

# Optimal Aggregate Production Planning with Fuzzy Data

Wen-Lung Huang and Shih-Pin Chen

**Abstract**—This paper investigates the optimization problem of multi-product aggregate production planning (APP) with fuzzy data. From a comprehensive viewpoint of conserving the fuzziness of input information, this paper proposes a method that can completely describe the membership function of the performance measure. The idea is based on the well-known Zadeh's extension principle which plays an important role in fuzzy theory. In the proposed solution procedure, a pair of mathematical programs parameterized by possibility level  $\alpha$  is formulated to calculate the bounds of the optimal performance measure at  $\alpha$ . Then the membership function of the optimal performance measure is constructed by enumerating different values of  $\alpha$ . Solutions obtained from the proposed method contain more information, and can offer more chance to achieve the feasible disaggregate plan. This is helpful to the decision-maker in practical applications.

**Keywords**—fuzzy data, aggregate production planning, membership function, parametric programming

## I. INTRODUCTION

ENTERPRISES around the world have increasingly emphasized aggregate production planning (APP) for deciding an appropriate way to match many controllable factors, such as capacity to forecast demand, varying customer orders over the medium term by adjusting regular and overtime production rates, and subcontracting and backordering rates [4,13,20]. Much effort has been expended in existing decision models for APP problem [21], such as linear decision rule (LDR), transportation method, linear programming (LP), management coefficient approach, simulation, and search decision rule. More recent studies include, for example, Jain and Palekar [9], Leung *et al.* [16], and Gomes da Silva *et al.* [8]. Most of the above models were developed under crisp environments, and can be categorized as deterministic optimization and stochastic programming models [6,19,20]. However, in many practical applications, the imprecise information embedded in APP can be obtained subjectively [10,27]. For example, the uncertain demand may be more suitably described by linguistic terms rather than by a probability distribution. Zadeh [28] introduced fuzzy set theory to handle uncertainties of this type. There have been relatively few studies of fuzzy APP problems [14]. For example, Tang *et al.* [22] proposed a method for multi-product APP problems with fuzzy demands and capacities.

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Afterward, Tang *et al.* [23] conducted the simulation study of the APP problems of this kind; and Fung *et al.* [7] additionally considered financial constraints in similar APP problems. There are several other related studies, including Wang and Fang [24,25], and Wang and Liang [26,27]. More recently, Aliev *et al.* [1] proposed a fuzzy integrated multi-period and multi-product aggregate production-distribution planning model in supply chain. Jamalnia and Soukhakian [10] proposed a hybrid fuzzy multi-objective nonlinear programming (H-FMONLP) model with different goal priorities for the APP problem. Leung and Chan [15] constructed a preemptive goal programming model to investigate the APP problem with different operational constraints.

It is clear that if there are some fuzzy parameters in an APP model, then its performance measures will be fuzzy. To completely conserve the fuzziness of input information of APP, they should be described by membership functions. If the membership function of performance measures for an APP model can be derived, more reasonable and realistic performance measures can be obtained because it maintains the fuzziness of input information which can be used to represent the fuzzy APP more accurately. Thus an effective APP and the feasible disaggregate plan can be obtained. In this paper, an APP model with fuzzy parameters is proposed, and a solution procedure that is able to find the membership function of the fuzzy objective value is developed. The basic idea is to apply the  $\alpha$ -cuts and Zadeh's extension principle [29,30] to transform the fuzzy APP into a family of crisp APPs. A pair of two-level mathematical programs is formulated to calculate the bounds of the  $\alpha$ -cut of the fuzzy minimum total cost, and then the membership function of the fuzzy minimum total cost is derived numerically by enumerating different values of  $\alpha$ . Since the proposed approach is based on the Zadeh's extension principle, it is significantly different from several related studies which may fail to compute the sets of possible values of the fuzzy minimum total cost such [14].

## II. MODELING APP WITH FUZZY DATA

First we address the issue of single-product fuzzy APP problem. Consider a company manufacturing one type of products to satisfy the market demand over a medium planning horizon  $T$ . For satisfying forecasted demands, the management adopts APP to find the most effective way to determine output rates, hiring and layoffs, inventory levels, overtime work, subcontracting, backorders and other controllable factors. In fuzzy environments, the problem is to determine a feasible way

of adjusting the appropriate amounts of the above factors to satisfy the demand at the right time that minimizes the total cost. Note that, as Lai and Hwang [14] pointed out, the forecasted demand in a period could either be met or backordered; however, the backorder must be fulfilled within the subsequent period. To solve this problem, a model is constructed under the assumptions following Lai and Hwang [14]:

- (1) Initial inventory is about the inventory level required for initiating an order.
- (2) The decision maker knows the initial workforce level for the production.
- (3) The quantity of the product includes that of regular-time production, overtime production, production due to hiring more employees, and to meet the demands.
- (4) The production costs excluding labor cost regarding to regular-time production and overtime production in any period are the same.
- (5) There exists no setup cost if successive orders are being processed.
- (6) Inventory storage space is large enough to store the finished goods in processes.
- (7) There exists a reliable work force pool. The new employees are assumed to be fully productive as are the old employees, when they begin to work.

Without loss of generality, assume that the maximum workforce ( $\tilde{W}_t^{max}$ ) is fuzzy. The parameters of the model are introduced in Table 1, and the decision variables are as follows:  $P_{rt}$ , the regular-time production in period  $t$  (units);  $P_{ot}$ , the overtime production in period  $t$  (units);  $I_t$ , the inventory level in period  $t$  (units);  $W_t$ , the workforce level in period  $t$  (man-day);  $B_t$ , the backorder level in period  $t$  (units);  $H_t$ , the worker hired in period  $t$  (man-day); and  $L_t$ , the worker layoff in period  $t$  (man-day).

TABLE I  
 PARAMETERS OF THE FUZZY APP MODEL

| Parameters | Definition   |
|------------|--|
| $C_{it}$   | inventory carrying cost in period $t$ (\$/unit-period)                     |
| $C_{ot}$   | overtime labor cost in period $t$ (\$/man-hour)                            |
| $C_{rt}$   | labor cost in period $t$ (\$/man-day)                                      |
| $C_{ht}$   | cost to hire one worker in period $t$ (\$/man-day)                         |
| $C_{lt}$   | cost to layoff one worker in period $t$ (\$/man-day)                       |
| $C_{bt}$   | unit backorder cost in period $t$ (\$/unit)                                |
| $k$        | conversion factor in hours of labor per unit of production (man-hour/unit) |
| $\delta$   | regular working hours per worker per day (man-hour/man-day)                |
| $\beta_t$  | fraction of working hours available for overtime production                |
| $T$        | planning horizon or number of periods                                      |
| $I_0$      | initial inventory level (units)  |

|                     |   |
|---------------------|---|
| $W_0$               | initial work force level (man-day)                  |
| $B_0$               | initial backorder level(units)                      |
| $\tilde{W}_t^{max}$ | maximum workforce available in period $t$ (man-day) |
| $F_t$               | forecasted demand in period $t$ (units)             |
| $F_{tmin}$          | minimum demand in period $t$ (units)                |

The objective function is to minimize the total cost that is the summation of the following terms: (1) the total production cost,

$$\sum_{t=1}^T C_{pt} (P_{rt} + P_{ot}) ; \quad (2) \quad \text{the total labor cost,}$$

$$\sum_{t=1}^T C_{rt} W_t + \sum_{t=1}^T C_{ot} (kP_{ot}), \text{ where } k \text{ is a conversion factor for}$$

transforming the unit of  $P_{ot}$  to man-hour (referring to Table 1);

$$(3) \quad \text{the total inventory carrying cost, } \sum_{t=1}^T C_{it} I_t ; (4) \quad \text{the total}$$

$$\text{backorder cost, } \sum_{t=1}^T C_{bt} B_t ; (5) \quad \text{the total costs of changes in labor}$$

levels, including the costs to hire and layoff workers,

$$\sum_{t=1}^T (C_{ht} H_t + C_{lt} L_t), \text{ where } H_t L_t = 0, \forall t, \text{ indicating that either}$$

net hiring or net firing of labor occurs during a period, but not both [14].

The following constraints are considered for each time period: (1) labor level constraints include (a)  $W_t \leq \tilde{W}_t^{max}, \forall t$ ,

indicating the workforce level should not be greater than the maximum available workforce level during any period; (b)  $W_t = W_{t-1} + H_t - L_t, \forall t$ ,

where  $H_t L_t = 0, \forall t$ , indicating the workforce level in period  $t$  should equal the workforce level in period  $(t-1)$  plus the new hires ( $H_t$ ) minus the layoffs ( $L_t$ ); (c)

$kP_{ot} \leq \delta W_t, \forall t$ , where  $k$  and  $\delta$  are conversion factors for transforming the unit of  $P_{ot}$  and  $W_t$  to man-hour, respectively

(referring to Table 1), indicating the regular time production capacity should not be greater than the available labor capacity;

(d)  $kP_{ot} \leq \beta_t \delta W_t, \forall t$ , indicating the variation of a workforce should not exceed the permitted level of a company's policy during any period. (2) capacity constraints include

$$P_{rt} + P_{ot} + I_{t-1} - B_{t-1} \geq F_t^{min}, \forall t, \text{ where } I_t B_t = 0, \forall t,$$

indicating that demand over a particular period can be either met or backordered, but not both. (3) inventory level constraints, according to a fundamental material quantity balance, include

$$I_{t-1} - B_{t-1} + P_{rt} + P_{ot} - I_t + B_t = F_t, \forall t, \text{ where}$$

$I_t B_t = 0, \forall t$ . This indicates that the inventory level or backorder level in period  $t$  is equal to what it was in period

$(t-1)$  plus the regular-time and overtime production and minus

the forecasted demand. This equation ensures that the amount sold of each product in a period plus the inventory (or backorder) at the end of the period equals the total supply consisting of inventory (or backorder) from the previous period plus the regular and overtime production in the current period [18]. (4) nonnegative constraints on decision variables,  $P_{rt}, P_{ot}, W_t, I_t, B_t, H_t, L_t \geq 0, \forall t$ .

By incorporating the objective function and all the above constraints, the fuzzy APP linear model can be formulated as follows [14]:

$$\begin{aligned} \tilde{Z} = \min. & \sum_{t=1}^T [C_{ot}(P_{rt} + P_{ot}) + C_{rt}W_t + C_{ot}(kP_{ot}) + C_{it}I_t + C_{bt}B_t + C_{ht}H_t + C_{lt}L_t] \\ \text{s.t. } & W_t \leq \tilde{W}_t^{\max}, \forall t; \\ & W_t - W_{t-1} - H_t + L_t = 0, \forall t; \\ & kP_{rt} - \delta W_t \leq 0, \forall t; \\ & kP_{ot} - \beta_t \delta W_t \leq 0, \forall t; \\ & P_{rt} + P_{ot} + I_{t-1} - B_{t-1} \geq F_t^{\min}, \forall t; \\ & I_{t-1} - B_{t-1} + P_{rt} + P_{ot} - I_t + B_t = F_t, \forall t; \\ & P_{rt}, P_{ot}, W_t, I_t, B_t, H_t, L_t \geq 0, \forall t. \end{aligned} \quad (1)$$

Note that it is not necessary to impose the two sets of constraints,  $I_t B_t = 0$  and  $H_t L_t = 0, \forall t$ , on Model (1), since according to the theory of LP, a basic solution to Model (1) having the special structure cannot include both  $I_t$  and  $B_t$ ,  $H_t$  and  $L_t$ , simultaneously [2].

### III. THE SOLUTION METHOD

In Model (1), the maximum workforce ( $\tilde{W}_t^{\max}$ ) is approximately known, and is represented by the following fuzzy number:

$$\tilde{W}_t^{\max} = \{(w_t^{\max}, \mu_{\tilde{W}_t^{\max}}(w_t^{\max}) \mid w_t^{\max} \in W_t^{\max}\}, \forall t,$$

where  $W_t^{\max}$  is the universal sets of the maximum workforce, and  $\mu_{\tilde{W}_t^{\max}}(w_t^{\max})$  is its corresponding membership function.

On the basis of Zadeh's extension principle [29,30], when  $\tilde{W}_t^{\max}$  is a fuzzy number, the minimum total cost  $\tilde{Z}$  is also a fuzzy number with membership function  $\mu_{\tilde{Z}}$  defined as follows:

$$\mu_{\tilde{Z}}(z) = \sup_{w_t^{\max} \in W_t^{\max}} \{\mu_{\tilde{W}_t^{\max}}(w_t^{\max}), \forall t \mid z = Z(w_t^{\max})\}, \quad (2)$$

where  $Z(w_t^{\max})$  is the crisp objective value of Model (1) with a fuzzy parameter degenerated to the crisp value  $w_t^{\max}$ . Although the membership function defined in (2) is theoretically correct, it is not in the usual form for practical use and is very complicated; it is even very difficult to imagine its shape.

In this paper the above difficult problem is tackled using parametric mathematical programming. A pair of mathematical programs is developed to find the  $\alpha$ -cuts of  $\mu_{\tilde{Z}}$  based on Zadeh's extension principle. This procedure is explained in the next section.

Clearly, from Definition (2) the membership function  $\mu_{\tilde{Z}}$  is also parameterized by  $\alpha$ . Thus one approach for constructing  $\mu_{\tilde{Z}}$  is to derive the  $\alpha$ -cuts of  $\tilde{Z}$ . The  $\alpha$ -cuts of  $\tilde{W}_t^{\max}$  are defined as follow [12,29]:

$$W_t^{\max}(\alpha) = \{w_t^{\max} \in W_t^{\max} \mid \mu_{\tilde{W}_t^{\max}}(w_t^{\max}) \geq \alpha\}, \quad \forall t. \quad (3)$$

Note that  $W_t^{\max}(\alpha)$  are crisp sets rather than fuzzy sets. On the basis of the convexity of fuzzy numbers, the  $\alpha$ -cuts defined in Equations (3) can be expressed in the following form:

$$\begin{aligned} W_t^{\max}(\alpha) = & [\min_{w_t^{\max} \in W_t^{\max}} \{w_t^{\max} \mid \mu_{\tilde{W}_t^{\max}}(w_t^{\max}) \geq \alpha\}, \\ & \max_{w_t^{\max} \in W_t^{\max}} \{w_t^{\max} \mid \mu_{\tilde{W}_t^{\max}}(w_t^{\max}) \geq \alpha\}] = [(w_t^{\max})_{\alpha}^L, (w_t^{\max})_{\alpha}^U]. \end{aligned} \quad (4)$$

These intervals respectively indicate where the forecasted demands and the maximum workforce lie at possibility level  $\alpha$ .

According to the Definition (2),  $\mu_{\tilde{Z}}$  is the supremum of  $\mu_{\tilde{W}_t^{\max}}(w_t^{\max})$  for all  $t$ . One needs  $\mu_{\tilde{W}_t^{\max}}(w_t^{\max}) \geq \alpha$ , and at least one of  $\mu_{\tilde{W}_t^{\max}}(w_t^{\max})$ , for all  $t$ , to be equal to  $\alpha$  such that  $z = Z(w_t^{\max})$  to satisfy  $\mu_{\tilde{Z}}(z) = \alpha$ . To derive the  $\alpha$ -cuts of  $\tilde{Z}$ , it is sufficient to find the left and right shape function of  $\mu_{\tilde{Z}}$ . This is equivalent to find the lower bound  $Z_{\alpha}^L$  and the upper bound  $Z_{\alpha}^U$  of  $\mu_{\tilde{Z}}$ . Clearly,  $Z_{\alpha}^L$  is the minimum of  $z = Z(w_t^{\max})$  and  $Z_{\alpha}^U$  is the maximum of  $z = Z(w_t^{\max})$ , which can be expressed as follows:

$$Z_{\alpha}^L = \min \{Z(w_t^{\max}) \mid (W_t^{\max})_{\alpha}^L \leq w_t^{\max} \leq (W_t^{\max})_{\alpha}^U\}, \quad (5a)$$

$$Z_{\alpha}^U = \max \{Z(w_t^{\max}) \mid (W_t^{\max})_{\alpha}^L \leq w_t^{\max} \leq (W_t^{\max})_{\alpha}^U\}. \quad (5b)$$

These two expressions can be formulated as a pair of two-level mathematical programs as follows:

$$Z_{\alpha}^L = \min_{(W_t^{\max})_{\alpha}^L \leq w_t^{\max} \leq (W_t^{\max})_{\alpha}^U} \left\{ \begin{array}{l} \min \sum_{t=1}^T [C_{pt} (P_{rt} + P_{ot}) + C_{rt} W_t \\ + C_{ot} (kP_{ot}) + C_{it} I_t \\ + C_{bt} B_t + C_{ht} H_t + C_{lt} L_t] \\ \text{s.t. } W_t - w_t^{\max} \leq 0, \quad \forall t, \\ W_t - W_{t-1} - H_t + L_t = 0, \quad \forall t, \\ kP_{rt} - \delta W_t \leq 0, \quad \forall t, \\ kP_{ot} - \beta_t \delta W_t \leq 0, \quad \forall t, \\ P_{rt} + P_{ot} + I_{t-1} - B_{t-1} \geq F_t^{\min}, \quad \forall t, \\ I_{t-1} - B_{t-1} + P_{rt} + P_{ot} - I_t + B_t = F_t, \quad \forall t, \\ P_{rt}, P_{ot}, W_t, I_t, B_t, H_t, L_t \geq 0, \quad \forall t. \end{array} \right. \quad (6a)$$

$$Z_{\alpha}^U = \max_{(W_t^{\max})_{\alpha}^L \leq w_t^{\max} \leq (W_t^{\max})_{\alpha}^U} \left\{ \begin{array}{l} \min \sum_{t=1}^T [C_{pt} (P_{rt} + P_{ot}) + C_{rt} W_t \\ + C_{ot} (kP_{ot}) + C_{it} I_t \\ + C_{bt} B_t + C_{ht} H_t + C_{lt} L_t] \\ \text{s.t. } W_t - w_t^{\max} \leq 0, \quad \forall t, \\ W_t - W_{t-1} - H_t + L_t = 0, \quad \forall t, \\ kP_{rt} - \delta W_t \leq 0, \quad \forall t, \\ kP_{ot} - \beta_t \delta W_t \leq 0, \quad \forall t, \\ P_{rt} + P_{ot} + I_{t-1} - B_{t-1} \geq F_t^{\min}, \quad \forall t, \\ I_{t-1} - B_{t-1} + P_{rt} + P_{ot} - I_t + B_t = F_t, \quad \forall t, \\ P_{rt}, P_{ot}, W_t, I_t, B_t, H_t, L_t \geq 0, \quad \forall t. \end{array} \right. \quad (6b)$$

Note that to satisfy  $\mu_z(z) = \alpha$  required by Definition (2), at least one of  $w_t^{\max}$  must hit the boundary of their  $\alpha$ -cuts.

Note that to satisfy  $\mu_z(z) = \alpha$  required by (2), at least one of  $f_t$  must hit the boundary of their  $\alpha$ -cuts.

To derive the lower bound of the objective value in Model (6a),  $w_t^{\max}$  can be directly set to its upper bound  $(W_t^{\max})_{\alpha}^U$  for all  $t$ , since it defines the largest feasible region as  $(W_t^{\max})_{\alpha}^L \leq w_t^{\max} \leq (W_t^{\max})_{\alpha}^U$ . Moreover, since the aim of this model is to find the minimum of all minimum objective values, the constraints of the first level can be inserted into the second level to transfer the two-level mathematical program of Model (6a) to the following classical one-level mathematical programs:

$$Z_{\alpha}^L = \min \sum_{t=1}^T [C_{pt} (P_{rt} + P_{ot}) + C_{rt} W_t + C_{ot} (kP_{ot}) + C_{it} I_t \\ + C_{bt} B_t + C_{ht} H_t + C_{lt} L_t] \\ \text{s.t. } W_t - (W_t^{\max})_{\alpha}^U \leq 0, \quad \forall t, \\ W_t - W_{t-1} - H_t + L_t = 0, \quad \forall t, \\ kP_{rt} - \delta W_t \leq 0, \quad \forall t, \\ kP_{ot} - \beta_t \delta W_t \leq 0, \quad \forall t, \\ P_{rt} + P_{ot} + I_{t-1} - B_{t-1} \geq F_t^{\min}, \quad \forall t, \\ I_{t-1} - B_{t-1} + P_{rt} + P_{ot} - I_t + B_t = F_t, \quad \forall t, \\ P_{rt}, P_{ot}, W_t, I_t, B_t, H_t, L_t \geq 0, \quad \forall t. \quad (7)$$

This model is a conventional linear program which can be solved easily. Since all  $w_t^{\max}$ ,  $\forall t$ , have been set to the upper bounds of their  $\alpha$ -cuts in this model,  $\mu_z(z) = \alpha$ , as required by (2) based on Zadeh's extension principle, is guaranteed.

On the other hand, since the second level of Model (6b) is a minimization problem which is inconsistent with the maximization operation of the first level, it is not possible to insert the constraints of the first level directly into the second level. To tackle this, the dual of the second-level problem, which is a maximization problem, is formulated to be consistent with the maximization operation of the first level. It is well-known from the duality theorem of linear programming that the primal model and the dual model have the same objective value [2]. Since  $(W_t^{\max})_{\alpha}^L \leq w_t^{\max} \leq (W_t^{\max})_{\alpha}^U$ ,  $\forall t$ , the upper bound of the objective value can be derived by setting  $w_t^{\max} = (W_t^{\max})_{\alpha}^L$ ,  $\forall t$ , which gives the largest feasible region. Moreover, since both the first level and the second level perform the same maximization operation, their constraints can be combined to form a classical mathematical program. Consequently, Model (6b) can be reformulated as:

$$Z_{\alpha}^U = \max \sum_{t=1}^T [- (W_t^{\max})_{\alpha}^L A_t - W_0 N_1 + F_t^{\min} G_t + F_t M_t] \\ \text{s.t. } kD_t + G_t + M_t \leq C_{pt}, \quad kE_t + G_t + M_t \leq C_{pt} + kC_{ot}, \quad \forall t, \\ -A_t - N_t + B_{t+1} + \delta D_t + \beta_t \delta E_t \leq C_{rt}, \quad t = 1, 2, \dots, T-1, \\ -A_T - N_T + \delta D_T + \beta_T \delta E_T \leq C_{rT}, \\ G_{t+1} - M_t + M_{t+1} \leq C_{it}, \quad t = 1, 2, \dots, T-1, \\ -M_T \leq C_{iT}, \quad G_T \leq C_{iT}, \\ -G_{t+1} + M_t - M_{t+1} \leq C_{bt}, \quad t = 1, 2, \dots, T-1, \\ N_t \leq C_{ht}, \quad -N_t \leq C_{lt}, \quad \forall t, \\ A_t, D_t, E_t, G_t \geq 0, \quad N_t, M_t \text{ unrestricted}, \quad \forall t, \quad (8)$$

where  $A_t, N_t, D_t, E_t, G_t$ , and  $M_t, t = 1, 2, \dots, T$ , are the sets of dual variables defined for the first to sixth sets of constraints in Model (6b), respectively. Model (8) is a linearly constrained nonlinear program that can be solved by several efficient and effective approaches [3]. Since all  $w_t^{\max}, \forall t$ , have been set to the lower bounds of their  $\alpha$ -cut in this model, this guarantees that  $\mu_z(z) = \alpha$  as required by (2).

Solving Models (7) and (8) gives the crisp interval  $[Z_\alpha^L, Z_\alpha^U]$ , which is the  $\alpha$ -cut of  $\tilde{Z}$ . An attractive feature of the  $\alpha$ -cut approach is that all  $\alpha$ -cuts form a nested structure with respect to  $\alpha$  [11]. According to Zadeh's extension principle [29,30],  $\tilde{Z}$  defined in (2) is a fuzzy number which possesses convexity [12]. That is, given  $\alpha_1, \alpha_2 \in [0, 1]$  with  $\alpha_1 > \alpha_2$ , the feasible regions defined by  $\alpha_1$  in Models (7a) and (7c) are smaller than those defined by  $\alpha_2$ . Consequently,  $Z_{\alpha_1}^L \geq Z_{\alpha_2}^L$  and  $Z_{\alpha_1}^U \geq Z_{\alpha_2}^U$ , i.e.,  $Z_\alpha^L$  is non-decreasing with respect to  $\alpha$ , and  $Z_\alpha^U$  is non-increasing with respect to  $\alpha$ . Consequently, the membership function  $\tilde{Z}$  can be constructed from the solutions of Models (7) and (8).

In most cases Models (7) and (8) are so complicated that they cannot be solved analytically, the set of intervals  $\{[Z_\alpha^L, Z_\alpha^U] | \alpha \in [0, 1]\}$ , which can be obtained numerically, shows the shape of  $\mu_z$  by enumerating different value of  $\alpha$ .

The modeling and solution procedure proposed in this paper can be expanded to multi-product fuzzy APP. Here we briefly describe the modeling. For example, the parameter of  $P_n$ , the regular-time production in period  $t$  (units) becomes  $P_{nt}^n$ , indicating the regular-time production of the  $n$ th product in period  $t$  (units). Consequently, Model (1) becomes

$$\tilde{Z} = \min. \sum_{n=1}^N \sum_{t=1}^T [C_{pn}^n (P_{nt}^n + P_{on}^n) + C_{wn}^n W_{nt}^n + C_{on}^n (k_n P_{on}^n) + C_{in}^n I_{nt}^n + C_{bn}^n B_{nt}^n + C_{hn}^n H_{nt}^n + C_{ln}^n L_{nt}^n]$$

s.t.  $W_{nt}^n \leq (\bar{W}_t^{\max})_n, \forall t, n;$   
 $W_{nt}^n - W_{n,t-1}^n - H_{nt}^n + L_{nt}^n = 0, \forall t, n;$   
 $\tilde{k}_n P_{on}^n - \delta W_{nt}^n \leq 0, \forall t, n;$   
 $\tilde{k}_n P_{on}^n - \beta W_{nt}^n \leq 0, \forall t, n;$   
 $P_{nt}^n + P_{on}^n + I_{n,t-1}^n - B_{n,t-1}^n \geq (F_t^{\min})_n, \forall t, n;$   
 $I_{n,t-1}^n - B_{n,t-1}^n + P_{nt}^n + P_{on}^n - I_{nt}^n + B_{nt}^n = F_{nt}^n, \forall t, n;$   
 $P_{nt}^n, P_{on}^n, W_{nt}^n, I_{nt}^n, B_{nt}^n, H_{nt}^n, L_{nt}^n \geq 0, \forall t, n.$

Then it can be solved by using the proposed approach.

Note that the minimum total cost derived by using the proposed approach is expressed by a membership function rather than by a crisp value; i.e., it is a fuzzy performance measure. The benefit and significance of such a fuzzy performance measure are that it maintains the fuzziness of input information completely, thus it can represent the vague systems more accurately. This indicates that the proposed approach can obtain more realistic performance measures when some data in the APP model are ambiguous. Furthermore, following Lai and

Hwang [14], only two parameters are assumed to be fuzzy; it is clear that the proposed approach is also applicable to APP problems with more fuzzy parameters.

Although both of the proposed approach and Chanas's approach are based on FLP, they are significantly different on methodology and obtained results. Aiming at completely conserving all the fuzziness of fuzzy parameters such as market demands and maximum workforces, the proposed approach is developed based on a combination of Zadeh's extension principle, LP formulation of the APP problem,  $\alpha$ -cut representation, two-level mathematical programming, and parametric programming. The results obtained from Chanas's approach only showed a crisp minimum total cost for each different possibility level, and the corresponding values of decision variables [14]. Since the crisp solution could be underestimated, it might be happened that the over-optimistic solution provided let the decision maker make a wrong decision, and vice versa. As Tang et al. [22] pointed out, the sole optimal solution to APP is no guarantee of achieving the feasible disaggregate plan. On the contrary, much more important information is obtained by the proposed approach, including the lower and upper bounds of the minimum total costs and the corresponding values of decision variables for each possibility level  $\alpha$ . Compared with Chanas's approach, the results shown in this paper are more reasonable according to fuzzy sets, and conserve the fuzziness of market demands and maximum workforces completely.

#### IV. CONCLUSION AND FUTURE WORK

From a different viewpoint, this paper proposes a method to finding the membership function of the fuzzy minimum total cost of APP problems with fuzzy parameters. The idea is based on the concept of  $\alpha$ -cuts and Zadeh's extension principle to transform the fuzzy APP model to a family of crisp APP models which can be described by a pair of mathematical programs. The bounds of the  $\alpha$ -cuts of the fuzzy minimum total cost for different possibility levels  $\alpha$  are calculated to derive the approximated membership function, and the corresponding optimal aggregate production plans are also provided. Compared with other studies, the proposed approach can obtain more reasonable solution for imprecise/fuzzy parameters, and so more wide-range decision information on alternative strategies for overtime, inventory, backorder, and hiring and layoffs workers is provided for decision-makers in response to variations in fuzzy environments. Thus the feasible disaggregate plan can be achieved afterwards. Although single-product APP models with one fuzzy parameter were investigated in this paper, it is clear that the proposed solution procedure is not confined to APP models of this type. Future research directions include applying this method to APP models with multiple products and more fuzzy parameters, and other APP models with different structures, such as multi-site, multiple objectives, different goal priorities, resource utilization constraint, aggregate production-distribution planning in SCM.

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