Abstract

In the academic world, two groups of academics are intensely interested in prime numbers. The mathematicians for obvious reasons and the computer scientists for a variety of reasons. The biggest one is in cryptography, which relies on things which are easy to do one way but difficult to do in the opposite direction. Multiplying two primes is easy but given the answer, finding the two primes that were multiplied is not easy. That is one major reasons why computer scientists are interested in prime numbers.

In this paper, I demostrate how to calculate the number of prime numbers below 1,000 using a new method that groups odd numbers according to their last digits. After numbers are grouped according to their last digits, a certain set of rules are used to separate prime numbers from non-prime numbers.

On a New Prime Number Sieve Model

A Monogragh

Chapter 1 Introduction

This monograph strictly talks about a new type of prime number sieve that is still under development but this author feels like it is the right time to publish the initial results. This monograph is also written in a lay-man's language and so it should be fairly easy for anyone to read and understand it.

In Marcus du Sautoy's "The music of the primes," at the beginning of the book he explains clearly what the sieve of Eratosthenes is. "The librarian of the great ancient Greek research institute in Alexandria was the first person we know to have produced tables of primes. Like some ancient mathematical Mendeleev, Eratosthenes in the third century bc discovered a reasonably painless procedure for determining which numbers are prime in a list of, say, the first 1,000 numbers. He began by writing out all the numbers from 1 to 1,000. He then took the first prime,2, and struck off every second number in the list. Since all these numbers were divisible by 2, they weren't prime.He then moved to the next number that hadn't been struck off, namely 3, they weren't prime either. He kept doing this, just picking up the next number that hadn't been struck off from the list and striking off all the numbers divisibly by the new prime. By this systematic process he produced tables of primes. The procedure was later christened the sieve of Eratosthenes. Each new creates a "sieve" which Eratosthenes uses to eliminate non-primes. The size of the sieve changes at each stage, but by the time he reaches 1,000 the only numbers to have made it through all the sieves are prime numbers." (du Sautoy, 2003)

My model

I propose a new model of prime number sieve. This sieve groups prime numbers according to their last digits. This would suggest that there would be four such groups.

Statements of fact about prime numbers

Proving the following two statements is a trivial matter. I don't think anyone can argue with these two statements

1 All prime numbers, except number 2, end with odd numbers
2 5 is the only prime number ending with 5, in the numberline.

Conclusion

Therefore the conclusion from the two statements above is that all prime numbers end in either 1,3,7 or 9 (except for numbers 2 and 5). Therefore in this model there will be four columns of prime numbers each ending in one of the four numbers given above.A simple example of such classification will be done on all prime numbers less than 50, just to demonstrate to readers what i am talking about. The table below classifies all prime numbers (except 2 and 5) below 50, into their respective endings.

Table 1(i)

Number endings	1	3	7	9
	11	13	17	19
	31	23	37	29
	41	43	47	

As you can in that table, all the prime numbers (except 2 and 5) are classified according to their endings. This is done so as to study the relationship between prime numbers with similar endings.

Let us pick one group and study it indepth, so as to get an idea of how prime numbers with similar endings can be studied and why it is important. Let us pick the group ending with 1 for example.

This group of primes ending with 1 can be derived from a larger group of numbers ending with 1. In other words, if you write down a list of numbers between 0 and 1,000 that end with 1, you can then use my model to derive from this list, a smaller list containing only prime numbers between 0 and 1,000 ending with 1. I will present below a table containing two columns. The first column contains numbers below 200 that end with 1. The second column contains a list of all numbers below 200 that end with 1 and are also prime.

Table 1(ii)

	Numbers ending with 1	Numbers ending with 1 and also prime
1	1	
2	11	11
3	21	
4	31	31
5	41	41
6	51	
7	61	61
8	71	71
9	81	
10	91	
11	101	101
12	111	
13	121	
14	131	131
15	141	
16	151	151
17	161	
18	171	
19	181	181
20	191	191

As you can see in the table above, the first thing that one notices, is the fact that the list of prime numbers ending with 1 has gaps in it. It has been my endevour in the last two years to understand those gaps and what they mean. I strongly believe that by understanding those gaps, we can seamlessly derive prime numbers on the right of the table above from the list on the left.

I want to say something small about the gaps. It is common knowledge that if you pick any number on the the numberline, that number will either be a prime number or a factor of a prime number. This is according to the fundamental theorem of arithmetic. The fundamental theorem of arithmetic states that every positive integer (except number 1) can be represented in exactly one way apart from rearrangement as a product of one or more primes (Hardy and Wright 1979, pp. 2-3)
This theorem is also called the unique factorization theorem. The fundamental theorem of arithmetic is a corollary of the first of Euclid's theorems (Hardy and Wright 1979)

Next i want to talk about semiprimes because they are a crucial element of my model. A semiprime, also called a 2-almost prime, biprime (Conway et al.2008), or pq-number, is a composite number that is the product of two (possibly equal) primes. The first few are 4,6,9,10,14,15,21,22,... (OEIS A001358).
The first few semiprimes whose factors are distinct (i.e, the squarefree semiprime) are 6,10,14,15,21,22,26,33,34,... (OEIS A006881).

As we have seen, a semiprime is itself not a prime but rather the product of two primes. Therefore a semiprime can either be an even number or an odd number. From table 1(ii), examples of semiprimes would be 21 (3×7), 51 (3×17), 111 (3×37), 141 (3×47), 171 (3×57).

In my quest to explain the gaps between primes on the right side of table 1(ii), I was able to explain some of those gaps as semiprimes (21, 51, 111, 141 and 171). However there are still some gaps that i have not explained What could they be? The gaps i haven't explained are as follows;
Numbers (1, 81, 91, 121 and 161). Now these numbers are neither semiprimes nor primes. First of all, I know that 1 does not belong to the same group as the rest of the numbers because it is not the result of multiplication of two numbers (except 1×1), yet the other numbers (81,91,121,161) are certainly the result of multiplication of a prime and another number. Therefore 1 does not belong to this list because it is not a result of multilication of a prime and another number, therefore i will remove it from this list and create for it its own special list. The final list looks like this (81, 91, 121 and 161).

Let us pick 81 and deconstruct it to its basic elements.

1 3×27=81
2 3×3×9=81
3 9×9=81

Looking at these 3 possible iterations of numbers multiplied to get 81, it becomes obvious that 81 is neither a prime nor a semiprime. This is because if 81 was to be prime then it would have to be divisible by 1 and itself only but this is not true for 81 because we see that it can be divided by 3 for example. Secondly, if 81 was to be a semiprime, then it would be the product of two prime numbers but this is not true for 81 because we see that in the three possible iterations above, none of those iterations represents a multiplication of two primes to get 81. This therefore means that we will create a third category of numbers that are neither primes nor semiprimes but rather "non-semiprimes multiples of primes."It is quite a mouthful and if there is a better word to use, the reader can suggest it. Non-semiprime multiples of prime simply refers to all multiples of primes that are not semiprime. Therefore in summary, I have shown that the entire list of numbers below 200 ending in 1 belong to four major categories (we will include 1 as i have already explained before that 1 does not fit into any of those three categories but belongs to its own unique category.) They are namely
1) Primes
2)Semiprimes
3) 1
4) Non-semiprime multiples of prime

Therefore i have successfully explained what the gap is.

Next i want to state a very important priciple of this model.
Before i state it, i must put out a disclaimer that i searched extensively online to see whether this

principle has already been stated in full by someone else but i could not find it so in case i am wrong please contact me and i will be glad to give credit to the author.

Fundamental principle 1: All odd numbers ending with the same number (except numeral 1) can be classified into three categories where they are either primes, semiprimes or non-semiprime multiples of primes. As for numbers ending with 1, they can be classified into 4 categories, that is, the three categories already stated plus a fourth category containing number 1 only.

The basic tenet in this model is that there are infinite prime numbers.
If there are infinite prime numbers then there are also infinite semiprimes. If there are infinite prime numbers then there are also infinite Non-semiprime multiples of primes.
Euclid states that the number of primes is infinite. This theorem also called the infinitude of primes theorem, was proved by Euclid in proposition IX.20 of the elements (Tietze 1965, pp. 7-9.)

Chapter 2 The Real Deal

This chapter is the real deal because we are no longer dealing with introductory concepts.
The previous chapter was just introduction and stating of principles. In this chapter i will introduce some novel ideas and some of them will not be novel and i will give credit where it is due.
My purpose for writing this monograph is not to fully prove all of these ideas but rather to bring these ideas out into the world while they are still under-development so as to get priority for my work but also enable other mathematicians to jump onboard and start working on this project.

The second principle : All numbers in the numberline ending with 9, except prime numbers, can be produced by engaging in four types of multiplications as follows. The first instance is where a number ending with 7 is multiplied by another number ending with 7. The second instance is where a number ending with 9 is multiplied by a number ending with 1. The third instance is where a number ending with 3 is multiplied by another number ending with 3. The fourth instance is where a number ending with 1 is multiplied by a number ending with 9

A brief example Table 2(i)

	A	B	C	D
1	7×7=49	9×11=99	3×3=9	11×9=99
2	7×17=119	9×21=189	3×13=39	21×9=189
3	7×27=189	9×31=279	3×23=69	31×9=279

The third principle : All numbers in the numberline ending with 3, except prime numbers, can be produced by engaging in three types of multiplications as follows. The first instance is where a number ending with 3 is multiplied by a number ending with 1. The second instance is where a number ending with 9 is multiplied by a number ending with 7. The third instance is where a number with 7 is multiplied with a number ending with 9.

Table 2(ii)

	A	B	C
1	3×11=33	9×7=63	7×9=63
2	3×21=63	9×17=153	7×19=133
3	3×31=93	9×27=243	7×29=203

The fourth principle: All numbers in the numberline ending with 1, except prime numbers can be produced by engaging in these four types of multiplications as follows. The first instance is where a number ending with 1 is
number ending with 9 is multiplied by another number ending with 9. The third instance is where a number ending with 3 is multiplied by a number ending with 7. The fourth instance is where a number ending with 7 is multiplied by a number ending with 3.

Table 2(iii)

	A	B	C	D
1	11×11=121	9×9=81	3×7=21	7×3=21
2	11×21=231	9×19=171	3×17=51	7×13=91
3	11×31=341	9×29=261	3×37=111	7×23=161

The fifth principle: All numers in the numberline ending with 7, except prime numbers, can be produced by engaging in three types of multiplications as follows.The first instance is where a number ending 7 is multiplied by a number ending with 1. The second instance is where a number ending with 9 is multiplied by a number ending with 3. The third instance is where a number ending with 3 is multiplied by a number ending with 9

Table 2(iv)

	A	B	C

1	7×11=77	9×3=27	3×9=27
2	7×21=147	9×13=117	3×19=57
3	7×31=217	9×23=207	3×29=87

To prove that the second upto the fifth principles are correct, we will be forced to multiples two odd number variables in all possible combinations.

<u>All possible combinations of odd number multiplied by another odd number.</u>

Table 2(v)

a	1×1=1	k	1×5=5	u	1×9=9
b	3×1=3	l	3×5=15	v	3×9=27
c	5×1=5	m	5×5=25	w	5×9=45
d	7×1=7	n	7×5=35	x	7×9=63
e	9×1=9	o	9×5=45	y	9×9=81
f	1×3=3	p	1×7=7		
g	3×3=9	q	3×7=21		
h	5×3=15	r	5×7=35		
i	7×3=21	s	7×7=49		
j	9×3=27	t	9×7=63		

Below is presented all possible iterations of multiplications that result in numbers ending with 1.
They are a) 1×1=1, (i) 7×3=21, (q) 3×7=21 (y) 9×9=81

Below is presented all possible iterations of multiplications that result in numbers ending with 3.
They are (b) 3×1=3 (f) 1×3=3 (t) 9×7=63 (x) 7×9=63

Below is presented all possible iterations for multiplications that result in numbers ending with 7
They are (d) 7×1=7 (j) 9×3=27 (p) 1×7=7 (v) 3×9=27

Below is presented all possible iterations for multiplications that result in numbers ending with 9
They are (e) 9×1=9 (g) 3×3=9 (s) 7×7=49 (u) 1×9=9

We know from the fundamental theorem of arithmetic that a number's prime factorization must be unique (Hardy and Wright 1979). That also means it does not matter the order in which they appear thus (i)7×3=21 is the same as (q) 3×7=21. It does not matter whether 7 starts or 3 starts ,as long as there is one 3 and one 7 we will always get 21 as By that same principle, i will argue that
(f) 1×3=3 can be substituted by (b) 3×1=3
Therefore we remove the following similar equations from table 2(v)
We remove (f) because it is similar to (b)
We remove (i) because it is similar to (q)
We remove (j) because it is similar to (v)
We remove (p) because it is similar to (e)
We remove (t) because it is similar to (x)
We also remove any multiple of 5 because there are no prime numbers that end with 5 (except numeral 5)

The final table looks like this:

Table 2(vi)

a	1×1=1

b	$3\times1=3$
d	$7\times1=7$
e	$9\times1=9$
g	$3\times3=9$
q	$3\times7=21$
s	$7\times7=49$
v	$3\times9=27$
w	$5\times9=45$
y	$9\times9=81$

Conclusion

We will use these ten multiplications to find all prime numbers . That will be done as from the next chapter. Remember these multiplications are of number endings and not necessarily numbers in themselves. I know things will become clearer as from the next chapter.

Chapter 3 Finding prime numbers ending with 9 from a list of all numbers ending with 9 between 0 and 1,000

Part A (i) I will start by multiplying number 7 by another set of numbers ending with 7 where the result falls between 0 and 1,000.

Table 3(i)

1	7×7 = 49
2	7×17 = 119
3	7×27 = 189
4	7×37 = 259
5	7×47 = 329
6	7×57 = 399
7	7×67 = 469
8	7×77 = 539
9	7×87 = 609
10	7×97 = 679
11	7×107 = 749
12	7×117 = 819
13	7×127 = 889
14	7×137 = 959

Table 3 (ii)

1	7×7 = 49
2	7×17 = 119
3	7×37 = 259
4	7×47 = 329
5	7×67 = 469
6	7×77 = 539
7	7×97 = 679
8	7×107 = 749
9	7×127 = 889
10	7×137 = 959

Next we remove from this list, numbers that are multiples of 3 because these numbers will be included later in another list that contains 3 as one of the variables in the multiplication.We are trying to avoid duplication of results in this modeĺ. For example 7×27 =189 and 3×63 = 189 all have the same result of 189 so here we have to pick one and ignore the other, because in this model all numbers below a certain value n must be accounted for in the calculations ONLY ONCE.This is a golden rule and i can't overemphasize it.

Avoiding duplication of results will be a central theme of this model. From table 3(i) above, all multiples of 3 are as follows :27,57,87,117. Removing them we are left with figures table 3(.Notice
that that table 3(ii) has 10 elements. This number is important and it will be used later somewhere, remeber it.

A(ii) Now let us take this a step further. Let us multiply number 17 by another set of numbers ending with 7 where the result falls between 0 and 1,000.

Table 3(iii)

1	17×17=289
2	17×27=459
3	17×37=629
4	17×47=799
5	17×57=969

Table 3(iv)

1	17×17=289
2	17×37=629
3	17×47=799

Next we remove from this list, numbers that are multiples of 3 for the same reasons explained above. From table 3(iii), multiples of 3 are 27 and 57.Notice that in table 3(iii) we have excluded 17×17=119 because it is already represented in table 3 as 7×17=119.Also notice that table 3(iv) has 3 elements. This number is important we will use it later.

A(iii) Now let us multiply 37 by a set of numbers ending with 7 where the results fall between 0 and 1,000.

(i) 37×7=259. Notice this number is already represented in table 3(i) as number 4.) 7×37=259, so we exclude it
(ii)37×17=629. Notice this number is already represented in table 3(iii) as number 3.)17×37=629, so we exclude it

(iii)37×27=999. Notice this number has a variable 27 which is a multiple of 3, so we exclude it
Therefore our table in this section will contain zero elements in it because every element in it is also found in other tables.

Part B(i) Here we will multiply 9 by a set of numbers ending with 1 where the result falls between 0 and 1,000.

	Table 3(v)		Table 3(vi)
1	9×11=99		
2	9×21=189		
3	9×31=279		
4	9×41=369		
5	9×51=459		
6	9×61=549		
7	9×71=639		
8	9×81=729		
9	9×91=819		
10	9×101=909		
11	9×111=999		

Next we remove from this list numbers that are multiples of 3 as explained before.
In table 3(v) above, you will notice that 9 is divisible by 3 therefore we will exclude all the elements in that table from our calculations. Therefore table 3(vi) is blank.

B(ii)
Here we multiply 19 by a set of numbers ending with 1 where the result falls between 0 and 1,000.

	Table 3(vii)		Table 3(viii)
1	19×11=209	1	19×11=209

2	19×21=399	2	19×31=589
3	19×31=589	3	19×41=779
4	19×41=779		
5	19×51=969		

Next we remove from this list, numbers that are multiples of 3 (namely, 21 and 51).After doing that, we are left with table 3(viii) which has 3 elements.

B(iii)

Here we multiply 29 by a set of numbers ending with 1 where the result falls between 0 and 1,000.

	Table 3(ix)		Table 3(x)
1	29×11=319	1	29×11=319
2	29×21=609	2	29×31=899
3	29×31=899		

Next we remove from this list,numbers that are multiples of 3 (namely 21). After doing that we are left with table 3(x)

B(iv) Here we multiply any number above 29 ending with 9 that when multiplied by a set of numbers ending with 1, gives us a result falling between 0 and 1,000 and where multiples of 3 are excluded in the result.

	Table 3(xi)
1	59×11=649
2	79×11=869
3	89×11=979

C (i) Here we multiply number 3 by a set of numbers ending with 3 where the result falls between 0 and 1,000.

	Table 3(xii)
1	3×3=9

2	3×13=39
3	3×23=69
4	3×33=99
5	3×43=129
6	3×53=159
7	3×63=189
8	3×73=219
9	3×83=249
10	3×93=279
11	3×103=309
12	3×113=339
13	3×123=369
14	3×133=399
15	3×143=429
16	3×153=459
17	3×163=489
18	3×173=519
19	3×183=549
20	3×193=579
21	3×203=609
22	3×213=639
23	3×223=669
24	3×233=699
25	3×243=729
26	3×253=759
27	3×263=789
28	3×273=819

29	3×283=849
30	3×293=879
31	3×303=909
32	3×313=939
33	3×323=969
34	3×333=999

Here we do not remove any element because this is the basic table of the model where we excluded some elements in some tables and even left other tables completely blank because those elements are also found in this primary or basic table.

To be fair to me, typing this thing out is quite tiresome for me so i will make the following changes to reduce the workload. From now on, i will create only final tables where multiples of 3 have already been removed. You should, by yourselves, try to find out which elements have been removed from the tables. Something important to note is that, most of the elements removed will either be multiples of 3 or 7. I know so far we have not encountered multiples of 7, but we will in future and they will also be removed from some of these tables to avoid duplication of data.

C(ii) Here we multiply 13 by a set of numbers ending with 3 where the result falls between 0 and 1,000.00

Table 3(xiii)

1	13×13=169
2	13×23=299
3	13×43=559
4	13×53=689
5	13×73=949

Notice that this table does not include any multiple of 3 or 7 and that is how we want it to be.
All tables from now on will be presented like that (without showing which multiples of 3 or 7 were removed, with the assumption that finding these multiples is a trivial affair for any serious mathematician.)

C(iii) Here we multiply 23 by a set of numbers ending with 3 where the result falls between 0 and 1,000.

Table 3(xiv)

1	23×23=529
2	23×43=989

Thus there are only two numbers ending with 3 that when multiplied by 23 give a result that falls between 0 and 1,000 and where these numbers are neither multiples of 3 nor 7.

C(iii) Here we are supporsed to multiply 33 by a set of numbers ending with 3 where the result falls between 0 and 1,000 but we will not do that because 33 is dividible by 3.

Analysis

We know that there are 100 numbers between 0 and 1,000 that end with 9. We want to find out how many of those numbers are prime by subtracting all non-prime numbers from 100.Therefore we want to find out the total non-prime numbers ending with 9 between 0 and 1,000.

These non-prime numbers ending with 9 between 0 and 1,000 are found in the following tables and their quantity is indicated besides the table

Table	Number
Table 3(ii)	10

Table 3(iv)	3
Table 3(viii)	3
Table 3(x)	2
Table 3(xi)	3
Table 3(xii)	34
Table 3(xiii)	5
Table 3(xiv)	2
Total	62

Therefore there are 62 non-prime numbers ending with 9 between 0 and 1,000. Subtracting this figure from 100 gives us 38 which suggests that there are 38 prime numbers ending with 9 between 0 and 1,000. To confirm whether this is true we have to look at the list of prime numbers as categorised by their endings at the back of the book. We find from that table that indeed there are 38 prime numbers ending with 9 between 0 and 1,000.

Chapter 4 Finding prime numbers ending with 1 from a list of all numbers ending with 1 between 0 and 1,000

A (i) Here we multiply 11 by a set of numbers ending with 1 where the result falls between 0 and 1,000.00

Table 4(i)

1 11×11=121
2 11×31=341
3 11×41=451
4 11×61=671
5 11×71=781

All numbers divisible by 3 have been removed. They 11×21=231, 11×51=561 and 11×81=891.

A(ii) Here we will skip 21 because 21 is divisible by 3

A(iii) Here we will multiply 31 by a set of numbers ending with 1 where the result falls between 1 and 1,000.00

Table 4(ii)

1 31×31=961

Notice we have not included 31×11=341 in this table because it is already in table 4(i) as the second element.

B(i) Here we are supposed to multiply 9 by a set of numbers ending with 9 where the result falls between 0 and 1,000 but we will not bother because 9 is divisible by 3.

B(ii) Here we will multiply 19 by a set of numbers ending with 9 where the result falls between 0 and 1,000.

Table 4(iii)

1 19×19=361

2 19×29=551

B(iii) Here we multiply 29 by a set of numbers ending with 9 where the result falls between 0 and 1,000.00

Table 4(iv)

1 29×29=849

Notice 29×19 is already represented in table 4(iii) as 19×29=551, that is why we did not include it in table 4(iv).

C(i) Here we multiply 3 with a set of numbers ending with 7 where the result falls between 0 and 1,000.00

Table 4(v)

1 3×7=21

2 3×17=51

3 3×27=81

4 3×37=111

5 3×47=141

6 3×57=171
7 3×67=201
8 3×77=231
9 3×87=261
10 3×97=291
11 3×107=321
12 3×117=351
13 3×127=381
14 3×137=411
15 3×147=441
16 3×157=471
17 3×167=501
18 3×177=531
19 3×187=561
20 3×197=591
21 3×207=621
22 3×217=651
23 3×227=681
24 3×237=711
25 3×247=741
26 3×257=771
27 3×267=801
28 3×277=831
29 3×287=861
30 3×297=891
31 3×307=921
32 3×317=951

33 3×327=981

Since this is a primary table, we will not remove any elements from it.

C(ii) Here we multiply 13 by a set of numbers ending with 7 where the results fall between 0 and 1,000.00

Table 4(vi)

1 13×7=91

2 13×17=221

3 13×37=481

4 13×47=611

5 13×67=871

C(iii) Here we multiply 23 by a set of numbers ending with 7 where the results fall between 0 and 1,000

Table 4(vii)

1 23×7=161

2 23×17=391

3 23×37=851

C(iv) Here we will not bother with 33 because 33 is divisible by 3.

C(v) Here we will multiply 43 by a set of numbers ending with 7 where the result falls between 0 and 1,000.

Table 4 (viii)

1 43×7=301

2 43×17=731

C(vi) Here we will multiply 53 by a set of numbers ending with 7 where the result falls between 0 and 1,000

Table 4 (ix)

1 53×7=371

2 53×17=901

C(vii) Here we multiply any number above 53 by a set of numbers ending with 7 where the result falls between 0 and 1,000.

Table (x)

1 73×7=511

2 83×7=581

3 103×7=721

4 113×7=791

5 133×7=931

Analysis

We know that there are 100 numbers below 1,000 that end with 1. To find the number of prime numbers below 1,000 ending with 1, we first have to calculate the number of non-prime numbers between 0 and 1,000 ending with 1 and then subtract this figure from 100.

Below is a list of tables and their corresponding number of prime numbers ending with 1 between 1 and 1,000

Table	Number
Table 4(i)	5
Table 4(ii)	1
Table 4(iii)	2

Table 4(iv)	1
Table 4(v)	33
Table 4(vi)	5
Table 4(vii)	3
Table 4(viii)	2
Table 4(ix)	5
Table 4(x)	1
Number 1	1
Total	60

The final element in the table written as "number 1" represents the numeral 1 which is not a prime neither is it a semi-prime nor a multiple of a prime. Therefore it deserves its own special category and that is why i have added it last. Anyway looking at the table we can see that we have 60 non-primes ending with 1 between 0 and 1,000. That means subtracting this figure from 100 gives us 40. This means that we have 40 prime numbers ending with 1 between 0 and 1,000. Indeed confirming with the prime number table at the back of the book gives us the same figure of 40.

Chapter 5 Finding prime numbers ending with 7 from a list of all numbers ending with 7 between 0 and 1,000.

A(i) Here we multiply 7 by a set of numbers ending with 1 where the result falls between 0 and 1,000 and excluding multiples of 3

Table 5(i)

1 7×11=77
2 7×31=217
3 7×41=287
4 7×61=427
5 7×71=497
6 7×91=637
7 7×101=707
8 7×121=847
9 7×131=917

A(ii) Here we multiply 17 by a set of numbers ending with 1 where the results fall between 0 and 1,000 and excluding multiples of 3.

Table 5(ii)

1 17×11=187
2 17×31=527
3 17×41=697

A(iii) Here we multiply all numbers ending with 7 above 17 with a set of numbers ending with 1 where the results fall between 0 and 1,000 and excluding multiples of 3

Table 5(iii)

1 37×11=407

2 47×11=517
3 67×11=737

B(i) Here we will multiply 3 by a set of numbers ending with 9 where the results fall between between 0 and 1,000

Table 5(iv)

1 3×9=27
2 3×19=57
3 3×29=87
4 3×39=117
5 3×49=147
6 3×59=177
7 3×69=207
8 3×79=237
9 3×89=267
10 3×99=297
11 3×109=327
12 3×119=357
13 3×129=387
14 3×139=417
15 3×149=447
16 3×159=477
17 3×169=507
18 3×179=537
19 3×189=567
20 3×199=597
21 3×209=627

22 3×219=657
23 3×229=687
24 3×239=717
25 3×249=747
26 3×259=777
27 3×269=807
28 3×279=837
29 3×289=867
30 3×299=897
31 3×309=927
32 3×319=957
33 3×329=987

B(ii) Here we multiply 13 by a set of numbers ending with 7 where the results fall between 0 and 1,000 and excluding multiples 3.

Table 5(v)

1 13×19=247
2 13×29=377
3 13×59=767

B(ii) Here we multiply 23 by a set of numbers ending with 9 where the result fall between 0 and 1,000 and excluding multiples of 3.

Table 5(vi)

1 23×19=437
2 23×29=667

B(iii) Here we multiply all numbers ending with 3 above 23 by a set of numbers ending with 9 where the result falls between 0 and 1,000

Table 5(vii)

1 43×19=817

Notice that for prime numbers ending with 7, there are only two phases A and B, there is no C due to repetition and this was proven in earlier chapters. That is, there are only two iterations of multiplications for prime numbers ending with 7.

Analysis

There are 100 numbers ending with 7 between 0 and 1,000. We need to find the number of non-primes ending with 7 between 0 and 1,000. That is documented below

Table	Number
Table 5(i)	9
Table 5(ii)	3
Table 5(iii)	3
Table 5(iv)	33
Table 5(v)	3
Table 5(vi)	2
Table 5(vii)	1
Total	54

Thus there are 54 non-prime numbers ending with 7 between 0 and 1,000.
This figure when subtracted from 100 gives 46. This figure is confirmed by checking the list of prime numbers at the back of the book that indeed there are 46 primes ending 7 between 0 and 1,000

Chapter 6 Finding prime numbers ending with 3 from a list of all numbers ending with 3 between 0 and 1,000.

A(i) Here we multiply 7 by a set of numbers ending with 9 where the result falls between 0 and 1,000 and excluding multiples of 3.

Table 6(i)

1	7×19=133
2	7×29=203
3	7×49=343
4	7×59=413
5	7×79=553
6	7×89=623
7	7×109=763
8	7×119=833
9	7×139=973

A(ii) Here we multiply 17 by a set of numbers ending with 9 where the result falls between 0 and 1,000 and excluding multiples of 3.

Table 6(ii)

1	17×19=323
2	17×29=493

A(iii) Here we multiply numbers ending with 7 greater than 17 by a set of numbers ending with 9 where the result falls between 0 and 1,000 excluding multiples of 3.

Table 6(iii)

1	37×19=703

2	47×19=893

B(i) Here we multiply 3 by a set of numbers ending with 1 where the result falls between 0 and 1,000

	Table 6(iv)
1	3×11=33
2	3×21=63
3	3×31=93
4	3×41=123
5	3×51=153
6	3×61=183
7	3×71=213
8	3×81=243
9	3×91=273
10	3×101=303
11	3×111=333
12	3×121=363
13	3×131=393
14	3×141=423
15	3×151=453
16	3×161=483
17	3×171=513
18	3×181=543
19	3×191=573
20	3×201=603
21	3×211=633

22	3×221=663
23	3×231=693
24	3×241=723
25	3×251=753
26	3×261=783
27	3×271=813
28	3×281=843
29	3×291=873
30	3×301=903
31	3×311=933
32	3×321=963
33	3×331=993

B(ii) Here we multiply 13 by a set of numbers ending with 1 where the result falls between 0 and 1,000 and excluding multiples of 3.

Table 6(v)

1	13×11=143
2	13×31=403
3	13×41=533
4	13×61=793
5	13×71=923

B(ii) Here we multiply 23 by a set of numbers ending with 1 where the result falls between 0 and 1,000 and excluding multiples of 3.

Table 6(vi)

1	23×11=253

2	23×31=713
3	23×41=943

B(iii) Here we multiply numbers ending with 3 greater than 23 by a set of numbers ending with 1 where the result falls between 0 and 1,000.

Table 6(vii)

1	43×11=473
2	53×11=583
3	73×11=803
4	83×11=913

Analysis

There are 100 numbers ending with 3 between 0 and 1,000. We must find the number of non-prime numbers ending with 3 between 0 and 1,000

Table	Number
Table 6(i)	9
Table 6(ii)	2
Table 6(iii)	2
Table 6(iv)	33
Table 6(v)	5
Table 6(vi)	3
Table 6(vii)	4
Total	58

This suggests there are 58 non-primes ending with 3 between 0 and 1,000 and therefore that there are 42 prime numbers ending with 3 between 0 and 1,000. Looking at the prime number table at the back of the book confirms that indeed there are 42 prime numbers ending with 3 between 0 and 1,000

Further Analysis

Analysing further all the results we have found so far from chapter three to six

We have found that between 0 and 1,000 there are 38 primes ending with 9, 40 primes ending with 1, 46 primes ending with 7 and 42 primes ending with 3. Adding them together 38+40+46+42=166. (Don't forget to add numbers 2 and 5 which appear only once)

This suggests there are 168 primes between 0 and 1,000 and looking at the prime number table at the back of the book confirms this to be true.

Chapter 7 Finding prime numbers ending with 9 from a list of all numbers ending with 9 between 0 and 2,000

A(i) Here we multiply 7 by a set of numbers ending with 7 where the result falls between 0 and 2,000 and excluding multiples of 3

Table 7(i)

1	7×7 = 49
2	7×17 = 119
3	7×37 = 259
4	7×47 = 329
5	7×67 = 469
6	7×77 = 539
7	7×97 = 679
8	7×107 = 749
9	7×127 = 889
10	7×137 = 959
11	7×157= 1,099
12	7×167=1,169
13	7×187=1309
14	7×197=1,379
15	7×217=1519
16	7×227=1,589
17	7×247=1,729
18	7×257=1,799
19	7×277=1,939

A(ii) Here we multiply 17 by a set of numbers ending with 7 where the result falls between 0 and

2,000 and excluding multiples of 3

Table 7(ii)

1	17×17=289
2	17×37=629
3	17×47=799
4	17×67=1,139
5	17×97=1,649
6	17×107=1,819

A(iii) Here we multiply 37 by a set of numbers ending with 7 where the result falls between 0 and 2,000 and excluding multiples of 3

Table 7(iii)

1	37×37=1,369
2	37×47=1,739

B(i) Here we multiply 19 by a set of numbers ending with 1 where the result falls between 0 and 2,000 and excluding multiples of 3 and 7

Table 7(iv)

1	19×11=209
2	19×31=589
3	19×41=779
4	19×61=1,159
5	19×71=1,349
6	19×101=1,919

B(ii) Here we multiply 29 by a set of numbers ending with 1 where the result falls between 0 and

2,000 and excluding multiples of 3 and 7

Table 7(v)

1	29×11=319
2	29×31=899
3	29×41=1,189
4	29×61=1,769

B(iii) Here we multiply numbers ending with 9 greater than 29 by a set of numbers ending with 1 where the result falls between 0 and 2,000 and excluding multiples of 3 and 7

Table 7(vi)

1	59×11=649
2	59×31=1,829
3	79×11=869
4	89×11=979
5	109×11=1,199
6	139×11=1,529
7	149×11=1,639
8	169×11=1,859
9	179×11=1,969

C(i) Here we multiply 3 by a set of numbers ending with 3 where the result falls between 0 and 2,000

Table 7(vii)

1	3×3=9
2	3×13=39

3	$3 \times 23 = 69$
4	$3 \times 33 = 99$
5	$3 \times 43 = 129$
6	$3 \times 53 = 159$
7	$3 \times 63 = 189$
8	$3 \times 73 = 219$
9	$3 \times 83 = 249$
10	$3 \times 93 = 279$
11	$3 \times 103 = 309$
12	$3 \times 113 = 339$
13	$3 \times 123 = 369$
14	$3 \times 133 = 399$
15	$3 \times 143 = 429$
16	$3 \times 153 = 459$
17	$3 \times 163 = 489$
18	$3 \times 173 = 519$
19	$3 \times 183 = 549$
20	$3 \times 193 = 579$
21	$3 \times 203 = 609$
22	$3 \times 213 = 639$
23	$3 \times 223 = 669$
24	$3 \times 233 = 699$
25	$3 \times 243 = 729$
26	$3 \times 253 = 759$
27	$3 \times 263 = 789$
28	$3 \times 273 = 819$
29	$3 \times 283 = 849$

30 3×293=879

31 3×303=909

32 3×313=939

33 3×323=969

34 3×333=999

35 3×343=1,029

36 3×353=1,059

37 3×363=1,089

38 3×373=1,119

39 3×383=1,149

40 3×393=1,178

41 3×403=1,209

42 3×413=1,239

43 3×423=1,269

44 3×433=1,299

45 3×443=1,329

46 3×453=1,359

47 3×463=1,389

48 3×473=1,419

49 3×483=1,449

50 3×493=1,479

51 3×503=1,509

52 3×513=1,539

53 3×523=1,569

54 3×533=1,599

55 3×543=1,629

56 3×553=1,659

57 3×563=1,689
58 3×573=1,719
59 3×583=1,749
60 3×593=1,779
61 3×603=1809
62 3×613=1,839
63 3×623=1,869
64 3×633=1,899
65 3×643=1,929
66 3×653=1,959
67 3×663=1,989

C(ii) Here we multiply 13 by a set of numbers ending with 3 where the result falls between 0 and 2,000 and excluding multiples of 3

Table 7(viii)

1	13×13=169
2	13×23=299
3	13×43=559
4	13×53=689
5	13×73=949
6	13×83=1,079
7	13×103=1,339
8	13×113=1,469

C(iii) Here we multiply 23 by a set of numbers ending with 3 where the result falls between 0 and

2,000 and excluding multiples of 3

Table 7(ix)

1	23×23=529
2	23×43=989
3	23×53=1,219
4	23×73=1,679
5	23×83=1,909

C(iv) Here we multiply 43 by a set of numbers ending with 3 where the result falls between 0 and 2,000 and excluding multiples of 3

Table 7(x)

1	43×43=1,849

Analysis

There are 200 numbers between 0 and 2000 ending with 9. We want to find the number of non-primes between 0 and 2,000.

Table	Number
Table 7(i)	19
Table 7(ii)	6
Table 7(iii)	2
Table 7(iv)	6
Table 7(v)	4
Table 7(vi)	9
Table 7(vii)	67

Table 7(viii)	8
Table 7(ix)	5
Table 7(x)	1
Total	127

This suggests that there are 127 non-prime numbers ending with 9 between 0 and 2,000, therefore also suggesting that there are 73 prime numbers ending with 9 between 0 and 2,000. Checking the prime number table at the end of the book confirms this to be true.

Prime numbers ending with 9

19
29
59
79
89
109
139
149
179
199
229
239
269
349
359
379
389
409
419
439
449
479
499
509
569
599

619
659
709
719
739
769
809
829
839
859
919
929
1009
1019
1039
1049
1069
1109
1129
1229
1249
1259
1279
1289
1319
1399
1409

1429
1439
1459
1489
1499
1549
1559
1579
1609
1619
1669
1699
1709
1759
1789
1879
1889
1949
1979
1999

Total=73

List of prime numbers below 1,000 as classified by their endings.

Primes ending with 1	Primes ending with 3	Primes ending with 7	Primes ending with 9
11	3	7	19
31	13	17	29
41	23	37	59
61	43	47	79
71	53	67	89
101	73	97	109
131	83	107	139
151	103	127	149
181	113	137	179
191	163	157	199
211	173	167	229
241	193	197	239
251	223	227	269
271	233	257	349
281	263	277	359
311	283	307	379
331	293	317	389
401	313	337	409
421	353	347	419
431	373	367	439
461	383	397	449
491	433	457	479
521	443	467	499
541	463	487	509

571	503	547	569
601	523	557	599
631	563	577	619
641	593	587	659
661	613	607	709
691	643	617	719
701	653	647	739
751	673	677	769
761	683	727	809
811	733	757	829
821	743	787	839
881	773	797	859
911	823	827	919
941	853	857	929
971	863	877	
991	883	887	
	953	907	
	983	937	
		947	
		967	
		977	
		997	
Total=40	Total=42	Total=46	Total=38

References

1.Hardy, G. H. and Wright, E. M. "Statement of the Fundamental Theorem of Arithmetic," "Proof of the Fundamental Theorem of Arithmetic," and "Another Proof of the Fundamental Theorem of Arithmetic." §1.3, 2.10 and 2.11 in An Introduction to the Theory of Numbers, 5th ed. Oxford, England: Clarendon Press, pp. 3 and 21, 1979.

2 Tietze, H.Famous Problems of Mathematics:
Solved and Unsolved Mathematics Problems from Antiquity to Modern Times. New york: Graylock Press, pp.7-9, 1965.

3.Marcus du Sautoy, The music of the Primes, 2003.

4. Conway, J. H.; Dietrich, H.; O'Brien, E. A. "Counting Groups: Gnus, Moas, and Other Exotica." Math. Intell. 30, 6-18, 2008.

5. Sloane, N.J.A. Sequences A001358 /M3274, A006881 in "The On-Line Encyclopedia of Integer Sequences."

6. Weisstein, Eric W. "Semiprime." From MathWorld-A Wolfram Web Resource. http://mathworld.wolfram.com/Semiprime.html