Analysis for a food chain model with Crowley–Martin functional response and time delay

Kejun Zhuang, Zhaohui Wen

Abstract—This paper is concerned with a nonautonomous three species food chain model with Crowley–Martin type functional response and time delay. Using the Mawhin's continuation theorem in theory of degree, sufficient conditions for existence of periodic solutions are obtained.

Keywords—Periodic solutions; coincidence degree; food chain model; Crowley–Martin functional response.

I. INTRODUCTION

THE dynamic behaviors of food chain systems have received more and more attention due to their universal existence and importance. Many kinds of these models have been extensively investigated [1], [2], [3], [4], [5], [6]. All these studies depend on the classical types of functional responses, such as Holling types, Mechaelies–Menten ratio–dependent type, Beddington–DeAngelis type, Hassell–Varley type and so on. As far as we know, there are very few literatures to discuss the population dynamics with Crowley–Martin type functional response [7], [8], [9], [10]. The Crowley–Martin type functional response is classified as one of predator–dependent functional response. It is assumed that predator–feeding rate decreases by higher predator density even when prey density is high, and therefore the effects of predator interference in feeding rate remain important all the time whether an individual predator is handling or searching for a prey at a given instant of time.

Recently, R.K. Upadhyay and R.K. Naji have studied a three species food chain model with Crowley–Martin type functional response in [7] in the form of

$$
\begin{cases}
\dot{X}(t) = a_1 X \left(1 - \frac{X}{K}\right) - \frac{wXY}{X+D}, \\
\dot{Y}(t) = -a_2 Y + \frac{w_1 XY}{X+D_1} - \frac{w_2 YZ}{1+dY+bZ+bdYZ}, \\
\dot{Z}(t) = -cZ + \frac{w_3 YZ}{1+dY+bZ+bdYZ},
\end{cases}
\tag{1}
$$

where all the parameters are positive constants. The prey X grows with intrinsic growth rate a_1 and carrying capacity K in the absence of predation; D and D_1 measure the extent to which environment provide protection to prey X and Y, respectively; w is the maximum value which per capita reduction rate of X can attain, w_1 has a similar meaning to w. The constants w_2, w_3, b and d are the saturating Crowley–Martin functional response parameters, in which b measures

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the magnitude of interference among predator. Besides, a_2 is the death rate of the intermediate predator and c is the death rate of the top predator.

For system (1), the stability and persistence conditions were established and bifurcation diagrams were obtained in [7]. Further, chaotic behaviors have been derived with the help of numerical results [8]. In addition, local and global stability for a predator–prey model with Crowley–Martin function and stage structure was explicitly discussed [9]. It is apparent to all that time delay is an important factor in biological systems. Also, the effect of environmental changes cannot be ignored.

In this paper, we mainly focus on the following nonautonomous food chain system with Crowley–Martin functional response and time delay:

$$
\begin{cases}
\dot{x}(t) = x(t)\left(a_1(t) - b_1(t)x(t) - \frac{w(t)y(t)}{x(t)+D(t)}\right), \\
\dot{y}(t) = y(t)\left(-a_2(t) + \frac{w_1(t)x(t)}{x(t)+D_1(t)}\right. \\
\qquad\qquad \left. - \frac{w_2(t)z(t)}{1+d(t)y(t)+b(t)z(t)+b(t)d(t)y(t)z(t)}\right), \\
\dot{z}(t) = -c(t)z(t) \\
\qquad + \frac{w_3(t)y(t-\tau)z(t)}{1+d(t)y(t-\tau)+b(t)z(t-\tau)+b(t)d(t)y(t-\tau)z(t-\tau)},
\end{cases}
\tag{2}
$$

here all the coefficients are the positive $\omega-$periodic functions and time delay τ is the positive constant. The main purpose of this paper is to explore the existence of periodic solutions for system (2).

II. PRELIMINARIES

For convenience, we first present the preliminary results we shall use, more details can be found in [11], [12]. From the main theorem in [11], we can easily obtain the following lemma.

Lemma 2.1. Let t_1, $t_2 \in [0, \omega]$ and $t \in \mathbf{R}$. If $g : \mathbf{R} \to \mathbf{R}$ is $\omega-$periodic, then

$$
g(t) \le g(t_1) + \frac{1}{2}\int_0^\omega |g'(t)|dt
$$

and

$$
g(t) \ge g(t_2) - \frac{1}{2}\int_0^\omega |g'(t)|dt,
$$

where the constant factor $1/2$ is the best possible.

For simplicity, we use the following notations throughout this paper:

$$
I_\omega = [0, \omega], \quad \bar{g} = \frac{1}{\omega}\int_{I_\omega} g(t)dt = \frac{1}{\omega}\int_0^\omega g(t)dt,
$$

$$
f^M = \max_{t\in I_\omega} f(t), \quad f^L = \min_{t\in I_\omega} f(t).
$$

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Now, we introduce some concepts and a useful result from [12]. Let X, Z be normed vector spaces, $L : \mathrm{Dom}\, L \subset X \to Z$ be a linear mapping, $N : X \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim \ker L = \mathrm{codim}\, \mathrm{Im}\, L < +\infty$ and $\mathrm{Im}\, L$ is closed in Z. If L is a Fredholm mapping of index zero and there exist continuous projections $P : X \to X$ and $Q : Z \to Z$ such that $\mathrm{Im}\, P = \ker L$, $\mathrm{Im}\, L = \ker Q = \mathrm{Im}(I - Q)$, then it follows that $L|\, \mathrm{Dom}\, L \cap \ker P : (I - P)X \to \mathrm{Im}\, L$ is invertible. We denote the inverse of that map by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I - Q)N : \bar{\Omega} \to X$ is compact. Since $\mathrm{Im}\, Q$ is isomorphic to $\ker L$, there exists an isomorphism $J : \mathrm{Im}\, Q \to \ker L$.

Next, we state the Mawhin's continuation theorem, which is a main tool in the proof of our theorem.

Lemma 2.2.[12] (Continuation Theorem) Let L be a Fredholm mapping of index zero and N be L-compact on $\bar{\Omega}$. Suppose

(a) for each $\lambda \in (0, 1)$, every solution u of $Lu = \lambda Nu$ is such that $u \notin \partial\Omega$;

(b) $QNu \neq 0$ for each $u \in \partial\Omega \cap \ker L$ and the Brouwer degree $\deg\{JQN, \Omega \cap \ker L, 0\} \neq 0$.

Then the operator equation $Lu = Nu$ has at least one solution lying in $\mathrm{Dom}\, L \cap \bar{\Omega}$.

III. EXISTENCE OF PERIODIC SOLUTIONS

Theorem 3.1. If the condition

$$w_1^L \exp\{L_1\}/(\exp\{M_1\} + D_1^M) > a_2^M$$

is satisfied, where $M_1 = \ln(a_1^M/b_1^L) + \omega\bar{a}_1$ and $L_1 = \ln(a_2^L D_1^L/w_1^M) - \omega\bar{a}_1$. Then system (2) has at least one $\omega-$periodic solution.

Proof Set $x(t) = \exp\{u_1(t)\}$, $y(t) = \exp\{u_2(t)\}$, $z(t) = \exp\{u_3(t)\}$, then system (2) can be reduced to the following form,

$$
\begin{cases}
\dot{u}_1(t) = a_1(t) - b_1(t)e^{u_1(t)} - \dfrac{w(t)e^{u_2(t)}}{e^{u_1(t)} + D(t)}, \\
\dot{u}_2(t) = -a_2(t) + \dfrac{w_1(t)e^{u_1(t)}}{e^{u_1(t)} + D_1(t)} \\
\qquad - \dfrac{w_2(t)e^{u_3(t)}}{1 + d(t)e^{u_2(t)} + b(t)e^{u_3(t)} + b(t)d(t)e^{u_2(t)+u_3(t)}}, \\
\dot{u}_3(t) = -c(t) + \\
\dfrac{w_3(t)e^{u_2(t-\tau)}}{1 + d(t)e^{u_2(t-\tau)} + b(t)e^{u_3(t-\tau)} + b(t)d(t)e^{u_2(t-\tau)+u_3(t-\tau)}}.
\end{cases}
$$
(3)

Then we only need to prove the existence of periodic solutions for system (3).

Let $X = Z = \{(u_1, u_2, u_3)^T \in C(\mathbf{R}, \mathbf{R}^3) : u_i(t + \omega) = u_i(t),\ i = 1, 2, 3, \forall t \in \mathbf{R}\}$, $\|(u_1, u_2, u_3)^T\| = \sum_{i=1}^{3} \max_{t \in I_\omega} |u_i(t)|$, $(u_1, u_2, u_3)^T \in X$ (or in Z). Then X and Z are both Banach spaces when they are endowed

with the above norm $\|\cdot\|$. Let

$$
N \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}
$$

$$
= \begin{bmatrix}
a_1(t) - b_1(t)e^{u_1(t)} - \dfrac{w(t)e^{u_2(t)}}{e^{u_1(t)} + D(t)} \\
-a_2(t) + \dfrac{w_1(t)e^{u_1(t)}}{e^{u_1(t)} + D_1(t)} - \dfrac{w_2(t)e^{u_3(t)}}{1 + d(t)e^{u_2(t)} + b(t)e^{u_3(t)} + b(t)d(t)e^{u_2(t)+u_3(t)}} \\
-c(t) + \dfrac{w_3(t)e^{u_2(t-\tau)}}{1 + d(t)e^{u_2(t-\tau)} + b(t)e^{u_3(t-\tau)} + b(t)d(t)e^{u_2(t-\tau)+u_3(t-\tau)}}
\end{bmatrix}
$$

$$
:= \begin{bmatrix} N_1(t) \\ N_2(t) \\ N_3(t) \end{bmatrix},
$$

$$
L \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \dot{u}_1(t) \\ \dot{u}_2(t) \\ \dot{u}_3(t) \end{bmatrix}, \quad
P \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = Q \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega}\int_0^\omega u_1(t)dt \\ \frac{1}{\omega}\int_0^\omega u_2(t)dt \\ \frac{1}{\omega}\int_0^\omega u_3(t)dt \end{bmatrix}.
$$

Then it follows that $\ker L = \big\{(u_1, u_2, u_3)^T \in X : (u_1(t), u_2(t), u_3(t))^T = (h_1, h_2, h_3)^T \in \mathbf{R}^3, t \in \mathbf{R}\big\}$, $\mathrm{Im}\, L = \big\{(u_1, u_2, u_3)^T \in Z : \bar{u}_1 = \bar{u}_2 = \bar{u}_3 = 0, t \in \mathbf{R}\big\}$, $\dim \ker L = 3 = \mathrm{codim}\, \mathrm{Im}\, L$.

Since $\mathrm{Im}\, L$ is closed in Z, then L is a Fredholm mapping of index zero. It is easy to show that P and Q are continuous projections such that $\mathrm{Im}\, P = \ker L$ and $\mathrm{Im}\, L = \ker Q = \mathrm{Im}(I - Q)$. Furthermore, the generalized inverse (of L) $K_P : \mathrm{Im}\, L \to \ker P \cap \mathrm{Dom}\, L$ exists and is given by

$$
K_P \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \int_0^t u_1(s)ds - \frac{1}{\omega}\int_0^\omega \int_0^t u_1(s)dsdt \\ \int_0^t u_2(s)ds - \frac{1}{\omega}\int_0^\omega \int_0^t u_2(s)dsdt \\ \int_0^t u_3(s)ds - \frac{1}{\omega}\int_0^\omega \int_0^t u_3(s)dsdt \end{bmatrix}.
$$

Thus

$$
QN \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega}\int_0^\omega (N_1(s))\, ds \\ \frac{1}{\omega}\int_0^\omega (N_2(s))\, ds \\ \frac{1}{\omega}\int_0^\omega (N_3(s))\, ds \end{bmatrix},
$$

and

$$
K_P(I - Q)N \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix}
\int_0^\omega N_1(s)ds - \frac{1}{\omega}\int_0^\omega \int_0^t N_1(s)dsdt \\ \qquad - \left(\frac{1}{2} - \frac{1}{\omega}\right)\int_0^t N_1(s)ds, \\
\int_0^\omega N_2(s)ds - \frac{1}{\omega}\int_0^\omega \int_0^t N_2(s)dsdt \\ \qquad - \left(\frac{1}{2} - \frac{1}{\omega}\right)\int_0^t N_2(s)ds, \\
\int_0^\omega N_3(s)ds - \frac{1}{\omega}\int_0^\omega \int_0^t N_3(s)dsdt \\ \qquad - \left(\frac{1}{2} - \frac{1}{\omega}\right)\int_0^t N_3(s)ds,
\end{bmatrix}.
$$

Clearly, QN and $K_P(I - Q)N$ are continuous. According to the Arzela-Ascoli theorem, it is not difficulty to show that $K_P(I - Q)N(\bar{\Omega})$ is compact for any open bounded set $\Omega \subset X$ and $QN(\bar{\Omega})$ is bounded. Thus, N is L-compact on $\bar{\Omega}$.

Now, we shall search an appropriate open bounded subset Ω for the application of the continuation theorem, Lemma 2.2. For the operator equation $Lu = \lambda Nu$, where $\lambda \in (0, 1)$, we have

$$
\begin{cases}
\dot{u}_1(t) = \lambda \left(a_1(t) - b_1(t)e^{u_1(t)} - \dfrac{w(t)e^{u_2(t)}}{e^{u_1(t)} + D(t)} \right), \\
\dot{u}_2(t) = \lambda \left(-a_2(t) + \dfrac{w_1(t)e^{u_1(t)}}{e^{u_1(t)} + D_1(t)} \right. \\
\qquad \left. - \dfrac{w_2(t)e^{u_3(t)}}{1 + d(t)e^{u_2(t)} + b(t)e^{u_3(t)} + b(t)d(t)e^{u_2(t)+u_3(t)}} \right), \\
\dot{u}_3(t) = -\lambda c(t) + \\
\dfrac{\lambda w_3(t)e^{u_2(t-\tau)}}{1 + d(t)e^{u_2(t-\tau)} + b(t)e^{u_3(t-\tau)} + b(t)d(t)e^{u_2(t-\tau)+u_3(t-\tau)}}.
\end{cases}
$$
(4)

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Assume that $(u_1, u_2, u_3)^T \in X$ is a solution of system (4) for a certain $\lambda \in (0,1)$. Integrating (4) on both sides from 0 to ω, we obtain

$$
\begin{cases}
\int_0^\omega \left(b_1(t)e^{u_1(t)}\right) dt + \int_0^\omega \frac{w(t)e^{u_2(t)}}{e^{u_1(t)}+D(t)} dt = \bar{a}_1\omega, \\
\int_0^\omega \frac{w_1(t)e^{u_1(t)}}{e^{u_1(t)}+D_1(t)} dt = \bar{a}_2\omega \\
\quad + \int_0^\omega \frac{w_2(t)e^{u_3(t)}}{1+d(t)e^{u_2(t)}+b(t)e^{u_3(t)}+b(t)d(t)e^{u_2(t)+u_3(t)}} dt, \\
\int_0^\omega \frac{w_3(t)e^{u_2(t-\tau)}}{1+d(t)e^{u_2(t-\tau)}+b(t)e^{u_3(t-\tau)}+b(t)d(t)e^{u_2(t-\tau)+u_3(t-\tau)}} dt = \bar{c}\omega.
\end{cases}
\tag{5}
$$

Since $(u_1, u_2, u_3)^T \in X$, there exist $\xi_i, \eta_i \in I_\omega$, $i = 1, 2, 3$, such that

$$
u_i(\xi_i) = \min_{t \in I_\omega}\{u_i(t)\}, u_i(\eta_i) = \max_{t \in I_\omega}\{u_i(t)\}, i = 1, 2, 3. \tag{6}
$$

From (4) and (5), we have

$$
\int_0^\omega |\dot{u}_1(t)|\, dt \leq 2\bar{a}_1\omega,
$$

$$
\int_0^\omega |\dot{u}_2(t)|\, dt \leq 2\bar{w}_1\omega,
$$

$$
\int_0^\omega |\dot{u}_3(t)|\, dt \leq 2\bar{c}\omega.
$$

By the third equation of (5) and (6), we have

$$
\omega c(\eta_3) \leq \int_0^\omega w_3(\eta_3)e^{u_2(\eta_3-\tau)} dt
$$

and

$$
\omega c(\xi_3) \leq \int_0^\omega \frac{w_3(\xi_3)e^{u_2(\xi_3-\tau)}}{b(\xi_3)d(\xi_3)e^{u_2(\xi_3-\tau)+u_3(\xi_3-\tau)}} dt,
$$

thus,

$$
u_2(\eta_2) \geq u_2(\eta_3 - \tau) \geq \ln \frac{c^L}{w_3^M}
$$

and

$$
u_3(\xi_3) \leq u_3(\xi_3 - \tau) \leq \ln \frac{w_3^M}{c^L b^L d^L}.
$$

According to Lemma 2.1, we have the following estimations:

$$
\begin{aligned}
u_2(t) &\geq u_2(\eta_2) - \frac{1}{2}\int_0^\omega |\dot{u}_2(t)|\, dt \\
&\geq \ln \frac{c^L}{w_3^M} - \omega\bar{w}_1 := L_2,
\end{aligned}
$$

and

$$
\begin{aligned}
u_3(t) &\leq u_3(\xi_3) + \frac{1}{2}\int_0^\omega |\dot{u}_3(t)|\, dt \\
&\leq \ln \frac{w_3^M}{c^L b^L d^L} + \omega\bar{c} := M_3.
\end{aligned}
$$

By the first equation of (5), it follows that

$$
b_1(\xi_1)e^{u_1(\xi_1)} \leq a_1(\xi_1)
$$

and

$$
\begin{aligned}
e^{u_2(\xi_2)} &\leq e^{u_2(\eta_1)} \\
&\leq \frac{a_1(\eta_1)(e^{u_1(\eta_1)}+D(\eta_1))}{w(\eta_1)},
\end{aligned}
$$

which imply

$$
u_1(\xi_1) \leq \ln \frac{a_1^M}{b_1^L}
$$

and

$$
u_2(\xi_2) \leq \ln \frac{a_1^M(e^{M_1}+D^M)}{w^L}.
$$

Therefore, we have

$$
\begin{aligned}
u_1(t) &\leq u_1(\xi_1) + \frac{1}{2}\int_0^\omega |\dot{u}_1(t)|\, dt \\
&\leq \ln \frac{a_1^M}{b_1^L} + \omega\bar{a}_1 := M_1.
\end{aligned}
$$

and

$$
\begin{aligned}
u_2(t) &\leq u_2(\xi_2) + \frac{1}{2}\int_0^\omega |\dot{u}_2(t)|\, dt \\
&\leq \ln \frac{a_1^M(e^{M_1}+D^M)}{w^L} + \omega\bar{w}_1 := M_2.
\end{aligned}
$$

From the second equation of (5), we obtain

$$
a_2(\eta_2) \leq \frac{w_1(\eta_2)e^{u_1(\eta_2)}}{e^{u_1(\eta_2)}+D_1(\eta_2)} < \frac{w_1(\eta)e^{u_1(\eta_2)}}{D_1(\eta_2)}
$$

and

$$
\frac{w_1(\xi_2)e^{u_1(\xi_2)}}{e^{u_1(\xi_2)}+D_1(\xi_2)} \leq a_2(\xi_2) + w_2(\xi_2)e^{u_3(\xi_2)},
$$

which reduce to

$$
u_1(\eta_1) \geq u_1(\eta_2) > \ln \frac{a_2^L D_1^L}{w_1^M}
$$

and

$$
u_3(\eta_3) \geq u_3(\xi_2) \geq \ln \frac{\frac{w_1^L e^{L_1}}{e^{M_1}+D_1^M} - a_2^M}{w_2^M}.
$$

Then we have

$$
\begin{aligned}
u_1(t) &\geq u_1(\eta_1) - \frac{1}{2}\int_0^\omega |\dot{u}_1(t)|\, dt \\
&\geq \ln \frac{a_2^L D_1^L}{w_1^M} - \omega\bar{a}_1 := L_1
\end{aligned}
$$

and

$$
\begin{aligned}
u_3(t) &\geq u_3(\eta_3) - \frac{1}{2}\int_0^\omega |\dot{u}_3(t)|\, dt \\
&\geq \ln \frac{\frac{w_1^L e^{L_1}}{e^{M_1}+D_1^M} - a_2^M}{w_2^M} - \omega\bar{c} := L_3,
\end{aligned}
$$

From above, we can get

$$
\max_{t \in [0,\omega]} |u_1(t)| \leq \max\{|M_1|, |L_1|\} := R_1,
$$
$$
\max_{t \in [0,\omega]} |u_2(t)| \leq \max\{|M_2|, |L_2|\} := R_2,
$$
$$
\max_{t \in [0,\omega]} |u_3(t)| \leq \max\{|M_3|, |L_3|\} := R_3.
$$

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Clearly, R_1, R_2 and R_3 are independent of λ. Let $R = R_1 + R_2 + R_3 + R_0$, where R_0 is taken sufficiently large such that for for the following algebraic equations:

$$
\begin{cases}
\bar{a}_1 - \bar{b}_1 e^{u_1} - \frac{1}{\omega} \int_0^\omega \frac{w(t)e^{u_2}}{e^{u_1} + D_1(t)} dt = 0, \\
\bar{a}_2 - \frac{1}{\omega} \int_0^\omega \frac{w_1(t)e^{u_1}}{e^{u_1} + D_1(t)} dt \\
\quad + \frac{1}{\omega} \int_0^\omega \frac{w_2(t)e^{u_3}}{1 + d(t)e^{u_2} + b(t)e^{u_3} + b(t)d(t)e^{u_2+u_3}} dt = 0, \\
\bar{c} - \frac{1}{\omega} \int_0^\omega \frac{w_3(t)e^{u_2}}{1 + d(t)e^{u_2} + b(t)e^{u_3} + b(t)d(t)e^{u_2+u_3}} dt = 0,
\end{cases}
\tag{7}
$$

every solution $(u_1^*, u_2^*, u_3^*)^T$ of (7) satisfies $\|(u_1^*, u_2^*, u_3^*)^T\| < R$. Now, we define

$$\Omega = \{(u_1, u_2, u_3)^T \in X : \|(u_1, u_2, u_3)^T\| < R\}.$$

Then it is clear that Ω verifies the requirement (a) of Lemma 2.2. If $(u_1, u_2, u_3)^T \in \partial\Omega \cap \ker L = \partial\Omega \cap \mathbb{R}^3$, then $(u_1, u_2, u_3)^T$ is a constant vector in $\mathbb{R}^3$ with $\|(u_1, u_2, u_3)^T\| = |u_1| + |u_2| + |u_3| = R$, so we have

$$
QN \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.
$$

By the assumption in Theorem 3.1 and the definition of topological degree, the invariance of homotopy produces $\deg(JQN, \Omega \cap \ker L, 0) \neq 0$. We have verified that Ω satisfies all requirements of Lemma 2.2; therefore, system (2) has at least one ω-periodic solution in $\operatorname{Dom} L \cap \bar{\Omega}$. This completes the proof.

IV. CONCLUSION

This paper has introduced a novel nonautonomous food chain system with Crowley–Martin type functional response and time delay. The existence of periodic solutions has been explored in detail, by means of coincidence degree theory. The main results show that the three species will vary periodically under certain conditions.

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