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# Stability Criteria for Neural Networks with Two Additive Time-varying Delay Components

Qingqing Wang, Shouming Zhong



*Abstract*—This paper is concerned with the stability problem with two additive time-varying delay components. By choosing one augmented Lyapunov-Krasovskii functional, using some new zero equalities, and combining linear matrix inequalities (LMI) techniques, two new sufficient criteria ensuring the global stability asymptotic stability of DNNs is obtained. These stability criteria are present in terms of linear matrix inequalities and can be easily checked. Finally, some examples are showed to demonstrate the effectiveness and less conservatism of the proposed method.

*Keywords*—Neural networks, Globally asymptotic stability, LMI approach, Additive time-varying delays.

## I. Introduction

In the past few decades, neural networks have found a way into many engineering and scientific areas such as model identification, optimization problem and pattern recognition. The existence of time delay may cause instability and oscillation of neural networks. Since stability is an important property to many systems, much effort has been done to analysis the stability problem of of neural networks with time delay [1-20].

It is known that, according to dependence on the size of the delays, the stability criteria for delayed neural networks can be classified into two types: delay-independent stability criteria [1-3] and delay-dependent stability criteria [4-25]. Generally speaking, the later one has less conservatism than the former one, especially when the delay size is small [23,24]. [26] point out that in some situations, signals transmissions may experience a few segments of networks. Since the conditions of networks transmission may be different, it can possibly induce successive delays with different properties. In [26] the model of neural networks with two additive time-varying delays. By constructing a new Lyapunov functional and using a convex polyhedron method to estimate the derivative of the Lyapunov functional,some new delay-dependent stability criteria are derived in [27,28].

In this paper, the problem of stability criteria of neural networks with two additive time-varying delays has been investigated. By choosing new Lyapunov-Krasovskii functional which contains some new integral terms and establishing some new zero equalities, two new sufficient criteria ensuring the global stability asymptotic stability of DNNs is obtained. Finally, some examples are showed to demonstrate the effectiveness and less conservatism of the proposed method.

Qingqing Wang and Shouming Zhong are with the School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, PR China.

Shouming Zhong is with Key Laboratory for NeuroInformation of Ministry of Education, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, PR China.

(e-mail address: wangqqchenbc@163.com).

## II. Problem Statement

Consider a class of delay neural networks described by the following equation:

$$\dot{x}(t)=-Ax(t)+Bg(x(t))+Dg(x(t-d_1(t)-d_2(t)))+\mu \tag{1}$$

where $x(t)=[x_1(t),x_2(t),\dots,x_n(t)]^T\in R^n$ is the neuron state vector. $g(x(t))=[g_1(x_1(t)),g_2(x_2(t)),\dots,g_n(x_n(t))]^T$ denotes the neuron activation function, and a constant input vector $\mu=[\mu_1,\mu_2,\dots,\mu_n]^T.A=diag\{a_i\}$ with $a_i>0,i=1,2,\dots,n$. $B,D\in R^{n\times n}$ are the connection weight matrix and the delayed connection weight matrix,respectively. The following assumptions are adopted throughout the paper.

**Assumption 1**: The delay $d_1(t),d_2(t)$ are time-varying continuous function and satisfy:

$$0\le d_1(t)\le d_1,\ \dot{d}_1(t)\le\mu_1,\ 0\le d_2(t)\le d_2,\ \dot{d}_2(t)\le\mu_2. \tag{2}$$

where $d_1,d_2$ and $\mu_1,\mu_2$ are constants.we denote

$$d(t)=d_1(t)+d_2(t),d=d_1+d_2,\mu=\mu_1+\mu_2 \tag{3}$$

**Assumption 2**: Each neuron activation function $g_i(\cdot),i=1,2,\dots,n$,in (1) satisfies the following condition:

$$0\le\frac{g_i(\alpha)-g_i(\beta)}{\alpha-\beta}\le l_i,\forall\alpha,\beta\in R,\alpha\ne\beta \tag{4}$$

where $l_i,i=1,2,\dots,n$ are constants,and denote matrix $L=diag\{l_i\}$.

Based on Assumption 1-2, it can be easily proven that there exists one equilibrium point for (1) by Brouwer‘s fixed-point theorem. Assuming that $x^*=[x_1^*,x_2^*,\dots,x_n^*]^T$is the equilibrium point of (1) and using the transformation $z(\cdot)=x(\cdot)-x^*$,system (1) can be converted to the following system :

$$\dot{z}(t)=-Az(t)+Bf(z(t))+Df(z(t-d_1(t)-d_2(t))) \tag{5}$$

where $z(t)=[z_1(t),z_2(t),\dots,z_n(t)]^T, f(z(t))=[f_1(z_1(t)), f_2(z_2(t)),\dots,f_n(z_n(t))]^T$, $f_i(z_i(\cdot))=g_i(x_i(\cdot)+x_i^*)-g_i(x_i^*)$, $i=1,2,\dots,n$.

From $Eq.(4)$,$f_i(\cdot)$ satisfies the following condition:

$$0\le\frac{f_i(\alpha)}{\alpha}\le l_i,\forall\alpha\ne0,i=1,2,\dots,n. \tag{6}$$

**Lemma 1** [29]. For any constant matrix $P = P^T > 0$ and $0 \leq h_1 < h_2$ such that the following integrations are well defined, then

$$-h_{12}\int_{t-h_2}^{t-h_1} x^T(s)Px(s)ds \leq -(\int_{t-h_2}^{t-h_1} x(s)ds)^T P(\int_{t-h_2}^{t-h_1} x(s)ds) \tag{7}$$

where $h_{12} = h_1 - h_2$.

**Lemma 2** [30].Let $\zeta \in R^n, \Gamma = \Gamma^T \in R^{n\times n}$, and $B \in R^{m\times n}$ such that $rank(G) < n$. Then, the following statements are equivalent:

$$\begin{aligned}&(1)\ \zeta^T\Gamma\zeta < 0,\ G\zeta = 0, \zeta \neq 0,\\ &(2)\ (G^{\perp})^T\Gamma G^{\perp} < 0,\end{aligned} \tag{8}$$

where $G^{\perp}$ is a right orthogonal complement of $G$.

## III. Main results

In this section,a new Lyapunov functional is constructed and a less conservative delay-dependent stability criterion is obtained.

**Theorem 1** Given that the Assumption 1-2 hold, the system (5) is globally asymptotic stability if there exist symmetric positive definite matrices $P, Q_i, i = 1, 2, \ldots, 7$, $\begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix}$, $R_j, j = 1, 2, \ldots, 6$,positive diagonal matrices $\Lambda = diag\{\lambda_i\}$, $T_1, T_2$,and any symmetric matrix $S_1, i = 1, 2, \ldots, n$, such that the following LMIs hold:

$$(\Gamma^{\perp})^T\Omega\Gamma^{\perp} < 0 \tag{9}$$

$$\begin{bmatrix} R_1 & S_i \\ * & \frac{d_1}{2}R_2 \end{bmatrix} > 0,\ \ i = 1, 2 \tag{10}$$

$$\begin{bmatrix} R_3 & S_i \\ * & \frac{d_2}{2}R_4 \end{bmatrix} > 0,\ \ i = 3, 4 \tag{11}$$

$$\begin{bmatrix} R_5 & S_i \\ * & \frac{d}{2}R_6 \end{bmatrix} > 0,\ \ i = 5, 6 \tag{12}$$

where

$$\Gamma = \begin{bmatrix} -A & O_{n\times 6n} & B & D \end{bmatrix}$$

$$\Omega = \begin{bmatrix}
\Omega_{11} & 0 & \frac{R_6}{2} & 0 & \frac{R_2}{2} & 0 & \Omega_{17} & \Omega_{18} & \Omega_{19} \\
* & \Omega_{22} & 0 & 0 & 0 & 0 & 0 & 0 & T_2L \\
* & * & \Omega_{33} & 0 & -G_{12} & 0 & 0 & 0 & 0 \\
* & * & * & \Omega_{44} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \Omega_{55} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Omega_{66} & 0 & 0 & 0 \\
* & * & * & * & * & * & \Omega_{77} & 0 & 0 \\
* & * & * & * & * & * & * & \Omega_{88} & \Omega_{89} \\
* & * & * & * & * & * & * & * & \Omega_{99}
\end{bmatrix}$$

$$\begin{aligned}\Omega_{11} &= -PA - AP + Q_1 + Q_3 + Q_4 + Q_5 + Q_6 + Q_7 \\ &\quad + G_{11} + d_1R_1 + d_2R_3 + dR_5 - S_1 - S_3 - S_5 \\ &\quad - \frac{1}{2}(R_2 + R_4 + R_6) + A^T\bar{R}A\end{aligned}$$

$$\Omega_{17} = \frac{R_4}{2} + G_{12}$$

$$\Omega_{18} = PB - A\Lambda - A^T\bar{R}B + T_1L$$

$$\Omega_{19} = PD - A^T\bar{R}D$$

$$\Omega_{22} = -(1-\mu)Q_1 + S_5 - S_6$$

$$\Omega_{33} = -Q_7 - G_{22} + S_6 - \frac{R_6}{2}$$

$$\Omega_{44} = -(1-\mu_1)Q_3 + S_1 - S_2$$

$$\Omega_{55} = -Q_4 - G_{11} + S_2 - \frac{R_2}{2}$$

$$\Omega_{66} = -(1-\mu_2)Q_5 + S_3 - S_4$$

$$\Omega_{77} = -Q_2 + G_{22} + S_4 - \frac{R_4}{2}$$

$$\Omega_{88} = \Lambda B + B^T\Lambda + Q_2 + B^T\bar{R}B - 2T_1$$

$$\Omega_{89} = \Lambda D + B^T\bar{R}D$$

$$\Omega_{99} = -(1-\mu)Q_2 + D^T\bar{R}D - 2T_2$$

$$\bar{R} = d_1^2R_2 + d_2^2R_4 + d^2R_6$$

*Proof:* Construct a new class of Lyapunov functional candidate as follow:

$$V(z_t) = \sum_{i=1}^{4} V_i(z_t)$$

with

$$V_1(z_t) = z^T(t)Pz(t) + 2\sum_{i=1}^{n}\lambda_i\int_0^{z_i(t)} f_i(s)ds$$

$$\begin{aligned}V_2(z_t) &= \int_{t-d(t)}^{t}(z^T(s)Q_1z(s) + f^T(z(s))Q_2f(z(s)))ds \\ &\quad + \int_{t-d_1(t)}^{t} z^T(s)Q_3z(s)ds + \int_{t-d_1}^{t} z^T(s)Q_4z(s)ds \\ &\quad + \int_{t-d_2(t)}^{t} z^T(s)Q_5z(s)ds + \int_{t-d_2}^{t} z^T(s)Q_6z(s)ds \\ &\quad + \int_{t-d}^{t} z^T(s)Q_7z(s)ds\end{aligned}$$

$$V_3(z_t) = \int_{t-d_1}^{t}\begin{bmatrix} z(s) \\ z(s-d_2)\end{bmatrix}^T\begin{bmatrix} G_{11} & G_{12} \\ * & G_{22}\end{bmatrix}\begin{bmatrix} z(s) \\ z(s-d_2)\end{bmatrix}ds$$

$$\begin{aligned}V_4(z_t) &= \int_{-d_1}^{0}\int_{t+\theta}^{t}(z^T(s)R_1z(s)+d_1\dot{z}^T(s)R_2\dot{z}(s))ds\\&+\int_{-d_2}^{0}\int_{t+\theta}^{t}(z^T(s)R_3z(s)+d_2\dot{z}^T(s)R_4\dot{z}(s))ds\\&+\int_{-d}^{0}\int_{t+\theta}^{t}(z^T(s)R_5z(s)+d\dot{z}^T(s)R_6\dot{z}(s))ds\end{aligned}$$

Then, taking the time derivative of $V(t)$ with respect to $t$ along the system (5) yield

$$\dot{V}(z_t)=\sum_{i=1}^{4}\dot{V}_i(z_t)$$

where

$$\begin{aligned}\dot{V}_1(z_t) &= 2z^T(t)P\dot{z}(t)+2\sum_{i=1}^{n}\lambda_i f_i(z_i(t))\dot{z}_i(t)\\&=2z^T(t)P\dot{z}(t)+2f^T(z(t))\Lambda\dot{z}(t)\end{aligned}\tag{13}$$

$$\begin{aligned}\dot{V}_2(z_t) \le\, & z^T(t)(Q_1+Q_3+Q_4+Q_5+Q_6+Q_7)z(t)\\&+f^T(z(t))Q_2f(z(t))-z^T(t-d_1)Q_4z(t-d_1)\\&-(1-\mu)z^T(t-d(t))Q_1z(t-d(t))\\&-(1-\mu)f^T(z(t-d(t)))Q_2f^T(z(t-d(t)))\\&-(1-\mu_1)z^T(t-d_1(t))Q_3z(t-d_1(t))\\&-(1-\mu_2)z^T(t-d_2(t))Q_5z(t-d_2(t))\\&-z^T(t-d_2)Q_2z(t-d_2)-z^T(t-d)Q_7z(t-d)\end{aligned}\tag{14}$$

$$\begin{aligned}\dot{V}_3(z_t) &= \begin{bmatrix}z(t)\\z(t-d_2)\end{bmatrix}^T\begin{bmatrix}G_{11}&G_{12}\\ *&G_{22}\end{bmatrix}\begin{bmatrix}z(t)\\z(t-d_2)\end{bmatrix}\\&-\begin{bmatrix}z(t-d_1)\\z(t-d)\end{bmatrix}^T\begin{bmatrix}G_{11}&G_{12}\\ *&G_{22}\end{bmatrix}\begin{bmatrix}z(t-d_1)\\z(t-d)\end{bmatrix}\end{aligned}\tag{15}$$

$$\begin{aligned}\dot{V}_4(z_t) &= z^T(t)(d_1R_1+d_2R_2+dR_5)z(t)+\dot{z}(t)\bar{R}\dot{z}(t)\\&-\int_{t-d_1}^{t}(z^T(s)R_1z(s)+d_1\dot{z}(s)R_2\dot{z}(s))ds\\&-\int_{t-d_2}^{t}(z^T(s)R_3z(s)+d_2\dot{z}(s)R_4\dot{z}(s))ds\\&-\int_{t-d}^{t}(z^T(s)R_5z(s)+d\dot{z}(s)R_6\dot{z}(s))ds\end{aligned}\tag{16}$$

Using Lemma 1, we can obtain that

$$\begin{aligned}&-d_1\int_{t-d_1}^{t}\dot{z}^T(s)\frac{R_2}{2}\dot{z}(s)ds\le\\&-(z(t)-z(t-d_1))^T\frac{R_2}{2}(z(t)-z(t-d_1))\end{aligned}\tag{17}$$

$$\begin{aligned}&-d_2\int_{t-d_2}^{t}\dot{z}^T(s)\frac{R_4}{2}\dot{z}(s)ds\le\\&-(z(t)-z(t-d_2))^T\frac{R_4}{2}(z(t)-z(t-d_2))\end{aligned}\tag{18}$$

$$\begin{aligned}&-d\int_{t-d}^{t}\dot{z}^T(s)\frac{R_6}{2}\dot{z}(s)ds\le\\&-(z(t)-z(t-d))^T\frac{R_6}{2}(z(t)-z(t-d))\end{aligned}\tag{19}$$

The following six zero equalities with any symmetric matrix $S_i, i=1,2,\dots,6$ are considered:

$$\begin{aligned}&z^T(t)S_1z(t)-z^T(t-d_1(t))S_1z(t-d_1(t))\\&\qquad-2\int_{t-d_1(t)}^{t}z^T(s)S_1\dot{z}(s)=0\end{aligned}\tag{20}$$

$$\begin{aligned}&z^T(t-d_1(t))S_2z(t-d_1(t))-z^T(t-d_1)S_2z(t-d_1)\\&\qquad-2\int_{t-d_1}^{t-d_1(t)}z^T(s)S_2\dot{z}(s)=0\end{aligned}\tag{21}$$

$$\begin{aligned}&z^T(t)S_3z(t)-z^T(t-d_2(t))S_3z(t-d_2(t))\\&\qquad-2\int_{t-d_2(t)}^{t}z^T(s)S_3\dot{z}(s)=0\end{aligned}\tag{22}$$

$$\begin{aligned}&z^T(t-d_2(t))S_4z(t-d_2(t))-z^T(t-d_2)S_4z(t-d_2)\\&\qquad-2\int_{t-d_2}^{t-d_2(t)}z^T(s)S_4\dot{z}(s)=0\end{aligned}\tag{23}$$

$$\begin{aligned}&z^T(t)S_5z(t)-z^T(t-d(t))S_5z(t-d(t))\\&\qquad-2\int_{t-d(t)}^{t}z^T(s)S_5\dot{z}(s)=0\end{aligned}\tag{24}$$

$$\begin{aligned}&z^T(t-d(t))S_6z(t-d(t))-z^T(t-d)S_6z(t-d)\\&\qquad-2\int_{t-d}^{t-d(t)}z^T(s)S_6\dot{z}(s)=0\end{aligned}\tag{25}$$

From (6), we can get that there exist positive diagonal matrices $T_1, T_2$, such that the following inequalities holds:

$$-2f^T(z(t))T_1f(z(t))+2z^T(t)T_1Lf(z(t))\ge 0\tag{26}$$

$$\begin{aligned}&-2f^T(z(t-d(t)))T_2f(z(t-d(t)))\\&+2z^T(t-d(t))T_2Lf(z(t-d(t)))\ge 0\end{aligned}\tag{27}$$

From (13)-(27),we can obtain that

$$\begin{aligned}\dot{V}(z_t)\le\,&\xi^T(t)\Omega\xi(t)-\int_{t-d_1(t)}^{t}\begin{bmatrix}z(s)\\\dot{z}(s)\end{bmatrix}^T\begin{bmatrix}R_1&S_1\\ *&\frac{d_1}{2}R_2\end{bmatrix}\begin{bmatrix}z(s)\\\dot{z}(s)\end{bmatrix}ds\\&-\int_{t-d_1}^{t-d_1(t)}\begin{bmatrix}z(s)\\\dot{z}(s)\end{bmatrix}^T\begin{bmatrix}R_1&S_2\\ *&\frac{d_1}{2}R_2\end{bmatrix}\begin{bmatrix}z(s)\\\dot{z}(s)\end{bmatrix}ds\\&-\int_{t-d_2(t)}^{t}\begin{bmatrix}z(s)\\\dot{z}(s)\end{bmatrix}^T\begin{bmatrix}R_3&S_3\\ *&\frac{d_2}{2}R_4\end{bmatrix}\begin{bmatrix}z(s)\\\dot{z}(s)\end{bmatrix}ds\\&-\int_{t-d_2}^{t-d_2(t)}\begin{bmatrix}z(s)\\\dot{z}(s)\end{bmatrix}^T\begin{bmatrix}R_3&S_4\\ *&\frac{d_2}{2}R_4\end{bmatrix}\begin{bmatrix}z(s)\\\dot{z}(s)\end{bmatrix}ds\\&-\int_{t-d(t)}^{t}\begin{bmatrix}z(s)\\\dot{z}(s)\end{bmatrix}^T\begin{bmatrix}R_5&S_5\\ *&\frac{d}{2}R_6\end{bmatrix}\begin{bmatrix}z(s)\\\dot{z}(s)\end{bmatrix}ds\\&-\int_{t-d}^{t-d(t)}\begin{bmatrix}z(s)\\\dot{z}(s)\end{bmatrix}^T\begin{bmatrix}R_5&S_6\\ *&\frac{d}{2}R_6\end{bmatrix}\begin{bmatrix}z(s)\\\dot{z}(s)\end{bmatrix}ds\end{aligned}\tag{28}$$

where

$$\xi^T(t) = [z^T(t), z^T(t-d(t)), z^T(t-d), z^T(t-d_1(t)), z^T(t-d_1), z^T(t-d_2(t)), z^T(t-d_2), f^T(z(t)), f^T(z(t-d(t)))]^T$$

By Lemma 2, $\xi^T(t)\Omega\xi(t) < 0$ with $\Gamma\xi(t) = 0$ is equivalent to $(\Gamma^\perp)^T\Omega\Gamma^\perp < 0$.Therefore, if LMIs (9)-(12) hold, we can obtain $\dot{V}(z_t) < 0$. then the neural networks (5) is asymptotically stable. This completes the proof. ■

**Remark 1** Theorem 1 require the upper bound $\mu_1, \mu_2$ of time-delay $d_1(t), d_2(t)$ to be known. if $\mu_1, \mu_2$ is unknown, by setting $Q_1 = Q_2 = Q_3 = Q_5 = 0$ in $V_2(z_t)$ and employing same methods in Theorem 1, we can derive the delay-dependent and delay-derivative-dependent stability criteria.

**Remark 2** It is noted that a novel term $V_4(z_t)$ is included in the Lyapunov functional $V(z_t)$, which plays an important role in reducing conservativeness of our results.

**Theorem 2** Given that the Assumption 1-2 hold, the system (5) is globally asymptotic stability if there exist symmetric positive definite matrices $\begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix}, R_j, j = 1, 2, \ldots, 6$, $P, Q_4, Q_6, Q_7$,positive diagonal matrices $\Lambda = diag\{\lambda_i\}, T_1$, $T_2$,and any symmetric matrix $S_1, i = 1, 2, \ldots, n$, such that the following LMIs hold:

$$(\Gamma^\perp)^T\Phi\Gamma^\perp < 0 \tag{29}$$

$$\begin{bmatrix} R_1 & S_i \\ * & \frac{d_1}{2}R_2 \end{bmatrix} > 0, \ \ i = 1, 2 \tag{30}$$

$$\begin{bmatrix} R_3 & S_i \\ * & \frac{d_2}{2}R_4 \end{bmatrix} > 0, \ \ i = 3, 4 \tag{31}$$

$$\begin{bmatrix} R_5 & S_i \\ * & \frac{d}{2}R_6 \end{bmatrix} > 0, \ \ i = 5, 6 \tag{32}$$

where

$$\Gamma = \begin{bmatrix} -A & O_{n\times 6n} & B & D \end{bmatrix}$$

$$\Phi = \begin{bmatrix} \Phi_{11} & 0 & \frac{R_6}{2} & 0 & \frac{R_2}{2} & 0 & \Phi_{17} & \Phi_{18} & \Phi_{19} \\ * & \Phi_{22} & 0 & 0 & 0 & 0 & 0 & 0 & T_2L \\ * & * & \Phi_{33} & 0 & -G_{12} & 0 & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Phi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Phi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Phi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \Phi_{88} & \Phi_{89} \\ * & * & * & * & * & * & * & * & \Phi_{99} \end{bmatrix}$$

$$\Phi_{11} = -PA - AP + Q_4 + Q_6 + Q_7 + G_{11} + d_1R_1 + d_2R_3 + dR_5 - S_1 - S_3 - S_5 + A^T\bar{R}A - \frac{1}{2}(R_2 + R_4 + R_6)$$

$$\Phi_{17} = \frac{R_4}{2} + G_{12}, \ \Phi_{18} = PB - A\Lambda - A^T\bar{R}B + T_1L$$

TABLE I
ADMISSIBLE UPPER BOUND $d_2$ FOR DIFFERENT $d_1$ WITH $\mu_1 = 0.7$ AND $\mu_2 = 0.1$.

| Method | $d_1 = 0.8$ | $d_1 = 1$ | $d_1 = 1.2$ |
|---|---|---|---|
| [26] | 0.8831 | 0.6832 | 0.4843 |
| [27] | 1.5666 | 1.3668 | 1.1664 |
| [31] | 0.8831 | 0.6831 | 0.4831 |
| Theorem 1 | 2.0214 | 1.9275 | 1.7816 |

TABLE II
ADMISSIBLE UPPER BOUND $d_1$ FOR DIFFERENT $d_2$ WITH $\mu_1 = 0.7$ AND $\mu_2 = 0.1$.

| Method | $d_2 = 0.8$ | $d_2 = 1$ | $d_2 = 1.2$ |
|---|---|---|---|
| [26] | 2.1564 | 1.6464 | 1.4365 |
| [27] | 2.6928 | 2.2389 | 2.0639 |
| [31] | 1.5831 | 1.4831 | 1.3831 |
| Theorem 1 | 3.0768 | 2.9013 | 2.5612 |

$$\Phi_{19} = PD - A^T\bar{R}D, \ \Phi_{22} = S_5 - S_6$$

$$\Phi_{33} = -Q_7 - G_{22} + S_6 - \frac{R_6}{2}, \ \Phi_{44} = S_1 - S_2$$

$$\Phi_{55} = -Q_4 - G_{11} + S_2 - \frac{R_2}{2}, \ \Phi_{66} = S_3 - S_4$$

$$\Phi_{77} = -Q_2 + G_{22} + S_4 - \frac{R_4}{2}$$

$$\Phi_{88} = \Lambda B + B^T\Lambda + Q_2 + B^T\bar{R}B - 2T_1$$

$$\Phi_{89} = \Lambda D + B^T\bar{R}D, \ \Phi_{99} = D^T\bar{R}D - 2T_2$$

$$\bar{R} = d_1^2R_2 + d_2^2R_4 + d^2R_6$$

*Proof:* The proof of the Theorem 2 is consequence of Theorem 1 by choosing $Q_1 = Q_2 = Q_3 = Q_5 = 0$ in $V(z_t)$. Hence the proof is omitted. ■

## IV. EXAMPLE

In this section,we provide a numerical examples to demonstrate the effectiveness and less conservatism of our delay-dependent stability criteria.

**Example 1** Consider the system (5) with the following parameters:

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, D = \begin{bmatrix} 0.88 & 1 \\ 1 & 1 \end{bmatrix}$$

$$f_1(s) = 0.4\tanh(s), f_2(s) = 0.8\tanh(s), L = diag\{0.4, 0.8\}.$$

According to Table I and Table II,we can see that Theorem 1 in our paper can indeed provide much larger admissible upper bounds than the stability criteria in [26,27,31]. In Table III, we consider the other case with different $d_2$, unknown $\mu_1, \mu_2$, according to this Table,we can see this example shows that the stability condition gives much less conservative results in this paper.

TABLE III
ADMISSIBLE UPPER BOUND $d_1$ FOR DIFFERENT $d_2$ WITH UNKNOWN $\mu_1, \mu_2$.

| Method | $d_2 = 0.8$ | $d_2 = 1$ | $d_2 = 1.2$ |
|---|---|---|---|
| Theorem 2 | 2.3147 | 2.02160 | 1.9856 |

## V. CONCLUSION

In this paper, the problem of stability analysis for delayed neural networks with two additive time-varying delay components has been investigated. By choosing new Lyapunov-Krasovskii functional, using some new zero equalities, and combining linear matrix inequalities (LMI) techniques, two new sufficient criteria ensuring the global stability asymptotic stability of DNNs is obtained. Finally, some examples are given to show the effectiveness of our obtained criteria.

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**Qingqing Wang** was born in Anhui Province, China,in 1989. She received the B.S. degree from Anqing University in 2012. She is currently pursuing the M.S.degree from University of Electronic Science and Technology of China. Her research interests include neural networks, switch and delay dynamic systems.

**Shouming Zhong** was born in 1955 in Sichuan, China. He received B.S. degree in applied mathematics from UESTC, Chengdu, China, in 1982. From 1984 to 1986, he studied at the Department of Mathematics in Sun Yatsen University, Guangzhou, China. From 2005 to 2006, he was a visiting research associate with the Department of Mathematics in University of Waterloo, Waterloo, Canada. He is currently as a full professor with School of Applied Mathematics, UESTC. His current research interests include differential equations, neural networks, biomathematics and robust control. He has authored more than 80 papers in reputed journals such as the International Journal of Systems Science, Applied Mathematics and Computation, Chaos, Solitons and Fractals, Dynamics of Continuous, Discrete and Impulsive Systems, Acta Automatica Sinica, Journal of Control Theory and Applications, Acta Electronica Sinica, Control and Decision, and Journal of Engineering Mathematics.