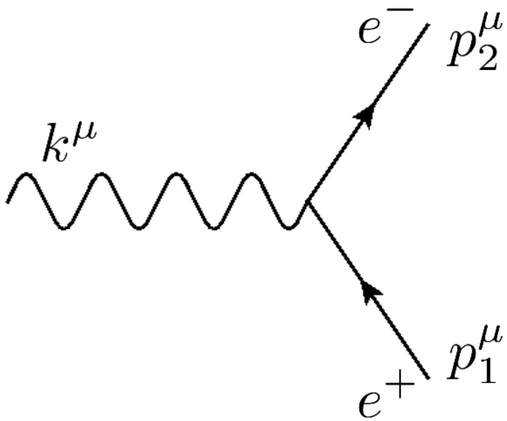
Pair creation on a nucleus in plasma induced by a strong laser field

P. Mati and S. Varró

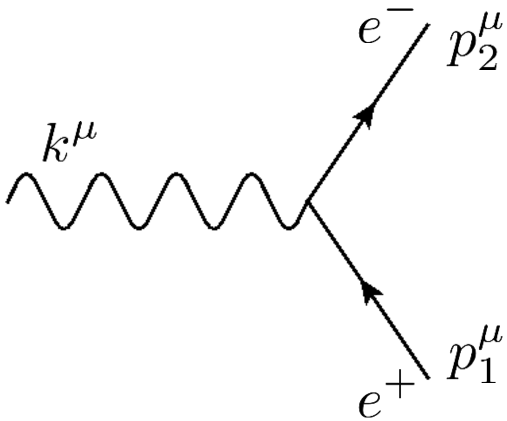
attosecond

Pair production in QED



$$\gamma \to e^+ + e^-$$

Pair production in QED



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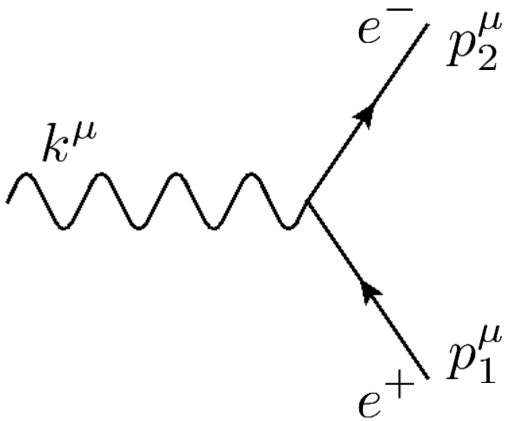
$$\hbar = c = 1$$

For the created pairs the four momentum $P_{e^-e^+} = \{2p_0, \mathbf{0}\}$

And the photon $k^\mu = \{k_0, \mathbf{k}\}$ with $k^2 = k_0^2 - \mathbf{k}^2 = 0$

Pair production in QED



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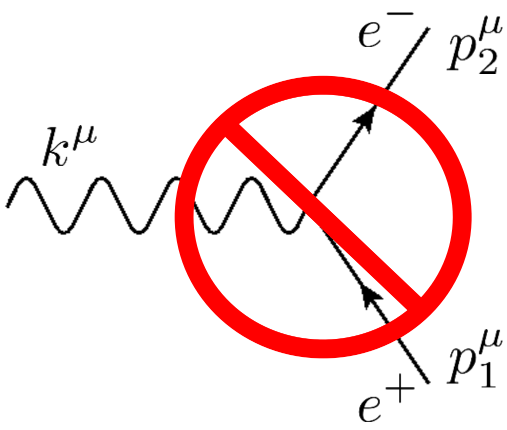

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And the photon $k^\mu = \{k_0, \mathbf{k}\}$ with $k^2 = k_0^2 - \mathbf{k}^2 = 0$

Due to the energy-momentum conservation $k_0 = 2p_0$ and $\mathbf{k} = \mathbf{0}$

This requirement contradicts with the photon's light-like nature!

Hence this process is forbidden within the framework of QED.

A second order process is the lowest for pair production: **the Breit-Wheeler process**.

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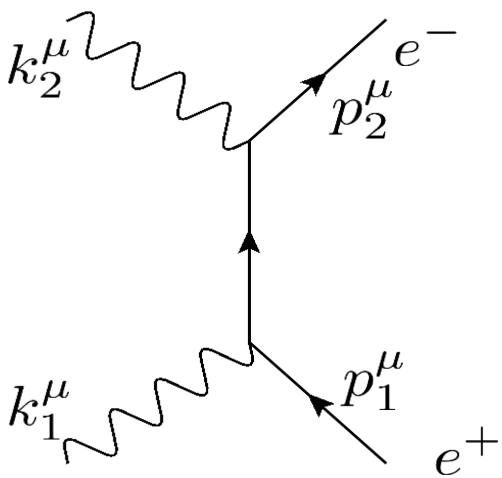


$$\gamma + \gamma' \rightarrow e^+ + e^-$$

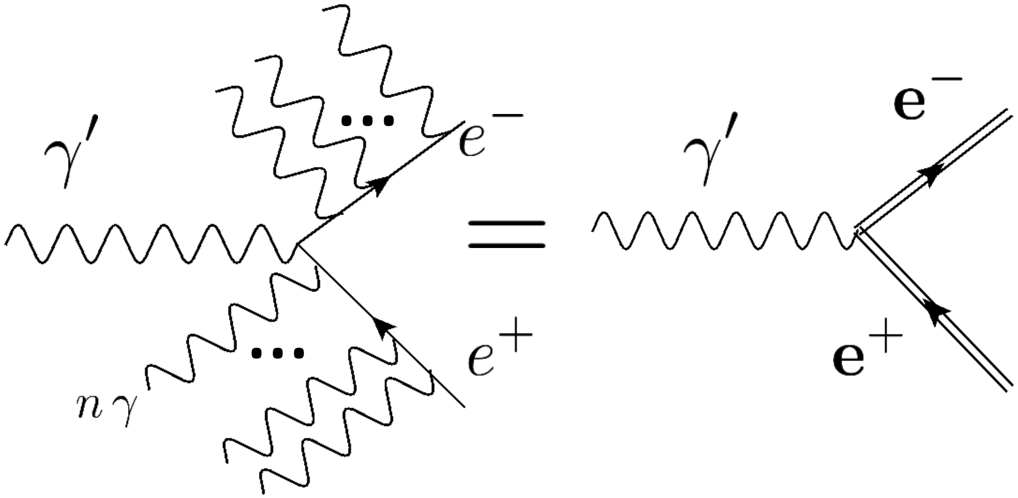
Never experienced

Pair production with background

Laser induced pair production: multiphoton Breit-Wheeler process

Pair production with background

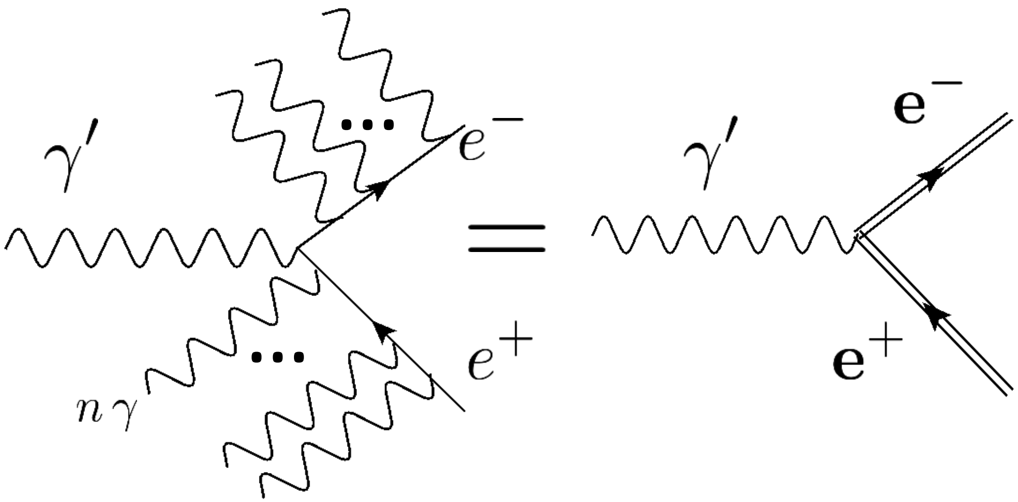
Laser induced pair production: multiphoton Breit-Wheeler process



The incident high energy photon splits into an electron-positron pair.

Pair production with background

Laser induced pair production: multiphoton Breit-Wheeler process



The incident high energy photon splits into an electron-positron pair.

Theory

H.R. Reiss, J. Math. Phys. 3, 59 (1962); A.I. Nikishov, V.I. Ritus, Sov. Phys. JETP 19, 529 (1964).

Experiments at SLAC

D.Burke etal., Phys.Rev. Lett.79,1626 (1997);C.Bamberetal., Phys.Rev.D60,092004 (1999).

The Dirac equation can be solved exactly in a laser field

$$\left(i\partial\!\!\!/ - eA\!\!\!/ - m\right)\psi_p(x) = 0$$

where the vector potential has the form $A^\mu = a(\varepsilon_1^\mu \cos(kx) + \varepsilon_2^\mu \sin(kx))$

And the Lorentz gauge condition is applied $k_\mu A^\mu = 0$

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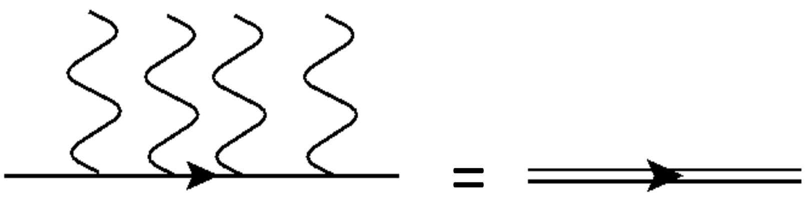
And the Lorentz gauge condition is applied $k_\mu A^\mu = 0$

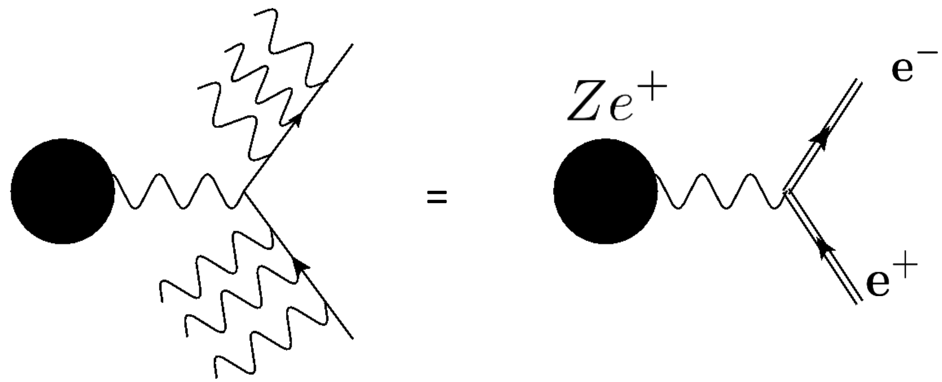
$$\psi_p(x) = \sqrt{\frac{m}{p_0 V}}\left(1 + \frac{e}{2kp}k\!\!\!/ A\!\!\!/\right) u(p) \exp\left(ie\frac{\cos(kx)}{kp}\epsilon_2 p - ie\frac{\sin(kx)}{kp}\epsilon_1 p - iqx\right)$$

“Laser dressing” of an electron



D.M. Volkov, Z. Physik94, 250 (1935).

However, one could replace the incident photon by a scattering potential



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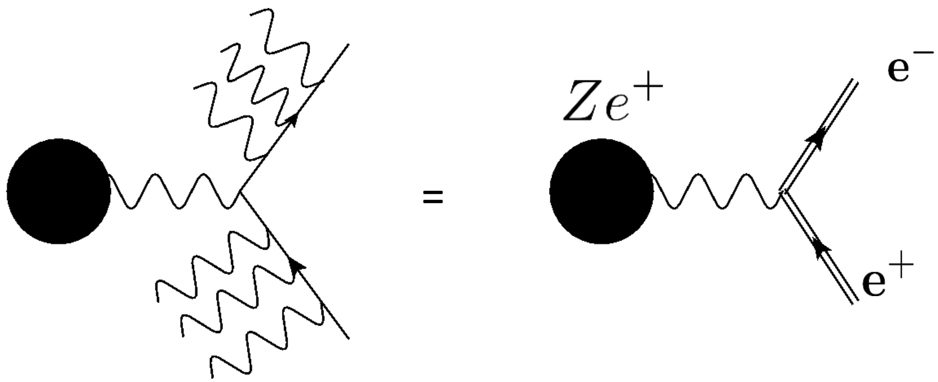
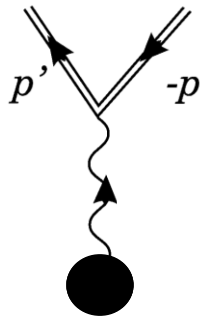


E.g. it can be the **Coulomb potential** of a point charge (nucleus). That is, this process describes the interaction of laser with a nucleus resulting in dressed electron-positron pairs.

General description: J Bergou and S Varró J. Phys. A: Math. Gen. 13 (1980) 2823-2837.

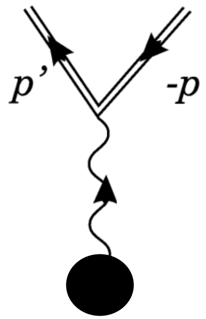
Application with linearly polarized light:
J. Z. Kamiński, K. Krajewska, and F. Ehlotzky, PHYSICAL REVIEW A 74, 033402 (2006)
P. Panek, P. Siecka J. Z. Kamiński, K. Krajewska, and F. Ehlotzky, PHYSICAL REVIEW A 73, 053409 (2006)

Pair creation on a nucleus in plasma



$$S_{fi} = -ie \int d^4x \bar{\psi}'_p(x) \not{A}(x) \psi_{-p}(x)$$

Pair creation on a nucleus in plasma



$$S_{fi} = -ie \int d^4x \bar{\psi}'_p(x) \not{A}(x) \psi_{-p}(x)$$

$$A_\mu = \left(\frac{Ze}{|\mathbf{x}|} e^{-|\mathbf{x}|/\lambda_\mathrm{D}}, \mathbf{0} \right)$$

$$\psi_{p'}(x) = \sqrt{\frac{m}{p'_0 V}} \bar{u}(p) \left(1 + \frac{e}{2kp'} \not{A} \not{k} \right) \exp\left(-ie \frac{\cos(kx)}{kp'} \epsilon_2 p' + ie \frac{\sin(kx)}{kp'} \epsilon_1 p' + iq'x \right)$$

$$\psi_{-p}(x) = \sqrt{\frac{m}{p_0 V}} \left(1 - \frac{e}{2kp} \not{k} \not{A} \right) v(p) \exp\left(ie \frac{\cos(kx)}{kp} \epsilon_2 p - ie \frac{\sin(kx)}{kp} \epsilon_1 p + iqx \right)$$

$$q^\mu = p^\mu + e^2 a^2 \frac{k^\mu}{2k \cdot p}$$

Pair creation on a nucleus in plasma

$$S_{fi} = -ie\frac{m}{\sqrt{p_0 p_0' }V}\int d^4x\,\bar{u}(p')\left(1+\frac{e}{2p'k}\not{A}\not{k}\right)\gamma^0\left(\frac{Ze}{|\mathbf{x}|}e^{-|\mathbf{x}|/\lambda_{\mathbf{D}}}\right)\left(1-\frac{e}{2pk}\not{k}\not{A}\right)v(p)$$

$$\times e^{iea\sin(kx)\left(\frac{\epsilon_1 p}{kp}-\frac{\epsilon_1 p'}{kp'}\right)+iea\cos(kx)\left(\frac{\epsilon_1 p'}{kp'}-\frac{\epsilon_2 p}{kp}\right)}e^{i(q+q')}$$

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$$S_{fi} = -ie\frac{m}{\sqrt{p_0 p_0' V}}\int d^4x\, \bar{u}(p')\left(1+\frac{e}{2p'k}\not{A}\not{k}\right)\gamma^0\left(\frac{Ze}{|\mathbf{x}|}e^{-|\mathbf{x}|/\lambda_\mathbf{D}}\right)\left(1-\frac{e}{2pk}\not{k}\not{A}\right)v(p)$$

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$$M = \left(1+\frac{e}{2p'k}\not{A}\not{k}\right)\gamma^0\left(1-\frac{e}{2pk}\not{k}\not{A}\right)$$

$$e^{iea\sin(kx)\left(\frac{\epsilon_1 p}{kp}-\frac{\epsilon_1 p'}{kp'}\right)+iea\cos(kx)\left(\frac{\epsilon_1 p'}{kp'}-\frac{\epsilon_2 p}{kp}\right)}e^{i(q+q')}$$

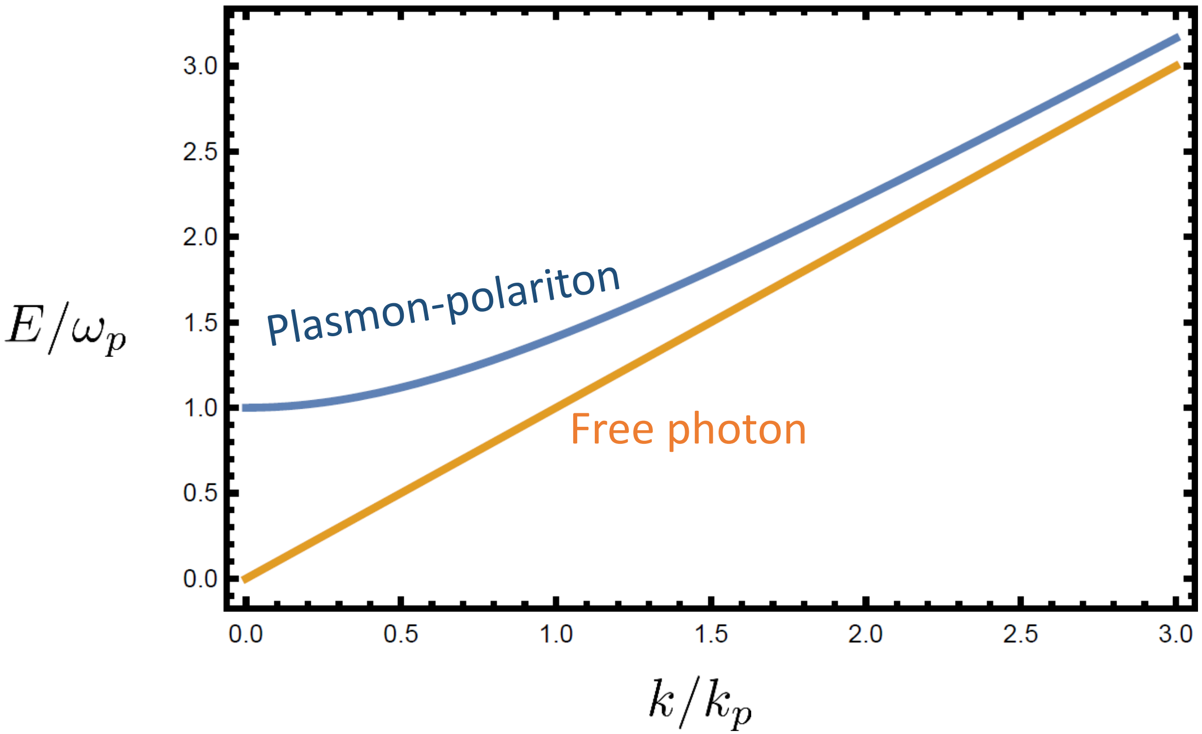
Pair creation on a nucleus in plasma

The effect of the plasma

For the laser we take into account the effect of the plasma: it alters the dispersion relations of the propagating EM waves

$$E = \sqrt{\omega^2 + \omega_p^2}$$

Hence $k^2 \neq 0$ but $k^2 = k_0^2(1 - n_m^2) = \omega_p^2 = k_p$ where te refraction $n_m < 1$



Pair creation on a nucleus in plasma

M matrix

$$M = \gamma^0 + ea\cos\phi\left(\frac{\not\epsilon_1 \not k \gamma^0}{2p'k} - \frac{\gamma^0 \not k \not\epsilon_1}{2pk}\right) + ea\sin\phi\left(\frac{\not\epsilon_2 \not k \gamma^0}{2p'k} - \frac{\gamma^0 \not k \not\epsilon_2}{2pk}\right)$$

$$\frac{e^2a^2}{4(p'k)(pk)}(k^2\gamma^0 - 2\not k k^0)$$

Pair creation on a nucleus in plasma

M matrix

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$$\frac{e^2a^2}{4(p'k)(pk)}(k^2\gamma^0 - 2\not{k}k^0)$$

Exponential

$$e^{iea\sin(kx)\left(\frac{\epsilon_1 p}{kp} - \frac{\epsilon_1 p'}{kp'}\right) + iea\cos(kx)\left(\frac{\epsilon_1 p'}{kp'} - \frac{\epsilon_2 p}{kp}\right)} e^{i(q+q')}$$

$$P_\mu = \frac{q_\mu}{kp} - \frac{q'_\mu}{kp}$$

$$z = ea\sqrt{\epsilon_1 P + \epsilon_2 P}$$

$$e^{z\left(\frac{eaP\epsilon_1}{z}\sin\phi - \frac{eaP\epsilon_2}{z}\cos\phi\right)} = e^{z\sin(\phi-\phi_0)}$$

Pair creation on a nucleus in plasma

$$e^{i\sin\phi-\phi_0} = \sum_{n=-\infty}^{\infty} B_n(z)e^{in\phi}$$

$$\cos\phi\, e^{i\sin\phi-\phi_0} = \sum_{n=-\infty}^{\infty} C_n(z)e^{in\phi}$$

Jacobi-Anger expansion

$$\sin\phi\, e^{i\sin\phi-\phi_0} = \sum_{n=-\infty}^{\infty} D_n(z)e^{in\phi}$$

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$$\sin\phi\, e^{i\sin\phi-\phi_0} = \sum_{n=-\infty}^{\infty} D_n(z)e^{in\phi}$$

$$B_n(z) = J_n(z)e^{-in\phi_0}$$

$$C_n(z) = 1/2\left(J_{n-1}(z)e^{-i(n-1)\phi_0} + J_{n+1}(z)e^{-i(n+1)\phi_0}\right)$$

$$D_n(z) = 1/2i\left(J_{n-1}(z)e^{-i(n-1)\phi_0} - J_{n+1}(z)e^{-i(n+1)\phi_0}\right)$$

Pair creation on a nucleus in plasma

$$
\begin{aligned}
M_n &= C_n e a \left(\frac{\not{\epsilon}_1 \not{k} \gamma^0}{2p'k} - \frac{\gamma^0 \not{k} \not{\epsilon}_1}{2pk} \right) + D_n e a \left(\frac{\not{\epsilon}_2 \not{k} \gamma^0}{2p'k} - \frac{\gamma^0 \not{k} \not{\epsilon}_2}{2pk} \right) \\
& \quad B_n \left(\gamma^0 + \frac{e^2 a^2}{4(p'k)(pk)} (k^2 \gamma^0 - 2\not{k} k^0) \right)
\end{aligned}
$$

Pair creation on a nucleus in plasma

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$$B_n \left(\gamma^0 + \frac{e^2 a^2}{4(p'k)(pk)} (k^2 \gamma^0 - 2\not{k} k^0) \right)$$

$$S_{fi} = i e^2 Z \frac{m}{\sqrt{p_0 p_0'} V} \sum_n \int d^4 x e^{i(q+q'+nk)x} \bar{u}(p') M_n v(p) \frac{e^{-|\mathbf{x}|/\lambda_{\mathbf{D}}}}{|\mathbf{x}|}$$

Pair creation on a nucleus in plasma

$$
\begin{aligned}
M_n &= C_n ea\left(\frac{\not{\epsilon}_1 \not{k}\gamma^0}{2p'k} - \frac{\gamma^0\not{k}\not{\epsilon}_1}{2pk}\right) + D_n ea\left(\frac{\not{\epsilon}_2 \not{k}\gamma^0}{2p'k} - \frac{\gamma^0\not{k}\not{\epsilon}_2}{2pk}\right) \\
& \quad B_n\left(\gamma^0 + \frac{e^2a^2}{4(p'k)(pk)}(k^2\gamma^0 - 2\not{k}k^0)\right)
\end{aligned}
$$

$$
S_{fi} = ie^2 Z \frac{m}{\sqrt{p_0 p_0' }V} \sum_n \int d^4x e^{i(q+q'+nk)x} \bar{u}(p') M_n v(p) \frac{e^{-|\mathbf{x}|/\lambda_{\mathbf{D}}}}{|\mathbf{x}|}
$$

$$
\int d^4x e^{i(q+q'+nk)x} \frac{e^{-|\mathbf{x}|/\lambda_{\mathbf{D}}}}{|\mathbf{x}|} = \frac{4\pi}{(\mathbf{q}+\mathbf{q}'+n\mathbf{k})^2 + 1/\lambda_D^2} \delta(q_0 + q_0' + nk_0)
$$

Only energy conserves

Pair creation on a nucleus in plasma

$$S_{fi} = ie^2 Z \frac{m}{\sqrt{p_0 p_0' V}} \sum_{n=-\infty}^{\infty} \frac{8\pi^2}{m_n^2 + 1/\lambda_D^2} \delta(q_0 + q_0' + nk_0) \bar{u}(p') M_n v(p)$$

Pair creation on a nucleus in plasma

$$S_{fi} = ie^2 Z \frac{m}{\sqrt{p_0 p_0'} V} \sum_{n=-\infty}^{\infty} \frac{8\pi^2}{m_n^2 + 1/\lambda_D^2} \delta(q_0 + q_0' + nk_0) \bar{u}(p') M_n v(p)$$

$$d\sigma = \frac{d^3 q}{(2\pi)^3} \frac{d^3 q'}{(2\pi)^3} \frac{m^2}{p_0 p_0'} \frac{4}{(2\pi)^2} |S_{fi}|^2$$

$$d\sigma = \frac{d^3 q}{(2\pi)^3} \frac{d^3 q'}{(2\pi)^3} \frac{m^2}{p_0 p_0'} \frac{4}{(2\pi)^2} Z^2 e^4 \sum_{n=1}^{\infty} \frac{1}{(m_n^2 + 1/\lambda_D^2)^2} \delta(q_0 + q_0' + nk_0) |\bar{u}(p') M_n v(p)|^2$$

Pair creation on a nucleus in plasma

$$S_{fi} = ie^2 Z \frac{m}{\sqrt{p_0 p_0' V}} \sum_{n=-\infty}^{\infty} \frac{8\pi^2}{m_n^2 + 1/\lambda_D^2} \delta(q_0 + q_0' + nk_0) \bar{u}(p') M_n v(p)$$

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Summing over spin states

$$\overline{|M_{fi,n}|^2} = \sum_{s,s'} |\bar{u}(p') M_n v(p)|^2 = \mathrm{Tr}\left[\frac{\not{p}' + m}{2m} M_n \frac{\not{p} - m}{2m} \gamma_0 M_n^\dagger \gamma_0 \right]$$

Pair creation on a nucleus in plasma

$$\overline{|M_{fi,n}|^2} = \frac{1}{8m^2}\mathrm{Tr}\left\{(\not p+m)\left[B_n\left(\gamma^0+\frac{e^2a^2}{4(p'k)(pk)}(k^2\gamma^0-2\not k k^0)\right)+ea\left(\frac{\not E\not k\gamma^0}{2kp'}-\frac{\gamma^0\not k\not E}{2kp}\right)\right]\right.$$
$$\left.(\not p'-m)\left[B_n^*\left(\gamma^0+\frac{e^2a^2}{4(p'k)(pk)}(k^2\gamma^0-2\not k k^0)\right)+ea\left(\frac{\gamma^0\not k\not E^*}{2kp'}-\frac{\not E^*\not k\gamma^0}{2kp}\right)\right]\right\}$$

$$E = C_n\not\epsilon_1 + D_n\not\epsilon_2$$

$$E^* = \gamma^0 E^\dagger\gamma^0 = C_n^*\not\epsilon_1 + D_n^*\not\epsilon_2$$

Evaluate the trace.

Pair creation on a nucleus in plasma

$$\overline{|M_{fi,n}|^2} =$$

$$\begin{aligned}
&-4\,\mathrm{B1}\,\mathrm{B2}\,m^2 + \mathrm{B1}\,\mathrm{B2}\,(-4\,\mathrm{P1P2} + 8\,\mathrm{P1}[0]\,\mathrm{P2}[0]) + \mathrm{EA}\Bigg(-\frac{\mathrm{B2}\,(4\,\mathrm{KP2}\,\mathrm{P1ES1} - 4\,\mathrm{KP1}\,\mathrm{P2ES1} + 8\,\mathrm{k0}\,\mathrm{P2ES1}\,\mathrm{P1}[0])}{2\,\mathrm{KP2}} - \\
&\quad \frac{\mathrm{B1}\,(4\,\mathrm{KP2}\,\mathrm{P1ES2} - 4\,\mathrm{KP1}\,\mathrm{P2ES2} + 8\,\mathrm{k0}\,\mathrm{P2ES2}\,\mathrm{P1}[0])}{2\,\mathrm{KP2}} + \frac{\mathrm{B2}\,(-4\,\mathrm{KP2}\,\mathrm{P1ES1} + 4\,\mathrm{KP1}\,\mathrm{P2ES1} + 8\,\mathrm{k0}\,\mathrm{P1ES1}\,\mathrm{P2}[0])}{2\,\mathrm{KP1}} + \frac{\mathrm{B1}\,(-4\,\mathrm{KP2}\,\mathrm{P1ES2} + 4\,\mathrm{KP1}\,\mathrm{P2ES2} + 8\,\mathrm{k0}\,\mathrm{P1ES2}\,\mathrm{P2}[0])}{2\,\mathrm{KP1}}\Bigg) + \\
&\mathrm{EA}^3\Bigg(-\frac{\mathrm{B2}\,k^2\,(4\,\mathrm{KP2}\,\mathrm{P1ES1} - 4\,\mathrm{KP1}\,\mathrm{P2ES1} + 8\,\mathrm{k0}\,\mathrm{P2ES1}\,\mathrm{P1}[0])}{8\,\mathrm{KP1}\,\mathrm{KP2}^2} - \frac{\mathrm{B1}\,k^2\,(4\,\mathrm{KP2}\,\mathrm{P1ES2} - 4\,\mathrm{KP1}\,\mathrm{P2ES2} + 8\,\mathrm{k0}\,\mathrm{P2ES2}\,\mathrm{P1}[0])}{8\,\mathrm{KP1}\,\mathrm{KP2}^2} + \\
&\quad \frac{\mathrm{B2}\,\mathrm{k0}\,(8\,\mathrm{k0}\,\mathrm{KP2}\,\mathrm{P1ES1} + 4\,k^2\,\mathrm{P2ES1}\,\mathrm{P1}[0] - 4\,k^2\,\mathrm{P1ES1}\,\mathrm{P2}[0])}{4\,\mathrm{KP1}\,\mathrm{KP2}^2} - \frac{\mathrm{B2}\,\mathrm{k0}\,(8\,\mathrm{k0}\,\mathrm{KP1}\,\mathrm{P2ES1} - 4\,k^2\,\mathrm{P2ES1}\,\mathrm{P1}[0] + 4\,k^2\,\mathrm{P1ES1}\,\mathrm{P2}[0])}{4\,\mathrm{KP1}^2\,\mathrm{KP2}} + \\
&\quad \frac{\mathrm{B2}\,k^2\,(-4\,\mathrm{KP2}\,\mathrm{P1ES1} + 4\,\mathrm{KP1}\,\mathrm{P2ES1} + 8\,\mathrm{k0}\,\mathrm{P1ES1}\,\mathrm{P2}[0])}{8\,\mathrm{KP1}^2\,\mathrm{KP2}} + \frac{\mathrm{B1}\,\mathrm{k0}\,(8\,\mathrm{k0}\,\mathrm{KP2}\,\mathrm{P1ES2} + 4\,k^2\,\mathrm{P2ES2}\,\mathrm{P1}[0] - 4\,k^2\,\mathrm{P1ES2}\,\mathrm{P2}[0])}{4\,\mathrm{KP1}\,\mathrm{KP2}^2} - \\
&\quad \frac{\mathrm{B1}\,\mathrm{k0}\,(8\,\mathrm{k0}\,\mathrm{KP1}\,\mathrm{P2ES2} - 4\,k^2\,\mathrm{P2ES2}\,\mathrm{P1}[0] + 4\,k^2\,\mathrm{P1ES2}\,\mathrm{P2}[0])}{4\,\mathrm{KP1}^2\,\mathrm{KP2}} + \frac{\mathrm{B1}\,k^2\,(-4\,\mathrm{KP2}\,\mathrm{P1ES2} + 4\,\mathrm{KP1}\,\mathrm{P2ES2} + 8\,\mathrm{k0}\,\mathrm{P1ES2}\,\mathrm{P2}[0])}{8\,\mathrm{KP1}^2\,\mathrm{KP2}}\Bigg) + \\
&\mathrm{EA}^4\Bigg(-\frac{\mathrm{B1}\,\mathrm{B2}\,k^4\,m^2}{4\,\mathrm{KP1}^2\,\mathrm{KP2}^2} + \frac{\mathrm{B1}\,\mathrm{B2}\,\mathrm{k0}^2\,(8\,\mathrm{KP1}\,\mathrm{KP2} - 4\,k^2\,\mathrm{P1P2})}{4\,\mathrm{KP1}^2\,\mathrm{KP2}^2} - \frac{\mathrm{B1}\,\mathrm{B2}\,k^2\,\mathrm{k0}\,(-4\,\mathrm{k0}\,\mathrm{P1P2} + 4\,\mathrm{KP2}\,\mathrm{P1}[0] + 4\,\mathrm{KP1}\,\mathrm{P2}[0])}{4\,\mathrm{KP1}^2\,\mathrm{KP2}^2} + \frac{\mathrm{B1}\,\mathrm{B2}\,k^4\,(-4\,\mathrm{P1P2} + 8\,\mathrm{P1}[0]\,\mathrm{P2}[0])}{16\,\mathrm{KP1}^2\,\mathrm{KP2}^2}\Bigg) + \\
&\mathrm{EA}^2\Bigg(-\frac{\mathrm{ES1ES2}\,k^2\,m^2}{\mathrm{KP1}^2} - \frac{\mathrm{ES1ES2}\,k^2\,m^2}{\mathrm{KP2}^2} - \frac{2\,\mathrm{B1}\,\mathrm{B2}\,k^2\,m^2}{\mathrm{KP1}\,\mathrm{KP2}} + \frac{4\,\mathrm{B1}\,\mathrm{B2}\,\mathrm{k0}^2\,m^2}{\mathrm{KP1}\,\mathrm{KP2}} + \frac{(-4\,\mathrm{ES1ES2}\,k^2 + 8\,\mathrm{ES1ES2}\,\mathrm{k0}^2)\,m^2}{2\,\mathrm{KP1}\,\mathrm{KP2}} - \frac{\mathrm{B1}\,\mathrm{B2}\,\mathrm{k0}\,(-4\,\mathrm{k0}\,\mathrm{P1P2} + 4\,\mathrm{KP2}\,\mathrm{P1}[0] + 4\,\mathrm{KP1}\,\mathrm{P2}[0])}{\mathrm{KP1}\,\mathrm{KP2}} + \\
&\quad \frac{\mathrm{B1}\,\mathrm{B2}\,k^2\,(-4\,\mathrm{P1P2} + 8\,\mathrm{P1}[0]\,\mathrm{P2}[0])}{2\,\mathrm{KP1}\,\mathrm{KP2}} - \frac{1}{2\,\mathrm{KP1}\,\mathrm{KP2}}\big(-8\,\mathrm{ES1ES2}\,\mathrm{KP1}\,\mathrm{KP2} + 4\,\mathrm{ES1ES2}\,k^2\,\mathrm{P1P2} - 8\,\mathrm{ES1ES2}\,\mathrm{k0}^2\,\mathrm{P1P2} - 4\,k^2\,\mathrm{P1ES2}\,\mathrm{P2ES1} + \\
&\qquad 8\,\mathrm{k0}^2\,\mathrm{P1ES2}\,\mathrm{P2ES1} - 4\,k^2\,\mathrm{P1ES1}\,\mathrm{P2ES2} + 8\,\mathrm{k0}^2\,\mathrm{P1ES1}\,\mathrm{P2ES2} + 8\,\mathrm{ES1ES2}\,\mathrm{k0}\,\mathrm{KP2}\,\mathrm{P1}[0] + 8\,\mathrm{ES1ES2}\,\mathrm{k0}\,\mathrm{KP1}\,\mathrm{P2}[0] - 8\,\mathrm{ES1ES2}\,k^2\,\mathrm{P1}[0]\,\mathrm{P2}[0]\big) + \\
&\quad \frac{8\,\mathrm{ES1ES2}\,\mathrm{KP1}\,\mathrm{KP2} - 4\,\mathrm{ES1ES2}\,k^2\,\mathrm{P1P2} + 4\,k^2\,\mathrm{P1ES2}\,\mathrm{P2ES1} + 4\,k^2\,\mathrm{P1ES1}\,\mathrm{P2ES2} - 16\,\mathrm{ES1ES2}\,\mathrm{k0}\,\mathrm{KP2}\,\mathrm{P1}[0] + 8\,\mathrm{ES1ES2}\,k^2\,\mathrm{P1}[0]\,\mathrm{P2}[0]}{4\,\mathrm{KP2}^2} + \\
&\quad \frac{8\,\mathrm{ES1ES2}\,\mathrm{KP1}\,\mathrm{KP2} - 4\,\mathrm{ES1ES2}\,k^2\,\mathrm{P1P2} + 4\,k^2\,\mathrm{P1ES2}\,\mathrm{P2ES1} + 4\,k^2\,\mathrm{P1ES1}\,\mathrm{P2ES2} - 16\,\mathrm{ES1ES2}\,\mathrm{k0}\,\mathrm{KP1}\,\mathrm{P2}[0] + 8\,\mathrm{ES1ES2}\,k^2\,\mathrm{P1}[0]\,\mathrm{P2}[0]}{4\,\mathrm{KP1}^2}\Bigg)
\end{aligned}$$

Pair creation on a nucleus in plasma

$$\overline{|M_{fi,n}|^2} =$$

$$\frac{2\,\mathrm{B1}\,\mathrm{B2}\,\mathrm{EA}^4\,\mathrm{k0}^2}{\mathrm{KP1}\,\mathrm{KP2}} - 4\,\mathrm{B1}\,\mathrm{B2}\,\mathrm{m}^2 + \mathrm{EA}^3\left(\frac{2\,\mathrm{B2}\,\mathrm{k0}^2\,\mathrm{P1ES1}}{\mathrm{KP1}\,\mathrm{KP2}} + \frac{2\,\mathrm{B1}\,\mathrm{k0}^2\,\mathrm{P1ES2}}{\mathrm{KP1}\,\mathrm{KP2}} - \frac{2\,\mathrm{B2}\,\mathrm{k0}^2\,\mathrm{P2ES1}}{\mathrm{KP1}\,\mathrm{KP2}} - \frac{2\,\mathrm{B1}\,\mathrm{k0}^2\,\mathrm{P2ES2}}{\mathrm{KP1}\,\mathrm{KP2}}\right) +$$

$$\mathrm{B1}\,\mathrm{B2}\,(-4\,\mathrm{P1P2} + 8\,\mathrm{P1}[0]\,\mathrm{P2}[0]) + \mathrm{EA}^2\left(\frac{4\,\mathrm{B1}\,\mathrm{B2}\,\mathrm{k0}^2\,\mathrm{m}^2}{\mathrm{KP1}\,\mathrm{KP2}} + \frac{4\,\mathrm{ES1ES2}\,\mathrm{k0}^2\,\mathrm{m}^2}{\mathrm{KP1}\,\mathrm{KP2}} + \frac{8\,\mathrm{ES1ES2}\,\mathrm{KP1}\,\mathrm{KP2} - 16\,\mathrm{ES1ES2}\,\mathrm{k0}\,\mathrm{KP2}\,\mathrm{P1}[0]}{4\,\mathrm{KP2}^2} - \right.$$

$$\frac{\mathrm{B1}\,\mathrm{B2}\,\mathrm{k0}\,(-4\,\mathrm{k0}\,\mathrm{P1P2} + 4\,\mathrm{KP2}\,\mathrm{P1}[0] + 4\,\mathrm{KP1}\,\mathrm{P2}[0])}{\mathrm{KP1}\,\mathrm{KP2}} + \frac{8\,\mathrm{ES1ES2}\,\mathrm{KP1}\,\mathrm{KP2} - 16\,\mathrm{ES1ES2}\,\mathrm{k0}\,\mathrm{KP1}\,\mathrm{P2}[0]}{4\,\mathrm{KP1}^2} -$$

$$\left.\frac{-8\,\mathrm{ES1ES2}\,\mathrm{KP1}\,\mathrm{KP2} - 8\,\mathrm{ES1ES2}\,\mathrm{k0}^2\,\mathrm{P1P2} + 8\,\mathrm{k0}^2\,\mathrm{P1ES2}\,\mathrm{P2ES1} + 8\,\mathrm{k0}^2\,\mathrm{P1ES1}\,\mathrm{P2ES2} + 8\,\mathrm{ES1ES2}\,\mathrm{k0}\,\mathrm{KP2}\,\mathrm{P1}[0] + 8\,\mathrm{ES1ES2}\,\mathrm{k0}\,\mathrm{KP1}\,\mathrm{P2}[0]}{2\,\mathrm{KP1}\,\mathrm{KP2}}\right) +$$

$$\mathrm{EA}\left(-\frac{\mathrm{B2}\,(4\,\mathrm{KP2}\,\mathrm{P1ES1} - 4\,\mathrm{KP1}\,\mathrm{P2ES1} + 8\,\mathrm{k0}\,\mathrm{P2ES1}\,\mathrm{P1}[0])}{2\,\mathrm{KP2}} - \frac{\mathrm{B1}\,(4\,\mathrm{KP2}\,\mathrm{P1ES2} - 4\,\mathrm{KP1}\,\mathrm{P2ES2} + 8\,\mathrm{k0}\,\mathrm{P2ES2}\,\mathrm{P1}[0])}{2\,\mathrm{KP2}} + \right.$$

$$\left.\frac{\mathrm{B2}\,(-4\,\mathrm{KP2}\,\mathrm{P1ES1} + 4\,\mathrm{KP1}\,\mathrm{P2ES1} + 8\,\mathrm{k0}\,\mathrm{P1ES1}\,\mathrm{P2}[0])}{2\,\mathrm{KP1}} + \frac{\mathrm{B1}\,(-4\,\mathrm{KP2}\,\mathrm{P1ES2} + 4\,\mathrm{KP1}\,\mathrm{P2ES2} + 8\,\mathrm{k0}\,\mathrm{P1ES2}\,\mathrm{P2}[0])}{2\,\mathrm{KP1}}\right)$$

For $k^2 = 0$

Pair creation on a nucleus in plasma

$$\sigma = \int\int \frac{d^3q}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}\frac{m^2}{p_0p_0'}\frac{4}{(2\pi)^2}Z^2e^4\sum_{n=1}^{\infty}\frac{1}{(m_n^2+1/\lambda_D^2)^2}\delta(q_0+q_0'+nk_0)|\bar{u}(p')M_n v(p)|^2$$

$$m_{e^+,e^-} = 0.511 \text{ MeV}$$

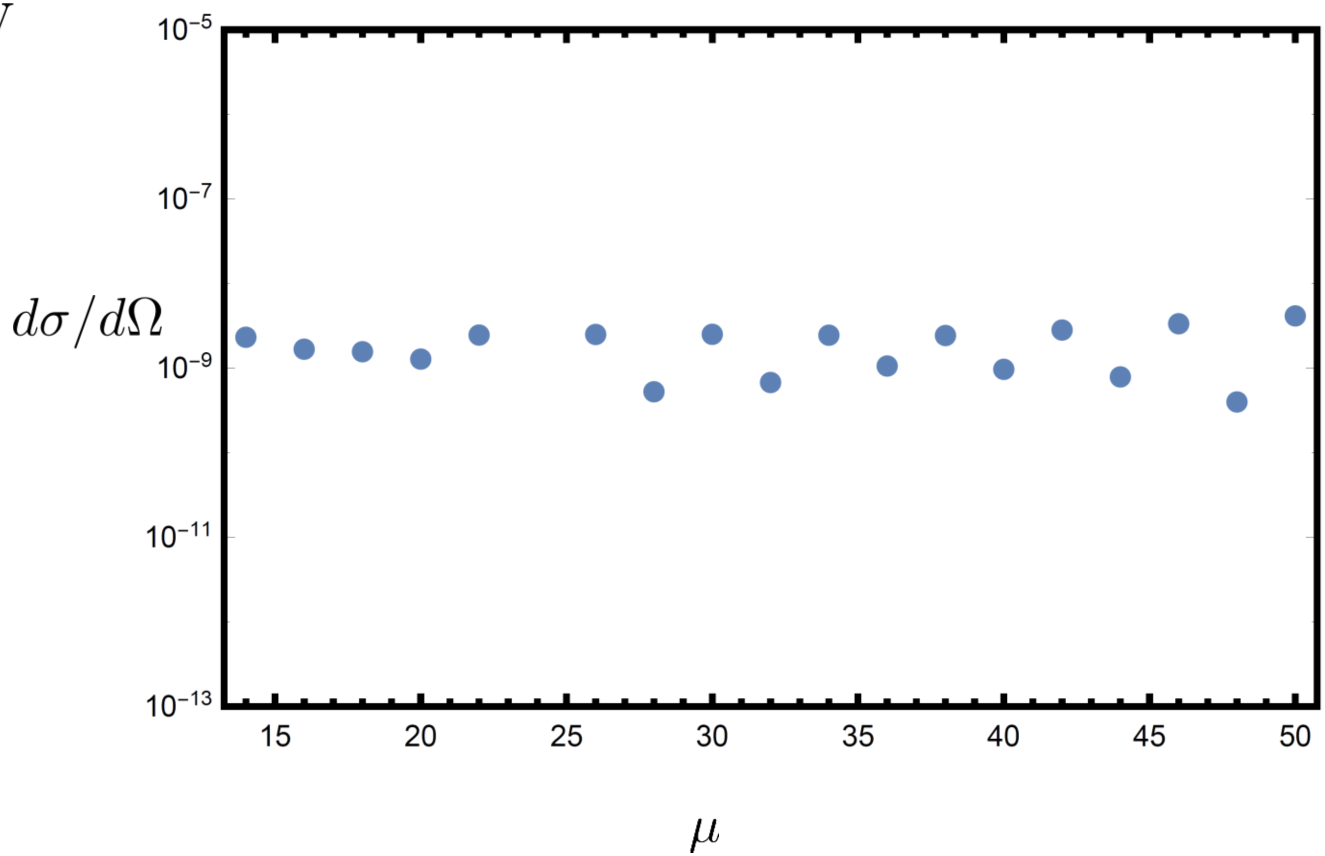
$$k^0 = 1.56 \text{ eV}$$

$$k_p = 1 \text{ eV}$$

$$\mu = \frac{ea}{m} = 1 - 50$$

$$\lambda_D = 10^{-10}\text{m}$$

The differential cross section



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