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Quadrature-based moment methods for polydisperse gas-liquid flows

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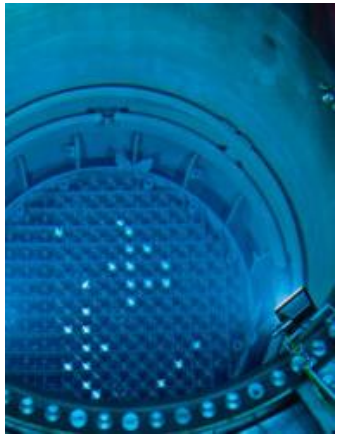
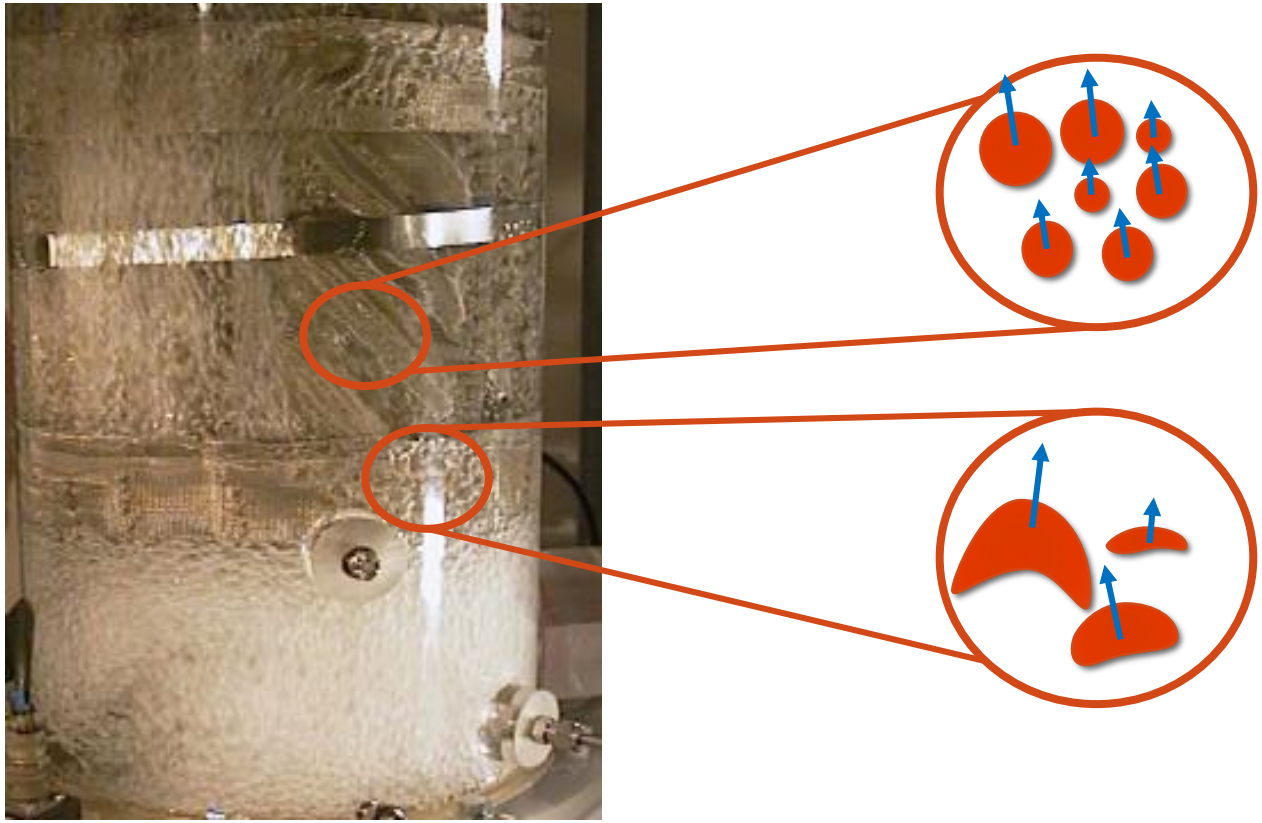


Motivation

Polydisperse gas-liquid flows are relevant to several engineering fields:

- Chemical
  - Bubble columns
  - Stirred-tank reactors
- Environment
  - Waste processing
- Energy
  - Boilers
  - Nuclear reactors
  - Production of biofuels
- Bioengineering
  - Fermentation

Challenges



Polydispersity
- Size distribution
- Shape distribution

Polycelerity
- Velocity of bubbles is a function of their size

Requirements of a CFD model for gas-liquid flows

Physical

- Account for the physically relevant force terms
- Describe the evolution of the bubble size distribution (polydispersity)
- Account for the difference in velocity for bubbles of different size (polycelerity)
- Must correctly degenerate in the monodisperse limit

Numerical

- Robustness in case of phase separation (regions with only one phase)
- Adaptivity to accurately capture significant changes in size distribution
- Moderate computational cost to allow adoption for engineering applications

Conventional Euler-Euler approaches

Two-fluid model + Population balance

- One continuity and momentum equation for each phase
- Accounts for bubble size distribution
- Assumes the same local velocity is shared by all sizes (no polycelerity)

Multi-fluid model + Population balance

- Multiple continuity and momentum equations for the gas phase
- One “virtual phase” per each size (or size group)
- Higher computational cost
- Exchange terms to account for size evolution

Kinetic equation for bubble populations [1]

The behavior of a population of bubbles can be described by means of a number density function (NDF) $f(\mathbf{x}, t, \xi, \mathbf{v})$:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{\partial}{\partial \mathbf{v}} \cdot \left[(\mathbf{A} + \mathbf{g}) f \; - \mathcal{D}_{\mathrm{dis}} \frac{\partial \ln(n)}{\partial \mathbf{x}} f \right] = \mathbb{C}(\xi, \mathbf{v})$$

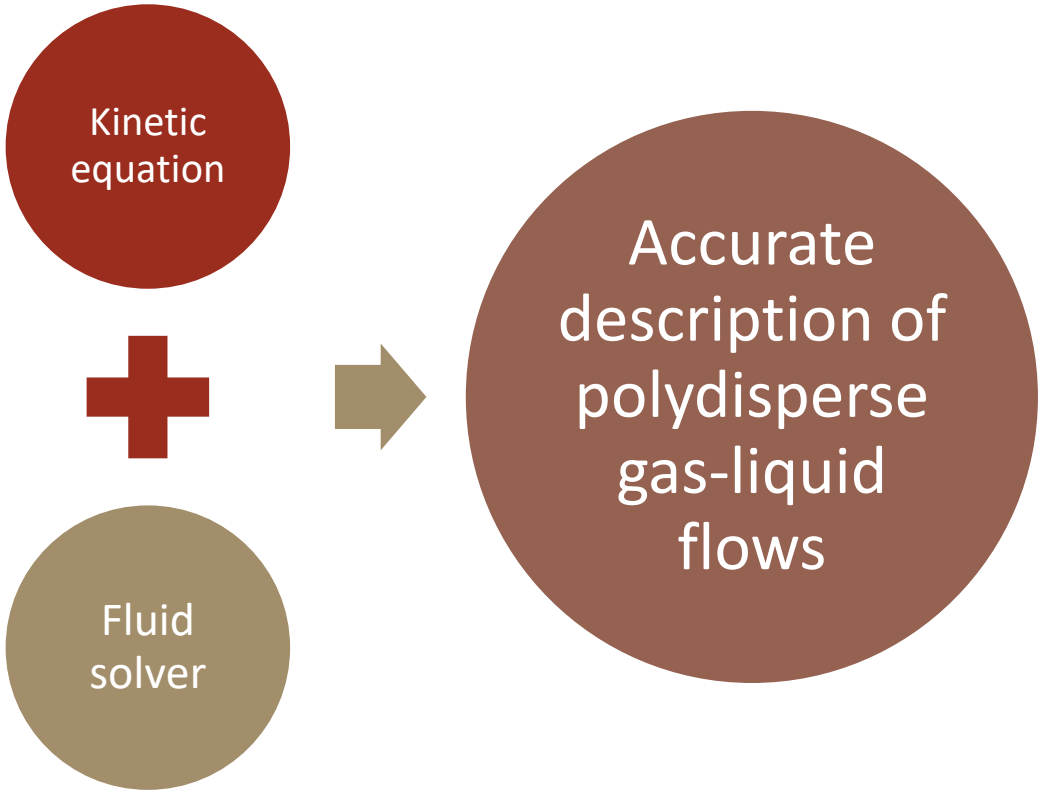
where:

- $\mathbf{A}$ is the acceleration due to forces (drag, lift, added mass, …) acting on bubbles, excluding the gravitational acceleration
- $\mathbf{g}$ is the gravitational acceleration
- $\mathcal{D}_{\mathrm{dis}}$ is the dispersion coefficient
- $\mathbb{C}(\xi, \mathbf{v})$ accounts for interactions between bubbles (i.e. breakup, coalescence)

The bubble size distribution is the marginal NDF:

$$n(\xi) = \int_{\mathbb{R}^3} f(\mathbf{x}, t, \xi, \mathbf{v}) \mathrm{d}\mathbf{v}$$

Kinetic equation for bubble populations [2]



However:

- The NDF $f(\mathbf{x}, t, \xi, \mathbf{v})$ has a high dimensionality
  - 3 spatial dimensions
  - 1 temporal dimension
  - 1 (at least) internal coordinate describing bubble mass or size
  - 3 velocity component
- Direct discretization is prohibitively expensive for applications: 7 dimensions + time
- Lagrangian description of bubbles
  - Accurate
  - Costly
  - Numerical difficulties due to strong coupling with the fluid phase
- In engineering applications accurately knowing some moments of the NDF is sufficient

Moment methods for gas-liquid flows – Basic concepts

In moment methods:

- The idea of solving for the NDF is abandoned
- Moments of the NDF are transported:

$$m_{p,i,j,k}(\mathbf{x},t) = \int_{\mathbb{R}^3}\int_{\mathbb{R}^+} \xi^p u_{\mathrm{x}}^i u_{\mathrm{y}}^j u_{\mathrm{z}}^k f(\mathbf{x},t,\xi,\mathbf{v})\mathrm{d}\xi\mathrm{d}\mathbf{U}$$

where

- p is the order of the moment with respect to the size coordinate ξ
- i, j, k are the orders of the moments with respect to the velocity components

Applying the definition of joint moment to the kinetic equation, partial differential equations for the spatio-temporal evolution of the moments of the NDF are found.

Mono-kinetic assumption

The Stokes number of bubbles is relatively small, even though not negligible

It is acceptable to assume that, locally, bubbles with the same size move with the same velocity

This corresponds to rewriting the NDF as

$$f(\mathbf{x}, t, \xi, \mathbf{v}) = n(\mathbf{x}, t, \xi)\delta(\mathbf{x}, t, \mathbf{v} - \mathbf{U}(\xi))$$

where:

- $n(\mathbf{x}, t, \xi)$ is the bubble mass distribution
- $\delta(\mathbf{x}, t, \mathbf{v} - \mathbf{U}(\xi))$ is a Dirac delta distribution
- $\mathbf{U}(\xi)$ is the bubble velocity conditioned on the bubble mass

We need to define a procedure to reconstruct $n(\mathbf{x}, t, \xi)$ and determine $\mathbf{U}(\xi)$ from the moments to close the moment evolution equations

Moment transport equations

With the mono-kinetic assumption, we can:

- Close the bubble mass distribution considering $2N$ moments in bubble mass

$$M_p = m_{p,0,00}, \qquad p \in \{0, 1, \dots, 2N-1\},$$

$$\frac{\partial M_p}{\partial t} + \nabla \cdot \mathcal{U}_p = \int_{\mathbb{R}^+} \xi^p \mathbb{C}(\xi) \mathrm{d}\xi$$

- Find the functional form of $\mathbf{U}(\xi)$ from the first-order velocity moments:

$$\boldsymbol{\mathcal{U}}_p = (m_{p,1,0,0}, m_{p,0,1,0}, m_{p,0,0,1}), \; p \in \{0, 1, \dots, N-1\}$$

$$\frac{\partial \boldsymbol{\mathcal{U}}_p}{\partial t} + \nabla \cdot \mathcal{P}_\mathrm{p} = M_p \mathbf{g} + \int_{\mathbb{R}^+} n(\xi)\xi^p \big(\mathbf{A}(\xi) - \mathcal{D}_\mathrm{dis} \nabla \ln\big(n(\xi)\big)\big) \mathrm{d}\xi$$

$$+ \frac{\alpha_\mathrm{b} g_0}{\tau_\mathrm{c}} \big(M_p \mathbf{U}_b - \mathcal{U}_p\big) + \int_{\mathbb{R}^+} n(\xi)\xi^p \mathbf{U}(\xi) \mathbb{C}(\xi) \, \mathrm{d}\xi$$

$$\mathcal{P}_p = \int_{\mathbb{R}^+} n(\xi)\xi^p \mathbf{U}(\xi) \otimes \mathbf{U}(\xi) \mathrm{d}\xi$$

Closure of the moment equations [1]

The moment equations are unclosed:

- Terms containing the NDF
- Moment fluxes

We close these terms by means of Gaussian quadrature:

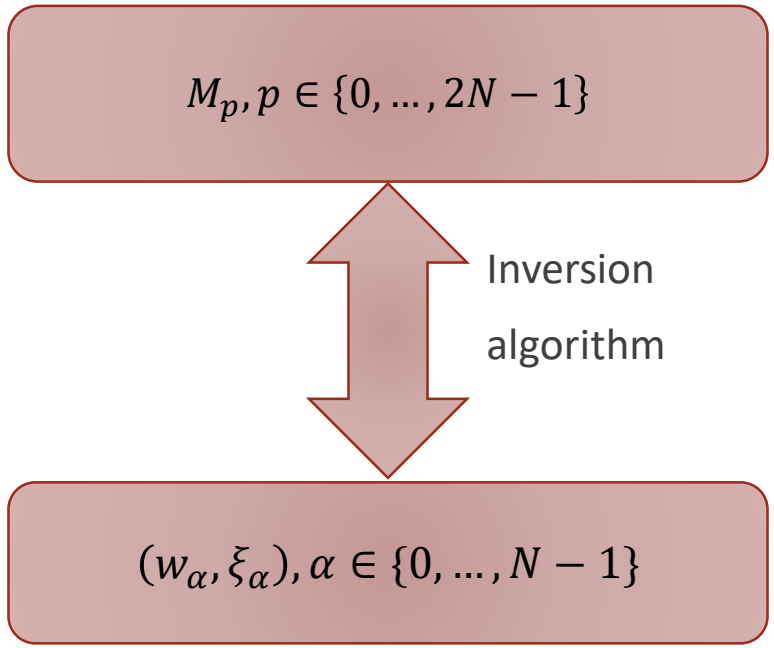
- The bubble mass NDF is written as weighted sum of Dirac delta distributions:

$$n(\xi) = \sum_{\alpha=0}^{N-1} w_\alpha \delta(\xi - \xi_\alpha)$$

which leads to:

$$M_p = \sum_{\alpha=0}^{N-1} w_\alpha \xi_\alpha^p$$

Weights and abscissae are found from the $2N$ moments $M_p, p \in \{0, \dots, 2N-1\}$.



J.C. Wheeler, Modified moments and Gaussian quadratures, Rocky Mountain J. Math. 4 (1974) 287–296. doi:10.1216/RMJ-1974-4-2-287.

Closure of the moment equations [2]

For each size node α, the velocity conditioned on size $\mathbf{U}_\alpha$ are found from the velocity moments $\mathcal{U}_p$ using the conditional quadrature method of moments by solving the family of linear systems:

$$\begin{bmatrix} \boldsymbol{\mathcal{U}}_0 \\ \boldsymbol{\mathcal{U}}_1 \\ \vdots \\ \boldsymbol{\mathcal{U}}_{N-1} \end{bmatrix} = \begin{bmatrix} \mathrm{w}_0 & 0 & \dots & 0 \\ 0 & \mathrm{w}_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathrm{w}_{N-1} \end{bmatrix} \begin{bmatrix} 1 & 1 & \dots & 1 \\ \xi_0 & \xi_1 & \dots & \xi_{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ \xi_0^{N-1} & \xi_1^{N} & \dots & \xi_{N-1}^{N-1} \end{bmatrix} \begin{bmatrix} \mathbf{U}_0 \\ \mathbf{U}_1 \\ \vdots \\ \mathbf{U}_{N-1} \end{bmatrix}$$

Reconstruction of $\mathbf{U}(\xi)$

C. Yuan, R.O. Fox, Conditional quadrature method of moments for kinetic equations, Journal of Computational Physics. 230 (2011) 8216–8246. doi:10.1016/j.jcp.2011.07.020.

Some considerations on the computational cost

We compare the cost of the solution of the proposed approach to a multi-fluid model.

We consider three size nodes ($N = 3$): generally sufficient to accurately predict the bubble size evolution (in some cases 4 nodes are needed)

The number of partial differential equations to solve for the bubble phase is:

- $2N$ pure size moments
- $3N$ velocity moments (one velocity per size)
- Total: $5N$ scalar PDEs
  - $\boldsymbol{N = 2}$**: 10 scalar PDEs**
  - $\boldsymbol{N = 3}$**: 15 scalar PDEs**
  - $\boldsymbol{N = 4}$**: 20 scalar PDEs**

For a multi-fluid model (liquid equations excluded):

- 3 bubble classes
  - 3 continuity equations (3 scalar PDEs)
  - 3 momentum equations (9 scalar PDEs)
  - **Total: 12 scalar PDEs**
- 4 bubble classes
  - 4 continuity equations (4 scalar PDEs)
  - 4 momentum equations (12 scalar PDEs)
  - **Total: 16 scalar PDEs**
- 5 bubble classes
  - 5 continuity equations (5 scalar PDEs)
  - 5 momentum equations (15 scalar PDEs)
  - **Total: 20 scalar PDEs**

The adaptivity intrinsic to quadrature-based moment methods contains the cost and helps preserving accuracy.

Numerical approach - Challenges

Robustness at gas-liquid “interfaces” and in absence of one phase	Pressure-based two-fluid solver with shared pressure
Boundedness of the volume fractions	Flux-corrected limited scheme
Moment realizability	Kinetic fluxes and realizable moment advection

Solution algorithm [1]

We leverage the existing structure of the two-fluid solver in OpenFOAM 5.x

- Pressure-based solver with shared pressure
- Flux-corrected scheme for the advection of the volume fraction

The moment equations need to be modified to be incorporated in the existing two-fluid framework.

We start from:

- Size moments transport

$$\frac{\partial M_p}{\partial t} + \nabla \cdot \boldsymbol{\mathcal{U}}_p = \int_{\mathbb{R}^+} \xi^p \mathbb{C}(\xi) \mathrm{d}\xi$$

- Velocity moment transport

$$\frac{\partial \boldsymbol{\mathcal{U}}_p}{\partial t} + \nabla \cdot \boldsymbol{\mathcal{P}}_p = M_p \mathbf{g} + \int_{\mathbb{R}^+} n(\xi)\xi^p \left(\mathbf{A}(\xi) - \mathcal{D}_{\mathrm{dis}} \nabla \ln\left(n(\xi)\right)\right) \mathrm{d}\xi + \frac{\varepsilon_b g_0}{\tau_c}\left(M_p \mathbf{U}_b - \boldsymbol{\mathcal{U}}_p\right) + \int_{\mathbb{R}^+} n(\xi)\xi^p \mathbf{U}(\xi)\mathbb{C}(\xi)\, \mathrm{d}\xi$$

with

$$\boldsymbol{\mathcal{P}}_p = \int_{\mathbb{R}^+} n(\xi)\xi^p \mathbf{U}(\xi) \otimes \mathbf{U}(\xi) \mathrm{d}\xi$$

Decomposition in mean and deviation transport

We introduce the relative velocity with respect to the mean:

$$\mathbf{V}_\alpha = \mathbf{U}_\alpha - \mathbf{U}_b$$

We then define the moments:

$$\boldsymbol{\mathcal{V}}_p = \int_{\mathbb{R}^+} n(\xi)\xi^p(\mathbf{U}(\xi) - \mathbf{U}_b)\,\mathrm{d}\xi$$

$$\boldsymbol{\mathcal{P}}_v = \int_{\mathbb{R}^+} n(\xi)\xi^{\mathrm{p}}\mathbf{U}(\xi) \otimes\ (\mathbf{U}(\xi) - \mathbf{U}_{\mathrm{b}})\mathrm{d}\xi$$

This allows the advection terms for both mass and velocity moments to be written as:

$$\nabla \cdot \mathcal{U}_p = \nabla \cdot \left(M_p \mathbf{U}_b\right) + \nabla \cdot \left(\mathcal{V}_p\right)$$

$$\nabla \cdot \mathcal{P}_p = \nabla \cdot \left(\mathcal{U}_p \otimes \mathbf{U}_b\right) + \nabla \cdot \left(\mathcal{P}_v\right)$$

Conventional advection schemes used in two-fluid solvers can be used for mean transport

Kinetic fluxes can be used to discretize the advection terms with respect to the mean velocity.

Kinetic fluxes

We indicate with $\varphi = (\mathbf{U})_{\mathrm{f}} \cdot \mathbf{S}_{\mathrm{f}}$ the face flux (dot product of velocity and cell face normal vector)

Size moment kinetic fluxes

$$\nabla \cdot (\boldsymbol{\nu}_p) = \sum_{\alpha} (w_\alpha \xi_\alpha^p)_{\mathrm{nei}} \min(\varphi_\alpha - \varphi_b, 0) + \left(w_\alpha \xi_\alpha^p\right)_{\mathrm{own}} \max(\varphi_\alpha - \varphi_b, 0)$$

Velocity moments kinetic fluxes

$$\nabla \cdot (\boldsymbol{\mathcal{P}}_v) = \sum_{\alpha} (w_\alpha \xi_\alpha^p \mathbf{U}_\alpha)_{\mathrm{nei}} \min(\varphi_\alpha - \varphi_b, 0) + \left(w_\alpha \xi_\alpha^p \mathbf{U}_\alpha\right)_{\mathrm{own}} \max(\varphi_\alpha - \varphi_b, 0)$$

Steps of the solution procedure [1]

We use an iterative, pressure-based approach and integrate it with the quadrature-based moment method.

1. Compute $w_\alpha, \xi_\alpha, \mathbf{U}_\alpha$ at cell centers using the inversion algorithm
2. Compute $\mathbf{V}_\alpha = \mathbf{U}_\alpha - \mathbf{U}_b$ using the value of $\mathbf{U}_b$ from the previous time-step
3. Define $\boldsymbol{\mathcal{V}}_p = \sum_\alpha w_\alpha \xi_\alpha^p \mathbf{V}_\alpha$ and $\boldsymbol{\mathcal{P}}_{p,v} = \sum_\alpha w_\alpha \xi_\alpha^p \mathbf{V}_\alpha \otimes \mathbf{U}_\alpha$
4. Advect size moments M_p and velocity moments $\boldsymbol{\mathcal{U}}_p$ with respect the mean, using **kinetic fluxes**, by solving:

$$\frac{\partial M_p}{\partial t} + \nabla \cdot \boldsymbol{\mathcal{V}}_p = 0 \Rightarrow M_p^*$$

$$\frac{\partial \boldsymbol{\mathcal{U}}_p}{\partial t} + \nabla \cdot \boldsymbol{\mathcal{P}}_{p,v} = 0 \Longrightarrow \boldsymbol{\mathcal{U}}_p^*$$

Note
M_1 is unchanged in this step

Steps of the solution procedure [2]

5. Recompute $w_\alpha, \xi_\alpha, \mathbf{U}_\alpha$ at cell centers using the inversion algorithm applied to M_p^* and $\boldsymbol{\mathcal{U}}_p^*$
6. Recalculate $\mathbf{V}_\alpha = \mathbf{U}_\alpha - \mathbf{U}_b$ noting that $\mathbf{U}_b = \boldsymbol{\mathcal{U}}_1^*/M_1$: **fluxes used in the two-fluid solver must be updated** (including the total flux)
7. Observe that $M_1 = \varepsilon_b \rho_b$ and $\boldsymbol{\mathcal{U}}_1 = \varepsilon_b \rho_b \mathbf{U}_b$ and use a **two-fluid solver** to solve

$$\frac{\partial}{\partial t}(\varepsilon_b \rho_b) + \nabla \cdot (\varepsilon_b \rho_b \mathbf{U}_b) = 0$$

$$\frac{\partial}{\partial t}(\varepsilon_b \rho_b \mathbf{U}_b) + \nabla \cdot [\varepsilon_b \rho_b \mathbf{U}_b \otimes \mathbf{U}_b + p_c \mathbf{I} + \boldsymbol{\tau}_c] = \varepsilon_b \rho_b \mathbf{g} + \int_{\mathbb{R}^+} \xi \{\mathbf{A}(\xi) - \mathcal{D}_{\text{dis}} \nabla \ln[n(\xi)]\} \mathrm{d}\xi$$

Momentum exchange term in the two-fluid model [1]

Buoyancy force

$$\int_{\mathbb{R}^+} \xi n(\xi) \left[-\frac{1}{\rho_b} \left(\nabla p_l - \nabla \tau_l^* \right) \right] \mathrm{d}\xi = -\sum_\alpha \frac{w_\alpha \xi_\alpha}{\rho_b} \left(\nabla p_l - \nabla \tau_l^* \right)$$

Drag force

$$\int_{\mathbb{R}^+} \xi n(\xi) \frac{K_{\mathrm{drag}}}{\rho_b} (\mathbf{U}_l - \mathbf{U}) \mathrm{d}\xi = \sum_\alpha \frac{w_\alpha \xi_\alpha}{\rho_b} K_{\mathrm{drag}}\left(\mathbf{U}_\alpha, d_{b,\alpha}\right)(\mathbf{U}_l - \mathbf{U}_\alpha)$$

$$= \sum_\alpha \frac{w_\alpha \xi_\alpha}{\rho_b} K_{\mathrm{drag}}\left(\mathbf{U}_\alpha, d_{b,\alpha}\right)(\mathbf{U}_l - \mathbf{U}_b) - \sum_\alpha \frac{w_\alpha \xi_\alpha}{\rho_b} K_{\mathrm{drag}}\left(\mathbf{U}_\alpha, d_{b,\alpha}\right)\mathbf{V}_\alpha$$

Implicit

Explicit

Lift force

$$\int_{\mathbb{R}^+} \xi n(\xi) \frac{C_L \rho_l}{\rho_b} (\mathbf{U}_l - \mathbf{U}) \times (\nabla \times \mathbf{U}_l) \mathrm{d}\xi$$

$$= \sum_\alpha \frac{w_\alpha \xi_\alpha}{\rho_b} C_{L,\alpha} \frac{\rho_l}{\rho_b} (\mathbf{U}_l - \mathbf{U}) \times (\nabla \times \mathbf{U}_l)$$

Momentum exchange term in the two-fluid model [2]

Dispersion

$$-\int_{\mathbb{R}^+} \xi n(\xi) \mathcal{D}_{\mathrm{dis}}(\xi) \nabla \ln \mathrm{n}(\xi) \mathrm{d}\xi = -\sum_{\alpha} \mathcal{D}_{\mathrm{dis}} \nabla \frac{w_\alpha \xi_\alpha}{\rho_l}$$

Virtual mass

$$\int_{\mathbb{R}^+} \xi n(\xi) \rho_l C_{VM} \left(\frac{d\mathbf{U}_l}{dt} - \frac{d\mathbf{U}}{dt} \right) \mathrm{d}\xi = \sum_{\alpha} \frac{w_\alpha \xi_\alpha}{\rho_b} C_{VM,\alpha} \rho_l \left[\left(\frac{d\mathbf{U}_l}{dt} - \frac{d\mathbf{U}_b}{dt} \right) - \left(\frac{d\mathbf{U}_\alpha}{dt} - \frac{d\mathbf{U}_b}{dt} \right) \right]$$

Steps of the solution procedure [3]

8. Start from M_p^* and $\mathcal{U}_p^*$ to finish updating M_p and $\mathcal{U}_p$

$$\frac{\partial M_p}{\partial t} + \nabla \cdot \mathbf{U}_{\mathrm{b}}^{\dagger} = \mathcal{C}_p$$

$$\frac{\partial \boldsymbol{\mathcal{U}}_p}{\partial t} + \nabla \cdot \boldsymbol{\mathcal{P}}_{p,b}^{\dagger} + \nabla \cdot \left[\frac{M_p}{M_1^{\dagger}} (p_c \mathbf{I} - \tau_c)^{\dagger} \right] = M_p \mathbf{g} + \int_{\mathbb{R}^+} \xi^p n(\xi) \left(\mathbf{A}(\xi) - \mathcal{D}_{\mathrm{dis}} \nabla \ln\left(n(\xi)\right) \right) \mathrm{d}\xi + \frac{\varepsilon_b g_0}{\tau_{\mathrm{c}}} \left(M_p \mathbf{U}_b - \boldsymbol{\mathcal{U}}_p \right) + \mathcal{C}_{p,i,j,k}$$

9. Update the total flux and $\mathbf{U}_b$
10. Return to step 1

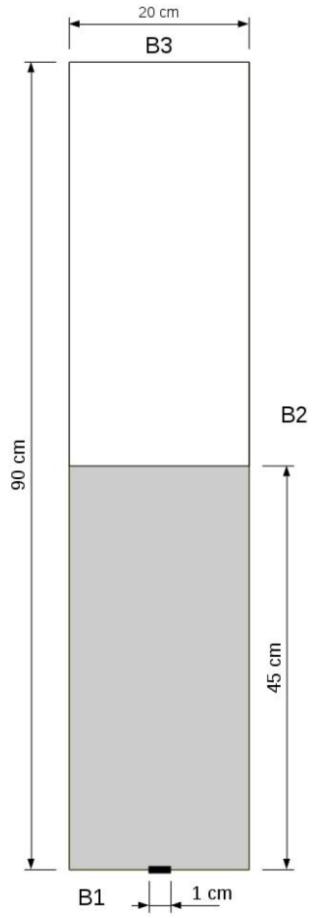
Verification in the monodisperse limit

Case setup

- Monodisperse flow in a bubble column with central injection
- Bubble size: 2 mm
- Gas flow rate: 48 l/h

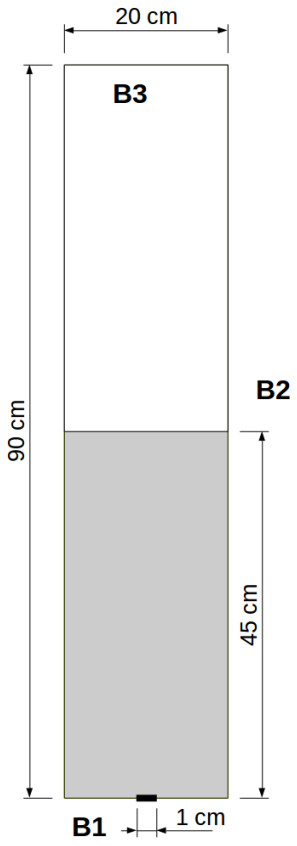
Objective

- Verify the QBMM model correctly degenerates in the two-fluid model in mono-disperse case
- Reference solver
  - twoPhaseEulerFoam (OpenFOAM-dev, OpenFOAM Foundation release).



Pfleger, S. Gomes, N. Gilbert, H.-G. Wagner, Hydrodynamic simulations of laboratory scale bubble columns fundamental studies of the Eulerian-Eulerian modelling approach, Chemical Engineering Science. 54 (1999) 5091–5099. doi:10.1016/S0009-2509(99)00261-4.

Case setup



Phases
- Gas: air, $\rho_g = 1.2\ \mathrm{kg\ m^{-3}}$
- Liquid: water, $\rho_l = 1000\ \mathrm{kg\ m^{-3}}$

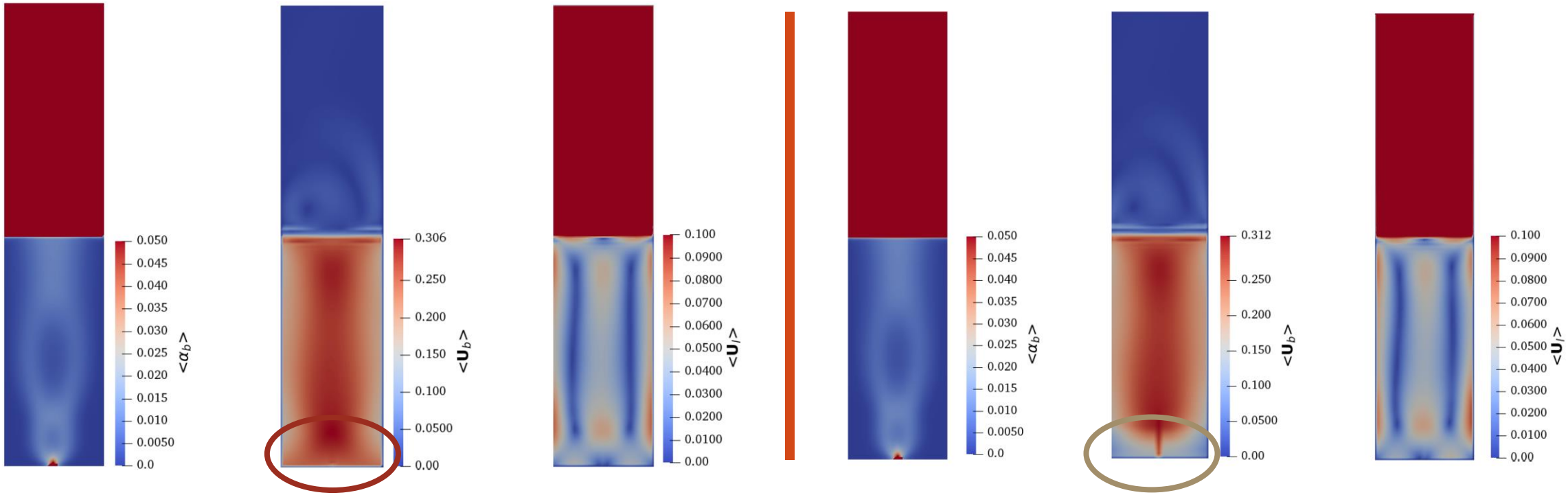
Isothermal system at room temperature

Numerical setup
- Mesh
  - Regular, hexahedral
  - $\Delta x = 5\ \mathrm{mm}$

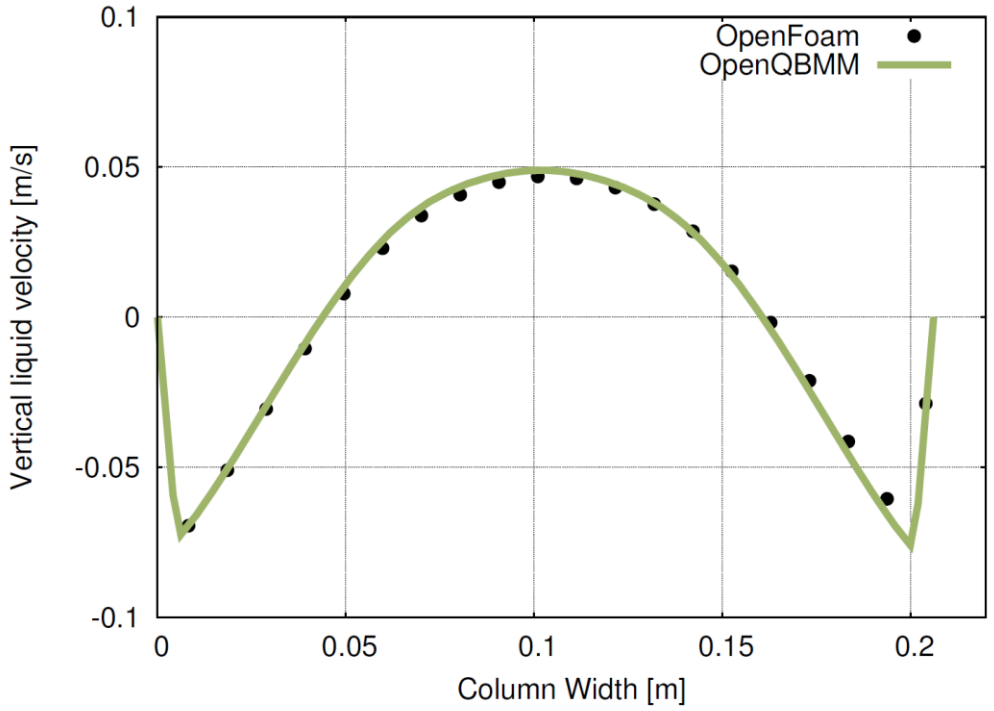
Boundary conditions
- See table

Boundary	p	$\mathbf{U}_b, \mathrm{U}_l$	α_b
Inlet	Neumann	Dirichlet	Neumann
Walls	Neumann	No-slip	Neumann
Outlet	Dirichlet	Neumann	Neumann

Submodels
- Drag and lift: Tomiyama (1998)
- Virtual mass: Constant coefficient
- Wall lubrication: Antal et al. (1991)
- Dispersion: Panicker et al. (2018)
- Bubble-pressure and viscosity: Biesheuvel and Gorissen (1990)



Comparison of QBMM with two-fluid model solver – Monodisperse limit



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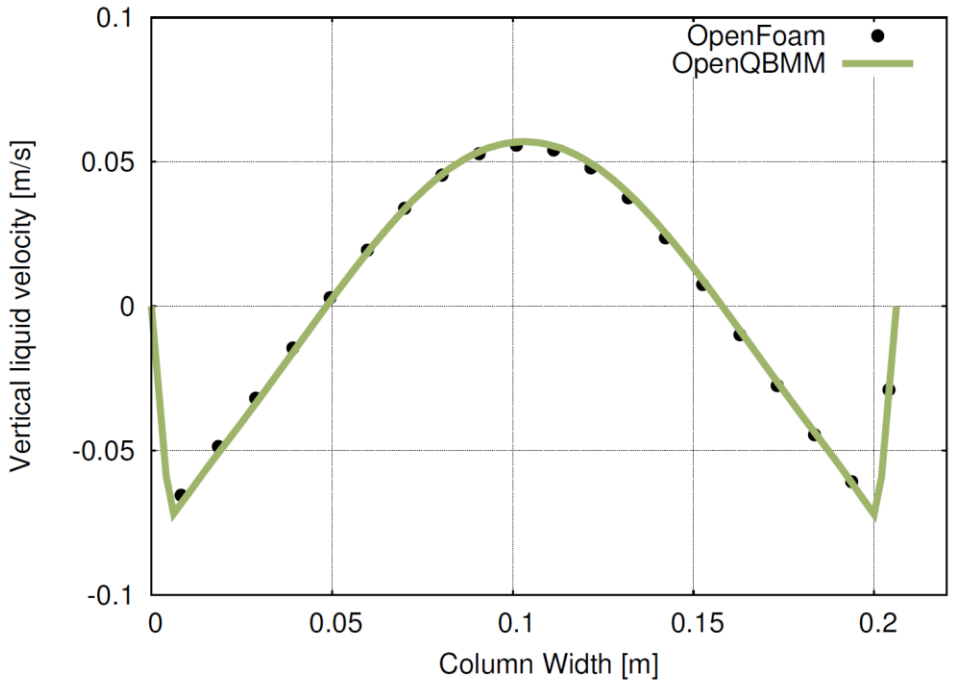
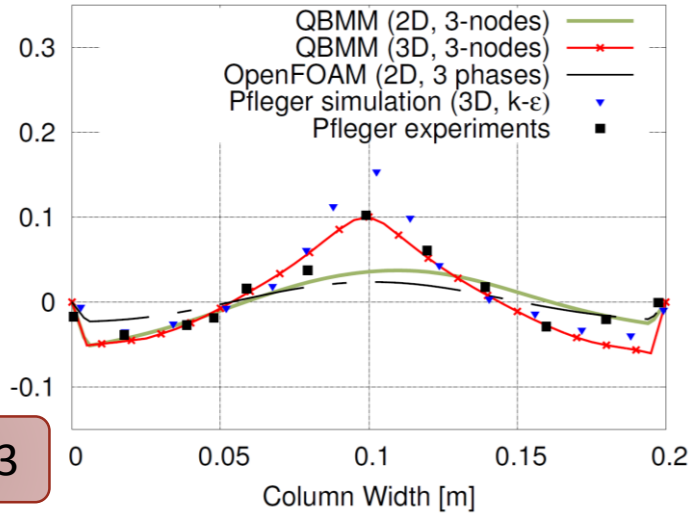


H = 0.37

Comparison of QBMM with two-fluid model solver – Monodisperse limit



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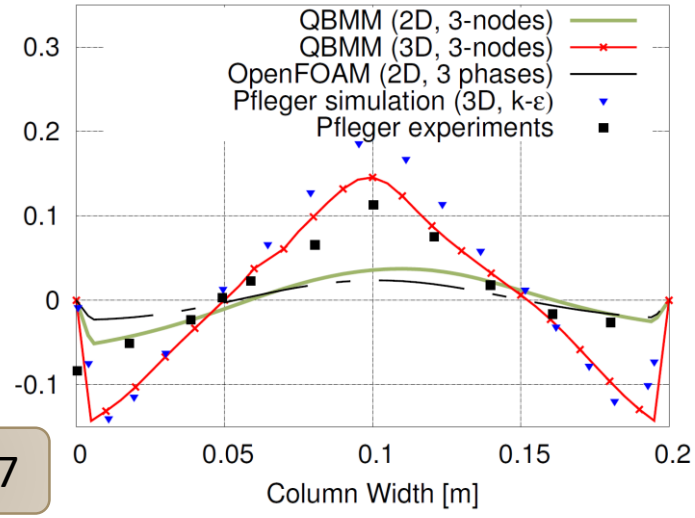


H = 0.37

Polydisperse case

Same bubble colum as in the monodisperse case

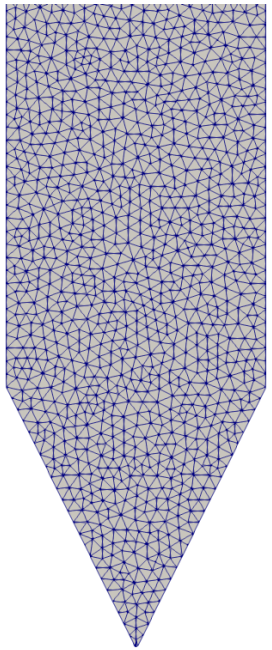
Three bubble sizes:

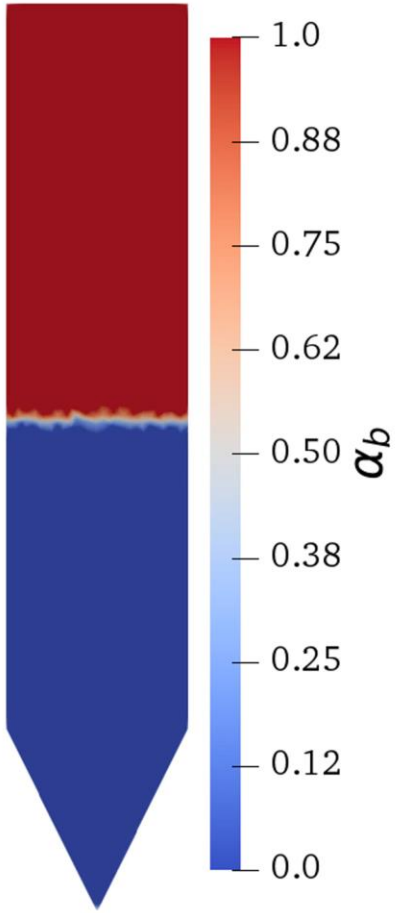
- 1.0 mm
- 2.5 mm
- 4.0 mm
- Equal volume fractions and inlet velocities to match the mean bubble diameter in experiments

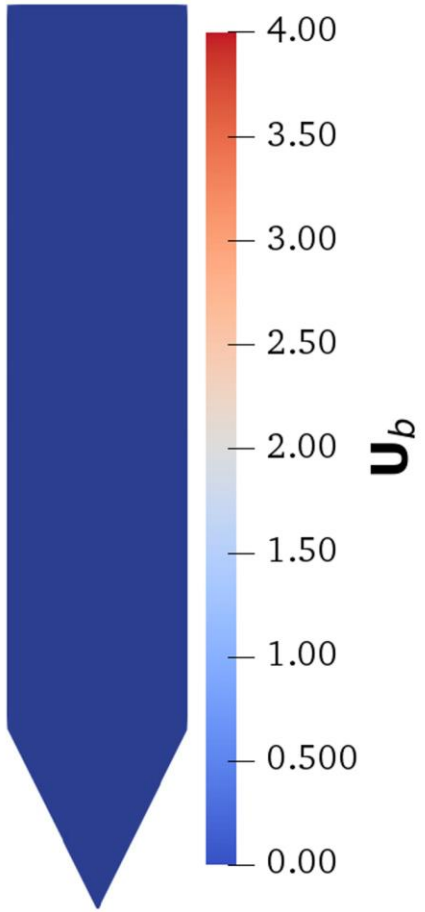
Test on unstructured mesh

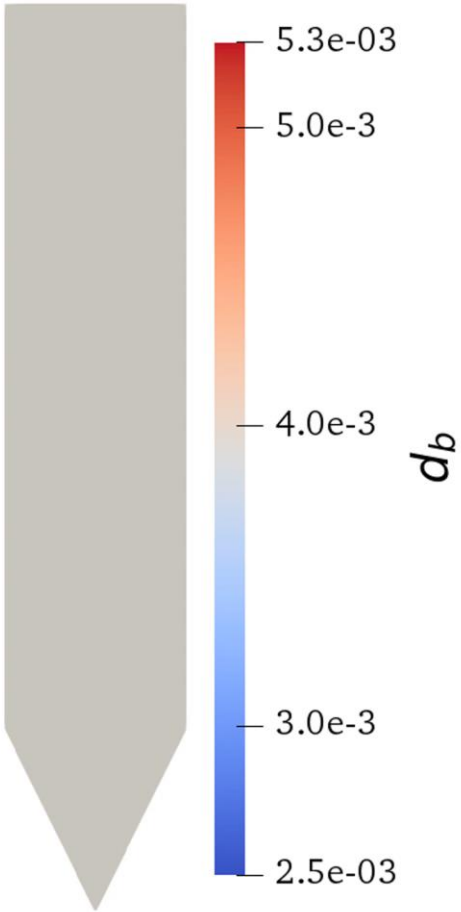
Mixing vessel

- Coalescence and breakup
- $U_{g,\text{inlet}} = 10\ \text{m/s}$









OpenQBMM

The source code of the solver used in this project is part of OpenQBMM, an open-source framework for quadrature-based moment methods based on OpenFOAM.

The code is available at:

- Website: www.openqbmm.org
- GitHub repository: https://github.com/OpenQBMM

If you use OpenQBMM, **cite it**! Software citations are as important as citations of papers.

- DOI for OpenQBMM: 10.5281/zenodo.591651 (all versions)
- Specific DOI for a release: https://github.com/OpenQBMM/OpenQBMM/releases

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Collaborators

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- Frédérique Laurent-Nègre – CentraleSupélec
- Bo Kong – US-DOE Ames Laboratory

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- Nithin Panicker – Ph.D. Student (graduated) – Now at Tenneco
- Ehsan Madadi-Kandjani – Ph.D. Student (graduated) – Now at U.T. Austin

Questions?

End

Interfacial terms

Tomiyama drag model

$$C_d = \max\left(\frac{24}{Re_b}\left(1 + 0.15Re_b^{0.687}\right), \frac{8}{3}\frac{Eo}{Eo + 4}, 0.44\right)$$

$$K_d = \frac{3C_d\rho_l|\mathbf{U}_l - \mathbf{U}_b|}{4d_b}$$

$$\mathbf{F}_d = \alpha_b K_d(\mathbf{U}_l - \mathbf{U}_b)$$

Constant coefficient virtual mass model

$$F_{VM} = \alpha_b\rho_l C_{VM}\left(\frac{d\mathbf{U}_l}{dt} - \frac{d\mathbf{U}_b}{dt}\right)$$

$$\frac{d\mathbf{U}}{dt} = \frac{\partial\mathbf{U}}{\partial t} + \mathbf{U}\cdot\nabla\mathbf{U}$$

A. Tomiyama, Struggle with computational bubble dynamics, Multiphase Science and Technology 10 (4) (1998) 369 – 405.

Interfacial terms

Tomiyama lift model

$$C_L = \begin{cases} \min[0.288\tanh(0.121\mathrm{Re_b}), f(\mathrm{Eo_d})] & \mathrm{Eo_d} < 4 \\ f(\mathrm{Eo_d}) & 4 \le \mathrm{Eo_d} \le 10.7 \\ -0.27 & \mathrm{Eo_d} > 10.7 \end{cases}$$

$$f(\mathrm{Eo_d}) = 0.00105\mathrm{Eo_d^3} - 0.0159\mathrm{Eo_d^2} - 0.0204\mathrm{Eo_d} + 0.474$$

$$Eo_d = \frac{(\rho_l - \rho_b) g d_h^2}{\sigma_b}$$

$$d_h = d_b\left(1 + 0.163 Eo_d^{0.757}\right)^{\frac{1}{3}}$$

A. Tomiyama, Struggle with computational bubble dynamics, Multiphase Science and Technology 10 (4) (1998) 369 – 405.

Interfacial terms

Panicker dispersion model

$$\mathbf{F}_{\mathrm{dis}} = \mathrm{C}_{\mathrm{dis}}\mathrm{K}_{\mathrm{d}}\left(1 - \mathrm{a}\alpha_{\mathrm{b}} + \mathrm{b}\alpha_{\mathrm{b}}^{2}\right)\nabla\alpha_{b}$$

$$\mathrm{C}_{\mathrm{dis}} = 4.544, \qquad \mathrm{b} = 0.5, \qquad \mathrm{a} = 1 + \mathrm{b} - \frac{1}{3}$$

Antal wall lubrication model

$$\mathbf{F}_{\mathrm{WL}} = -\alpha_{\mathrm{b}}\nabla\mathrm{p}_{\mathrm{w}}$$

$$\mathrm{p}_{\mathrm{w}} = \rho_{\mathrm{l}}|\mathbf{U}_{\mathrm{l}} - \mathbf{U}_{\mathrm{b}}|^{2}\frac{\mathrm{d}_{\mathrm{b}}}{|\mathbf{x} - \mathbf{x}_{\mathrm{w}}|}\left[\mathrm{a}_{0} - \mathrm{a}_{1}\frac{\mathrm{d}_{\mathrm{b}}}{|\mathbf{x} - \mathbf{x}_{\mathrm{w}}|} + \mathrm{a}_{2}\frac{\mathrm{d}_{\mathrm{b}}^{2}}{|\mathbf{x} - \mathbf{x}_{\mathrm{w}}|^{2}}\right]$$

$$\mathrm{C}_{\mathrm{w}} = 0.0217, \quad \mathrm{a}_{0} = \mathrm{C}_{\mathrm{w}}, \quad \mathrm{a}_{1} = \frac{\mathrm{C}_{\mathrm{w}}}{2}, \quad \mathrm{a}_{2} = \frac{\mathrm{C}_{w}}{4}$$

S. P. Antal, R. T. Lahey Jr, J. E. Flaherty, Analysis of phase distribution in fully developed laminar bubbly two-phase ow, International Journal of Multiphase Flow 17 (5) (1991) 635 – 652.

N. Panicker, A. Passalacqua, R.O. Fox, On the hyperbolicity of the two-fluid model for gas–liquid bubbly flows, Applied Mathematical Modelling. 57 (2018) 432 – 447. doi:10.1016/j.apm.2018.01.011.

Interfacial terms

Biesheuvel and Gorissen bubble pressure

$$\mathrm{F_{BP}} = -\mathrm{C_{bp}}\alpha_\mathrm{b}(\nabla \mathrm{p_b} - \nabla \cdot \boldsymbol{\tau}_\mathrm{l})$$

$$p_\mathrm{b} = \mathrm{C_{bp}}(\rho_\mathrm{b} + \rho_\mathrm{l}\mathrm{C_{VM}})\alpha_\mathrm{b}|\mathbf{U}_\mathrm{l} - \mathbf{U}_\mathrm{b}|^2\alpha_\mathrm{b}\alpha_\mathrm{l}$$

Biesheuvel and Gorissen bubble induced viscosity

$$\nu_\mathrm{b} = \mathrm{C_b}\mathrm{d_b}|\mathbf{U}_\mathrm{l} - \mathbf{U}_\mathrm{b}|\sqrt{\alpha_\mathrm{b}\alpha_\mathrm{l}}$$

$$\nu_{\mathrm{l,eff}} = \nu_\mathrm{l} + \frac{\alpha_\mathrm{b}}{\alpha_\mathrm{l}}\left(\frac{\rho_\mathrm{b}}{\rho_\mathrm{l}} + \mathrm{C_{VM}}\right)\nu_\mathrm{b}$$

A. Biesheuvel, W. C. M. Gorissen, Void fraction disturbances in a uniform bubbly fluid, International Journal of Multiphase Flow 16 (2) (1990) 211 – 231, ISSN 0301-9322.