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Efficient Detection Using Sequential Probability Ratio Test in Mobile Cognitive Radio Systems

Yeon-Jea Cho, Sang-Uk Park, Won-Chul Choi and Dong-Jo Park

Abstract—This paper proposes a smart design strategy for a sequential detector to reliably detect the primary user's signal, especially in fast fading environments. We study the computation of the log-likelihood ratio for coping with a fast changing received signal and noise sample variances, which are considered random variables. First, we analyze the detectability of the conventional generalized log-likelihood ratio (GLLR) scheme when considering fast changing statistics of unknown parameters caused by fast fading effects. Secondly, we propose an efficient sensing algorithm for performing the sequential probability ratio test in a robust and efficient manner when the channel statistics are unknown. Finally, the proposed scheme is compared to the conventional method with simulation results with respect to the average number of samples required to reach a detection decision.

Keywords—Cognitive radio, fast fading, sequential detection, spectrum sensing.

I. Introduction

SPECTRUM sensing is a core concept of cognitive radio networks [3], [4] for ensuring that cognitive radios do not cause harmful interference to primary user networks. Much of the related research on sensing and detection methods [5], [6] has already been conducted including sequential method devised by Wald [2]. In this paper, we consider the sequential method to reduce the average number of samples required to reach a detection decision by using the sequential probability ratio test (SPRT) and a practical energy detector. Spectrum sensing for cognitive radio systems considering fast fading is of interest in mobile communication research. The cooperative sequential detection scheme for reducing the average sensing time in cognitive radio networks has been well studied by Qiyue Zou [1]; composite hypotheses using the generalized log-likelihood ratio (GLLR) are well handled in [1] in terms of independent and identically distributed (i.i.d.) samples acquired by the detector.

In this paper, we use an energy detector for SPRT; the sensing is based on the difference between the received signal and noise variances. Therefore, the statistics of the acquired signal and noise samples depend on the received signal and noise power level, respectively. Here, we design the sequential detector for spectrum sensing when the samples acquired by cognitive radios are independent but not identically distributed (i.n.i.d.). This is called fast fading and is caused by the effect of fast changing channel characteristics in mobile cognitive radio systems. More generally, we allow not only the received signal statistics but also the noise statistics to be fast changing due to noise power uncertainty and characteristics of the mobile radio system.

Throughout this paper, we adopt a system model similar to that in [1] that consists of M cognitive radios and assume that the nth acquired sample by the mth (m=1,2,...,M) cognitive radio is a zero mean Gaussian random variable with received signal and noise variances $v_{1,m}(n)$ and $v_{0,m}(n)$ ($v_{1,m}(n) > v_{0,m}(n)$), i.e.,

$$H_i : \mathbf{x}_m[n] \sim \frac{1}{\sqrt{2\pi v_{i,m}(n)}} \exp\left(-\frac{(\mathbf{x}_m[n])^2}{2v_{i,m}(n)}\right), \quad i = 0, 1, \tag{1}$$

where the two hypotheses are defined as

$$H_0 : \text{target signal is absent}$$
$$H_1 : \text{target signal is present.}$$

The nth instantaneous sample variances under H_0 and H_1 are $v_{0,m}(n)$ and $v_{1,m}(n)$, respectively. In this fast fading environment, unknown instantaneous variances $v_{0,m}(n) \in V_{0,m}$ and $v_{1,m}(n) \in V_{1,m}$ are considered to be random variables for n=1,2,...,N and m=1,2,...,M whose statistics are determined from the channel characteristics, and $p(v_{0,m})$ and $p(v_{1,m})$ are the probability density functions of $v_{0,m}(n)$ and $v_{1,m}(n)$, respectively.

We assume that the parameter spaces $V_{0,m}$ and $V_{1,m}$ are disjoint, where

$$V_{0,m} = \{x | L_{0,m} \leq x \leq U_{0,m}\} \tag{2}$$

and

$$V_{1,m} = \{y | L_{1,m} \leq y \leq U_{1,m}\}. \tag{3}$$

The distributions of $v_{i,m}(n)$ (n=1,2,...,N) by the same cognitive radio are the same over the long term. But, the distributions of $v_{i,m}(n)$ (m=1,2,...,M) between the other cognitive radios can be different from each other. Under H_0 and H_1, the distributions of the acquired signal at the mth radio are characterized by the probability density functions $p_{0,m}(\mathbf{x}_m[n]; v_{0,m}(n))$ and $p_{1,m}(\mathbf{x}_m[n]; v_{1,m}(n))$, respectively.

II. Sequential Detection in Fast Fading Environments

A. Sequential Detection using GLLR

As explained in Section I, the received signal and noise variances change continuously during sensing. In this fast fading environment, an ideal sequential probability ratio test

The authors are with the Department of Electrical Engineering, Korea Advanced Institute of Science and Technology (KAIST), Daejeon 305-701, Korea (e-mail: yeonjeacho@kaist.ac.kr; sangukpark@kaist.ac.kr; wc-choi@kaist.ac.kr; djpark@kaist.ac.kr).

(SPRT) must perform the following test:

1) The mth(m=1,2,...,M) cognitive radio acquires sample $\mathbf{x}_m[N]$ and computes $\ln\left(\frac{p_{1,m}(\mathbf{x}_m[N];v_{0,m}(N))}{p_{0,m}(\mathbf{x}_m[N];v_{0,m}(N))}\right)$.

2) The base station updates the sequential log-likelihood ratio LLR_N^{ideal} according to

$$\begin{aligned} &LLR_N^{ideal} \\ &= LLR_{N-1}^{ideal} + \sum_{m=1}^{M} \ln\left(\frac{p_{1,m}(\mathbf{x}_m[N];v_{0,m}(N))}{p_{0,m}(\mathbf{x}_m[N];v_{0,m}(N))}\right). \end{aligned} \tag{4}$$

3) If $LLR_N^{ideal} \le \delta_0$, H_0 is accepted; if $LLR_N^{ideal} \ge \delta_1$, H_1 is accepted, where δ_0 and δ_1 are conceptual thresholds.

4) Otherwise, take one more sample and repeat 1) to 4).

However, $v_{i,m}(n)$ (i=0,1, m=1,2,...,M, n=1,2,...,N) is not a deterministic value but a random variable, so an exact computation of LLR_N^{ideal} is impossible. Thus, we try to perform the SPRT using the GLLR by replacing $v_{i,m}(n)$ with their maximum likelihood estimates and we analyze how this scheme works with respect to the robustness of this sensing method. We have

$$GLLR_N = \sum_{n=1}^{N}\sum_{m=1}^{M} \ln\left(\frac{p_{1,m}\left(\mathbf{x}_m[n];\hat{y}_m^{(N)}\right)}{p_{0,m}\left(\mathbf{x}_m[n];\hat{x}_m^{(N)}\right)}\right), \tag{5}$$

where $\hat{x}_m^{(N)}$ and $\hat{y}_m^{(N)}$ are the maximum likelihood estimates of x_m and y_m, i.e.,

$$\begin{aligned} \hat{x}_m^{(N)} &= \arg\max_{x_m \in V_{0,m}} \sum_{n=1}^{N} \ln p_{0,m}(\mathbf{x}_m[n];x_m) \\ \hat{y}_m^{(N)} &= \arg\max_{y_m \in V_{1,m}} \sum_{n=1}^{N} \ln p_{1,m}(\mathbf{x}_m[n];y_m). \end{aligned}$$

Although the acquired samples are not identically distributed (received signal and noise variances are not fixed) in fast fading environments, the maximum likelihood estimates converge to the following values:

1) under H_0, $\hat{x}_m^{(N)}$ converges to $\mathbf{E}\{v_{0,m}(n)\}$ and $\hat{y}_m^{(N)}$ converges to $L_{1,m}$.
2) under H_1, $\hat{x}_m^{(N)}$ converges to $U_{0,m}$ and $\hat{y}_m^{(N)}$ converges to $\mathbf{E}\{v_{1,m}(n)\}$.

For example, under H_0, Fig. 1 shows the convergence of the maximum likelihood estimate of x_m. The first figure shows that the instantaneous noise variance $v_{0,m}(n)$ for n=1,2,...,N is fast changing according to the each sample time n. The second figure shows that, in this environment, the maximum likelihood estimate $\hat{x}_m^{(N)}$ converges to $\mathbf{E}\{v_{0,m}\}$. The corresponding proof of the above concepts is following:

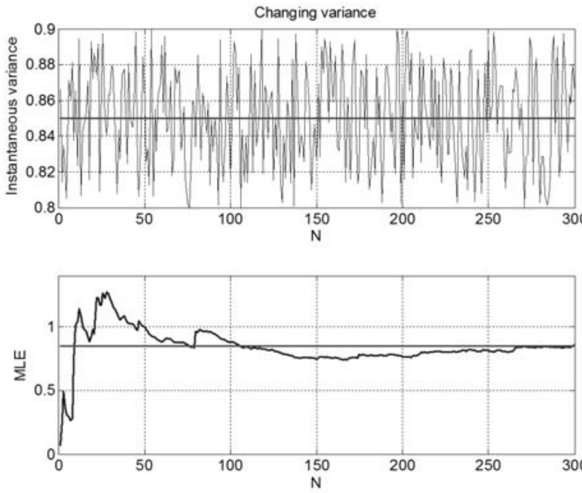

Fig. 1. The convergence of the maximum likelihood estimate of x_m. The first figure shows that the instantaneous noise variance $v_{0,m}(n)$ for n=1,2,...,N is fast changing and the second figure shows that the maximum likelihood estimate $\hat{x}_m^{(N)}$ converges to $\mathbf{E}\{v_{0,m}(n)\}$. $v_{0,m}(n)$ has a uniform distribution and the above grey horizontal line indicates $\mathbf{E}\{v_{0,m}(n)\}(=0.85)$. $V_{0,m} = \{x|0.8 \le x \le 0.9\}$.

$$\begin{aligned} \hat{x}_m^{(N)} &= \arg\max_{x_m \in V_{0,m}} \sum_{n=1}^{N} \ln p_{0,m}(\mathbf{x}_m[n];x_m) \\ &= \arg\max_{x_m \in V_{0,m}} \mathbf{E}_{v_{0,m}}\left\{\frac{1}{N}\sum_{n=1}^{N}\frac{\ln p_{0,m}(\mathbf{x}_m[n];x_m)}{\ln p_{0,m}(\mathbf{x}_m[n];v_{0,m}(n))}\right\} \end{aligned} \tag{6}$$

The distributions of $\mathbf{x}_m[n]$ (n=1,2,...,N) depend on the nth instantaneous signal variance $v_{0,m}(n)$, respectively. By the law of large numbers, as $N \to \infty$,

$$\begin{aligned} &\mathbf{E}_{v_{0,m}}\left\{\frac{1}{N}\sum_{n=1}^{N}\ln\left(\frac{p_{0,m}(\mathbf{x}_m[n];x_m)}{p_{0,m}(\mathbf{x}_m[n];v_{0,m}(n))}\right)\right\} \\ &= \mathbf{E}_{v_{0,m}}\left\{\int p_{0,m}(\mathbf{x}_m[n];v_{0,m}(n)) \right. \\ &\qquad \left. \times \ln\left(\frac{p_{1,m}(\mathbf{x}_m[n];x_m)}{p_{0,m}(\mathbf{x}_m[n];v_{0,m}(n))}\right) d\mathbf{x}_m[n]\right\} \\ &= \frac{1}{2}\left(\mathbf{E}\{\ln v_{0,m}(n)\} - \frac{\mathbf{E}\{v_{0,m}(n)\}}{x_m} - \ln x_m + 1\right). \end{aligned} \tag{7}$$

Then, we differentiate the above result with respect to x_m to find $\hat{x}_m^{(N)}$.

$$\begin{aligned} &\frac{d}{dx_m}\mathbf{E}_{v_{0,m}}\left\{\frac{1}{N}\sum_{n=1}^{N}\ln\left(\frac{p_{0,m}(\mathbf{x}_m[n];x_m)}{p_{0,m}(\mathbf{x}_m[n];v_{0,m}(n))}\right)\right\} \\ &\qquad = \frac{\mathbf{E}\{v_{0,m}(n)\}}{2{x_m}^2} - \frac{1}{2x_m} = 0, \end{aligned} \tag{8}$$

From the above equation, the solution is $\mathbf{E}\{v_{0,m}(n)\}$. Therefore, under H_0, $\hat{x}_m^{(N)}$ converges to $\mathbf{E}\{v_{0,m}(n)\}$, $\hat{y}_m^{(N)}$ converges to $L_{1,m}$ by the pre-known bound from the parameter spaces, and the corresponding $GLLR_{N,m}$ is expressed by $\sum_{n=1}^{N} \ln\left(\frac{p_{1,m}(\mathbf{x}_m[n];L_{1,m})}{p_{0,m}(\mathbf{x}_m[n];E\{v_{0,m}\})}\right)$. Similarly, under H_1, $\hat{x}_m^{(N)}$ converges to $U_{0,m}$, $\hat{y}_m^{(N)}$ converges to $\mathbf{E}\{v_{1,m}(n)\}$, and the corresponding $GLLR_{N,m}$ is expressed by $\sum_{n=1}^{N} \ln\left(\frac{p_{1,m}(\mathbf{x}_m[n];E\{v_{1,m}\})}{p_{0,m}(\mathbf{x}_m[n];U_{0,m})}\right)$. Thus, we can compute the expectation of the related log-likelihood ratio to verify the distinguishability of two hypotheses as in the following:

$$\begin{aligned}
&\mathbf{E}_{H_0}\left\{\ln\left(\frac{p_{1,m}(\mathbf{x}_m[n];L_{1,m})}{p_{0,m}(\mathbf{x}_m[n];\mathbf{E}\{v_{0,m}\})}\right)\right\} \\
&= \int p(v_{0,m})\cdot\int p_{0,m}(\mathbf{x}_m[n];v_{0,m}(n)) \\
&\qquad \times \ln\left(\frac{p_{1,m}(\mathbf{x}_m[n];L_{1,m})}{p_{0,m}(\mathbf{x}_m[n];\mathbf{E}\{v_{0,m}\})}\right) d\mathbf{x}_m[n]\,dv_{0,m} \\
&= \frac{1}{2}\left(\ln\frac{\mathbf{E}\{v_{0,m}\}}{L_{1,m}} - \frac{\mathbf{E}\{v_{0,m}\}}{L_{1,m}} + 1\right) < 0,
\end{aligned} \tag{9}$$

where $\mathbf{E}\{v_{0,m}\} < L_{1,m}$.

Similarly,

$$\begin{aligned}
&\mathbf{E}_{H_1}\left\{\ln\left(\frac{p_{1,m}(\mathbf{x}_m[n];\mathbf{E}\{v_{1,m}\})}{p_{0,m}(\mathbf{x}_m[n];U_{0,m})}\right)\right\} \\
&= \int p(v_{1,m})\cdot\int p_{1,m}(\mathbf{x}_m[n];v_{1,m}(n)) \\
&\qquad \times \ln\left(\frac{p_{1,m}(\mathbf{x}_m[n];\mathbf{E}\{v_{1,m}\})}{p_{0,m}(\mathbf{x}_m[n];U_{0,m})}\right) d\mathbf{x}_m[n]\,dv_{0,m} \\
&= \frac{1}{2}\left(-\ln\frac{\mathbf{E}\{v_{1,m}\}}{U_{0,m}} + \frac{\mathbf{E}\{v_{1,m}\}}{U_{0,m}} - 1\right) > 0,
\end{aligned} \tag{10}$$

where $\mathbf{E}\{v_{1,m}\} > U_{0,m}$.

The conditions (9) and (10) guarantee that the two hypotheses are also distinguishable by using the conventional GLLR scheme in fast fading environments and also ensure the detectability by [1, *Lemma 2.1*].

B. Efficient detection with unknown $p(v_{0,m})$ and $p(v_{1,m})$

It is known that we do no use the conventional GLLR scheme, to select the proper received signal and noise variances, which are used to compute $LLR^*_{N,m}$ for the SPRT to also ensure conditions (9) and (10). Here, we define

$$LLR^*_{N,m} = \sum_{n=1}^{N} \ln\left(\frac{p_{1,m}(\mathbf{x}_m[n];y^*_m)}{p_{0,m}(\mathbf{x}_m[n];x^*_m)}\right), \tag{11}$$

where the representative values x^*_m and y^*_m are set by force. If $p(v_{0,m})$ and $p(v_{1,m})$ are known, it may be possible to choose optimal x^*_m and y^*_m values with respect to the average number of samples required to reach a detection decision. In real mobile systems, it can be very difficult to find the exact probability density functions of $v_{0,m}$ and $v_{1,m}$. Therefore, we propose a practical sensing algorithm to efficiently cope with unknown $p(v_{0,m})$ and $p(v_{1,m})$.

The proposed sequential sensing algorithm is efficiently applicable to fast fading environment and is summarized in **Algorithm 1**. We verified that the conventional SPRT

Algorithm 1 The Proposed Cooperative Sequential Sensing Algorithm in Fast Fading Environment

0: **Mode1(initial state)**
1: Set η_0 and η_1 are pre-defined values.
2: The mth (m=1,2,…,M) cognitive radio performs the SPRT by using the GLLR.
3: When H_0 is accepted, store the samples acquired by the mth (m=1,2,…,M) cognitive radio and accumulate these samples in $tmp_m^{(0)}$.
4: When H_1 is accepted, store the samples acquired by the mth (m=1,2,…,M) cognitive radio and accumulate these samples in $tmp_m^{(1)}$.
5: **If** the total number of samples in $tmp_m^{(0)}$ is larger than η_0 and the total number of samples in $tmp_m^{(1)}$ is larger than η_1, **go to Mode2**.
6: Otherwise, **repeat** steps 2 to 6.
7: **Mode2(The use of $LLR^*_{N,m}$)**
8: Compute $\hat{x}_m^{(\eta_0)} = \underset{x_m \in V_{0,m}}{\arg\max} \sum_{n=1}^{\eta_0} \ln p_{0,m}\left(\mathbf{x}_{tmp_0}^{(m)}[n];x_m\right)$ by using the samples in $tmp_m^{(0)}$.
9: Compute $\hat{y}_m^{(\eta_1)} = \underset{y_m \in V_{1,m}}{\arg\max} \sum_{n=1}^{\eta_1} \ln p_{1,m}\left(\mathbf{x}_{tmp_1}^{(m)}[n];y_m\right)$ by using the samples in $tmp_m^{(1)}$.
10: Use the values x^*_m and y^*_m to compute $LLR^*_{N,m} = \sum_{n=1}^{N} \ln\left(\frac{p_{1,m}(\mathbf{x}_m[n];y^*_m)}{p_{0,m}(\mathbf{x}_m[n];x^*_m)}\right)$, where $x^*_m = \hat{x}_m^{(\eta_0)}$ and $y^*_m = \hat{y}_m^{(\eta_1)}$.
11: The mth (m=1,2,…,M) cognitive radio performs the SPRT by using the $LLR^*_{N,m}$.
12: When H_0 is accepted, update the samples in $tmp_m^{(0)}$.
13: When H_1 is accepted, update the samples in $tmp_m^{(1)}$.
14: **If** initialization command is executed, go to **Mode1**.
15: Otherwise, **repeat** steps 8 to 15.

using the GLLR is a detectable scheme especially in fast fading environments in Section II. A, from that fact, it is possible to perform the SPRT in the initial state by using the GLLR. In **Mode1**, $tmp_m^{(0)}$ and $tmp_m^{(1)}$ play an important role of accumulating samples needed to obtain the maximum likelihood estimates $\hat{x}_m^{(\eta_0)}$ and $\hat{y}_m^{(\eta_1)}$, where both η_0 and η_1 are the number of samples needed to obtain highly accurate estimates. In **Mode2**, $\hat{x}_m^{(\eta_0)}$ and $\hat{y}_m^{(\eta_1)}$ are obtained from the samples in $tmp_m^{(0)}$ and $tmp_m^{(1)}$, respectively, where

$$\hat{x}_m^{(\eta_0)} = \max\left\{\min\left\{\frac{1}{\eta_0}\sum_{n=1}^{\eta_0}\left(x_{tmp_0}^{(m)}[n]\right)^2, U_{0,m}\right\}, L_{0,m}\right\}, \tag{12}$$

$$\hat{y}_m^{(\eta_1)} = \max\left\{\min\left\{\frac{1}{\eta_1}\sum_{n=1}^{\eta_1}\left(x_{tmp_1}^{(m)}[n]\right)^2, U_{1,m}\right\}, L_{1,m}\right\}. \tag{13}$$

If η_0 and η_1 are large enough, $\hat{x}_m^{(\eta_0)}$ and $\hat{y}_m^{(\eta_1)}$ converge to $\mathbf{E}\{v_{0,m}(n)\}$ and $\mathbf{E}\{v_{0,m}(n)\}$, respectively. This fact is also

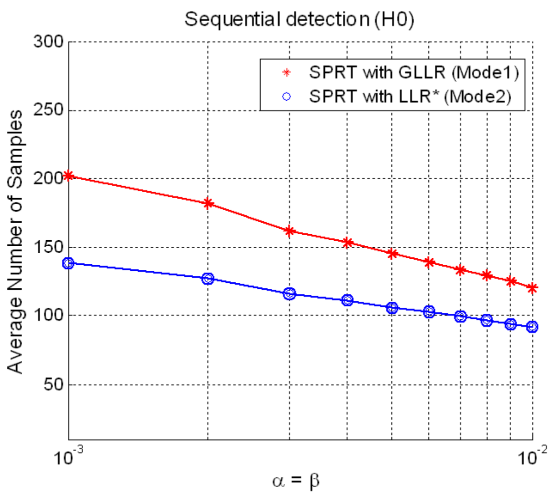

Fig. 2. Sequential detection in fast fading environments under H_0. The simulation parameters are listed in Table I and the detection thresholds δ_0 and δ_1 in the sequential method are determined through computer experiments.

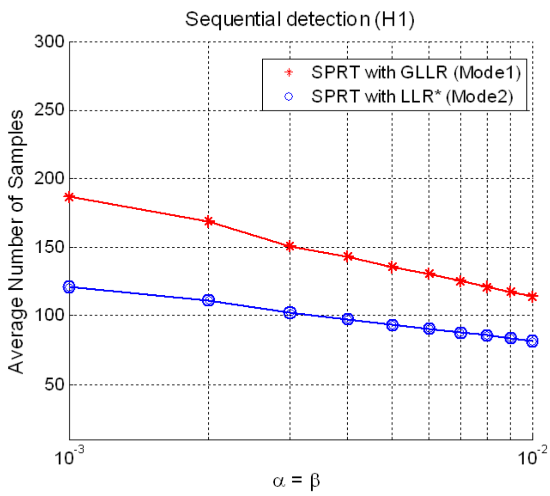

Fig. 3. Sequential detection in fast fading environments under H_1. The simulation parameters are listed in Table I and the detection thresholds δ_0 and δ_1 in the sequential method are determined through computer experiments.

proved in Section II. A. That is,

$$LLR^{*}_{N,m} \approx \sum_{n=1}^{N} \ln\left(\frac{p_{1,m}\left(\mathbf{x}_m\left[n\right];\mathbf{E}\left\{v_{1,m}\right\}\right)}{p_{0,m}\left(\mathbf{x}_m\left[n\right];\mathbf{E}\left\{v_{0,m}\right\}\right)}\right). \tag{14}$$

Basically, the mobile radio signal consists of a fast fading signal superimposed on a local mean value which remains constant over a small area. However, when the local mean value varies slowly, the samples in $tmp_m^{(0)}$ and $tmp_m^{(1)}$ are continuously updated as in steps 12 and 13 in Algorithm 1. In this proposed algorithm, we assume that false alarm and miss detection probabilities are sufficiently small such that the effect of unreliable samples induced by a false alarm and miss detection is negligible on calculating $\hat{x}_m^{(\eta_0)}$ and $\hat{y}_m^{(\eta_1)}$.

III. Simulation Results

To illustrate the efficiency of the proposed algorithm, simulations are conducted with respect to the average number of samples required to reach a detection decision. In this simulation, cooperative sensing using a sequential method is performed with four cognitive radios $(M = 4)$. The simulation parameters needed to describe the fast fading environments and used to determine η_0 and η_1 values are listed in Table I, and the received signal and noise variances $v_{1,m}\left(n\right)$ and $v_{0,m}\left(n\right)$ have a uniform distribution. Monte Carlo simulations are performed in both **Mode1** and **Mode2** for various values of δ_0 and δ_1 to find the thresholds that guarantee the pre-defined false alarm and miss detection constraints α and β. We use different values of δ_0 and δ_1 for different $\alpha = \beta$ because δ_0 and δ_1 depend on α and β [1]. The simulation results are shown in Fig. 2 and Fig. 3 under H_0 and H_1, respectively. Figure 2 and Figure 3 show that **Mode2** using $LLR^{*}_{N,m}$ is more efficient than **Mode1** using $GLLR_N$ [1] in fast fading environments with respect to average number of samples required to reach a detection decision. In addition, **Mode2** in Algorithm 1 has less computational complexity namely, $O\left(MN\right)$ than **Mode1**, which has computational complexity $O\left(MN^2\right)$.

TABLE I
THE SIMULATION PARAMETERS

	m=1	m=2	m=3	m=4
$L_{0,m}$	0.64	0.75	0.72	0.69
$U_{0,m}$	0.90	0.87	0.82	0.85
$L_{1,m}$	0.90	0.90	0.87	0.86
$U_{1,m}$	1.15	1.18	1.02	0.99
η_0	1000	1000	1000	1000
η_1	1000	1000	1000	1000

IV. Conclusion

We have proposed an efficient sensing algorithm to cope with fast fading by the mode change method. In Section II, we analyzed how the SPRT using the GLLR works with respect to the robustness of a sensing method, and then we proposed a more efficient sensing algorithm applicable to cognitive radio networks considering fast fading effect. Our simulation results show that **Mode2** using $LLR^{*}_{N,m}$ is more efficient than **Mode1** using $GLLR_N$ in fast fading environments with respect to average number of samples required to reach a detection decision. This paper can also give some intuition when studying a similar system model.

Acknowledgment

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