} = 1.5^2\alpha^{10}/2 = 4.8E\text{-}22 = \alpha_{Pl} \approx \frac{W_e}{W_{Pl}} \tag{23}$$

The additional factor of ≈ 3/2 of the electron might be related to this difference in α_0 and α_{Pl}. The electron coefficient in the exponential of Φ and the energy term equ. (13) would be given as: $\alpha_e \approx (3/2)^3\alpha^9$. ρ_n may be given as (δ = 1 for electron, = 0 otherwise; n = {0;1;2;..}):

$$\rho_n^3 \approx (1.5)^\delta\,\sigma_0\,\boldsymbol{\alpha_{\lim}^{-1}/2\ 1.5^3\,\alpha^9}\,\alpha^{4.5}/\alpha\wedge(4.5/3^n)\ \ (e_c/(4\pi\varepsilon_c))^3 \approx (1.5)^\delta\,\sigma_0\,\boldsymbol{\alpha_{Pl}}\,\alpha^{4.5}/\alpha\wedge(4.5/3^n)\ \ (e_c/(4\pi\varepsilon_c))^3 \tag{24}$$

2.6.3 Relationship with Lorentz boost

Interpreting the difference in wavelength of different states as a length contraction due to a Lorentz boost and calculating the necessary velocity according to $l = l_o(1-v^2/c_0^2)^{0.5}$, the ratio of 2 consecutive steps will converge to $v_n/v_{n+1} = 3^{0.5}$. This is the ratio of the sum of 3 orthogonal vectors of equal length to a single vector, a simple

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[12] If the term of (18) is interpreted as a (cube of a) volume parameter, a term of the kind of (20) would represent the (cube of a) 1D parameter.

[13] $\sigma_0 \approx (\alpha_{\lim}\, 2\Gamma_{-1/3}/3)^3 \approx (\Gamma_{-1/3}/\alpha)^3$

[14] cf. $W_n^2 \sim \left(\alpha_0^{1/3}\alpha_0^{1/9}....\alpha_0^{1/(3\wedge(n-1))}\boldsymbol{\alpha_0^{1/(3\wedge n)}}\right) / \left(\boldsymbol{\alpha_0^{1}}\alpha_0^{1/3}\alpha_0^{1/9}....\alpha_0^{1/(3\wedge(n-1))}\right) = \boldsymbol{\alpha_0^{1/(3\wedge n)}/\alpha_0}$

vector addition that corresponds to a Wigner rotation in 3D for the non-relativistic limit [8]. By adding again 3 orthogonal vectors of the resulting vector sum (i.e. of length $3^{0.5}$ of the original vector) one may construct an infinite series of connected states.

The associated angle of arccos($3^{-0.5}$) is the same as that between total spin J and its z-component for $J_z = 1/2$, indicating that alignment of magnetic moment / spin of sub-units of particle states may play a role.

2.7 Non-spherical symmetric states

Assuming the angular part to be related to spherical harmonics and exhibiting the corresponding nodes would give the analog of an atomic p-state for the 1st angular state, y_1^0. With the additional assumption that $W_{n,l} \sim 1/r_{n,l} \sim 1/V_{n,l}^{1/3} \sim (2l+1)^{1/3}$ ($V_{n,l}$ = volume) is applicable for non-spherically symmetric states [15], this would give $W_1^0/W_0^0 = 3^{1/3} = 1.44$ and $\sigma^{1/3} = 3^{-1/3}\,\sigma_0^{1/3}$. The considerations of chpt. 3.1 support that a y_1^0-like symmetry for particle states has to be considered and a second partial product series of energies in addition to (22) corresponding to these values approximately fits the data, see tab. 1.

A change in angular momentum has to be expected for a transition from spherical symmetric states, y_0^0, to y_1^0 which is actually observed with $\Delta J = \pm 1$ except for the pair μ/π with $\Delta J = 1/2$.

With $\sigma^{1/3}/2 = \{4\pi\Gamma_{-1/3}{}^3/3\ (y_0^0);\ \ 3^{-1/3}4\pi\Gamma_{-1/3}{}^3/3\ (y_1^0);\ \Gamma_{-1/3}/3\ (\max)\}$ energy relative to the electron state may be given as:

$$W_n/W_e \approx 3/2\frac{\alpha\wedge\left(1.5/3^n\right)}{\alpha^{1.5}}\frac{\sigma_0^{1/3}}{\sigma^{1/3}} = 3/2\,\Pi_{k=0}^{n}\alpha\wedge(-3/3^k)\ \frac{\sigma_0^{1/3}}{\sigma^{1/3}} \qquad n = \{1;2;..\} \tag{25}$$

According to the variable part in σ, (21), the maximum additional contribution to W_{max} with respect to a spherical symmetric state would be:

$$\Delta W_{max} \approx 3/2\ \alpha^{-1} \tag{26}$$

With (22) and (26) the total maximum energy will be $W_{max} \approx W_e\ 9/4\ \alpha^{-2.5}$ = 4.05E-8 [J] (= 1.03 Higgs vacuum expectation value, VEV = 246GeV = 3.941E-8 [J] [7]) [16].

2.8 Results of energy calculation

Table 1 presents the results of the energy calculation according to (13), (24) for y_0^0 (bold), y_1^0. Only states given in [7] as 4-star, characterized as *„Existence certain, properties at least fairly well explored"*, are included, up to Σ'^0 all states given in [7] are listed. Coefficients given in col. 4 refer to (22), (24), starting with the electron coefficient in W_e, including its extra term of 2/3. Exponents of -9/2 for Δ and tau are equal to the limit of the partial product of α(n), including the electron coefficient. The term [$3/2\alpha^{-1}$] represents (26).

In col. 5 equ. (13) and (24) are used to calculate energy with σ_0 according to the value of the *fit for $J_Z = 1/2$ and α_{Pl} given by W_e/W_{Pl} according to the experimental value* of the electron and definition (40) for Planck energy.

$$W_n = 2b_0\int_0^{r_n}\exp\left(-\left(1.5^{3\delta}\sigma_0\alpha_{Pl}\frac{\boldsymbol{\alpha}^{\mathbf{4.5}}}{\boldsymbol{\alpha}^{\left(\mathbf{4.5/3^n}\right)}}\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right)\right)r^{-2}\,dr \quad => \quad W_\mu = \frac{2}{3}\frac{\Gamma_{+1/3}\boldsymbol{\alpha}^{\mathbf{-1}}}{\left(\sigma_0\alpha_{Pl}\right)^{1/3}}\ e_c \tag{27}$$

(n = {0;1;2;..}; 1.5^δ = extra coefficient for the electron only, δ = δ(0,n); bold: particle coefficient; muon given as example[17]). In col. 6 an alternate version for calculating σ_0 according to (62)f of [A3.3] is given for comparison. Additional particle states and blanks in the table are discussed in [A6]. The values of physical constants are taken from [7].

To illustrate possible QED-effects and the non-linearity of the Γ-functions, a calculation of σ_0 with values of (16)f varying within +/-1.00116 gives a range of energy values of +/-1.006, varying within +/-1.00116^2 gives a range of energy values of +/-1.013 compared to the values given in table 1. Additional effects due to e.g. different charge in particle pairs of same isospin have to be expected.

The accuracy of ~1% of the values calculated for leptons, mesons and baryons is comparable to that of LQCD calculations for baryons [9].

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[15] Interpreting the 3rd power relationship of chpt. 2.6 as one between dimensionless coefficients attributable to volume / length;

[16] For the Higgs boson see [A6.3].

[17] The term for the muon is given as reference to avoid ambiguities due to extra term ≈ 3/2 of the electron.

	n, l	$W_{n,Lit}$ [MeV]	α-coefficient in W_n $\alpha(n)^{-1/3}$ [f(l)]	W_{calc}/W_{lit} Equ.(27)	W_{calc}/W_{Lit} Equ.(63)	J
ν	**-1***	**'~ E-7**	**0**	-		-
e^{+-}	**0, 0**	**0.51**	$2/3\ \alpha^{-3}$	**1.014**	**1.002**	**1/2**
μ^{+-}	**1, 0**	**105.66**	$\alpha^{-3}\alpha^{-1}$	**0.000**	**0.996**	**1/2**
π^{+-}	1, 1	139.57	$\alpha^{-3}\alpha^{-1}$ $[3^{1/3}]$	1.101	1.088	0
K		495	[A6]			0
η^0	**2, 0**	**547.86**	$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}$	**1.002**	**0.990**	**0**
ρ^0	2, 1	775.26	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})$ $[3^{1/3}]$	1.022	1.009	1
ω^0	2, 1	782.65	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})$ $[3^{1/3}]$	1.012	1.000	1
K*		894	[A6]			1
p^{+-}	**3, 0**	**938.27**	$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$	**1.011**	**0.999**	**1/2**
n	**3, 0**	**939.57**	$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$	**1.010**	**0.998**	**1/2**
η'		958	[A6]			0
Φ^0		1019	[A6]			1
Λ^0	**4, 0**	**1115.68**	$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}$	**1.020**	**1.008**	**1/2**
Σ^0	**5, 0**	**1192.62**	$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}$	**1.014**	**1.002**	**1/2**
Δ	**∞, 0**	**1232.00**	$\alpha^{-9/2}$	**1.012**	**1.000**	**3/2**
Ξ		1318				1/2
Σ^{*0}	3, 1	1383.70	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9})$ $[3^{1/3}]$	0.989	0.977	3/2
Ω^-	4, 1	1672.45	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27})$ $[3^{1/3}]$	0.982	0.970	3/2
N(1720)	5, 1	1720.00	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81})$ $[3^{1/3}]$	1.014	1.002	3/2
tau^{+-}	∞, 1	1776.82	$(\alpha^{-9/2})$ $[3^{1/3}]$	1.012	1.000	1/2
Higgs	∞,∞ **	1.25 E+5	$(\alpha^{-9/2})$ $[3/2\ \alpha^{-1}]$ /2	1.042	1.066	0
VEV	∞,∞**	2.46 E+5	$(\alpha^{-9/2})$ $[3/2\ \alpha^{-1}]$	1.059	1.083	

Table 1: Particle energies; col.2: radial, angular quantum number; col.4: α-coefficient in W_n according to (27), n = {0;1;2;..}; col.5,6: ratio of calculated energy, W_{calc} according to (27), (63) and literature value [7] (* see (19), ** chpt. 2.7, [A6.3]); col.7: angular momentum J_z [ħ];

2.9 Photon energy

In the following a term for length expressed via the Euler integral of (11) will be introduced for $\lambda_{C,n}$:

$$r_x = \int_0^{r_x} e^{-\left(\frac{\rho}{r}\right)^3} dr \quad = \quad \rho_n/3 \int_{(\rho_n/r_x)^3}^{\infty} t^{-4/3} e^{-t}\, dt \quad \approx \Gamma(-1/3, (\rho_n/r_x)^3)\ \rho_n/3 \tag{28}$$

In the limit $(\rho_x/r_x)^N$ -> 0

$$\Gamma(-1/N, (\rho_x/r_x)^N) = \int_{(\rho_x/r_x)^N}^{\infty} t^{-(1/N+1)} e^{-t}\, dt \quad \approx N\,(\rho_x/r_x)^{-1} = N\,\sigma^{1/3}/2 \tag{29}$$

holds. Equation (29) inserted in the right side of (28) gives back r_x, however, (28)f may be seen as expressing r_x in terms useful for this model, i.e. ρ_n, σ_0 and Γ-functions. Using equ. (29) for the incomplete Γ-function and multiplying r_x in the integration limit $(\rho_n/r_x)^3$ by √3, the ratio of total angular momentum and its z-component (see [A4, (64)]), gives in good approximation (using (18)):

$$\lambda_{C,n} \approx 3^{1.5}\,\sigma_0^{1/3}/2\ \rho_n/3 \approx 3^{0.5}\,4\pi\,\Gamma_{-1/3}^{3}/3\,\rho_n \tag{30}$$

With (30) energy of a photon may be expressed as:

$$W_{Phot,n} = hc_0/\lambda_{C,n} \quad = \quad hc_0 \Big/ \int^{\lambda_{C,n}} e^{-\left(\frac{\rho}{r}\right)^3} dr \quad = \quad \frac{2\,hc_0}{3^{0.5}\rho_n\sigma_0^{1/3}} \quad \approx \quad \frac{3\,hc_0}{3^{0.5}\,4\,\pi\,\Gamma_{-1/3}^{3}\rho_n} \tag{31}$$

2.10 Fine-structure constant, α

The energy of a particle is assumed to be the same in both photon and point charge description. Equating (13) with (31) gives:

$$W_{pc,n} = W_{Phot,n} = 2b_0\,\Gamma_{+1/3}\,\rho_n^{-1}/3 \approx \frac{2hc_0}{3^{0.5}\rho_n\sigma_0^{1/3}} \approx \frac{3hc_0}{3^{0.5}4\pi\Gamma^3_{-1/3}\rho_n} \qquad (32)$$

Solving equ. (32) for α involves a term of two Γ-functions with an argument of same value and opposite sign for which the relation Γ(+x)Γ (-x) = π /(x sin(πx) holds [10], giving for the Γ-functions of (13) and (31):

$$\Gamma_{+1/3}\Gamma_{-1/3} = 3^{0.5}2\pi \qquad (33)$$

Using equation (32) with (33) will give (note: h => ħ):

$$\alpha^{-1} = \frac{hc_0}{2\pi b_0} \approx \left(\frac{2\Gamma_{+1/3}}{3^{0.5}2\pi}\right)\left(\frac{4\pi}{3}\Gamma^3_{-1/3}\right) \approx \frac{2}{3}\frac{\Gamma_{-1/3}}{\Gamma_{+1/3}}4\pi\Gamma_{+1/3}\Gamma_{-1/3} \approx \boldsymbol{4\pi\Gamma_{+1/3}\Gamma_{-1/3}} \qquad (34)$$

The last expression is emphasized since it has a simple interpretation in terms of the coefficients of the integrals over exp(-(ρ/r)^N). Equations (32)ff are based on the integral over a 3-dimensional point charge term modified by the exponential term according to (4) with N = 3, and a complementary integral - in 3D for length, λ_C - to yield a dimensionless constant. This may be generalized to N dimensions (N ={3; 4}), to give a point charge term (S_N = geometric factor for N-dimensional surface, in case of 3D: 4π; 4D: 2π^2):

$$\int_0^r \exp(-(\rho/r)^N)r^{-2(N-1)}\,d^N r = S_N\int_0^r \exp(-(\rho/r)^N)\,r^{-(N-1)}dr \qquad (35)$$

that has to be multiplied by a complementary integral

$$\int_0^r \exp(-(\rho_n/r)^N)\,r^{(N-3)}dr \qquad (36)$$

The exact result depends on the integration limit of the second integral, cf. [A4]. However, in terms of the Γ-functions both electroweak coupling constants can be given in 1st approximation as

$$\alpha_N^{-1} = S_N\,\frac{\Gamma(+m/N)\Gamma(-m/N)}{m^2} = S_N\,\frac{\Gamma(+(N-2)/N)\Gamma(-(N-2)/N)}{(N-2)^2} \qquad \text{(m = N-2, cf. (11))} \qquad (37)$$

Table 2 shows the calculated results.

Dimension – space	coupling constant	Value of *inverse* of coupling constant, α_N^{-1}	
4D	$\alpha_4 = \alpha_{weak}$	$2\pi^2\,\Gamma_{+1/2}\,\Gamma_{-1/2}/4 = \pi^3 =$	31.0
3D	$\alpha_3 = \alpha$	$4\pi\,\Gamma_{+1/3}\Gamma_{-1/3} = 4\pi\,\Gamma_{+1/3}\Gamma_{-1/3} =$	136.8

Table 2: Calculation of electroweak coupling constants [18]

The ratio of α and α_{weak} represents the weak mixing angle, θ_W, and may be expressed as:

$$\sin^2\theta_W = \frac{\alpha}{\alpha_{weak}} = \frac{\pi^3}{4\pi\Gamma_{+1/3}\Gamma_{-1/3}} = 0.227 \qquad (38)$$

(Experimental values: PDG [7]: $\sin^2\theta_W = 0.231$, CODATA [11]: $\sin^2\theta_W = 0.222$). The mass ratio of the W- and Z-bosons will be given by $\cos\theta_{W,calc} = (m_W/m_Z)_{calc} = 0.879 = 0.998(m_W/m_Z)_{exp}$ [7].

3 Quaternion ansatz

3.1 Basic approach

The model as described above emphasizes a Kaluza-like ansatz with spin as boundary condition. Reversing the main focus, emphasizing angular momentum and implicitly assuming curvature of space as necessary boundary condition for localization is a straight forward alternate way to get additional information about the states of this model [9], details are given in [A5].

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[18] Values of coupling constants refer to a rest frame, α_{weak} is defined by the weak charge, g, not via lifetime.

A circular polarized photon with its intrinsic angular momentum interpreted as having its E- and B-vectors rotating around a central axis of propagation, C, will be transformed into an object of SO(3)-type symmetry where the center of rotation is the origin of a triple of EBC-vectors, supposed to be locally orthogonal [19]. This has the following qualitative consequences:

1) Such a rotation is related to the group *SO(3)* and *SU(2)* as important special case. In the following a quaternion ansatz will be used for modeling the respective rotations.
2) E-vector constantly oriented to a fixed point implies *charge*. As implicitly assumed above, neutral particles are supposed to exhibit nodes separating corresponding equal volume elements of reversed E-vector orientation and opposite polarity.
3) A local coordinate system = rest frame implies *mass*.
4) In case of any lateral extension of the E-field, for r -> 0 the overlap of a rotating E-vector implies rising energy density, resulting in *rising curvature of space-time* according to GR or its modification as of equ. (7).
5) The EBC-triple can be given in 2 different *chiral* states (left- right-handed).
6) As essentially electromagnetic waves such states are consistent with a “point-like” structure function on the other hand imply a spatial distribution of energy density and angular momentum / spin.
7) Antiparticles may be constructed by reversing orientation of the fields.

For quantitative results 3 orthonormal vectors E, B, C, each described as imaginary part of a quaternion with real part 0, will be subject to alternate, incremental rotations around the axes E, B and C. In the following only solutions where one of the incremental angles of rotation has half the value of the other two will be considered. This may serve as a primitive model for spin $J_Z = 1/2$. There are 3 possible solutions corresponding to half the angular velocity for each of the components E, B, or C. The trajectory of the E-vector encloses a spherical cone, the spherical cap of the cone encompasses a fraction of the area of a hemisphere of 2/3, 1/3 and 1/3, respectively. Mirroring at the center of rotation gives the equivalent double cone (dark grey in fig.1), the fractions of both caps in relation to the surface of the total sphere may be interpreted to give partial charges of 2/3, 1/3 and 1/3 according to Gauss’ law. In the following such components will be assigned to uds-quark-like entities, the assignment (half-frequency-E-rotation, charge +2/3, U), (half-B, charge -1/3, D), (half-C, charge -1/3, S) will be used.

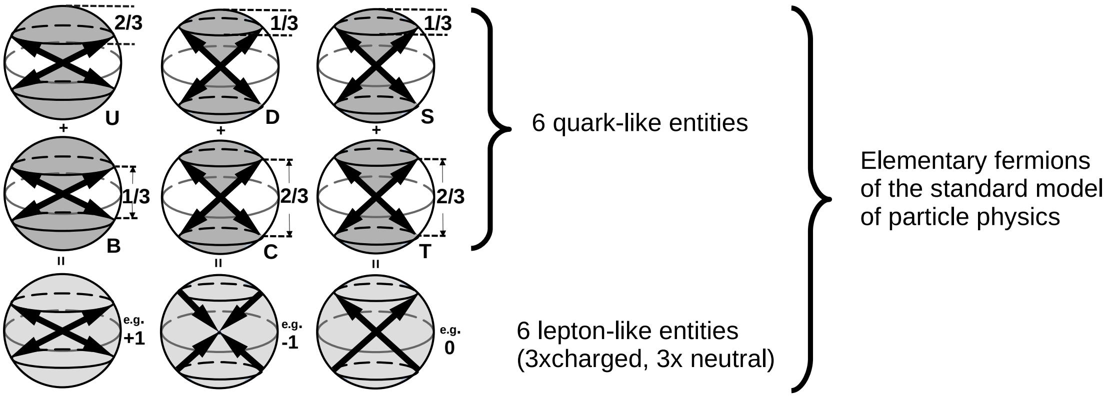

Fig.1: Trajectories of the E-vector, enclosing spherical cones and spherical wedges

The E-vector might as well be interpreted to enclose the complement of the double cone of a 3D-ball (white in fig.1), to be called a spherical wedge in the following. This gives the objects complement-U, complement-D, complement-S with charges 1/3, 2/3, 2/3. These objects may be assigned to cbt-quark-like entities.
A combination of two cones to give a double cone will always give a valid solution with any spin or chirality and may be considered to correspond to the y_1^0 solutions of chpt. 2.7 [20].
The simplest combination of 2 entities (grey + white) of fig.1 will consist of 2 complementary segments of same charge etc., to recover a simple sphere with no nodal planes (last row of fig. 1). Such particles should represent the lowest possible energy state, $J_Z = 1/2$ should still be valid and charge could have values of +/-1 or 0. An electron might be considered e.g. as an (anti-U + (U-Complement = B)) particle, however, unlike a

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[19] In the limit r -> λ_C => |C| -> c_0;

[20] Composite objects - in particular if composed of 3 UDS-components – may feature sufficient spherical symmetry to conform to the respective energy equation (27). The spherical symmetry of nucleons as assumed in chpt. 2 may be given by a suitable linear combinations of the states discussed in [A5], cf. [A6.2, η].

B-meson with spin 1/2. While this is not possible with quarks, i.e. objects with particle character, it is the simplest solution for such a type of an electromagnetic wave.

The neutral configuration will have to be distinct from all other particles by representing a state where the center of rotation is not at the "tip" of an E-vector, but at its "center", see last row right in figure 1. This will be an "intrinsically" neutral particle unlike particles consisting of components of opposite charge, such as the neutron and a unique solution that for spatial reasons is not suited as component to build other particles. It will not be subject to the conditions of 2.6.1, 2.6.2 and α_{Pl}, which are related to "charge".

3.2 Magnetic moments of baryons

There is a crucial test for the applicability of such a quaternion ansatz: calculation of magnetic moments of uds-baryons. Though it is possible to give values for all combinations of the uds-octet of spin 1/2 that match the experiment within a few percent they have to be selected from a large set of solutions. Unique solutions require additional boundary conditions. For nucleons this will be isospin. Exchanging U- and D-components results in switching the values for magnetic moment of p and n.

In the quaternion model both E- and B-fields are oriented to the center (magnetic monopole character on particle level) and will feature average fields of 1/3 and 2/3 for quark-like objects. The B-field for u- and d-entities will have Cartesian components of ± 2/9, ± 2/9, ± 1/9 (d) and ± 4/9, ± 4/9, ± 2/9 (u). Permutations of these values give a large set of solutions, isospin will be used to select the nucleons. Unique solutions (except for arbitrary orientation in space) for B-field components of nucleons will be e.g. ($B_{avg}=((\sum x_i)^2+(\sum y_i)^2+(\sum z_i)^2)^{0.5}/3$):

proton - uud -4/9, -4/9, -2/9 / -2/9, -4/9, -4/9 / +2/9, -2/9, +1/9 $B_{avg} = 141^{0.5}/27 \approx 0.440$

neutron - ddu -2/9, -2/9, -1/9 / -1/9, -2/9, -2/9 / +4/9, -4/9, +2/9 $B_{avg} = 66^{0.5}/27 \approx 0.301$

To get absolute values one has to multiply by $ec_0\lambda_C/2 = 2\pi\mu_B$ (λ_C = Compton wavelength, μ_B = Bohr magneton), see tab. 3 [22]. The ratio of both values is $(141/66)^{0.5} = 1.461631$, which compared to the ratio from experiments [7] gives 1.461631 / 1.459898 = 1.001187.

		λ_C	$e\ c_0 * \lambda_C /2$	B_Avg	$ec_0\lambda_C\ B_{avg}/2$	$\lvert M\rvert Exp[Am^2]$	$\lvert M\rvert Calc/ \lvert M\rvert Exp$
p^{+-}	UUD	1.32E-15	3.17E-26	0.440	1.39E-26	1.41E-26	0.988
n	DDU	1.32E-15	3.17E-26	0.301	9.55E-27	9.66E-27	0.988

Table 3: Magnetic moments for proton and neutron; For greater accuracy values of λ_C according to [7] are used;

These solutions are distinguished by one U and one D-component being collinear [23], indicating a particular stable configuration involving oppositely charged components (see [A7]).

Table 4 compares some ratios of baryon isospin pairs for calculations with the average of the B-field as calculated above, i.e. geometry only, and calculations of the actual moment, using the experimental value of the Compton wavelength. A simple analysis for particles with S-components is not possible due to differences in symmetry (cf. tab. 5 in [A5]).

	U,D,S-components	$\lvert M\rvert Calc\ (\lambda_C\ exp)$	B_avg
M(p/n)_Calc/M(p/n)_Exp	UUD/DDU	0.999809	1.001187
$M(\Sigma^+/\Sigma^-)$_Calc/$M(\Sigma^+/\Sigma^-)$_Exp	UUS/DDS	1.007813	1.001111
$M(\Xi^0/\Xi^-)$_Calc/$M(\Xi^0/\Xi^-)$_Exp	USS/DSS	0.974652	0.969601

Table 4: Ratio of particle magnetic moments of baryon isospin pairs compared for calculated and experimental values [7] (col.3: geometry only, B_avg; col.2 inc. exp. particle energy);

3.3 Chirality / Color

The orthonormal EBC-vectors feature two possible chiral configurations, right-handed "R" and left-handed "L", suggesting to be a possible source for a factor 3 frequently appearing in the quantitative interpretation of processes involving a quark-antiquark-pair, such as the decay, e.g. of the W- or Z-boson, or in the coefficient R of electron-positron-annihilation. While this is attributed to the 3 "colors" of quarks in the SM, the same factor would result for any pair of quark-like states having the possibility to exist in triplet-like states, "LL", "RR" and $(1/\sqrt{2})$ (LR+RL) [24] (referring to an axial vector representing the EBC-configuration).

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[22] Note: to allow for comparison with tabulated values of M in units of [Am^2] the calculations in this chapter and in [A5.2] use e [C] not e_c [J], conversion factor: [m^2C/s] /[m^2 J/s] = e/e_c = 1/19.4 [C/J].

[23] Time average! All E,B-components involved are orthogonal at any given point in time.

[24] With a singlet state corresponding to destructive interference; alternatively: 3 simple combinations RR, LL, RL;

4 Gravitation

Simplifying Kaluzas ansatz by considering only the leading terms in the metric would give the standard 4D-metric in cases where EM-effects are negligible, ie. on the large scales of astrophysics and cosmology.

$$g_{AB} = \begin{bmatrix} -\delta^2 \Phi^2 A_\alpha A_\beta & -1 \\ -1 & -1 \end{bmatrix} \;<\; g_{AB} = \begin{bmatrix} \left(g_{\alpha\beta} - \delta^2 \Phi^2 A_\alpha A_\beta\right) & -\delta \Phi^2 A_\alpha \\ -\delta \Phi^2 A_\beta & -\Phi^2 \end{bmatrix} \;>\; g_{AB} = \begin{bmatrix} g_{\alpha\beta} & -1 \\ -1 & -1 \end{bmatrix} \tag{39}$$

Some of the minor terms set to 1 in (39), in particular $g_{55} = \Phi^2$, may still play some role for phenomena that are not fully understood, such as dark energy or dark matter.

Apart from that, a model for elementary mass, based on the concepts of GR and having the Planck-term as one of its central parameters, might well be expected to provide a direct link to the interaction of masses.

4.1 Planck scale

In this work the expression

$$b_0 = G\, m_{Pl}^2 = G\, W_{Pl}^2 / c_0^4 \tag{40}$$

is used as definition for Planck terms, giving for the Planck energy, W_{Pl} :

$$W_{Pl} = c_0^2 \,(b_0 / G)^{0.5} = c_0^2 \,(\alpha\hbar c_0/G)^{0.5} = 1.671 \text{ E+8 [J]} \tag{41}$$

The value of W_{Pl} according to definition (41) allows to identify the ratio of W_e and W_{Pl} with the α-terms given in (23), i.e. the relation between W_e and W_{Pl} is given by $\alpha_e \approx (3/2)^3 \alpha^9$, the electron coefficient in the exponential part of Φ, divided by two times the limit factor, α_{lim}, according to (21) [25]. The constant G may be given as:

$$G \approx \frac{\alpha_{Pl}^2 c_0^4 b_0}{W_e^2} \tag{42}$$

Since α_{Pl} and W_e may be expressed as function of π, $\Gamma_{+1/3}$, $\Gamma_{-1/3}$ and e_c only, (23), (34) and (27) / (63), G may be expressed as a coefficient based on EM constants only, $G \approx 2/3\; c_0^4 \alpha^{24}/(4\pi\varepsilon_c) \approx 2/3\; c_0^4\, (4\pi\Gamma_{+1/3}\Gamma_{-1/3})^{24}/(4\pi\varepsilon_c)$.

4.2 Gravitation from series expansion of exponential function

Terms for gravitation may be recovered via a series expansion of either $\Gamma(+1/3, (\rho_n/r_n)^3)$ of (12) [12] or the exponential part of Φ in any suitable expression, e.g. potential ρ_0/r, resulting in a general term such as:

$$\frac{\rho_0}{r}\left[1 - \sigma\alpha_{Pl}\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right] \approx \text{Coulomb-term}\left[1 - \sigma\alpha_{Pl}\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right] \tag{43}$$

which is a very good approximation for $r > \alpha\lambda_C$. The 1st term is the classical Coulomb term, the 2nd term contains by definition the ratio between Coulomb and gravitational terms for *one* electron, α_{Pl}. To turn this into the exact Coulomb / gravitation relationship requires

1) coefficient σ to approach unity, which may be approximately justified by considering the limit of chpt. 2.5 or [A3.2],
2) parameter r in $e_c /(4\pi\varepsilon_c r)$ to turn into a constant,
3) parameter r to approach the value $e_c/(4\pi\varepsilon_c)$.

For condition 2) one has to consider that r *in the exponential* may not be considered to be a free parameter for $r > \lambda_C$, the limit of a real solution for an equation such as (56). Inserting the Compton wavelength of the electron in (43) would give a value in the parentheses two orders of magnitude off to yield the expected value for the electrostatic / gravitation ratio. Since σ is essentially related to spin of a particle and it has to be assumed that spin does not play a role for $r > \lambda_C$, one might omit the related coefficient in (43) as well as in the term for λ_C [26] and thus by definition of α_{Pl} recover the exact gravitational term.

The general expression for the series expansion would be:

Coulomb-term $(1 - \alpha_{Pl})$.

Coefficient α_{Pl} would be subject to the considerations of chpt. 2.6.2. Particle interaction would be given by

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[25] A factor 2 might correspond to relate only the electrostatic contributions of (13) for the electron with the electrostatically defined value of a Planck state.

[26] I.e. condition 2.6.1 would not have to apply with respect to wavelength anymore, while the more general condition 2.6.2, 1st term ≥ 2nd term, would still require α_{Pl} in the series expansion;

the square of the α_{Pl} term multiplied by appropriate coefficients from the α-series according to (22) for particles of spherical symmetry in a rest frame.

The approach using assumptions 1) - 3), is supported by the considerations of chpt. 4.3, yielding a term for the cosmological constant in the correct order of magnitude.

4.3 Cosmological constant Λ

A metric of the generic type described in [A2] will in general produce minor terms that might be considered as a natural candidate for the cosmological constant term, $g_{\alpha\beta}\Lambda$. In particular terms such as $\rho_n^3/\mathbf{r}^5$ or $\rho_n^6/\mathbf{r}^8$ with all r originating from derivatives of the exponential only [27] will yield approximate values in the order of magnitude of critical, vacuum density, ρ_c, ρ_{vac} if setting $\mathbf{r} = e_c/(4\pi\varepsilon_c)$ as upper bound of r, as suggested in 4.2:

$$\frac{\Phi''}{\Phi} \approx \frac{\rho^3}{\mathbf{r}^5} \approx \frac{\alpha_{Pl}}{\left(e_c/(4\pi\varepsilon_c)\right)^5}\left(\frac{e_c}{4\pi\varepsilon_c}\right)^3 = \alpha_{Pl}\left(\frac{4\pi\varepsilon_c}{e_c}\right)^2 = 0.089\ [\mathrm{m}^{-2}] \tag{44}$$

Multiplied by ε_c this gives an energy density of 2.97E-10 [J/m^3].

Multiplied by the conversion factor for the electromagnetic and gravitational equations, equ. (6), $8\pi\varepsilon_c G/c_0^4$, equ. (44) gives as estimate for the cosmological constant, Λ:

$$\alpha_{Pl}\frac{(4\pi)^2\varepsilon_c^3}{e_c^2}\frac{8\pi G}{c_0^4} \approx \quad 6.17\text{E-}53\ [\mathrm{m}^{-2}] \quad ^{28} \tag{45}$$

5 Discussion

Theory of everything is a somewhat ironic and pompous term and maybe an unachievable goal. At the time Theodor Kaluza's unification of general relativity and electromagnetism was conceived, it came pretty close, yet the emerging theory of quantum mechanics (QM) moved the finish line. It is a common thought ever since that the theory of GR somehow has to be unified with QM. The model presented here suggests that the ansatz of Kaluza is sufficient to give an excellent model for particles, in particular in combination with the boundary condition spin 1/2, bypassing QM in 1st approximation [29]. The major deviation from conventional GR is dropping the constant of gravitation in the field equations, a minor thing from a mathematical point of view. The resulting objects of interest are waves only, which naturally fits basic concepts of QM. General features of quantum mechanics that emerge from such an ansatz include quantization of energy or the pivotal constant of quantum mechanics, Planck's constant, h, that may be derived from the electromagnetic constants and geometry as expressed in the derivation of α.

The results of the quaternion ansatz of chpt. 3 reproduce the set of elementary fermions of the standard model of particle physics (SM). The number of 6 basic building blocks of matter can be traced back to the 3 possibilities to single out one of the orthogonal EBC-vectors and in a broad sense is a consequence of the 3 space dimensions in 4D space-time. While the properties of quarks, such as partial charges, are deduced from experimental particle data, in particular symmetry, they can be *derived* in the quaternion ansatz. Leptons are an integral part of the particle classification scheme.

There are several features of the model that indicate a close relationship with electroweak theory. In addition to the obvious common root in EM there are: SO(3)/SU(2) symmetry, the energy of the Higgs boson /VEV as upper limit for particle energy [30] and the possibility to calculate the IR-limits of the electroweak coupling constants. As for chirality the inherent chiral character of a circular polarized EM-wave is transferred via the orthogonal EBC-triple of the quaternion model to particles.

On the other hand, though the ansatz of this model yields basic features that match those of valence quarks, there seems to be no deeper connection with the concepts of QCD, such as color [31] or gluons. Properties such as confinement or the need for adhering to the Pauli principle in e.g. the Δ^{++} are obsolete for an object that is a (5D-) electromagnetic wave. The development of the SM from constituent quarks towards QCD, based on

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[27] Such as $\rho^3/\mathbf{r}^5$ in [A2.1] though this term cancels in the specific example for G_{00}; The 1st term of (55), representing field-free space, i.e. vacuum, might be a suitable starting point as well.

[28] $\Lambda \approx 1.11\text{E-}52\ [\mathrm{m}^{-2}]$ with Hubble constant $H_0 = 67.66$ [km/s/Mpc] [13]

[29] QED terms are considered to be a necessary correction for the results of this model.

[30] Some speculative energy relation for electroweak bosons is given in [A6.3].

[31] Whose role in the production of quark-antiquark pairs is replaced by chiral pairs, see chpt. 3.3.

valence and sea quarks plus gluons, was in part required by the limitations in explaining some scattering experiments with 3 point-like objects only. The waves of this model are consistent with a point like structure function and still feature spatial extension.

Thus not all the details of the SM are reproduced by the particle model presented here. However, the relevant benchmark is the agreement with experiments anyway. Though there are several open questions left, the model provides a solid framework of very basic particle features using essentially no free parameters and a minimal set of assumptions. Several assumptions of Kaluzas ansatz are not needed for this modification of his work. Concerning the introduction of a 5th dimension, a flat 5D space-time is mathematically equivalent to curved 4D space-time due to Campbells theorem. Apart from consistency in the units used no particular assumption about the constant in the metric has to be made, i.e. the constant of gravitation does not have to be introduced ad hoc. Kaluzas "cylinder condition" [32] is obsolete. The major specific assumptions actually needed are:

1.) spin 1/2, used for calculation of an integration limit and in the quaternion model of chpt. 3,

2.) assumptions concerning the ground state term, α_0, α_{Pl} - in the simplest case α_{Pl} may be introduced as W_e/W_{Pl}, a necessary limit in a series expansion.

Last but not least, a description of particles based on the laws of GR and EM, which have to be an indispensable part of any approach for a TOE, is a step towards a theory that is as concise as possible.

Conclusion

A formalism based on 5D-differential geometry and electromagnetic concepts, with spin 1/2 as boundary condition, provides a simple, coherent, comprehensive and first of all quantitative description of phenomena related to particles, such as

- an energy range defined by the limits of ~ 0,1 eV, the expected range for neutrino energy, and the Higgs VEV energy, including a convergent series of particle energies quantized as a function of the fine-structure constant, α, equ. (25),
- a single expression for the values of electroweak coupling constants, equ. (37),
- 3D-space and spin 1/2 define a set of 6 lepton-like and 6 quark-like objects with the associated charges,
- basic properties of the nucleons, including their magnetic moments.

A series expansion links electromagnetic and gravitational terms with a cosmological constant in the correct order of magnitude.

The model works *ab initio* without free parameter and allows to remove some values from the set of fundamental constants:

electromagnetic constants, h, G, α, α_{weak}, energies of elementary particles => electromagnetic constants.

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Appendix

In the following the exponential part of Φ^2 is abbreviated as e^v.

[A1] Scalar potential Φ

Equ. (4) may in general be interpreted to refer to the highest order terms of exponential N in Φ":

$$\Phi_N'' \sim \left(\frac{\rho^{3N-1}}{r^{3N+1}}\right) e^{v/2} \sim \boldsymbol{\Phi_N^3}\ e^{-v}\left(\boldsymbol{A_{el}}'\right)^{\boldsymbol{2}} \approx \left[\left(\frac{\boldsymbol{\rho}}{\boldsymbol{r}}\right)^{\boldsymbol{N-1}} \boldsymbol{e^{v/2}}\right]^{\boldsymbol{3}} e^{-v}\left(\frac{\boldsymbol{\rho}}{\boldsymbol{r^2}}\right)^{\boldsymbol{2}} \ = \ \left(\frac{\rho}{r}\right)^{3N-3} e^{v/2} \left(\frac{\rho}{r^2}\right)^2 \tag{46}$$

The solutions for the scalar Φ depend on the complete metric used. The easiest method to get a solution of order N is to use spherical coordinates of dimension N+1. Using e.g. the line element for a 4D metric of [4, equ. 6.76]

$$ds^2 = e^v dt^2 - e^\lambda dr^2 - e^\mu r^2 \left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right) \tag{47}$$

and $A_\alpha = (A_{el}, 0, 0, 0)$ gives as solution for equ.(3) (cf. [4, equ. 6.77], prime corresponds to derivatives with respect to r):

$$\Phi'' + \left(\frac{v' - \lambda' + 2\mu'}{2} + \frac{2}{r}\right)\Phi' - \frac{1}{2}\Phi^3 e^{-v}\left(A_{el}'\right)^2 = 0 \tag{48}$$

This can be solved with a function of type (4) for N = 2:

$$\Phi_2' = \left[-\left(\frac{\rho}{r^2}\right) + 2\left(\frac{\rho^3}{r^4}\right)\right] e^v \tag{49}$$

and

$$\Phi_2'' = \left[2\left(\frac{\rho}{r^3}\right) - 10\left(\frac{\rho^3}{r^5}\right) + 4\left(\frac{\rho^5}{r^7}\right)\right] e^v \tag{50}$$

The ρ^1 terms cancel in (48), the ρ^3 terms can be eliminated by appropriate choice of v', λ' and μ', a remaining factor in the ρ^5 term could be compensated by assuming a corresponding factor in A_{el}. For N = 3 hyperspherical coordinates with the line element

$$ds^2 = e^v dt^2 - e^\lambda dr^2 - e^\mu r^2 \left(d\psi^2 + \sin^2\psi \left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)\right) \tag{51}$$

may be used. A more complex metric of the kind given in [A2] may be used as well to solve equation (48).

[A2.1] Metric / point charge

Equ. (9)f. would be the result of the following metric:

$$\begin{aligned} g_{\mu\mu} &= \left(\frac{\rho_0}{r}\right)^2 \exp\left(-\left(\frac{\rho}{r}\right)^3\right) A_{el}^2,\ -\left(\frac{\rho_0}{r}\right)^2 \exp\left(\left(\frac{\rho}{r}\right)^3\right) A_{el}^2,\ -r^2 A_{el}^2,\ -r^2 \sin^2\vartheta\, A_{el}^2 = \\ g_{\mu\mu} &= \left(\frac{\rho_0}{r}\right)^4 \exp\left(-\left(\frac{\rho}{r}\right)^3\right),\ -\left(\frac{\rho_0}{r}\right)^4 \exp\left(\left(\frac{\rho}{r}\right)^3\right),\ -\rho_0^2,\ -\rho_0^2 \sin^2\vartheta \end{aligned} \tag{52}$$

The following gives an alternate metric in some detail to illustrate the significance and order of magnitude of the relevant terms [33]:

$$g_{\alpha\alpha} = \left(\frac{\rho_0}{r}\right)^2 \exp\left(-\left(\frac{\rho}{r}\right)^3\right),\ -\left(\frac{\rho_0}{r}\right)^2 \exp\left(\left(\frac{\rho}{r}\right)^3\right),\ -r^2,\ -r^2 \sin^2\vartheta \tag{53}$$

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[33] The sign in the argument of the exponentials is chosen to enable Schwarzschild-like solutions in a series expansion.

The variable r is marked bold if originating from the exponential term to facilitate a discussion of the implications of its restricted range of validity.

$\Gamma_{01}^{0} = \Gamma_{10}^{0} = - 1/r^1 + 3/2\ \rho^3/\mathbf{r}^4 \qquad \Gamma_{00}^{1} = - 1/r^1 e^{-2v} + 3/2\ \rho^3/\mathbf{r}^4 e^{-2v}$

$\Gamma_{11}^{1} = - 1/r^1 - 3/2\ \rho^3/\mathbf{r}^4$

$\Gamma_{12}^{2} = \Gamma_{21}^{2} = \Gamma_{13}^{3} = \Gamma_{31}^{3} = + 1/r^1 \qquad \Gamma_{22}^{1} = - r^3/\rho_0^2\, e^{-v} = \Gamma_{33}^{1}/\sin^2\vartheta$

$\Gamma_{23}^{3} = \Gamma_{32}^{3} = \cot\vartheta \qquad \Gamma_{33}^{2} = - \sin\vartheta\cos\vartheta$

$R_{00} = e^{2v}\, [+1/r^2 + 6\,\rho^3/\mathbf{r}^5 - 9/2\ \rho^6/\mathbf{r}^8)]$

$R_{11} = +3/r^2 - 6\,\rho^3/\,\mathbf{r}^5 + 9/2\rho^6/\mathbf{r}^8$

$R_{22} = - 1 + e^{+v}\, [+ r^2/\rho_0^2 + 3\rho^3 r^3/(\rho_0^2\mathbf{r}^4)]$

$R = + 2/r^2 + e^v\, [(- 4/\rho_0^2 - 6\rho^3 r/(\rho_0^2\mathbf{r}^4) + 12\, a\, \rho^3\, r^2/(\rho_0^2\,\mathbf{r}^5) - 9\, a^2\, \rho^6\, r^2/(\rho_0^2\,\mathbf{r}^8)]$

$G_{00} = e^{2v}\, [+1/r^2 + 6\rho^3/\mathbf{r}^5 - 9/2\rho^6/\mathbf{r}^8)] - e^v \rho_0^2/r^4 + e^{2v}\, [2/r^2 + 3\rho^3/(r\mathbf{r}^4) - 6\, a\, \rho^3/\,\mathbf{r}^5 + 9/2\ \rho^6/(\rho_0^2\,\mathbf{r}^8)] =$

$- e^v \rho_0^2/r^4 + e^{2v}\, [3/r^2 + 3\rho^3/(r\mathbf{r}^4)]$

Volume integrals over any ρ^n/r^{n+2} terms will yield energy results $\varepsilon_c \int e^v\, \rho^n/r^{n+2}\, d^3r \approx \varepsilon_c\, \rho \approx$ 1E-22 [J] compared to the term $\varepsilon_c \int e^v\, \rho_0^2/r^4\, d^3r \approx \varepsilon_c\, \rho_0^2\, \rho^{-1} \approx$ 1E-13 [J] (both with coefficients for the electron, $\sigma_0\alpha_{Pl}$) giving negligible contributions to particle energy within the parameter range discussed here. This leaves the first term as leading order: $G_{00} = - e^v\, \rho_0^2/r^4$

[A2.2] General solution N = {1; 2; 3}

This article has a focus on a solution of (4) with N = 3. However, all solutions in a 5D space-time according to [A1], i.e. up to using hyperspherical coordinates, N = {1; 2; 3}, might be used for the ansatz of a metric such as

$$g_{00} = \sum_{N=1}^{3} \left(\frac{\rho_0}{r}\right)^{N-1} \exp\left(-\left(\frac{\rho}{r}\right)^{N}\right) \tag{54}$$

With the approximation σ ≈ 1 this gives for g_{00}:

$$g_{00} = \exp\left(-\alpha_{Pl}\left(\frac{\rho_0}{\mathbf{r}}\right)\right) + \left(\frac{\rho_0}{r}\right)\exp\left(-\alpha_{Pl}\left(\frac{\rho_0}{\mathbf{r}}\right)^2\right) + \left(\frac{\rho_0}{r}\right)^2\exp\left(-\alpha_{Pl}\left(\frac{\rho_0}{\mathbf{r}}\right)^3\right) \tag{55}$$

Each term might be expanded and split in EM and gravitational part as suggested in chpt. 4.2.
The 3rd term corresponds to the case discussed above, resulting in terms giving the square of the E-field in G_{00} and eventually particle energy as a kind of self energy as well as an equivalent term for gravitation from the series expansion. The second term is the linear version and might be used to construct a Schwarzschild-like solution for potential terms. The first term might represent a general vacuum solution, i.e. without presence of any field ρ_0/r. A series expansion would give the 1 for flat space-time, while the minor terms of G_{00} could give Λ-like orders of magnitude equivalent to the reasoning of chpt. 4.3.

[A3] Model coefficients

[A3.1] Coefficient σ as component in ρ

The exponential term, $\exp(-\rho^3/r^3)$, together with the r^{-2} dependence of the field of a point charge define a maximum of particle energy near $r_{W(max)} \approx \rho$, rapidly approaching 0 for $r_{W(max)} > \rho$, effectively allowing to calculate energy terms without using a specific upper integration limit, r_n[34]. On the other hand the weaker r-dependence of angular momentum, ~1/r results in the calculated values being completely dominated by an integration limit. The limit of the Euler integral is given by ρ_n^3/r_n^3, a constant which will be denoted 8/σ in this work.
A general exponential function of radius featuring a limit radius, assumed to correspond to a damped oscillator-like solution, may be given in 1st approximation as:

$$e^{v'} = \exp\left(-\left(\frac{\text{ß}\rho'^3}{2r^3} + \left[\left(\frac{\text{ß}\rho'^3}{2r^3}\right)^2 - 4\,\frac{\rho'^3}{2r^3}\right]^{0.5}\right)\right) \tag{56}$$

ß being some general coefficient. At the limit r_n of the real solution (56)

$$\left(\text{ß}\rho'^3/r_n^3\right)^2 = 8\,\rho'^3/r_n^3 \quad => \quad \text{ß} = 8\left(\frac{r}{\rho}'\right)^3 = \sigma \tag{57}$$

holds, reproducing the definition of σ (14). Within the parameter range of this work the function $e^{v'} \approx \exp(-(\text{ß}\rho'^3/r^3))$ is a very good approximation of an equation of the kind of (56) and coefficient σ will have to be part of the exponential.

[A3.2] Coefficient σ, coefficient 1.5x

The basic relation of α(n) and σ with the fine-structure constant α and coefficient $\Gamma_{-1/3}/3$ is due to the considerations of chpt. 2.4ff. To get a more detailed description in a range of 1 percent precision is difficult since there are several options

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[34] For an upper limit $r_n \geq 10\rho$ other limitations supersede the attainable precision.

conceivable and in this range of accuracy, QED and other minor effects may be expected which might be amplified due to the non-linear nature of the Γ-functions involved. A factor ≈ 3/2 appears in several terms such as $\sigma_0 \sim 1.5\alpha^{-1}$ of (18), the ratio of electron and muon energy =1.5088, $\Gamma_{-1/3}/\Gamma_{+1/3}$ =1.516, π/2 = 1.5707 and the irregular electron coefficient in the power series that is part of α_{Pl} as well. The following discusses some relevant aspects with a focus on identifying possible underlying relationships while minimizing assumptions about the term ≈ 3/2 in particular.

In this model elementary charge may be given as $b_0\int \exp(-(e_c/(4\pi\varepsilon_c r))^3)r^{-2}dr \approx e_c$, the corresponding radial distribution of energy has its maximum at $r_c \approx e_c/(4\pi\varepsilon_c)$. To get the exact value of e_c coefficient Γ(+1/3)/3 is required to appear as a term in $W(e_c)$ due to the Euler integral, thus a counter term has to be part of ρ in (12)f:

$$W(e_c) = \frac{e_c^2}{4\pi\varepsilon_c}\int \exp\left(\frac{-\Gamma_{+1/3}}{3}\frac{e_c}{4\pi\varepsilon_c}\right)^3 r^{-2}dr = \frac{e_c^2}{4\pi\varepsilon_c}\frac{\Gamma_{+1/3}}{3}\left(\frac{\Gamma_{+1/3}}{3}\frac{e_c}{4\pi\varepsilon_c}\right)^{-1} = e_c \tag{58}$$

For r_c follows, considering the basic coefficients only, using (30), (33)

$$\lambda_C \sim 3^{0.5}\int \exp-\left(\frac{\Gamma_{+1/3}}{3}\frac{e_c}{4\pi\varepsilon_c}\right)^3 dr \sim \frac{\Gamma_{-1/3}\Gamma_{+1/3}}{3^{0.5}}\;\frac{e_c}{4\pi\varepsilon_c} = \frac{e_c}{2\varepsilon_c} \tag{59}$$

again removing all coefficients that are not part of a Coulomb-expression and suggesting an additional term of 2π in the denominator of ρ (note: for elementary charge σ = 1 has to be assumed; otherwise one gets (60)).

Looking only at the basic mathematical coefficients entering the equation (28)ff (i.e. σ -> $2\Gamma_{-1/3}/3$) an additional term $((2\pi)^{-1}\Gamma_{+1/3}/\Gamma_{-1/3})^3$ (bold in (60)) in ρ would cancel redundant $\Gamma_{-1/3}/3$ terms in the length expression as well:

$$\lambda_C \sim 3^{0.5}\frac{\Gamma_{-1/3}}{3}\;\frac{\sigma^{1/3}}{2}\rho \sim 3^{0.5}\Gamma_{-1/3}\frac{\Gamma_{-1/3}}{3}\frac{2\Gamma_{-1/3}}{3}\;\frac{\boldsymbol{\Gamma_{+1/3}}}{\boldsymbol{2\pi\Gamma_{-1/3}}} = \frac{2\Gamma_{-1/3}}{3} \tag{60}$$

The term $((2\pi)^{-1}\Gamma_{+1/3}/\Gamma_{-1/3})^3$ consists of components related to angular momentum and (with an additional factor 2) seems to be a suitable replacement for $1/(2\alpha_{\lim})$ e.g. in (23) and may thus be used in expressions such as (61)ff [35].

Using these coefficients considered essential for yielding basic quantities such as e_c, including the term 2π associated with angular momentum, and corresponding to the 3rd power structure of the equations best would give for σ_0:

$$\sigma_0 = \left[\frac{1}{4}\left(\frac{\Gamma_{-1/3}2\pi}{\Gamma_{+1/3}}\right)^3\frac{2\Gamma_{-1/3}}{3}\right]^3 = \left[\left(\frac{\Gamma_{-1/3}\pi}{\Gamma_{+1/3}}\right)^3\frac{4\Gamma_{-1/3}}{3}\right]^3 \quad = 2.008\text{E+8 [-]} \tag{61}$$

[A3.3] Model calculations for e^v

In col. 6 of tab. 1 equ. (13) and (24) are used with σ_0 according to (61), α_{Pl} will be replaced by $\alpha_{\lim}^{-1}/2\ (3/2\ \alpha^9)$ with $\alpha_{\lim}$ being recalculated from $\alpha_{\lim}^{-1} = \sigma_0^{-1/3}\,2\Gamma_{-1/3}/3$. This gives (62) as expression for e^v [36].

$$\begin{aligned}&\exp\left(-\left[\left(\rho_n/r\right)^3\right]\right) \approx \exp\left(-\left[1.5^{3\delta}\sigma_0\alpha_{Pl}\alpha(n+1)\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right]\right) \approx \exp\left(-\left[1.5^{3\delta}\left[\left(\frac{\Gamma_{-1/3}\pi}{\Gamma_{+1/3}}\right)^3\frac{4\Gamma_{-1/3}}{3}\right]^3\frac{\alpha(n)}{2\alpha_{\lim}}\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right]\right)\\ &\approx \exp\left(-\left[1.5^{3\delta}\left[\left(\frac{\Gamma_{-1/3}\pi}{\Gamma_{+1/3}}\right)^3\frac{4\Gamma_{-1/3}}{3}\right]^3 2\left(\frac{\Gamma_{+1/3}}{\Gamma_{-1/3}2\pi}\right)^3\left(\frac{3}{2}\right)^3\boldsymbol{\Pi_{k=0}^{n}\alpha\wedge(9/3^k)}\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right]\right) \approx \\ &\left(\exp\left(-\left[1.5^{3\delta}\frac{\pi^2\Gamma_{-1/3}^{\;3}}{\Gamma_{+1/3}^{\;2}}\boldsymbol{\Pi_{k=0}^{n}\alpha\wedge(3/3^k)}\frac{e_c}{4\pi\varepsilon_c r}\right]^3\right)\right)^2 \qquad\qquad n = \{0;1;2;..\}\end{aligned} \tag{62}$$

Inserted in the equation for energy, (12)f, follows:

$$\begin{aligned}W_n &= 2b_0\int_0^{r_n}\left(\exp\left(-\left[1.5^{3\delta}\frac{\pi^2\Gamma_{-1/3}^{\;3}}{\Gamma_{+1/3}^{\;2}}\boldsymbol{\Pi_{k=0}^{n}\alpha\wedge(3/3^k)}\frac{e_c}{4\pi\varepsilon_c r}\right]^3\right)\right)^2 r^{-2}\ dr \quad => \qquad n = \{0;1;2;..\}\\ W_\mu &= 2e_c\frac{\Gamma_{+1/3}}{3}\;2^{-1/3}\left[\frac{\Gamma_{+1/3}^{\;2}}{\pi^2\Gamma_{-1/3}^{\;3}}\boldsymbol{\alpha^{-4}}\right] = \frac{2^{2/3}}{3\pi^2}\left(\frac{\Gamma_{+1/3}}{\Gamma_{-1/3}}\right)^3\boldsymbol{\alpha^{-4}}\;e_c\end{aligned} \tag{63}$$

(1.5^δ = extra coefficient for the electron only, δ = δ(0,n); bold: particle coefficient; muon given as example)

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[35] The need of $\Gamma_{+1/3}/\Gamma_{-1/3}$ to appear in (58)ff and its more pronounced relationship with angular terms is the reason to prefer $(2\alpha_{\lim})^{-1} \approx 2\ ((2\pi)^{-1}\ \Gamma_{+1/3}/\Gamma_{-1/3})^3$ over $(2\alpha_{\lim})^{-1} \approx 2((2\pi)^{-1}\ 2/3)^3$ which would give σ_0=1.821E+8[-], i.e. a term very close to the value of σ_0 fitted to J_z.

[36] Expression intended to emphasize 3rd power relationship, a remaining factor of 2 is attributed to $e^{v/2}$ being squared.

[A4] Coupling constant in N dimensions

The integration limits for calculating angular momentum in z-direction, r_n of J_z, (15)ff, and (Compton-)wavelength, λ_C, supposed to represent the rotating E-vector and in turn total angular momentum J should be related by the factor √3 of the ratio J/J_z:

$$\lambda_C / r_n = (1/2(1/2 + 1))^{0.5} / (1/2) = \sqrt{3} \tag{64}$$ [37]

The 3D case of the coupling constant is easy to interpret, for the 4D-case some assumptions have to be made concerning the integration limit. The following gives an alternative, more detailed interpretation than 2.10 ($e^{v(N)} = \exp(-(\rho/r)^N)$).

3D case:

The exact value of the product of the integrals (35)f, depends on the integration limit relevant for the second integral, i.e. the lower integration limit of the Euler integrals, which can be expressed as 3D volume with $\Gamma_{-1/3}$ as radius (18):

$$\rho_n^3/\lambda_{C,n}^3 = 8/(3^{1.5}\sigma_0) = \left(3^{0.5}\frac{4\pi}{3}\ \Gamma_{-1/3}{}^3\right)^{-3} \tag{65}$$

The additional factor $3^{0.5}$ may be interpreted as the ratio between r_n of equ. (14) and $\lambda_{C,n}$ as required in the expression for photon *energy*. This gives $\Gamma(-1/3, 1/\sigma_0) \approx 36\pi^2\Gamma_{-1/3}$ and

$$2\int_0^r e^{v(3)} r^{-2} dr \int_0^r e^{v(3)} dr \approx 2\left[\frac{\Gamma_{1/3}}{3}\right]\left[2\pi 2\pi 9\frac{\Gamma_{-1/3}}{3}\right] = 4\pi\,\Gamma_{1/3}\Gamma_{-1/3}\ \mathbf{2\pi} = \mathbf{2\pi}\ \alpha^{-1} \tag{66}$$ [38]

The result of (66) yields a dimensionless constant $\alpha' = h\, c_0\, 4\pi\, \varepsilon/e^2$ and it is a matter of choice to include 2π in the dimensionless coupling constant. Factor 9 cancels the corresponding factors from the Euler integrals. The remaining factor of 4π is needed to yield the correct value of α.

A general N-dimensional version of (65) may be given as:

$$8/\sigma_N = \left(3^{0.5\delta} V_N\ (\Gamma(-1/\mathrm{N}))^N\right)^{-N/(N-2)} \tag{67}$$

V_N is the coefficient for volume in N-D, coefficient $3^{0.5}$ will be omitted in 4D where coordinate r is considered to be directly related to energy via $r_n \sim 1/W_n$ and r_n might be directly identified with $\lambda_{C,n}$; subscript in σ_N corresponds to dimension in the following.

4D case:

Using $e^{v(4)}$ according to the definition (4) and (67) for 4D:

$$\rho_n^4/r_n^4 = 8/\sigma_4 = \left(\frac{\pi^2}{2}\ (\Gamma_{-1/4})^4\right)^{-2} = 1.232\text{E-}7 \tag{68}$$

as integration limit, with (11) the non-point-charge integral in 4D will be given by:

$$\int_0^r e^{v(4)} r\, dr \sim \Gamma(-1/2, 8/\sigma_4) = \int_{8/\sigma_4}^{\infty} t^{-1.5} e^{-t}\, dt = 5687 \approx 16\pi^4\Gamma_{-1/2} \tag{69}$$

The 4D equivalent of (66) will be:

$$2\int_0^r e^{v(4)} r^{-3} dr \int_0^r e^{v(4)} r\, dr \approx 2\left[\frac{\Gamma_{1/2}}{4}\right]\left[16\pi^4\frac{\Gamma_{-1/2}}{4}\right] = \frac{\pi^2}{2}\Gamma_{1/2}\Gamma_{-1/2}\ \mathbf{4\pi^2} = \pi^3\,\mathbf{4\pi^2} = \alpha_{weak}^{-1}\mathbf{4\pi^2} \tag{70}$$

The interpretation is the same as in the 3D-case:
A $4\pi^2$ term originating from the second integral of equation (70) is required for turning h^2 into $\hbar^2$ since the integral refers to $\rho_n{}^2$ and thus to the square of energy and h, ħ. Factor 16 cancels the corresponding factors from the Euler integrals. The remaining factor of $\pi^2/2$ is needed to yield the correct value of α_{weak}.

2D case:

the 2D case is not as straightforward as the 4D case. The integral over the 1D point charge

$$\int_0^r e^{v(2)} r^{-1} dr = \Gamma(0, \rho_n^2/r_2^2)\ /2 \tag{71}$$

features Γ(0, x), with Γ(0, x) -> ∞ for x -> 0 and m = N-2 = 0 in the equations above. Setting nevertheless m=1 in the 2D equivalent of the integration limit

$$\rho_n^2/\lambda_{C,n}^2 = 8/(\sigma_2) = \left(3^{0.5}\pi\ \Gamma_{-1/2}{}^2\right)^{-2} \approx 1/4676 \tag{72}$$

and calculating $\Gamma(0, \rho_2{}^2/r_2{}^2)$ numerically gives $\int e^{v(2)} r^{-1}\, dr \approx \Gamma(0, \rho_2{}^2/r_2{}^2)/2 = 7.872/2$. In the 2D case the complementary integral would be identical to the point charge integral, giving $2(\int e^{v(2)} r^{-1}\, dr)^2 \approx 4\pi^3/4 = \pi^3$, i.e. the same value as 4D, maybe giving an alternate candidate for α_{weak}.

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[37] Alternatively: $\lambda_{C,n} = 3\rho h c_0/(2b_0\Gamma_{+1/3}) = 3\pi\,\alpha^{-1}\,\rho/\Gamma_{+1/3}$; $r_n = 3/2\ \alpha^{-1}\ \rho\ \Gamma_{-1/3}/3$ => $\lambda_{C,n}/r_n = 6\pi/(\Gamma_{+1/3}\Gamma_{-1/3}) = 6\pi/(2\pi\sqrt{3}) = 3^{0.5}$

[38] Factor 2 from adding electric and magnetic contributions to energy;

[A5] Quaternion-based quark-like model

[A5.1] Quaternion UDS-components

In the following the model described in chpt. 3 will be explained in some more detail. A standard algorithm for rotation with quaternions will be used.

Three orthonormal vectors E, B, C described as imaginary part of a quaternion with real parts set to 0, will be subject to alternate, incremental rotations around the axes E, B and C. For each E, B and C the following variables will be defined:

- de, db, dc: incremental step for rotation angle,
- de_sum, db_sum, dc_sum: total rotation angle,
- ex, ey, ez, bx, by, bz, cx, cy, cz: Cartesian components of the respective vectors,
- eex, eey, eez, bbx, bby, bbz, ccx, ccy, ccz: Cartesian components of the respective vectors to be buffered until rotation around the axes E, B and C is complete,
- sih, qw, qx, qy, qz: internal variables for quaternion-rotation calculation.

The following part of the algorithm gives the rotation of B around the E axis for an incremental step de:

```
de_sum = de_sum + de;   sih = sin(de / 2);   qw = cos(de / 2);    qx = ex * sih    qy = ey * sih;    qz = ez * sih;
bx = bbx;    by = bby;    bz = bbz;
bxx = bx * (qx * qx + qw * qw - qy * qy - qz * qz)  + by * (2 * qx * qy - 2 * qw * qz) + bz * (2 * qx * qz + 2 * qw * qy);
byy = bx * (2 * qw * qz + 2 * qx * qy)   + by * (qw * qw - qx * qx + qy * qy - qz * qz) + bz * (-2 * qw * qx + 2 * qy * qz);
bzz = bx * (-2 * qw * qy + 2 * qx * qz)  + by * (2 * qw * qx + 2 * qy * qz) + bz * (qw * qw - qx * qx - qy * qy + qz * qz);
bx = bxx;  by = byy;  bz = bzz;
```

This has to be followed by rotation of C around the E axis; and equivalent routines for the rotation of E, B around the C axis and the rotation of E, C around the B axis. After each incremental step for de, db, dc the Cartesian components of the E, B, C vectors may be stored in a list.

The vectors are thought to indicate spatial orientation only, *polarity (sign) of E and B has to be considered in the analysis of the results*. Orientation of angular momentum remains a free parameter.

In the following only solutions where one of the incremental angles of rotation has half the value of the other two will be considered. This may serve as a primitive model for spin J = 1/2.

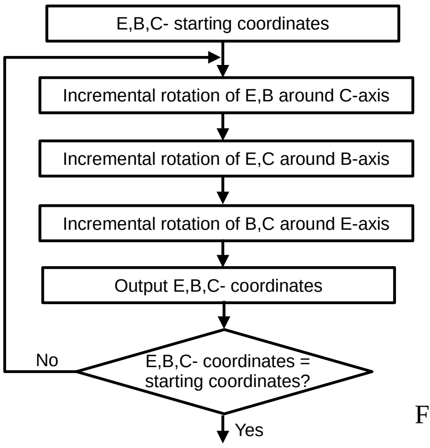

Fig. 2: Flowchart quaternion calculation

There are 6 possible solutions for de, db and dv, respectively, to be called U, D, S, C, B, T:

	de = 0.5 db = 0.5 dc				de = 0.5 db = 0.5 dc				de = 0.5 db = 0.5 dc			
	E-comp	E-avg	B-comp	B-avg	E-comp	E-avg	B-comp	B-avg	E-comp	E-avg	B-comp	B-avg
Spherical cone	2/9, 2/9, 1/9	1/3	4/9, 4/9, 2/9	2/3	4/9, 4/9, 2/9	2/3	2/9, 2/9, 1/9	1/3	4/9, 4/9, 2/9	2/3	4/9, 4/9, 2/9	2/3
	U				D				S			
Toroidal wedge	4/9, 4/9, 2/9	2/3	2/9, 2/9, 1/9	1/3	2/9, 2/9, 1/9	1/3	4/9, 4/9, 2/9	2/3	2/9, 2/9, 1/9	1/3	2/9, 2/9, 1/9	1/3
	C				B				T			

Tab.5: Average of x,y,z-components (E,B-comp) and total average (E,B-avg) of E-and B-field for complete rotation;

The average of the x, y, z-components of the fields are multiples of 1/9th of the original vector length, the average total sum of E- and B-fields is 1/3 or 2/3, respectively. Surface area / fractional charge of 1/3 and 2/3 correspond to an average of the E-field of 2/3 and 1/3.

The diagram for the E,B, C-components as function of the angle dc_sum is given in fig. 3a for a U-entity.

From a coordinate transformation to a representation with one Cartesian coordinate as axis of rotation (in fig. 3b transformation of z-axis +26,6°, x-axis -41,8°, to give y-axis as axis of rotation) one can infer that the E-vector circumvents a spherical cap of area 2πr 2/3r. Mirroring at the center of rotation gives a value of 2/3 of the surface of a sphere, which according to Gauss’ law may represent 2/3 of a full point charge. The analogue procedure yields a value of 1/3 of a point charge for D and S-rotations.

Fig. 3.: a) E-components for Cartesian starting values b) E-components after coordinate transformation

[A5.2] Magnetic moments of baryons from U, D, S-components

To calculate magnetic moments of uds-baryons three components of U,D,S will be combined that represent orthonormal starting conditions for E, B. Spin/angular moment of the 3 components has to add up to $J_Z = 1/2$. Within this model this is not an assumption but may be calculated in principle in detail. In the following it will be sufficient to have two components sharing the same orientation of the axis of rotation, i.e. both can be transformed according to fig. 3 above with the same set of rotation angles, or - in a trivial case – to have 2 identical components. Together with the freedom in choosing direction of rotation, allowing for canceling or adding up spin as needed, this will be sufficient to model $J_Z = 1/2$ baryons. Table 6 gives an example for UUD and DDU.

	UUD	**Proton**			**DDU**	**Neutron**	
	U_1				**D_1**		
Start value	-Ez	-Bx	Cy		-Ex	-Bz	Cy
Bx, By, Bz	**-0.444444**	**0.444444**	**-0.222222**		**-0.222222**	**0.222222**	**-0.111111**
	E	B			E	B	
Rot_Z_axis	-45	135			-45	135	
Rot_X_axis	19.47	19.47			19.47	19.47	
	U_2				**D_2**		
Start value	-Ex	By	-Cz		Ey	-Bx	-Cz
Bx, By, Bz	**-0.222222**	**0.444444**	**-0.444444**		**-0.111111**	**0.222222**	**-0.222222**
	E	B			E	B	
Rot_Z_axis	-26.57	116.56			-26.57	116.56	
Rot_X_axis	41.82	41.81			41.82	41.81	
E, B inverted	**D_inv**				**U_inv**		
Start value	-Ey	-Bz	Cx		-Ez	-By	Cx
	E	B			E	B	
Rot_Z_axis	-45	135			-26.57	116.56	
Rot_X_axis	19.47	19.47			41.82	41.82	
	D				**U**		
Start value	Ey	Bz	Cx		Ez	By	Cx
Bx, By, Bz	**0.222222**	**0.222222**	**0.111111**		**0.444444**	**0.444444**	**0.222222**
Bx, By, Bz Avg(UUD)	-0.148148	0.37037	-0.185185		0.037037	0.296296	-0.037037
B_Avg			**0.439790**				**0.300890**

Table 6: Example for appropriate combinations of U- and D-components for proton and neutron;

In D_inv and U_inv the sign of E- and B-components is inverted. The D and U for calculation of the effective B-field include the appropriate sign from their charge while U_inv, D_inv components represent the actual geometric orientation of the E, B-vector only, which is needed for calculation of the angular momentum J from the square of the electromagnetic fields. In table 6 ”Rot_X_axis” and ”Rot_Z_axis” give the angle of rotation needed to transform to a representation with y-coordinate as axis of rotation for the B-field. For U_1 and D_inv of the proton as well as for D_2 and U_inv of the neutron the angles of transformation are identical, so is their transformed y-axis, i.e. they posses identical orientation of spin (average of B) while still maintaining their orthonormal relationship (B(t)). Since orientation of rotation is a free parameter opposite spin may cancel both contributions, leaving the 3rd component’s spin of $J_Z = 1/2$ as total spin of the nucleon.

The U and D components of proton / neutron have equal sign and relative value of the components of the E- and B-fields (given in tab. 6 only for the Bx, By, Bz-components (bold) relevant for calculating a geometry-based average value of B, B_Avg). The starting values of E, B, C are given for reference only.

The results for U and D are exceptional in regard to the exchangeability of U and D-components. For other particle pairs this is difficult to asses due to identical B-field components of U and S and the different internal symmetry of S-components compared to U and D [39].

In the case of the solutions examined, compliance with condition $J_Z = 1/2$ for the lambda-particle (UDS) can be maintained by using a spin-cancelling UD-solution in combination with an S-component, for UUS, DDS, USS-combinations trivial solutions with two identical components exist, in the case of DSS, Xi^-, one can resort to the method used for the nucleons to find a $J_Z = 1/2$ solution. Results for the best fitting appropriate UDS-combinations are shown in tab. 7.

---

[39] U and D are symmetric in their E and B-fields while in S-components E- and B-fields are symmetric to each other.

	USD	Lambda		UUS	Sigma +		DDS	Sigma -		USS	Xi 0		DSS	Xi -	
	U			**U**			**D**			**S**			**S**		
Bx, By, Bz	-0.444	0.444	-0.222	-0.222	0.4444	-0.444	-0.111	-0.222	0.222	-0.222	-0.444	-0.444	0.444	-0.222	0.444
	S			**U**			**D**			**S**			**S**		
Bx, By, Bz	0.444	-0.444	0.222	-0.222	0.4444	-0.444	-0.111	-0.222	0.222	-0.222	-0.444	-0.444	-0.444	0.444	-0.222
	D			**S**			**S**			**U**			**D**		
Bx, By, Bz	0.222	0.222	0.111	0.4444	0.4444	0.222	0.444	0.444	0.222	0.444	0.444	0.222	0.222	-0.222	0.111
Bx, By, Bz Avg(UUD)	0.074	0.074	0.037	0.000	0.444	-0.222	0.074	0.000	0.222	0.000	-0.148	-0.222	0.074	0.000	0.111
B_Avg			**0.111**			**0.497**			**0.234**			**0.267**			**0.134**

Table 7: Combinations of UDS-components for calculating magnetic moments of baryons.

To calculate magnetic moments, above factors of B_avg, derived from the purely geometric quaternion model, have to be multiplied by a factor considering the absolute strength of fields. Using a simple model for a current loop, M = I*S (current * area), gives equ. (73) for magnetic moments of baryons with $J_Z = 1/2$.

$$\boldsymbol{M_n} \approx e c_0 \lambda_C / 2 \; * \; \text{B\_avg} \quad (= 2\pi \mu_{Bohr} * \text{B\_avg}) \qquad (73)$$

see tab. 8. Factor 2π of the Bohr magneton, μ_{Bohr}, applicable for the electron and muon, is considered to represent a degree of rotational freedom of simple particles that more complex structures composed of several U, D, S-components do not exhibit.

		λ_C	$e\,c_0 * \lambda_C / 2$	B_Avg	|M|Calc = $e c_0 \lambda_C B_{avg}/2$	|M|Exp[Am^2]	|M|Calc/ |M|Exp	|M|Calc/|M|Exp Const. quark
p^{+-}	UUD	1.32E-15	3.17E-26	0.440	1.39E-26	1.41E-26	0.988	-
n	DDU	1.32E-15	3.17E-26	0.301	9.55E-27	9.66E-27	0.988	0.973*
Λ^0	UDS	1.10E-15	2.64E-26	0.111	2.94E-27	3.10E-27	0.949	-
Σ^+	UUS	1.04E-15	2.50E-26	0.497	1.24E-26	1.24E-26	1.002	1.090
Σ^-	DDS	1.04E-15	2.50E-26	0.234	5.83E-27	5.86E-27	0.994	0.897
Ξ^0	USS	9.43E-16	2.26E-26	0.267	6.05E-27	6.31E-27	0.958	1.152
Ξ^-	DSS	9.38E-16	2.25E-26	0.134	3.01E-27	3.06E-27	0.983	0.784

Table 8: Magnetic moments for UDS-Baryons; col.3: Compton wavelength [7]; col.4: magnetic moment for current loop; col.5: average B-component from quaternion calc.; col.6: calculated magnetic moments; col.7: values from experiment [7]; col.8: ratio calculated / experiment value; col.9: ratio (calculated constituent quark model, [7]) / experiment [7]), *calc. via Clebsch-Gordan coefficients relative to p; Σ, Ξ via fit based on p, n, Λ^0.

[A6] Additional particle states

Assignment of more particle states will not be obvious. The following gives some possible approaches.

[A6.1] Partial products

One more partial product might be inferred from considering the next spherical harmonic, y_2^0 with a factor of $(2l+1)^{1/3} = 5^{1/3}$ as energy ratio relative to η, giving the start of an additional partial product series at $5^{1/3}$ W(η) = 937MeV i.e. close to energy values of the first particles available as starting point, η', Φ^0. However, in general it is not expected that partial products can explain all values of particle energies.

[A6.2] Linear combinations

Though the model reproduces basic properties of the quarks the fundamental differences might offer some alternate interpretations based on extended, non-point-like objects.

Linear combination states of the kaons, the first particle family that does not fit to the partial product series scheme, and the η-particle might be an example for such an interpretation:

The kaons are designated to the linear combination of $(d\bar{s} +/- \bar{d}s)/\sqrt{2}$ in the SM. They might be considered to be a linear combination of 2 extended y_1^0 states (double cones of $s|\bar{d}$, $\bar{s}|d$, etc., composition with 1 angular node) similar to the linear combination of 2 atomic p-orbitals, assumed to exhibit 2 angular nodes. A linear combination which would yield the basic symmetry properties of the 2 neutral kaons would be a planar structure such as:

$\bar{s}$ d
K_S^o d   s K_L^o s $\bar{s}$
$\bar{d}$ $\bar{d}$

providing two neutral kaons of different structure and parity (considering either flavour or chirality), implying a decay with different parity and MLT values.

A linear combination of 3 such states i.e. 3 orthogonal y_1^0 states would imply an essentially spherical symmetric object which might be attributable to the η-particle $((u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6})$.

[A6.3] Electroweak bosons

The considerations of chpt. 2.7 give the Higgs VEV as upper limit, the Higgs boson has about half its energy value.

The "rotating E-vector" of chpt. 3 may be interpreted to cover the whole angular range in the case of y_0^0 while a y_1^0 object might be interpreted as forming a double cone. Increasing the number of angular nodes would close the angle of the cone leaving in the limit l -> ∞, a state of minimal angular extension representing the original (E-) vector. This may imply that essentially no space is left for rotation (i.e. Spin = 0) and a vanishing contribution of the magnetic field to total particle energy according to (13), resulting in a factor 1/2 and giving the Higgs *boson* as alternate upper limit of energy.

Using the alternate definition of the Higgs VEV as <VEV> = VEV/√2 [7] would relate a Higgs boson to 〈VEV〉 through the "1D"-term, $\Gamma_{-1/3}/3$. Moreover, the Z boson would correspond to a 2D-object, the W bosons to a 3D-object (if the inverse of the coefficient of the integral for energy, $3/\Gamma_{+1/3}$, is considered to represent a length parameter attributed to λ_C). Except for a factor of 2 the volume term of (20) would give the Δ-particle as starting point of the energy series from the high energy side. This seems to be another hint that some aspects of this model might be expressible in terms of Euclidean geometry.

Electroweak bosons + VEV/√2	W [GeV]	Γ-coefficient relative to VEV/√2	VEV/√2 divided by Γ-coeff. [GeV]	W(calc)/ W(Lit.)
VEV/√2	174.1			
Higgs	125.4	$\Gamma_{-1/3}/3$	128.6	1.026
Z^0	91.2	$(\Gamma_{-1/3}/3)^2$	95.0	1.041
$W^{+/-}$	80.4	$(\Gamma_{-1/3})^2 \ /(3\Gamma_{+1/3})$	84.8	1.055
Δ	1.232	$4\pi/3\ (\Gamma_{-1/3})^3$	1.24/2	1.006

Tab. 9: Electroweak bosons and Δ-particle relative to the Higgs VEV/√2

[A7] Nucleons – stability, bonding in nuclei, scattering

Apart from the quantitative results for partial charges and magnetic moments some qualitative trends for nucleon properties may be inferred from the quaternion-based model.

The spin-cancelling of a UD-unit involves 2 collinear components with opposite charges occupying approximately the same spatial area (fig.4), which is energetically favorable. This suggests among other things:

1) Comparatively lower energy for particles with UD-component;
2) High stability / life time of the nucleons;
3) A possible contribution to bonding in nuclei via UD-U—D-UD, a direct U-D-bond even without meson intermediate;
4) If such an inter-nucleon UD-bond plays a role in bonding in nuclei this would suggest a significant change in UD-structure between isolated and bound nucleons, which might play a role in the "EMC-effect" [14];
5) In DIS-experiments the ratio of the structure functions of neutron and proton, $F_2^n(x)/F_2^p(x)$ approaches 1 for x -> 0 (x = Bjorken-scale) which would be in agreement with a supposed identical field distribution of E and B-fields in the nucleons. For x -> 1 this model predicts the ratio $F_2^n(x)/F_2^p(x)$ to approach

$(z(UD)^2 + Z(D)^2)/(z(UD)^2 + Z(U)^2) = ((+1/3)^2 + (-1/3)^2)/((+1/3)^2 + (+2/3)^2) = 2/5$

which is in good agreement with high precision scattering experiments which yield values in the range 0.4 – 0.5 [15].

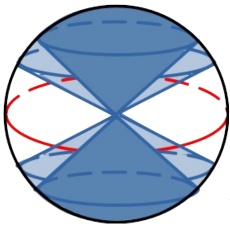

Fig. 4: Illustration of a UD-unit