

Modified Fast and Exact Algorithm for Fast Haar Transform

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Abstract—Wavelet transform or wavelet analysis is a recently developed mathematical tool in applied mathematics. In numerical analysis, wavelets also serve as a Galerkin basis to solve partial differential equations. Haar transform or Haar wavelet transform has been used as a simplest and earliest example for orthonormal wavelet transform. Since its popularity in wavelet analysis, there are several definitions and various generalizations or algorithms for calculating Haar transform. Fast Haar transform, FHT, is one of the algorithms which can reduce the tedious calculation works in Haar transform. In this paper, we present a modified fast and exact algorithm for FHT, namely Modified Fast Haar Transform, MFHT. The algorithm or procedure proposed allows certain calculation in the process decomposition be ignored without affecting the results.

Keywords—Fast Haar Transform, Haar transform, Wavelet analysis.

I. INTRODUCTION

WAVELET transform or wavelet analysis is a recently developed mathematical tool for signal analysis. To date, wavelets have been applied in numerous disciplines such as image compression, data compression, denoising data and many more [1]. In numerical analysis, wavelets also serve as a Galerkin basis to solve partial differential equations. Wavelet analysis involves tedious calculations. Practically, the calculation is done by using software with certain commands or special toolboxes. It may make the beginner feel intimidate [2]. Hence, Haar function always has been choose for educational purpose, especially in many papers or books written on topic of introduction to wavelets [3],[4]. Anyway, for simple data or signal, there are sufficient if ones use Haar function as the basis function for decomposition, threshold and reconstruction process. The process is so called Haar transform.

In this paper, we will look into the Haar transform and Fast Haar Transform, FHT. Section 2 will provide some of the related previous works. Section 3 describes about Haar transform and Fast Haar Transform, FHT. In Section IV, the proposed MFHT will be described and its properties will be

compared to existing method. Conclusion will be presented in Section V.

II. RELATED PREVIOUS WORKS

Since Haar transform has its great popularity in wavelet analysis, there are several definitions and various generalizations or algorithms have been used. Fast Haar transform, FHT is one of the algorithms to reduce the tedious calculation works in Haar transform. One of the earliest versions of FHT is in place Haar transform [5]. It is work with trying to reduce the memory requirements of the transform and the amount of inefficient movement of Haar coefficients. For this algorithm, there are necessary to consider the different ordering from the usual one. As shown in reference [6], multi-terminal binary decision diagrams, MTBDD, has been used to present the calculation of Haar transform. A decision nodes diagram has been used to explain the calculations. Basically, the result of calculation at each node is stored in two fields assigned to each non-terminal node, which we will refer the nodes as data and the fields are approximation and detail coefficients. In this works, calculation has been done by taking the summation and difference from two nodes. A similar FHT which involves addition, subtraction and division by 2 was presented and its application in atmospheric turbulence analysis, image analysis, signal and image compression has been discussed in [7].

With the purpose of remaining its property of reducing the memory requirements for FHT, in this paper, we present a modified fast and exact algorithm for FHT. We show the algorithm or may call as procedure through the decision diagram, MTBDD, as well. The algorithm or procedure proposed allows certain amount of inefficient movement of Haar coefficients in the process decomposition been ignored. Meanwhile, based on the MTBDD mentioned above, we present the modified method by using summation and difference from four nodes.

III. HAAR TRANSFORM AND FAST HAAR TRANSFORM

Haar wavelet is the simplest wavelet. Haar transform or Haar wavelet transform has been used as an earliest example for orthonormal wavelet transform with compact support. The Haar wavelet transform is the first known wavelet and was proposed in 1909 by Alfred Haar. The Haar function can be described as a step function $\psi(t)$ and in Fig. 1 as follow:

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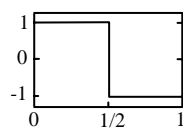
$$\psi(t) = \begin{cases} 1 & 0 \leq t \leq 0.5 \\ -1 & 0.5 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$


Fig. 1 Haar function

This is also called mother wavelet. In order to perform wavelet transform, Haar wavelet uses translations and dilations of the function, i.e. the transform make use of following function:

$$\psi_{ab}(t) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right) \text{ with } a, b \in \mathbf{R}, a \neq 0. \quad (2)$$

Translation / shifting $\psi(t) = \psi(t-b)$

Dilation / scaling $\psi(t) = \psi(t/a)$

where this is the basic works for wavelet expansion.

In Haar transform, basically, 2^n data (or we refer as nodes) has been used at n level. By taking average and difference from two nodes from previous level, approximate coefficients and detail coefficients for next level, $n-1, n-2, n-3, \dots$, of decomposition nodes are counted. The process is called Fast Haar Transform, FHT.

Example 1: Fig 2. shows calculation of the typical Fast Haar Transform, FHT, for $n=4$, given by the data

$$F = [2 \ 4 \ 6 \ 3 \ 1 \ 2 \ 5 \ 9 \ 8 \ 13 \ 15 \ 7 \ 8 \ 9 \ 0 \ 4]^T$$

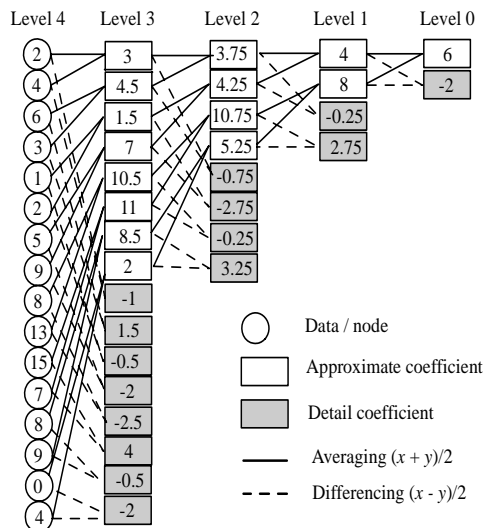


Fig. 2 Typical Fast Haar Transform, FHT

In Fig. 2, generally, the process is called wavelet decomposition and the detail coefficients are normally called as wavelet transform coefficients where these nodes will be considered in threshold process as well as reconstruction

works in multi-resolution wavelet. In many applications especially signal processing, threshold wavelet coefficients can be done to clean out “unnecessary” details which are consider as noise. Then, the data can be obtained again through wavelet reconstruction. For the multi-resolution wavelet, the detail coefficients (wavelet transform coefficients) are needed to reconstruction the original data while the approximation coefficients are not necessary involved. Mathematically, we regard wavelet decomposition (analysis) and reconstruction (synthesis) as wavelet transform and inverse of wavelet transform.

IV. MODIFIED FAST AND EXACT ALGORITHM FOR FAST HAAR TRANSFORM

Since the reconstruction process in multi-resolution wavelet are not require approximation coefficients, except for the level 0. The coefficients can be ignored to reduce the memory requirements of the transform and the amount of inefficient movement of Haar coefficients. As FHT, we use 2^N data.

Example 2: Fig 3. shows the calculation of the Modified Fast Haar Transform, MFHT, for $N=4$, given by the data

$$F = [2 \ 4 \ 6 \ 3 \ 1 \ 2 \ 5 \ 9 \ 8 \ 13 \ 15 \ 7 \ 8 \ 9 \ 0 \ 4]^T$$

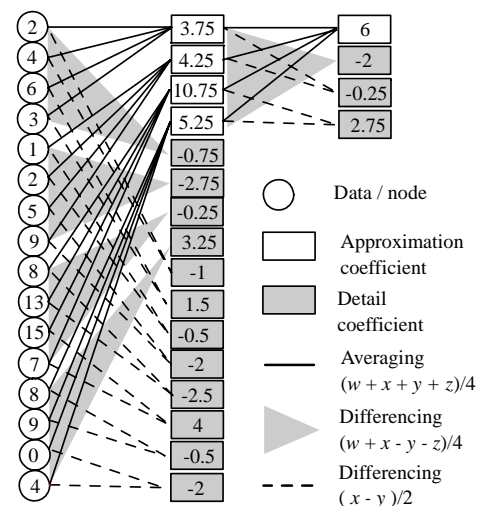


Fig. 3 Proposed Modified Fast Haar Transform, MFHT

For Modified Fast Haar Transform, MFHT, its can be done by just taking $(w+x+y+z)/4$ instead of $(x+y)/2$ for approximation and $(w+x-y-z)/4$ instead of $(x-y)/2$ for differencing process. 4 nodes have been considered at once time. Notice that the calculation for $(w+x-y-z)/4$ will yield the detail coefficients in the level of $n-2$. (Fig. 3)

For the purpose of getting detail coefficients, differencing process $(x-y)/2$ still need to be done. The decomposition step can be done by using matrix formulation as well (in (3)).

(3)

For coefficients at level $N - \theta$, where $\theta = 1, \dots, \text{int}(\frac{N}{2})$

(approximate coefficients),

(detail coefficients at level $n-2$),

(detail coefficients at level $n-1$).

Table I shows the comparison between typical Fast Haar transform, FHT and proposed Modified Fast Haar Transform, MFHT for $N = 4$.

Procedure	Number of operation		Movement	Approximate coefficients
	Addition / subtraction	Multiplication /division		
FHT	30	30	30	15
MFHT	40	20	20	5

to reduce the memory requirements of the transform and the amount of inefficient movement of Haar coefficients. The drawback in the number of addition and subtraction operation can be balanced by the decreasing in number of division operation. In fact, the total number of operation for 16 data is still remains the same. Table II shows the comparison between FHT and MFHT for $N = 8$.

Procedure	Number of operation		Number of	
	Addition / subtraction	Multiplication /division	Movement	Approximate coefficients
FHT	510	510	510	255
MFHT	850	340	340	85

The step plot illustrates the distribution of the number of non-zero elements in the product of two sparse matrices. The x-axis represents the number of non-zero elements, and the y-axis represents the cumulative count. The plot shows a series of steps, with the largest step occurring at 256 non-zero elements.

Fig. 4 Block diagram for FHT and MFHT for 256 data

For the reconstruction process without involving multi-resolution wavelet analysis, the fear of neglecting the approximation coefficients can be overcome by applying a

different procedure as in FHT. Example 3 shows how its work.

Example 3: Refer Fig. 2, reconstruct part of wavelet coefficients, $[6 \ -2 \ -0.25 \ 2.75]$.

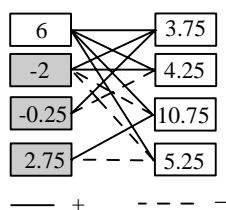


Fig. 5 Part of the reconstruction process

From the Fig. 5, the reconstruction process special deal with the MFHT shows that there are not necessary to consider all the approximate coefficients as in FHT. The similar way can be conducted to obtain the original data, without affecting the results.

V. CONCLUSION

The proposed MFHT not only allows certain calculation in the process decomposition be ignored without affecting the results, but also still remain in simple form of calculation as for FHT. The proposed MFHT works based on the idea that approximate coefficients can be neglected since it is not involve in the reconstruction work as well as threshold process in multi-resolution wavelet analysis.

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