Order in the particle zoo

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Abstract

The standard model of physics classifies particles into elementary leptons and hadrons composed of quarks. In this article the existence of an alternate ordering principle will be demonstrated giving particle energies to be quantized as a function of the fine-structure constant, α. The quantization can be derived using an appropriate wave function that acts as a probability amplitude on the electric field. The value of α itself can be approximated numerically by the gamma functions of the integrals for calculating particle energy.

The model may be used to calculate other particle properties as well, in particular particle interaction. The expansion of the gamma function provides quantitative terms for strong, Coulomb and gravitational interaction.

Necessary input parameters for all calculations can be reduced to elementary charge and electric constant.

1 Introduction

Particle zoo is the informal though fairly common nickname to describe what was formerly known as "elementary particles". The standard model of physics [1] divides these particles into leptons, considered to be fundamental "elementary particles" and the hadrons, composed of two (mesons) or three (baryons) quarks. Well hidden in the data of particle energies lies another ordering principle, based on a description of particles as electromagnetic objects.

Particles are interpreted as some kind of standing electromagnetic wave originating from a rotating electromagnetic field with the E-vector pointing towards the origin. Neutral particles are supposed to exhibit nodes separating corresponding equal volume elements of opposite polarity. To obtain quantifiable results, the electromagnetic field will be modified with an appropriate exponential function, Ψ(r, ϑ, φ), serving as probability amplitude of the field. The two integrals needed to calculate energy in point charge and photon representation exhibit the following two relations:

1) Their product - resulting from energy conservation - is characterized by containing the product of the two gamma functions $\Gamma_{+1/3}\,|\Gamma_{-1/3}| \approx \alpha^{-1}/(4\pi)$,

2) their ratio features a quantization of energy states with powers of $1/3^n$ over some base α_0, a relation that can be found in the particle data with $\alpha_0 = \alpha$ as:

$$W_n/W_e \approx 3/2\left(y_l^m\right)^{-1/3}\Pi_{k=0}^{n}\alpha\text{\^{}}(-1/3^k) \qquad n = \{0;1;2;..\} \quad ^1 \qquad (1)$$

with W_e = energy of electron, W_n = energy of particle n and y_l^m representing the angular part of Ψ(r, ϑ, φ). For spherical symmetry $y_0^0 = 1$ holds, corresponding particles are e, μ, η, p/n, Λ, Σ and Δ [2]. The factor 3/2 is related to angular momentum |J| = 1/2.

Apart from calculating energies the model may be used to describe other particle properties. At distances comparable to particle size, typically femtometer for hadrons, direct interaction of particle wave functions has to be expected. Interpreting this as strong interaction and considering the basic spatial characteristics of the functions may provide a possible explanation why leptons, in particular the tauon, are not subject to this interaction. Expanding the incomplete gamma function appearing in the integrals for calculating particle energy gives quantitative terms for the strong and Coulomb interaction and a possibility to derive a quantitative expression for gravitational attraction as well, suggesting a common base for all three forces.

2 Results

2.1 Basic calculations

The model is essentially based on a single assumption:

Particles can be described by using an appropriate exponential wave function, Ψ(r), that acts as a probability amplitude on an electromagnetic field.

An appropriate form of Ψ can be deduced from three boundary conditions:

1 Results of table 1 are calculated with coefficients according to 2.6 including minor correction factors of order ≤ 1.005.

2 The relation of the e, μ, π masses with α was noted in 1952 by Y.Nambu [2]. M.MacGregor calculated particle and constituent quark mass as multiples of α and related parameters [3]. This article is a shortened + revised version of [4].

1.) To be able to apply Ψ to a point charge Ψ(r = 0) = 0 is required, this gives

$$\Psi(r) \sim \exp\left(\frac{-\beta/2}{r^y}\right) \tag{2}$$

2.) To ensure integrability an integration limit is needed. This may be achieved by Ψ(r) being the solution of a 2nd order differential equation of general form

$$-\Delta\Psi(r) + \frac{\beta/2}{r^x}\nabla\Psi(r) - \frac{\beta/2}{\sigma r^y}\Psi(r) = 0 \tag{3}$$

giving a general solution for particle n as:

$$\Psi_n(r)=\exp\left(-\left(\frac{\beta_n/2}{r^x}+\left[\left(\frac{\beta_n/2}{r^x}\right)^2-4\frac{\beta_n/2}{\sigma r^y}\right]^{0.5}\right)/2\right) \tag{4}$$

3.) Ψ should be applicable regardless of the expression chosen to describe the electromagnetic object. In particular requiring a point charge and a photon representation of a localized electromagnetic field (particle) to have the same energy, the exponent of r is required to be x=y=3 (see (14)), giving finally:

$$\Psi_n(r)=\exp\left(-\left(\frac{\beta_n/2}{r^3}+\left[\left(\frac{\beta_n/2}{r^3}\right)^2-4\frac{\beta_n/2}{\sigma r^3}\right]^{0.5}\right)/2\right) \tag{5}$$

In all integrals over Ψ(r) given below equ. (6) may be used as approximation for (5) up to r = r_l:

$$\Psi(r<r_l) \approx \exp\left(\frac{-\beta_n/2}{r^3}\right) \tag{6}$$

Phase will be neglected on this approximation level, properties of particles will be calculated by the integral over $\Psi(r)^2$ (hence factor 2 in (2)ff) times some function of r and can be given in very good approximation by:

$$\int_0^{r_l}\Psi(r)^2 r^{-(m+1)}dr \approx \int_0^{r_l}\exp(-\beta/r_l^3)r^{-(m+1)}dr = \Gamma(m/3,\beta/r_l^3)\,\frac{\beta^{-m/3}}{3} = \int_{\beta/r_l^3}^{\infty} t^{\frac{m}{3}-1}e^{-t}dt\,\frac{\beta^{-m/3}}{3} \tag{7}$$

with m = {..-1;0;1;..}. The term $\Gamma(m/3, \beta/r_l^3)$) denotes the upper incomplete gamma function, given by the Euler integral of the second kind with β/r_l^3 as lower integration limit. For m ≥ 1 the complete gamma function $\Gamma_{m/3}$ is a sufficient approximation, for m ≤ 0 the integrals have to be integrated numerically. Coefficient β_n is a particle specific factor, for particle n it may be given as partial product of a starting value for a reference particle carrying the dimensions, β_e, chosen to be the electron, times dimensionless coefficients, α_n, of succeeding particles representing the ratio of β_n and β_{n+1}:

$$\beta_n = \beta_e\,\alpha_1\alpha_2..\,\alpha_n = \beta_e\,\Pi_{k=0}^{n}\alpha_k = \beta_e\,\Pi_n = \beta_{dim}\,\alpha_e\,\Pi_n \tag{8}$$ [3]

Coefficient σ is a constant (σ = 1.772E+8[-]) related to J= 1/2 [4] and can be calculated from setting the root term in (5) to zero and using the relation between radial coordinate and Euler integral, equation (7) for m= -1

$$r_l \approx 1.51\,|\Gamma_{-1/3}|\,\beta_n^{1/3}/(3\alpha) \tag{9}$$

to replace r_l, giving:

$$\sigma = 8\,r_{l,n}^3/\beta_n = 8(1.51\,|\Gamma_{-1/3}|\,\beta_n^{1/3}/(3\alpha))^3 \tag{10}$$

Particle energy is expected to be equally divided into electric and magnetic part, $W_n = 2W_{n,el} = 2W_{n,mag}$. To calculate energy the integral over the electrical field E(r) of a point charge is used as a first approximation. Using (7) for m = 1 gives:

$$W_{pc,n} = 2\varepsilon_0\int_0^{\infty}E(r)^2\Psi_n(r)^2 d^3r = 2b_0\int_0^{r_{l,n}}\Psi_n(r)^2 r^{-2}dr = 2\,b_0\,\Gamma_{1/3}\,\beta_n^{-1/3}/3 \tag{11}$$ [5]

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3 β_e will be further split into a dimension-attatched part β_{dim} and a dimensionless part α_e by (28).

4 Semi-classically, angular momentum J = r x p ≈ rW/c_0, requires an integral over $\Psi^2 r^{-1}$, yielding a constant sensitive to variations of the integration limit of the Euler integral, (7): $\beta/r_l^3 = 8/\sigma$ rather than of equally sized terms in Ψ.

Using equation (7) for m = -1 to calculate the Compton wavelength, λ_C, in the expression for the energy of a photon, hc_0/λ_C, gives:

$$\lambda_{C,n} = \int_0^{\lambda_{C,n}} \Psi_n(r)^2 dr \quad = \int_{\beta/\lambda^3_{C,n}}^{\infty} t^{-4/3} e^{-t} dt \;\; \beta_n^{1/3}/3 \quad = 36\,\pi^2\; |\Gamma_{-1/3}|\; \beta_n^{1/3}/3 \tag{12}$$

$$W_{Phot,n} = hc_0/\lambda_{C,n} \;=\; \frac{hc_0}{\int^{\lambda_{C,n}} \Psi_n(r)^2 dr} \;=\; \frac{3\,hc_0}{36\,\pi^2 |\Gamma_{-1/3}|\,\beta_n^{1/3}} \tag{13}$$

The energy of a particle has to be the same in both photon and point charge description. Equating (11) with (13) and rearranging to emphasize the relationship of α with the gamma functions ($\Gamma_{1/3}$ = 2.679; $|\Gamma_{-1/3}|$ = 4.062) gives (note: h => ħ):

$$\frac{4\,\pi\,\Gamma_{1/3}|\Gamma_{-1/3}|}{0.998} \;=\; \frac{9\,hc_0}{18\,\pi\,b_0} \;=\; \frac{\hbar\, c_0}{b_0} \;=\; \alpha^{-1} \tag{14}$$

2.2 Quantization with powers of $1/3^n$ over α

Inserting (8) in the product of the point charge and photon expression of energy, W_n^2, gives:

$$W_n^2 \;=\; 2\,b_0 hc_0 \;\frac{\int^{r_{l,n}} \Psi_n(r)^2 r^{-2}\, dr}{\int^{\lambda_{C,n}} \Psi_n(r)^2\, dr} \;\sim\; \frac{1}{\beta_n^{2/3}} \;\sim\; \frac{\alpha_0^{1/3}\,\alpha_1^{1/3}.....\alpha_n^{1/3}}{\alpha_0\,\alpha_1....\alpha_n} \qquad n = \{0;1;2;..\} \tag{15}$$

The last expression of (15) is obtained by expanding the product $\Pi_{\tau,n}^{-2/3}$ included in $\beta_n^{-2/3}$ with $\Pi_n^{1/3}$ From this term it is obvious that a relation $\alpha_{n+1} = \alpha_n^{1/3}$ such as given by equation (1) yields the only non-trivial solution for W_n^2 where all intermediate particle coefficients cancel out and W_n becomes a function of coefficient α_0 only. By comparison with experimental data α_0 may be identified as $\alpha_0 = \alpha_e \approx \alpha^9$ and the α-product can in general be given by:

$$\frac{\alpha^3\alpha^1....\alpha\wedge(9/3^n)\alpha\wedge(3/3^n)}{\alpha^9\,\alpha^3\,\alpha^1....\alpha\wedge(9/3^n)} \;=\; \alpha\wedge(3/3^n)/\alpha^9 \qquad n = \{0;1;2;..\} \tag{16}$$

The corresponding term for particle energies will be given by (using (14)):

$$W_n \;=\; \frac{4\,\pi\,b_0^2}{\alpha}\;\frac{\int^{r_{l,n}} \Psi_n(r)^2 r^{-2} dr}{\int^{\lambda_{C,n}} \Psi_n(r)^2 dr} \;=\; \left(\frac{(2\,b_0)^2\,\Gamma_{1/3}^2}{9\,[\alpha\, 4\,\pi\;\; |\Gamma_{-1/3}|\Gamma_{1/3}]\,\beta_n^{2/3}}\right)^{0.5} \;=$$

$$=\; 2\,b_0\frac{\Gamma_{1/3}}{3\,\beta_n^{1/3}} \;=\; 2\,b_0\frac{\Gamma_{1/3}}{3\,\beta_{dim}^{1/3}}\,\alpha\wedge(1.5/3^n)/\alpha^{4.5} \;=\; W_e\;\frac{3}{2}\;\;\Pi_{k=0}^{n}\,\alpha\wedge(-1/3^k) \qquad n = \{0;1;2;..\} \tag{17}$$

giving equation (1). For factor ≈ 3/2 see 2.6.

Extending the model to energies below the electron with a coefficient of α^3 in (1) gives a state with energy ~ 0.2eV which is roughly in a range expected for a neutrino [5].

Up to here only spherical symmetry and Ψ(r) is considered. The ratio of the volume integrals attributed to Y_1^0 and Y_0^0 gives a factor of 1/3. Assuming Y_1^0 to be a sufficient approximation for the next angular term and $W_n \sim 1/r_n \sim 1/V_n^{1/3}$ (V = volume) to be applicable for non-spherically symmetric states as well, will give $W_1^0/W_0^0 = 3^{1/3}$ =1.44 . A change in angular momentum is expected for this transition which is actually observed with ΔJ = ± 1 except for the pair μ/π with Δ J = 1/2.

Results for particles assigned to y_0^0, y_1^0 are presented in table 1.

2.3 Additional particle states

In general it is not expected that partial products can explain all values of particle energies and linear combination states have to be considered.

5 $b_0 = e^2/(4\pi\varepsilon)$ to be used as abbreviation in the following

	n	$W_{n,Lit}$ [MeV]	$\Pi_{k=0}^{n}\ \alpha^{(-1/3^k)}$ equ (1)	$\Pi_{\tau,n}$ equ (34)	W_{calc}/W_{Lit}	J	r_l [fm]
ν	**-1**	**2E-7 ***	$\boldsymbol{\alpha^{+3}}$		**-**	**1/2**	**3.6E+9**
e^{+-}	**0**	**0.51**	Reference	$\boldsymbol{\alpha^9}$	**1.0001**	**1/2**	**1413**
μ^{+-}	**1**	**105.66**	$\boldsymbol{\alpha^{-1}}$	$\boldsymbol{\alpha^9\alpha^3}$	**1.0001**	**1/2**	**6.84**
π^{+-}	1	139.57	1.44 α^{-1}	$\alpha^9\alpha^3/3$	1.0920	0	5.17
K		495				0	
η^0	**2**	**547.86**	$\boldsymbol{\alpha^{-1}\alpha^{-1/3}}$	$\boldsymbol{\alpha^9\alpha^3\alpha^1}$	**0.9934**	**0**	**1.32**
ρ^0	2	775.26	1.44 $(\alpha^{-1}\alpha^{-1/3})$	$\alpha^9\alpha^3\alpha^1/3$	1.0195	1	0.93
ω^0	2	782.65	1.44 $(\alpha^{-1}\alpha^{-1/3})$	$\alpha^9\alpha^3\alpha^1/3$	1.0031	1	0.93
K*		894				1	
p^{+-}	**3**	**938.27**	$\boldsymbol{\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}}$	$\boldsymbol{\alpha^9\alpha^3\alpha^1\alpha^{1/3}}$	**1.0018**	**1/2**	**0.77**
n	**3**	**939.57**	$\boldsymbol{\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}}$	$\boldsymbol{\alpha^9\alpha^3\alpha^1\alpha^{1/3}}$	**1.0004**	**1/2**	**0.77**
η'		958				0	
Φ^0		1019				1	
Λ^0	**4**	**1115.68**	$\boldsymbol{\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}}$	$\boldsymbol{\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}}$	**1.0108**	**1/2**	**0.65**
Σ^0	**5**	**1192.62**	$\boldsymbol{\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}}$	$\boldsymbol{\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}\alpha^{1/27}}$	**1.0048**	**1/2**	**0.61**
Δ	∞	**1232.00**	$\boldsymbol{\alpha^{-3/2}}$	$\boldsymbol{\alpha^{27/2}}$	**1.0027**	**3/2**	**0.59**
Ξ		1318				1/2	
Σ^{*0}	3	1383.70	1.44 $(\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9})$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}/3$	0.9798	3/2	0.52
Ω^-	4	1672.45	1.44 $(\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27})$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}/3$	0.9725	3/2	0.43
N(1720)	5	1720.00	1.44 $(\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81})$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}\alpha^{1/27}/3$	1.0048	3/2	0.42
tau^{+-}	∞	1776.82	1.44 $(\alpha^{-3/2})$	$\alpha^{27/2}/3$	1.0026	1/2	0.41

Table 1: Particles up to tauon energy[6]; values for $\mathbf{y_0^0}$ **(bold)**, y_1^0; col. 3: energy values of [6] except*: calculated from (1); electron value** calculated with (28);

The first particle family that does not fit to the partial product scheme are the kaons at ~ 495MeV. Assuming them to be a linear combination of two π-states with a supposed charge distribution of +|+, -|- and +|- would yield the basic symmetry properties of the four kaons as given below, providing two neutral kaons of different structure and parity:

		+				-				-				+		
K^+	+		+	K^-	-		-	K_S^o	+		+	K_L^o	+		-	(+/- = charge)
		+				-				-				-		

Analogous, for the charged kaons, K^+, K^-, a configuration for wave function sign equal to the configuration for charge of K_S^o and K_L^o might be possible, giving two analogous variants of + and - parity of otherwise identical particles. Such configurations for the kaons might give a simple explanation for the unusual decay modes observed in the experiments.

2.4 Expansion of the incomplete gamma function $\Gamma(1/3,\beta_n/r^3)$

The series expansion of $\Gamma(1/3,\beta_n/r^3)$ in the equation for calculating particle energy (11) gives [7]:

$$\Gamma(1/3,\ \beta_n/(r^3)) \approx \Gamma_{1/3} - 3\left(\frac{\beta_n}{r^3}\right)^{1/3} + \frac{3}{4}\left(\frac{\beta_n}{r^3}\right)^{4/3} = \Gamma_{1/3} - 3\frac{\beta_n^{1/3}}{r} + \frac{3}{4}\frac{\beta_n^{4/3}}{r^4} \tag{18}$$

and for the potential energy part of $W_n(r)$, $W_{n,pot}(r) = W_n(r)/2$:

$$W_{n,pot}(r) \approx W_n/2 - b_0\frac{3\beta_n^{1/3}}{3\beta_n^{1/3}r} + b_0\frac{3}{4}\frac{\beta_n^{4/3}}{3\beta_n^{1/3}r^4} = W_n/2 - \frac{b_0}{r} + b_0\frac{\beta_n}{4r^4} \tag{19}$$

The 2nd term in (19) drops the particle specific factor β_n and gives the electrostatic energy of two elementary charges at distance r. The 3rd term is an appropriate choice for the 0th order term of the differential equation

6 up to Σ'^0 all resonance states given in [6] as **** included; Exponent of -3/2, 27/2 for Δ and tau is equal to the limit of the partial products in (1) and (34); r_l calculated with (9);

below and is supposed to be responsible for the localized character of an electromagnetic object. In chpt. 3.1.2 some arguments are given that demonstrate a relationship of the properties of the wave functions used in this model with the "strong force" of the standard model. It may be assumed that the 3rd term of (19) represents this strong force.

2.5 Differential equation

The approximation $\underline{\Psi(r<r_l)}$ of equation $\underline{(6)}$ provides a solution to a differential equation of type

$$-\frac{r}{6}\frac{d^2\Psi(r)}{dr^2} + \frac{\beta/2}{2r^3}\frac{d\Psi(r)}{dr} - \frac{\beta/2}{r^4}\Psi(r) = 0 \tag{20}$$

However the correct discriminant form of $\underline{\Psi(r)}$ of equ. $\underline{(5)}$ would be provided by a slightly different equation (revised by 6 in 2^{nd}, 2 in 1^{st} and σ in 0^{th} order term) :

$$-r\frac{d^2\Psi(r)}{dr^2} + \frac{\beta/2}{r^3}\frac{d\Psi(r)}{dr} - \frac{\beta/2}{\sigma\ r^4}\Psi(r) = 0 \tag{21}$$

To proceed from the heuristic mathematical approach of equation (20) to one based more on physics, the second order term is expected to represent a quantum mechanical term for kinetic energy including the impulse operator. Mass may be replaced by the term $W_e/(2\ c_0^2) = W_{e,kin}/c_0^2$, giving

$$W_{kin} = \left(\frac{2\hbar^2 c_0^2}{2\ W_e}\right)\frac{d^2\Psi(r)}{dr^2} \tag{22}$$

To recover (21) the following procedures are used as approximation

1.) $W_e \Rightarrow \Gamma_- \Gamma_+ 2\ b_0/(9\ r)$ which is an approximation for $r \approx r_m$ [7];

2.) The 3rd term of equ. (19), $[b_0\ \beta\ r^{-4}/4]$, modified by 1/σ, and equivalently $[b_0\ \beta\ r^{-3}/4]$ will be chosen for the 0^{th} and 1^{st} order terms of the differential equation;

3.) Since β, technically β_e, of the resulting expression has to match (21), β_e may be redefined as β_e^*.

This gives as differential equation (using (14)):

$$-\left(\frac{9\hbar^2 c_0^2 r}{\Gamma_{-1/3}\Gamma_{+1/3} 2 b_0}\right)\frac{d^2\Psi(r)}{dr^2} + \frac{b_0\beta_e^*}{4r^3}\frac{d\Psi(r)}{dr} - \frac{b_0\beta_e^*}{4\sigma\ r^4}\Psi(r) = 0 \tag{23}$$

$$\begin{aligned} &-\frac{d^2\Psi(r)}{dr^2} + \frac{[\Gamma_{-1/3}\Gamma_{+1/3}4\pi]b_0^2\beta_e^*}{72\pi\hbar^2 c_0^2 r^4}\frac{d\Psi(r)}{dr} - \frac{[\Gamma_{-1/3}\Gamma_{+1/3}4\pi]b_0^2\beta_e^*}{72\pi\hbar^2 c_0^2\sigma\ r^5}\Psi(r) \approx \\ &-\frac{d^2\Psi(r)}{dr^2} + \frac{\alpha\beta_e^*}{72\pi r^4}\frac{d\Psi(r)}{dr} - \frac{\alpha\beta_e^*}{72\pi\sigma\ r^5}\Psi(r) = 0 \end{aligned} \tag{24}$$

This gives $\beta_e^* \sim 72\pi/\alpha\ \beta_e \approx (2\pi)^3/\alpha\ \beta_e$. Factor $(2\pi)^3$ will be canceled in (25) below leaving ~ α unaccounted for, giving evidence that the quantum mechanical operator of kinetic energy and the 3rd term of the expansion of (19) for potential energy are approximately appropriate terms for the differential equation.

2.6 Model coefficients σ and τ

To get additional insight into the relationships of this model, β_n may be expressed as

$$\beta_n = 2\ (2\pi)^{-3}\ \sigma\ \tau_n\ b_0^2 \tag{25}$$

with parameter τ_n taking the role of the particle specific coefficient.

In the following electromagnetic units are required that have to be based on their relation to c_0, such as given e.g. in Planck or cgs units. In this work SI units are kept with the modification:

$$c_0^2 = (\varepsilon_c\ \mu_c)^{-1} \tag{26}$$

with $\varepsilon_c = (2.998E+8\ [m^2/Jm]\ )^{-1} = (2.998E+8)^{-1}\ [J/m]$
$\mu_c = (2.998E+8\ [Jm/s^2]\ )^{-1} = (2.998E+8)^{-1}\ [s^2/Jm]$

i.e. the numerical values for c_0, $1/\varepsilon_c$, $1/\mu_c$ are identical, the units of ε_c, μ_c are expanded by [Jm], allowing to give τ_e as ab initio term. From b_0 follows for the square of the elementary charge: $e_c^2 = 9{,}67E-36\ [J^2]$.

7 β_n in (11) replaced via term $r_{max,n} \approx \Gamma_-\ \beta_n^{1/3}/3$

2.6.1 τ_e

In chapter 2.2 it has been noted that τ_e is proportional to α^9 and using α^9 as starting value α_0 produces a series that describes particle energies reasonably well.
Using constants e_c and ε_c as defined above, τ_e may be defined as :

$$\tau_e' = \left(\frac{2}{3}\right)^3 \frac{\alpha^9}{e_c \varepsilon_c} = 1.676\text{E+6 [m/J}^2] \tag{27}$$

To account for the W_μ/W_e relationship, factor 2/3 in (27) should be replaced by 3/2, i.e. in the following

$$\tau_e = \left(\frac{3}{2}\right)^3 \frac{\alpha^9}{e_c \varepsilon_c} = \frac{\alpha_e}{e_c \varepsilon_c} = 1.909\text{E+7 [m/J}^2] \tag{28}$$

will be used, giving ψ as

$$\Psi_n(r) = \exp\left(-\left\{\left(\left(\frac{2}{3}\right)^6 \frac{\sigma \tau_n b_0^2}{(2\pi)^3 r^3}\right) + \left[\left(\left(\frac{2}{3}\right)^6 \frac{\sigma \tau_n b_0^2}{(2\pi)^3 r^3}\right)^2 - \left(\frac{2}{3}\right)^6 \frac{4 \tau_n b_0^2}{(2\pi)^3 r^3}\right]^{0.5}\right\}/2\right) \tag{29}$$

With (28) W_e may be expressed as

$$W_e = \frac{e_c^2}{2\pi \varepsilon_c} \int_0^{r_{l,n}} \Psi_e(r)^2 r^{-2} dr = \frac{\pi^{2/3} \Gamma_+}{k_\tau \Gamma_-} \frac{e_c}{\alpha^2} \approx \frac{2^{0.5} e_c}{\alpha^2} \tag{30}$$

2.6.2 σ

The value of 1.51 in (9)f is close to the ratio of Γ_- / Γ_- = 1.516 suggesting to give the term ~1.51 α^{-1} Γ_- /3 a geometrical interpretation (using (14)):

$$1.516\ \boldsymbol{\alpha^{-1}}\ \Gamma_- /3 = \Gamma_- /\Gamma_+\ \boldsymbol{4\pi\, \Gamma_-\, \Gamma_+ / k_a}\ \Gamma_- /3 \approx 4\pi/3\ \Gamma_-^3 \tag{31}$$

i.e. giving a dimensionless representation of particle volume.
In the following 1.516*k_a = 1.5133 will be used as parameter for σ:

$$\sigma = 8(4\pi\, \Gamma_-^3/3\,)^{\mathbf{3}} = (1.5133\ \alpha^{-1}\ \Gamma_-\ 2/3)^3 = (k_s\ \alpha^{-1}\ \Gamma_-\,)^3 = 1.772\text{E+8}[-] \tag{32}$$

2.6.3 Factor 1.5088 of W_μ/W_e, τ_n

The factor of W_μ/W_e = 1.5088 relates to 1.533 via:

$$\left(\frac{1.5133}{1.5088}\right)^3 = \left(\frac{1.5133}{1.5}\right) = k_s \tag{33}$$

indicating that the particle specific term, τ, and the components of σ are not correctly separated yet. This minor term will be considered by introducing appropriate terms of k_s in the expression for the partial products of τ_n and W_n, to give:

$$\tau_n = y_l^m\ \tau_e\, k_s\ \ \Pi_{k=0}^{n}\ \left(k_s^{-1}\alpha\right)\wedge\left(3/3^{\,k}\right) \qquad n = \{0;1;2;..\} \tag{34}$$

$$W_n/W_e = (y_l^m)^{-1/3}\, 1.533\ \ \Pi_{k=0}^{n}\ \left(k_s^{1/3}\alpha\right)\wedge\left(-1/3^{\,k}\right) \qquad n = \{0;1;2;..\} \tag{35}$$

2.7 Gravitation

Expressing energy/mass in essentially electromagnetic terms suggests to test if mass interaction i.e. gravitational attraction can be derived from the corresponding terms. The differential equation of (20)ff has an imaginary solution for r > r_l providing a principal source for interaction. A suitable starting point not only for Coulomb but gravitational interaction as well is the second term of the expansion of $\Gamma(1/3,\beta_n/r^3)$ from the integral over r^{-2}, featuring r^{-1} and implying that the Coulomb term b_0 will be part of the expression for F_G, i.e. the ratio between Coulomb and gravitational force has to be a term that can be given as completely separate, self-contained expression.
The following is based on 3 assumptions:
1.) The second term of the incomplete gamma function $\Gamma(1/3, \beta/r_l^3)$ is a suitable starting point, implying an r^{-1} potential and $F_G \sim b_0\, m_1 m_2$.

2.) The term for mass in F_G will be replaced by W_e/c_0^2. *To get a dimensionless term the units of e_c and c_0 (or ε_c) have to be canceled. This can be done by using model parameters* [8] *though not non-ambiguously. Since the terms of these model parameters contain e_c and ε_c anyway, the most unbiased approach for canceling is using e_c and c_0 (or ε_c) as reference for canceling directly, effectively dropping them in the equations below.*

3.) The model provides parameters linking electromagnetic constants with particle energy /mass. These, i.e. in particular the one of the electron, σ and τ_e, α_e, *have to be part of the equation for F_G* [9].

(All equations for two electrons while not indicated otherwise.)

Starting from the second term of the expansion of the incomplete gamma function $\Gamma(1/3, \beta/r_l^3)$ in the energy form, (19) and including $(W_e/c_0^2)^2$, gives $\beta_e^{1/3}/\beta_e^{1/3}(W_e/c_0^2)^2$. W_e will be replaced by equ. (30), giving:

$$\frac{\beta_e^{1/3}}{\beta_e^{1/3}}\left(\frac{[W_e]}{c_0^2}\right)^2 = \left[\frac{\pi^{2/3}\Gamma_+}{k_s\Gamma_-\alpha^2}\right]^2 \frac{e_c^2}{c_0^4}\left[\frac{J^2 s^2}{m^2}\right] \Rightarrow \left[\frac{\pi^{2/3}\Gamma_+}{k_s\Gamma_-\alpha^2}\right]^2 [-] = X^2 * F_G/F_C \tag{36}$$

A rough estimation according to the example in note 8 suggests to try α_e/σ as first approximation for 1/X:

$$\left(\left[\frac{\pi^{2/3}\Gamma_+}{k_s\Gamma_-\alpha^2}\right]\frac{\alpha_e}{\sigma}\right)^2 [-] = 0.060^2 * F_G/F_C \tag{37}$$

Parameter σ contains the parameter for length, Γ_-, raised to 3. Assuming that for r^{-2} in the force term only two Γ_- are needed in the denominator, the excess Γ_- will be dropped, as well as the minor factor k_s, giving:

$$\left(\left[\frac{\pi^{2/3}\Gamma_+}{k_s\Gamma_-\alpha^2}\right]\frac{k_s\alpha_e\Gamma_-^2}{\sigma}\right)^2 [-] = [W_e[-]]^2\ \gamma = 1.00022^2 * F_G/F_C \tag{38}$$

With $\gamma = (k_s\alpha_e\Gamma_-^2/\sigma)^2 = 3.46$ E-52, the classical constant $G = r^2 F_G/(m_1 m_2)$ may be expressed as:

$$G = b_0\frac{\gamma c^4}{e_c^2} = \frac{\gamma c^4}{4\pi\varepsilon_c} = 6.676\text{E-}11\left[\frac{m^5}{Js^4}\right] = 1.00024\, G_{\exp} = 1.00024\, 6.67408(31)E-11\left[\frac{m^5}{Js^4}\right] \quad [6]\ [10] \tag{39}$$

The long range forces between particles will be given as:

$$F_{m,n} = \frac{b_0}{r^2}\left[1 - \gamma\frac{W_m W_n}{e_c^2}\right] = \frac{1}{4\pi\varepsilon_c}\left[e_c^2 - \gamma W_m W_n\right] \tag{40}$$

3.Discussion

3.1 Particle interaction

3.1.1 Gravitation

The results above might be interpreted in terms of superposition of particle states.

Coulomb's equation for interaction of charge, which in QED is attributed to a one photon process, could be an appropriate starting point, where an additional contribution to the one photon interaction might be proportional to the mass $m_i = W_i/c_0^2$ of the particles involved. Equation (40) is already in such a form.

However, even simpler, within this model particles might interact via direct contact in place of photon-mediated interaction. Though using parameters such as r_l all over, the particles are not at all expected to exhibit a rigid radius. Within the limits of charge and energy conservation a superposition of many states might be conceivable, extending the particle in space with radius r_l appropriate for energy of each superposition state, enabling interaction at distance r_l. The “source” particle could provide the energy for a number of $n = W_{source}/W_{superpos}$ superposition states, creating a gravitational potential proportional to W_{source} at position $r_{l,superpos}$ [11].

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8 E.g.: $\tau_e \sim 1/(e_c\,\varepsilon_c)$ is the obvious choice to cancel $e_c\,\varepsilon_c$ in $W_e\,\varepsilon_c^2$ (replacing W_e/c_0^2); ε_c may be cancelled by $(\tau_e/\beta^{1/3})^{0.5}$; considering dimensionless contributions of $r^{-1} = \alpha^2\,\Gamma_-^2/(3(4\pi)^{2/3})$ gives (adding $(2/3)^3$ and dropping Γ_-^2): $\tau_e\,(\tau_e/\beta^{1/3})^{0.5}\,3(4\pi)^{2/3}/\alpha^2\,(2/3)^3([W_e]\,\varepsilon_c^2) = 0.993^2\,F_G/F_C \approx ([\alpha^{-2}]\,\alpha^{12})^2$.

9 Effectively this reintroduces the parameters used in the last note, yet is considered to be less biased.

10 The precision for calculating G is higher since (38) includes the error of calculating electron energy with (30).

11 For example: a proton at distance $r_{l,e}$ could not only create one electron superposition state and thus recreate the gravitational field of the electron itself but have the capacity of producing ~1836 electron states giving the gravitational

As for the attractive character of gravitational force, charge does cancel in (39) and the *e* in (30) obviously has to be unsigned to avoid negative particle energy. Since for the energy of a particle $W \sim \int E^2\Psi^2 d^3r$ holds and $\Psi \leq 1$, the wave function of a particle might contribute an additional factor to lower total Ψ values on site of a second particle thereby reducing particle energy and resulting in an attractive force.

Though being quite speculative the mechanism sketched above fits to the expansion of the Γ-function in the term of particle energy and the calculation of 2.7 demonstrates that it is possible to obtain a result for F_G in the correct order of magnitude.

3.1.2 Short range interaction - strong force

In this model, on the length scale of particle radius, the wave functions of two particles should start to overlap and exert some kind of direct interaction. As demonstrated in table 1, last column, for hadrons the model yields particle radius in the range of femtometer, the characteristic scale for strong interaction and it seems likely to identify strong interaction with the interaction of wave functions. Interaction via overlapping of wave functions constitutes the basis of chemical bonding and has been examined extensively [8]. In general wave functions are signed (not to be confused with electrical charge), for particles above the ground state regions of different sign exist, separated by nodes. There are two major requirements for effective interaction:

1) Comparable size and energy of wave functions,

2) sufficient net overlap: In the overlap region of two interacting wave functions sign should be the same (bonding) or opposite (antibonding) in all overlapping regions. If regions with same and opposite sign balance to give zero net overlap, no interaction results.

From condition 1) and the data of table 1 it is obvious that the wave functions of neutrino and electron/positron will not show effective interaction with hadrons due to mismatch of size and energy. In the case of the tauon the second rule is crucial. According to this model the tauon is at the end of the partial product series for y_1^0 and should consequently exhibit a high, potentially infinite number of nodes, separating densely spaced volume elements of alternating wave function sign. Though having particle size and energy in the same order of magnitude as other hadrons, such as the proton, the frequent change of sign of the tauon wave function will prohibit net overlap and effective interaction.

3.2 Relation to standard model of particle physics and quantum mechanics

The standard model classifies particles into leptons and hadrons, composed of two (mesons) or three (baryons) quarks. The classification into the three groups may be reproduced by this model.

Mesons constitute a distinct group of particles due to their integer angular momentum which is considered to be a combination of half-integer contributions in both models. In the standard model leptons are defined as being particles not subject to strong interaction, being essentially point like. Neutrinos, electron and muon are the particles of lowest mass which in itself might provide an explanation for this quality. The tauon however is outstanding in possessing a mass almost twice that of the proton and major decay channels involving hadrons. The considerations in chpt. 3.1.2 about overlap and wave function symmetry might provide a consistent explanation for all leptons not to be subject to strong interaction with hadrons which in turn should prohibit detection of internal structure of these particles.

In the model presented the y_0^0 and y_1^0 groups each include all three particle types. The possibility to calculate particle energies with a single model using a uniform set of parameters does not support to identify a special set of particles as more "elementary" than others. However, the standard model of particle physics distinguishes quite rigidly between leptons and hadrons, postulating that a set of physical objects characterized by an almost identical set of experimental observables - such as mass, charge, spin, magnetic moment, well defined mean life time and the effects of electromagnetism, weak interaction and gravitation - is based on completely different physical principles. This is quite an extraordinary claim, is it covered by extraordinary evidence ?

The postulate of leptons not being subject to strong interaction is not verifiable beyond experimental accuracy. Neutrino mass is a precedent for the fallacy to confuse a very small value with zero.

The three generation model, attributing a neutrino to each charged lepton, is a more severe argument.

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field of the proton at $r_{l,e}$.

The superposition states might have any energy, not restricted to those of particles. The life time of such states might be given by the uncertainty principle though the states would not be virtual in the sense of violating energy conservation.

However, the total number of neutrinos is not beyond doubt (cosmic neutrinos [5], MiniBoone [9]) and neutrino oscillation obscures the earlier assumption of clearly distinct particles. Last not least, a distinctive interaction of neutrinos with the charged leptons might simply be due to the very weak strong interaction of the particles involved not requiring any assumption beyond that.
The standard model describes very successfully hadron properties and the reliability of the model presented here will depend crucially on reproducing the symmetry properties as represented by the various quarks. On a rudimentary level this is the case as demonstrated above.

The relation of this model to classical quantum mechanics may be given by interpreting Ψ(r) as probability amplitude that is the solution of a simple single 2nd order differential equation applied to a field instead of a particle. This implies that concepts such as orthonormalization and calculation of eigenvalues may not be applicable. Properties have to be calculated by integration over the spatial extent of the field.
As a consequence the quantization condition given in 2.2 is not exclusive. The solution of (16)f relates to a set of rest mass of particles of sufficient stability to be observable experimentally but does not prohibit the existence of particles with any other mass. As demonstrated in chpt. 2.5 a quantum mechanical approach for W_{kin} yields acceptable results.
As for the number of parameters needed to calculate energy states, the model resembles the simplicity of ab initio quantum mechanical models, relying essentially on e_c and ε_c as input parameters.

Conclusion

Using the exponential function $\Psi(r,\vartheta,\varphi,e_c,\varepsilon_c)$ as probability amplitude for the electric field E(r) gives the following results:

- a numerical approximation for the value of the fine-structure constant α,
- a quantization of energy levels given by a partial product of terms α^$(-1/3^n)$,
- qualitative explanations for particle properties such as the lepton character of the tauon or the decay of kaons,
- a possibility to quantitatively express gravitational force entirely in electromagnetic terms,
- an indication of a common base for strong force, electromagnetism and mass/gravitation, given by a common set of -electromagnetic- coefficients and the expansion of the incomplete gamma function.

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