

# Acceleration Analysis of a Rotating Body

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**Abstract**—The velocity of a moving point in a general path is the vector quantity, which has both magnitude and direction. The magnitude or the direction of the velocity vector can change over time as a result of acceleration that the time rate of velocity changes. Acceleration analysis is important because inertial forces and inertial torques are proportional to rectilinear and angular accelerations accordingly. The loads must be determined in advance to ensure that a machine is adequately designed to handle these dynamic loads. For planar motion, the vector direction of acceleration is commonly separated into two elements: tangential and centripetal or radial components of a point on a rotating body. All textbooks in physics, kinematics and dynamics of machinery consider the magnitude of a radial acceleration at condition when a point rotates with a constant angular velocity and it means without acceleration. The magnitude of the tangential acceleration considered on a basis of acceleration for a rotating point. Such condition of presentation of magnitudes for two components of acceleration logically and mathematically is not correct and may cause further confusion in calculation. This paper presents new analytical expressions of the radial and absolute accelerations of a rotating point with acceleration and covers the gap in theoretical study of acceleration analysis.

**Keywords**—acceleration analysis, kinematics of mechanisms.

## I. INTRODUCTION

Machines and mechanisms running at a high speed are considered by a kinematic analysis of their velocities and accelerations, and subsequently analyzed as a dynamic system in which forces due to accelerations are analyzed using the principles of kinetics. Force and stress analysis is commonly based on the acceleration analysis of links and points of interest in the mechanism or machine.

Many methods and approaches exist to find accelerations in mechanisms [1]–[8]. Curvilinear motion of a body with a variable speed is going on with a linear acceleration, which is commonly separated into two elements: tangential and centripetal or radial components of acceleration. The tangential component is formed as a result of a change in the magnitude of the velocity vector and its direction is perpendicular to the line that connects the point with the center rotation. The radial component is created as result of a change in the direction of the velocity vector and its direction is always toward the center of a point rotation. The expression of an absolute acceleration,  $\alpha$  of a point is the vector sum of the tangential,  $\alpha_t$  and radial,  $\alpha_r$  components as shown by equation [1]–[8]:

$$\bar{\alpha} = \bar{\alpha}_t + \bar{\alpha}_r \quad (1)$$

where  $\alpha$  – absolute acceleration,  $\alpha_t$  – tangential acceleration, and  $\alpha_r$  – radial acceleration.

The tangential acceleration of a rotating point has the following expression [1]–[8]:

$$\alpha_t = \varepsilon r \quad (2)$$

where  $\varepsilon$  – an angular acceleration of rotating point,  $r$  – a radius of rotation of a point.

The radial acceleration of a rotating point has the following expression [1]–[8]:

$$\alpha_r = r\omega^2 \quad (3)$$

where  $\omega$  – an angular velocity of rotation of a point considered, other component presented above.

The expression of an absolute acceleration of a rotating point has the following equation [3], [4]:

$$\alpha = r\sqrt{\varepsilon^2 + \omega^4} \quad (4)$$

where components presented above.

Known expressions of a tangential acceleration (Eq. 2) and a radial one (Eq. 3) is presented in textbooks and handbooks [1]–[8]. In physics, an angular velocity of a rotating point with acceleration is variable at the time. Expression of a tangential acceleration (Eq. 2) produced at condition when a point rotates with an angular acceleration,  $\varepsilon$ . Eq. (3) of a radial acceleration produced at condition when an angular velocity,  $\omega$  of a rotating point is constant [1]–[8], however the angular velocity of a rotating point is variable due to its angular acceleration. At condition when an angular velocity is constant, the rotating point does not have a tangential acceleration.

This situation confuses users of textbooks and contains logical and mathematical error that results on the magnitude of centrifugal and inertial forces and on the magnitude of an absolute acceleration of a rotating point. Some textbooks remark that for calculation of acceleration is accepted an instant angular velocity of a rotating point [2], [4], However, this explanation does not give more clarity.

It is known that a rotation of a point around a center with an angular acceleration gives constantly increasing angular velocity of rotation one [1]–[8]. The magnitude of a centrifugal force and other forces will increase with increasing of an angular velocity. Incorrect calculation of magnitudes of



$$\alpha_r = \alpha_{r(ad)} + \alpha_{r(dc)} \quad (14)$$

The first component of Eq. (13) as derivative of the radial vector velocity with respect to time is the expression of the first component of radial acceleration of a rotating point. In Fig. 1, because the angle  $\theta$  is small, the following relationship can be stated

$$\frac{dV_{ad}}{dt} = V_a \frac{d\theta}{dt} \quad (15)$$

The second component of Eq. (13) one is additional expression of the radial acceleration of the rotating point, due to acceleration one. The equation for the second component of the radial acceleration (Eq. 13) is expressed in the following computation.

The second component of the radial velocity vector  $V_{dc}$  expressed from the vector triangle,  $dbc$  (Fig. 1).

$$V_{dc} = V_{db} \sin(\theta/2), \quad (16)$$

where  $V_{db} = \varepsilon r t$  from Eq. (11), and  $\theta/2$  is infinitely small.

After substituting and transformation, the second component of the radial velocity vector  $V_{dc}$  will have the following expression with two variables  $t$  and  $\theta$ .

$$V_{dc} = \varepsilon r t \theta / 2 \quad (17)$$

The first derivative of Eq. (17), with respect to time, gives expression of the second component of the radial acceleration

$$\frac{dV_{dc}}{dt} = \frac{d(\varepsilon r t \theta / 2)}{dt} \quad (18)$$

After substituting Eq. (15), Eq. (18) to Eq. (13) and transformation, the expression of the change of radial velocity vector  $V_r$  will have the following expression with two variables  $t$  and  $\theta$ .

$$\frac{dV_r}{dt} = V_a \frac{d\theta}{dt} + \frac{d(\varepsilon r \theta / 2)}{dt} \quad (19)$$

Using the relationships between linear velocity and angular velocity, the following equation for the radial acceleration of a rotating point can be derived

$$\alpha_r = r \omega_{in}^2 + \varepsilon r (\theta + \omega_e t) / 2 \quad (20)$$

where  $\omega_{in}$  - an initial angular velocity of a rotating point,  $\omega_e = d\theta/dt$  - an extra angular velocity of a rotating point due to acceleration one, and  $\theta = \omega_e t$

The first component of Eq. (20) is known expression of the radial acceleration of a rotating point at condition when an angular velocity is constant  $\alpha_{r(ad)} = r \omega_{in}^2$  [1]–[8].

The second component of Eq. (20), after substituting and transformation will have the following expression:  $\alpha_{r(dc)} = r \omega_e \varepsilon t$ , where  $\omega_e = \varepsilon t$ , then

$$a_{r(dc)} = r (\varepsilon t)^2 \quad (21)$$

The total expression of the radial acceleration of a rotating point is obtained after substituting Eq. (21) into Eq. (20), and following transformation gives the equation:

$$a_r = r [\omega_{in}^2 + (\varepsilon t)^2] \quad (22)$$

Obtained Eq. (22) of the total radial acceleration involves the initial angular velocity,  $\omega_{in}$  of a rotating point, the angular acceleration one,  $\varepsilon$ , a time  $t$  of rotation, and radius  $r$  of rotation of a point  $P$  (Fig. 1). A new formula will not confuse users in the calculation of the radial acceleration of a rotating point with a variable angular velocity. When an angular acceleration of a rotating point,  $\varepsilon = 0$ , the equation of a radial acceleration has expression of Eq. (3).

The equation of the absolute acceleration of a rotating point, with variable angular velocity, obtained after substituting Eq. (2) of the tangential acceleration and Eq. (22) of the radial acceleration into Eq. (1). Converting this equation into algebraic form and transformation gives the following expression:

$$a = r \sqrt{\varepsilon^2 + [\omega_{in}^2 + (\varepsilon t)^2]^2} \quad (23)$$

The new equation of an absolute acceleration of a rotating point is quite different from the known expression that is presented in textbooks, (Eq. 4[1]–[8]).

### III. CASE STUDY

The rotating point shown in Fig.1 has the initial angular velocity of 10 rad/s, accelerates at the rate of 3 rad/s<sup>2</sup> and has the radius of rotation of 0.1 m. It is necessary to determine the magnitudes of the tangential component and the radial component of accelerations and absolute one of the point  $P$  after 5 seconds of rotation.

Eq. (2) used to determine the tangential component of acceleration is:

$$a_t = \varepsilon r = (3)(0.1) = 0.3 \text{ m/s}^2$$

The new Eq. (22) used to determine the radial component of acceleration is:

$$a_r = r [\omega_{in}^2 + (\varepsilon t)^2] = 0.1 [10^2 + \{(3)(5)\}^2] = 32.5 \text{ m/s}^2$$

The absolute acceleration of the rotating point according to new Eq. (23) is:

$$a = r \sqrt{\varepsilon^2 + [\omega_{in}^2 + (\varepsilon t)^2]^2} = 0.1 \sqrt{3^2 + [10^2 + \{(3)(5)\}^2]^2} = 32.501 \text{ m/s}^2$$

The result of the radial acceleration obtained by the new Eq. (22) is based on real conditions of mechanism work and include changed an angular velocity of the rotating point. Known Eq. (3) for calculation of a radial acceleration, which is presented in any textbooks, includes initial value of the angular velocity of rotating point. This is difference in approach for calculation of the radial acceleration.

#### IV. RESULT AND DISCUSSION

Relevant equations presented for radial and absolute accelerations of a rotating point will enable the calculation of more accurate results as function of a radius of a rotating point, angular velocity, and angular acceleration one. These equations are different from known equations of radial and absolute accelerations presented in textbooks of physics, kinematics and dynamics of machinery. In engineering, all machines work with a variable velocity of rotating elements that is real condition of functioning of any mechanism. It is very important to calculate an exact result for rotating components with acceleration for a machine. New equations of radial and absolute accelerations give correct result compared to those presented in the textbooks [1]–[8].

#### V. SUMMARY

Analytical solution of acceleration analysis of a rotation point that is presented in textbooks is not perfect and cannot give reliable equations of radial and absolute accelerations. The main contribution of this research paper is in the development of analytical equations that give a correct result for calculation of the radial and absolute accelerations of a rotating point as function of a radius, angular velocity, and angular acceleration.

New equation of the radial acceleration of a rotating point and hence new equation of an absolute acceleration have been obtained. These equations will be useful in calculating the correct magnitude of the normal acceleration and absolute one for a rotating point. New expressions of the radial and absolute accelerations of a rotating point developed in this paper can be used by designers and manufacturers for the creation of reliable machines that are guaranteed to perform better and will not have unpredictable failure. New acceleration analysis of a rotating point should be used in textbooks of physics, kinematics and dynamics of machinery to give accurate mathematical method for producing equation of the radial and absolute accelerations and accurate results in calculation.

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