Quantum description of relativistic charged particles interacting with a strong laser field in a plasma, represented by Lánczos-Proca vector bosons.

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SZÉCHENYI 2020

Outline of the talk.

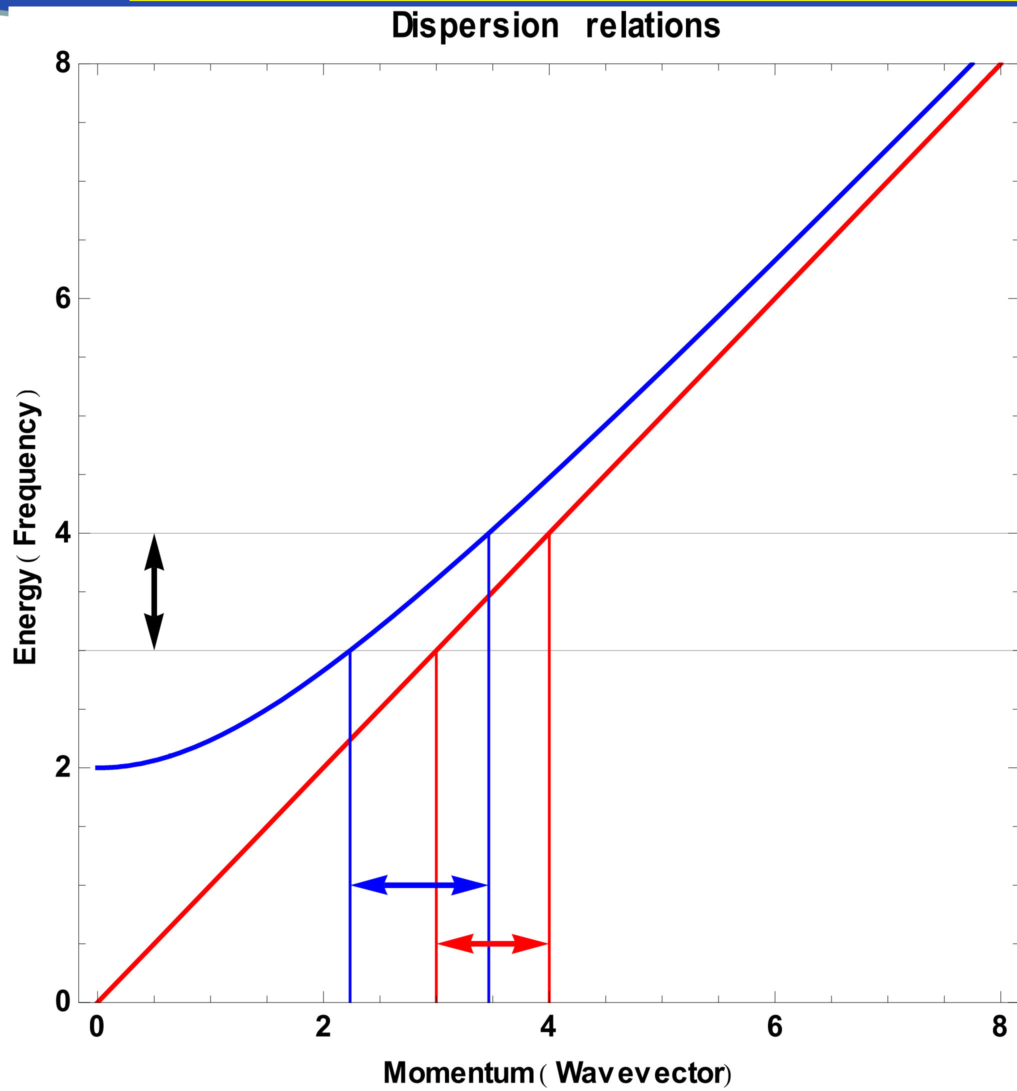
- **Introduction; the Gordon–Volkov solutions and generalizations.**

- **New exact closed form solutions of the Dirac and Klein–Gordon equations in a plane wave propagating in a medium.**

- **Some (unusual) properties of the new solutions, and the boundary-value problem at the vacuum-plasma transition regions.**

$h\nu - A$

„The impossibility of net photon absorption (and emission) by a free electron”. Energy and momentum conservations cannot be satisfied simultaneously.



$$E_{el} = \hbar c\sqrt{\kappa^2 + |\boldsymbol{k}_{el}|^2}$$

$$E_{ph} = \hbar c\sqrt{0^2 + |\boldsymbol{k}_{ph}|^2}$$

$h\nu - A$

Non-perturbative solutions; Gordon–Volkov states. I. „Compton effect treated on the basis of Schrödinger's theory"

Der Comptoneffekt nach der Schrödingerschen Theorie.

Von **W. Gordon** in Berlin.

(Eingegangen am 29. September 1926.)

Die beim Comptoneffekt ausgestrahlten Frequenzen und Intensitäten werden nach der Schrödingerschen Theorie berechnet. Die quantentheoretischen Größen ergeben sich als geometrische Mittel aus den klassischen Größen des Anfangs- und Endzustandes des Prozesses.

1. Aufstellung der Differentialgleichung für ψ. Heisenberg und Schrödinger haben Methoden angegeben zur Bestimmung der Quantenfrequenzen und Intensitäten. Der Comptoneffekt ist bereits von Dirac[1]) nach der Heisenbergschen Methode gerechnet worden. Hier soll dasselbe Problem nach Schrödinger behandelt werden. Das Verfahren von Schrödinger hat den Vorzug, sich der gebräuchlichen mathematischen Hilfsmittel zu bedienen. Es beruht auf der Ermittlung einer Größe ψ, die für ein einzelnes Elektron eine Funktion der kartesischen Raumkoordinaten x_1, x_2, x_3 und der Zeit t ist. Schrödinger hat zwei Regeln aufgestellt zur Gewinnung der linearen partiellen Differentialgleichung zweiter Ordnung, der ψ zu genügen hat. Beide

$$A(k \cdot x), \quad k \cdot A = 0, \quad k^2 = 0$$

$$\Phi_p = e^{-\frac{i}{\hbar} S_p}$$

$$(\partial S - \tfrac{e}{c} A)^2 = (mc)^2$$

Gordon W, Der Comptoneffekt nach der Schrödingerschen Theorie. *Zeitschrift für Physik* 40, 117-133 (1927). [Application to strong-field: ~1960..]

$h\nu - A$

Non-perturbative solutions; Gordon–Volkov states. II. „On a class of solutions of the Dirac equation”

Über eine Klasse von Lösungen der Diracschen Gleichung.

Von **D. M. Wolkow** in Leningrad.

(Eingegangen am 12. Februar 1935.)

1. Der Fall eines sinusoidalen Feldes. — 2. Lösung für den Fall, daß das äußere Feld aus polarisierten, in einer bestimmten Richtung fortschreitenden Wellen besteht, die ein *abzählbares* Spektrum nach Frequenz und Anfangsphasen h

1. Der Fall des sinusoidalen Feldes.

Es sei das skalare Potential des auf das relativistische Quantenelektron wirkenden äußeren Feldes gleich Null, das Vektorpotential sei

$$A = a\cos 2\pi\nu\left[t - \frac{n\,x}{c} + \alpha\right] = a\cos\varphi \quad \text{mit} \quad \varphi = 2\pi\nu\left[t - \frac{n\,x}{c} + \alpha\right],$$

ν ist hier eine konstante Zahl (die Frequenz), t die Zeit, c die Lichtgeschwindigkeit, n ein Einheitsvektor (der die Richtung der Ausbreitung einer dem gegebenen A zugeordneten elektromagnetischen Welle anzeigt); x bedeutet den Vektor, der vom Anfangspunkt des fest gewählten rechtwinkligen Cartesischen Koordinatensystems nach dem veränderlichen Punkt geht,

$$A(k\cdot x), \quad k\cdot A = 0, \quad k^2 = 0$$

$$\Psi_{ps} = \left(1 + \frac{\varepsilon\not{k}A}{2k\cdot p}\right)u_{ps}\,e^{-\frac{i}{\hbar}S_p}$$

$$(\not{p} + \kappa)u_{ps} = 0$$

$$\left(\partial S - \tfrac{e}{c}A\right)^2 = (mc)^2$$

Wolkow D M, Über eine Klasse von Lösungen der Diracschen Gleichung. *Zeitschrift für Physik* 94, 250-260 (1935). [Application to strong-field: ~1960..]

attosecond

High-intensity Compton scattering (HHG) beyond the semiclassical description (1981). The generalization of the Klein–Nishina formula. The effect of depletion of the laser field; e.g. decrease of high-harmonic cross-section.

J. Phys. A: Math. Gen. **14** (1981) 2281–2303. Printed

$$r_0^2 \to \propto r_0^2 e^{-n\nu}$$

$$\omega'_n = \frac{n\omega_0 + \omega_C \mu_0^2 \Delta}{1 + \left(2\frac{n\omega_0}{\omega_C} + \frac{\mu_0^2}{2}\right) \sin^2\frac{\theta}{2}}$$

Nonlinear scattering processes [...] quantised radiation field: II. Relativistic treatment

J Bergou and S Varró

Central Research Institute for Physics, H-1525 Budapest 114, POB 49, Hungary

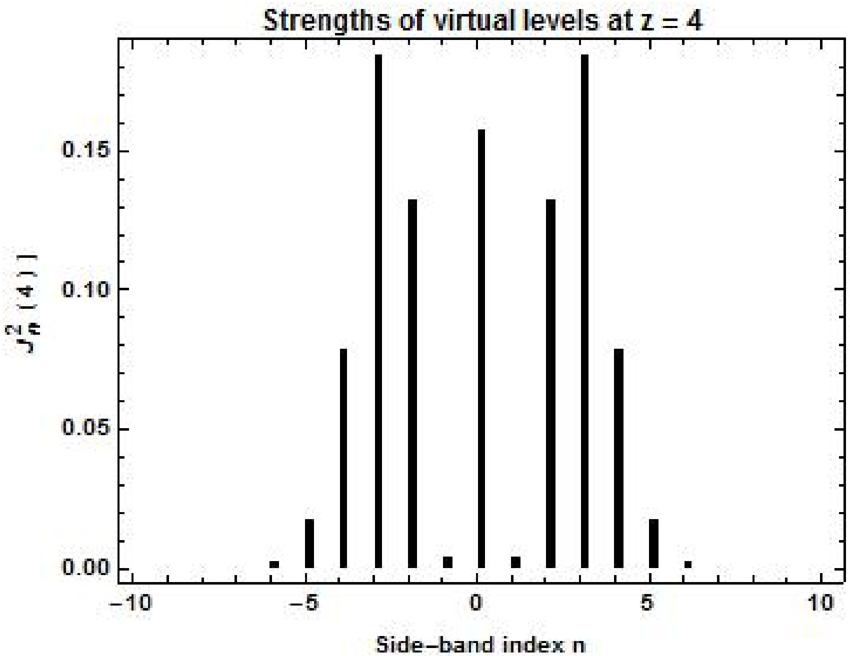
$$|t_{fi}^{(n)}|_{av}^2 = \frac{1}{4}\left[\frac{n\omega}{\omega'} + \frac{\omega'}{n\omega} - 2 + 4(\boldsymbol{\varepsilon}\cdot\boldsymbol{\varepsilon}')^2\right]\frac{(n\nu)^n}{n!}e^{-n\nu}$$

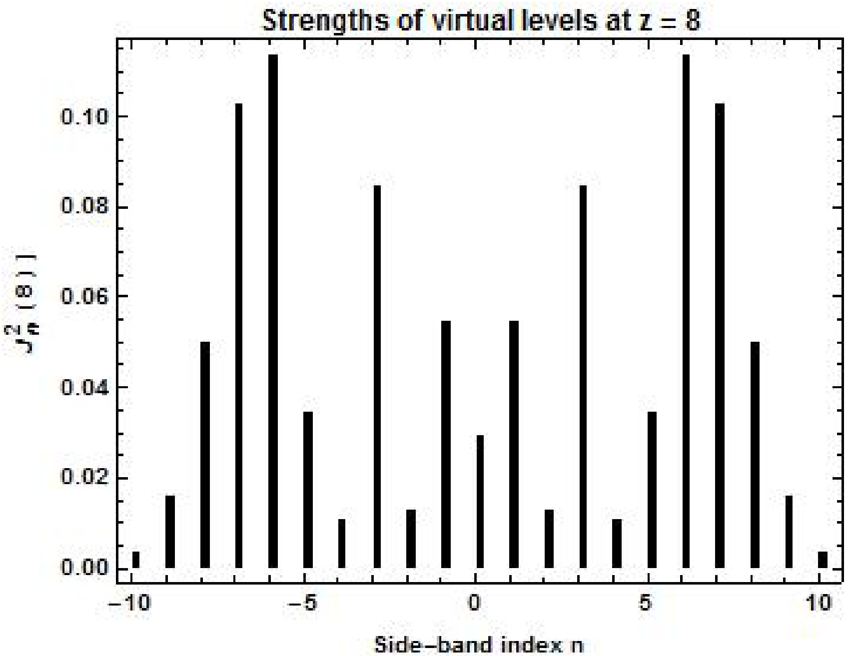
$$\nu = \alpha(\lambda\lambda_C^2\rho)(\hat{\boldsymbol{k}}'\cdot\boldsymbol{\varepsilon})$$

Multiphoton absorption and stimulated emission of a charged particle during scattering of an <u>inhomogeneous</u> laser field. [Illustration based on an approximate wave function.]

$$w_n = J_n^2\left(\frac{eF_0}{\hbar\omega_0}\left|\int_{-\infty}^{+\infty} dx f(x) e^{\frac{i}{\hbar}qx}\right|\right)$$

$$n_{\max} \approx \frac{eF_0 d}{\hbar\omega_0}$$





J. Bergou, S. Varró, Gy. Farkas and M.V. Fedorov: Absorption and induced emission of quanta in an external inhomogeneous electromagnetic field by a free electron.. Zh. Eksp. Teor. Fiz. 85, 57-69 (1983) [1.378]. English translation: Sov. Phys. JETP 58(1), July 1983. pp. 33-39. The figures are illustrations for the Bessel distribution for two arguments.

Gordon–Volkov states: Modulated de Broglie plane waves.

$$\Phi = \Phi_p(\xi) e^{-ip\cdot x}$$

$$\xi = k_\mu x^\mu = \omega_0 (t - y/c)$$

$$-k^2 \frac{d^2\Phi_p}{d\xi^2} + 2ik\cdot p \frac{d\Phi_p}{d\xi} + (p^2 - \kappa^2 - 2\varepsilon p\cdot A + \varepsilon^2 A^2)\Phi_p = 0$$

In vacuum: $k^2 = 0$

First-order ordinary differential equation for $\Phi \mathbf{p}$.

Immediately integrable, yielding the Gordon-Volkov solutions.

In a medium: $k^2 = (\omega_0 / c)^2 (1 - n_m^2) \neq 0$

Second-order ordinary differential equation for $\Phi \mathbf{p}$. E.g. Mathieu eq.

Laser field in a homogeneous underdense plasma: Lánczos-Proca vector boson. „Massive photon”

$$k^2 = (\omega_0 / c)^2 (1 - n_m^2) \neq 0$$

$$n_m^2 = \varepsilon_m = 1 - \omega_p^2 / \omega^2$$

$$\omega_p^2 = \frac{4\pi e^2 n_e}{m_e} \longrightarrow \mu = \hbar \omega_p / c^2$$

Effective mass [„Higgs-mechanism”]

$$L(x) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2}\mu^2 A_\nu A^\nu$$

$$F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$$

Proca equation:

$$\partial_\mu F^{\mu\nu} + \mu^2 A^\nu = 0$$

$$(\partial^2 + \mu^2) A_\nu = 0$$

(3 polarization!)

Lánczos C, Die tensoranalytischen Beziehungen der Diracschen Gleichung. *Zeitschrift für Physik* 57, 447 (1929).
Proca A, Sur la théorie ondulatoire desélectrons positifs et négatifs. J. Phys. Radium 7, 347 (1936).

Gordon-Volkov states (1927, 1935): **Exact solutions of the Klein–Gordon and Dirac equations of an electron in an arbitrary intense 'laser field' propagating in vacuum. After ~ 80 years; the only new exact, closed form solutions for the same problem in a medium [S. V. (2013, 2014)].**

Der Comptoneffekt nach der Schrödingerschen Theorie.

Von **W. Gordon** in Berlin.

(Eingegangen am 29. September 1926.)

Über eine Klasse von Lösungen der Diracschen Gleichung.

Von **D. M. Wolkow** in Leningrad.

(Eingegangen am 12. Februar 1935.)

$$e^{-iz\sin\omega_0 t} = \sum_n J_n(z) e^{-in\omega_0 t}$$

$$e^{\frac{i}{\hbar}(E_f - E_i)t - in\omega_0 t} = e^{\frac{i}{\hbar}(E_f - E_i - n\hbar\omega_0)t}$$

IOP Publishing — Laser Physics Letters

Laser Phys. Lett. 10 (2013) 095301 (13pp) doi:10.1088/1612-2011/10/9/095301

LETTER

New exact solutions of the Dirac equation of a charged particle interacting with an electromagnetic plane wave in a medium

Sándor Varró

Nuclear Instruments and Methods in Physics Research A 740 (2014) 280–283

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ELSEVIER

New exact solutions of the Dirac and Klein–Gordon equation of a charged particle propagating in a strong laser field in an underdense plasma

Sándor Varró

IOP Publishing | Astro Ltd — Laser Physics Letters

Laser Phys. Lett. 11 (2014) 016001 (14pp) doi:10.1088/1612-2011/11/1/016001

Letter

A new class of exact solutions of the Klein–Gordon equation of a charged particle interacting with an electromagnetic plane wave in a medium

Sándor Varró

Gordon W, *Zeitschrift für Physik* 40, 117-133 (1927). Volkov D M, ibid. 94, 250-260 (1935). [„Volkov solutions": Non-perturbative application to strong-field processes: from ~1960... E.g. Keldish (1964); ionization HHG, etc.]

New exact, closed form solutions of the Dirac and Klein–Gordon equations in a linearly polarized plane e.m. wave propagating in a medium (n_m < 1). III.

$$\xi = k_\mu x^\mu = \omega(t - n_m y / c)$$

$$k^2 > 0$$

$$[(i\partial - \varepsilon A)^2 - \kappa^2]\Phi = 0$$

$$\Phi_p = e^{-ip\cdot x + i(k\cdot p/k^2)\xi} g e^{-a\cos\xi}$$

Hill equation. Narozhny and Nikishov (1974) for n_m=0.

Ince's transformation (SV, 2013).

$$a \equiv 4\varepsilon A_0 / k_p = 4eF_0\lambda_p / \hbar\omega_0$$

Ince equation (SV, 2013). [Has a well depeloped theory in mathematics.]

$$\frac{d^2 g}{dz^2} + a\sin 2z\frac{dg}{dz} + (\eta - qa\cos 2z)g = 0$$

$$2z \equiv \xi$$

$$2p_x \equiv (q+1)k_p$$

$$k_p \equiv k_0\sqrt{1 - n_m^2}$$

$$2(k\cdot p)^2 / k_p^4 \equiv \eta$$

S. V., *Laser Physics Letters* 10 095301 (2013). S. V. *ibid.* 11 (2014). S. V,. *Nucl. Instr. Meth. in Physics Res.* A 740 (2014) 280-283.

$h\nu - A$

New exact, closed form solutions of the [Dirac and] Klein–Gordon equations in a linearly polarized plane e.m. wave propagating in a medium (n_m < 1). IV.

$$p_x = nk_p \quad (n = 1,2,\ldots) \qquad p_x = (n' + 1/2)k_p \quad (n' = 0,1,\ldots) \qquad (0 \le k \le n)$$

$$g = \varphi_n^k(\xi, a\,|+) = \sum_{r=0}^{n} A_r^{(k)}(a\,|\,n+1)\cos(r\xi)$$

Even cosine and sine type

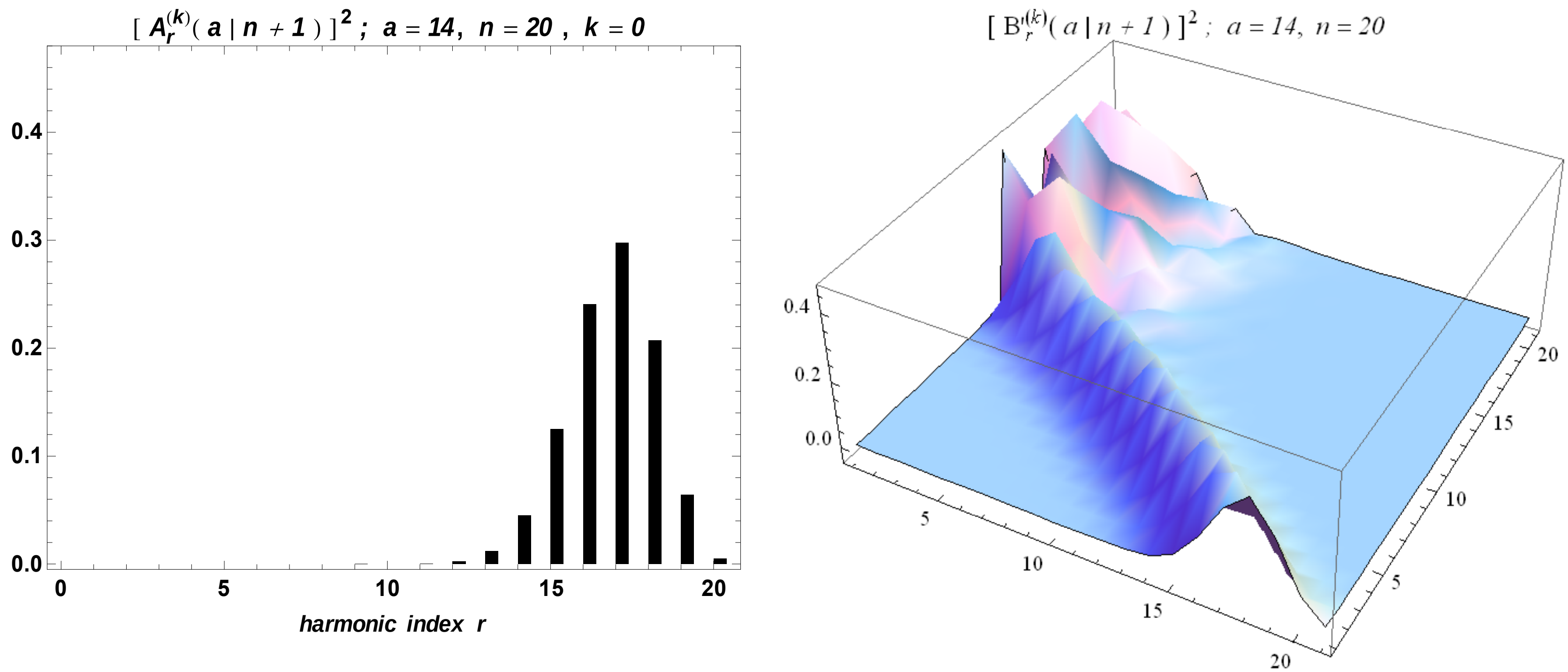
Odd cosine and sine type

$$\begin{bmatrix} 0 & (n+1)a & 0 & 0 & 0 \\ 2na & 4 & (n+2)a & \ldots & 0 \\ 0 & (n-1)a & 4\cdot 2^2 & \ldots & 0 \\ 0 & 0 & \cdots & 4(n-1)^2 & 2na \\ 0 & 0 & 0 & (n-(n-1))a & 4n^2 \end{bmatrix} \cdot \begin{bmatrix} A_0 \\ A_1 \\ \vdots \\ \vdots \\ A_n \end{bmatrix} = \eta_n^{(k)} \cdot \begin{bmatrix} A_0 \\ A_1 \\ \vdots \\ \vdots \\ A_n \end{bmatrix}$$

[S. V., New exact solutions of the Dirac equation of a charged particle interacting with an electromagnetic plane wave in a medium. *Laser Physics Letters* 10 095301 (2013).]; A new class of exact solutions of the Klein-Gordon equation of a charged particle interacting with an electromagnetic wave in a medium. In press *Laser Physics Letters* 11 (2014). Varró S, New exact solutions of the Klein-Gordon and Dirac equations of a charged particle propagating in a strong laser field in an underdense plasma. *Nuclear Instruments and Methods in Physics Research* A 740 (2014) 280-283.

Examples for the harmonic spectra.

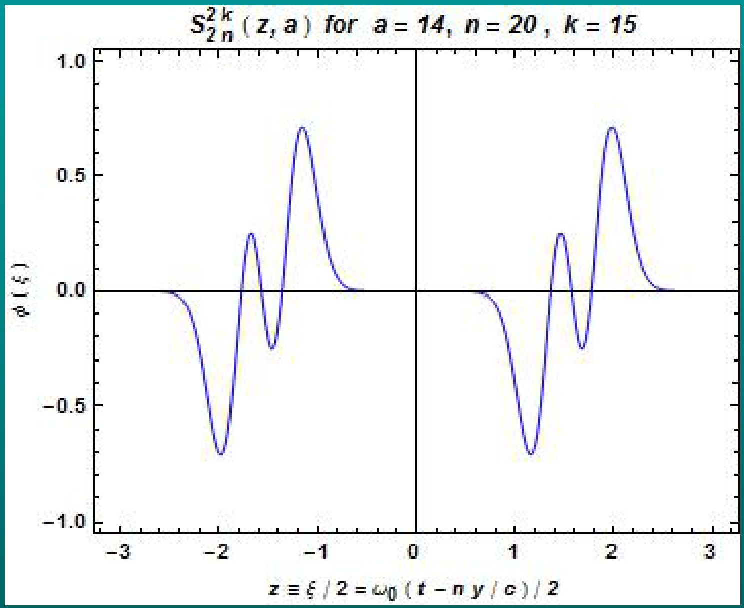
$\boxed{\cos(r\xi),}$ $\boxed{\cos((r+\tfrac{1}{2})\xi),}$

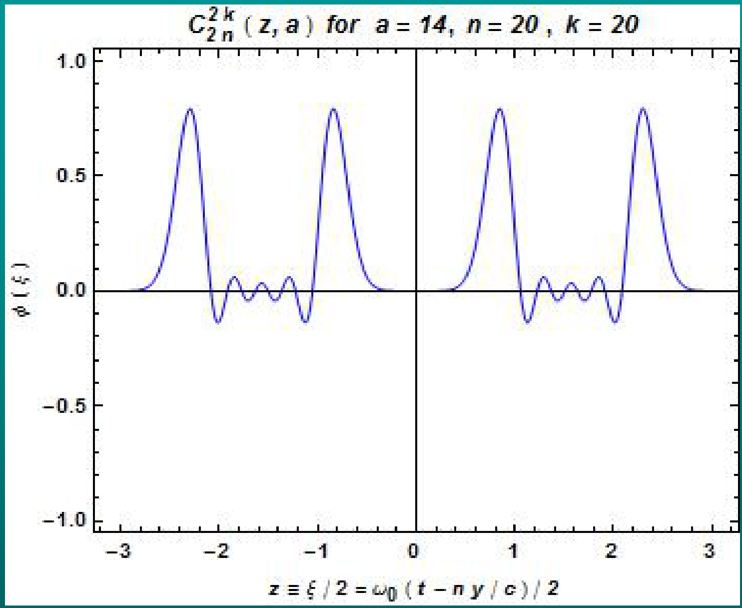


The strength of the harmonic coefficients of the polynomial for two eigenvalues labelled by the upper index k=0 and k=9. Here $p_x = 20 \times h\omega_p$, i.e. n=20, $h\omega_p$=1eV. The intensity for a Ti:Sa laser field ($h\omega$=1.56eV) I0= 100 MW/cm^2, i.e. a=14.

S. V., *Laser Physics Letters* 10 095301 (2013). S. V. *ibid.* 11 (2014). S. V,. *Nucl. Instr. Meth. in Physics Res.* A 740 (2014) 280-283.

K-G case. Ince polynomials.





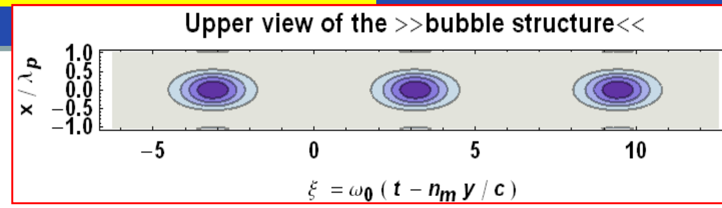
Klein-Gordon. k = 15.

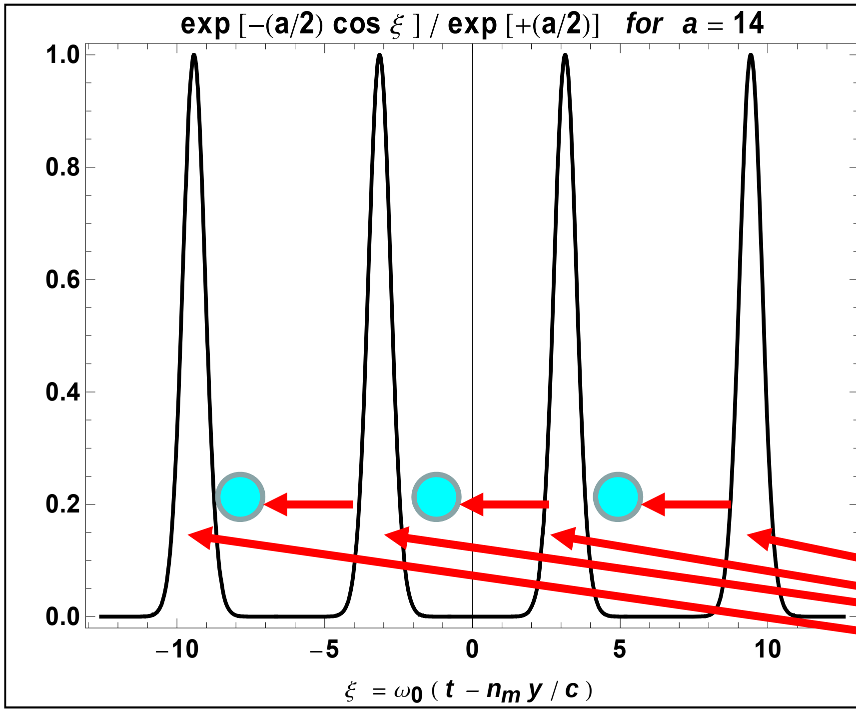
Klein-Gordon. k = 20.

Wave functions with negative eigenvalues (imaginary longitudinal momentum)

S. V., *Laser Physics Letters* 10 095301 (2013). S. V. *ibid.* 11 (2014). S. V,. *Nucl. Instr. Meth. in Physics Res.* A 740 (2014) 280-283.

Formation of a high-contrast, longitudinal periodic density structure (‘quantum bubble’?). A mechanism in laser-induced proton wake-field accelerator? Half-integer harmonics.





$$a = 4\frac{eF_0\lambda_p}{\hbar\omega_0} = 2\mu_0\frac{2mc^2}{\hbar\omega_p}$$

$$\mu_0 = \frac{eF_0}{mc\omega_0} = 8.5\times10^{-10}\sqrt{I_0}\lambda_0$$

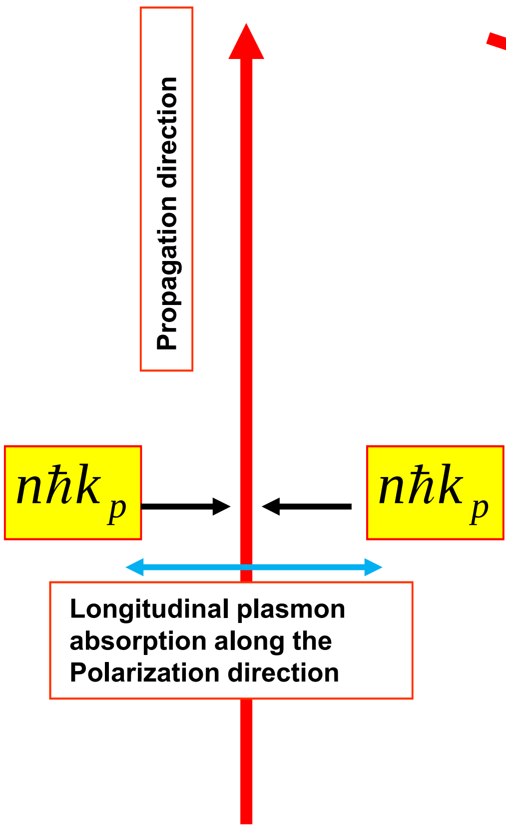
Plasmon-induced quantum modulation of the proton (electron) density on the plasma length scale [‘quantum bubble’]. Regular structure may be suitable for electron acceleration.

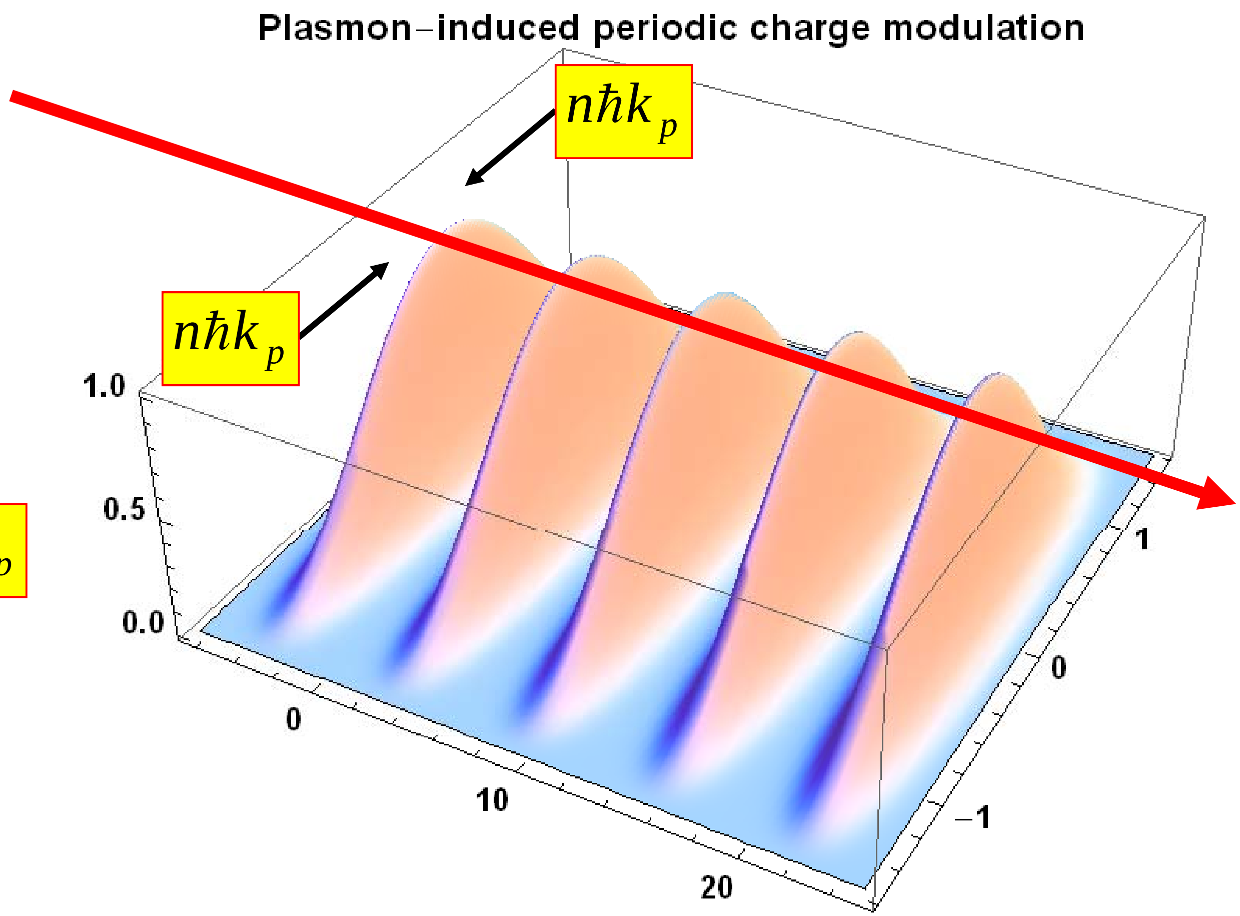
Integer and half – integer higher harmonics:

$\cos(r\xi)$, $\sin(r\xi)$, $\cos((r+\tfrac{1}{2})\xi)$, $\sin((r+\tfrac{1}{2})\xi)$,

S. V., *Laser Physics Letters* 10 095301 (2013). S. V. *ibid.* 11 (2014). S. V,. *Nucl. Instr. Meth. in Physics Res.* A 740 (2014) 280-283.

(Longitudinal) plasmon absorption along the (transverse) polarization (electric field) direction induces a high – contrast charge modulation along the propagation direction. [Ince polynomials with an exponential envelope]





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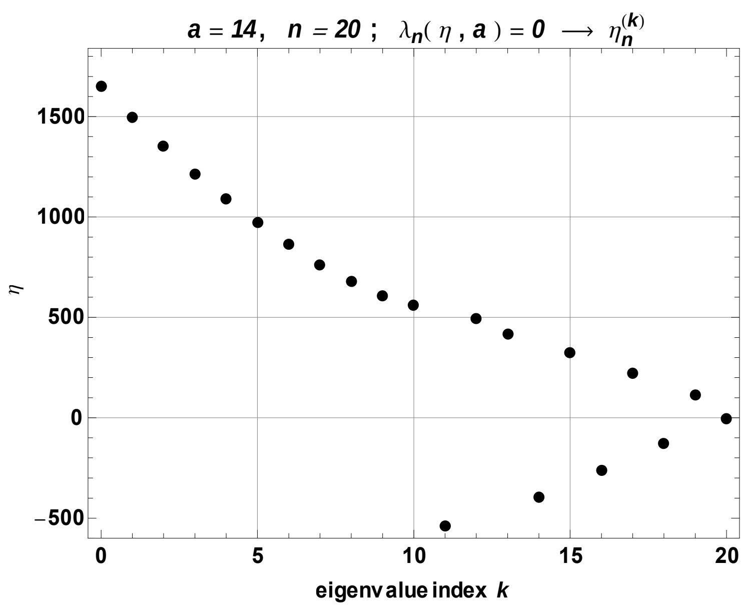
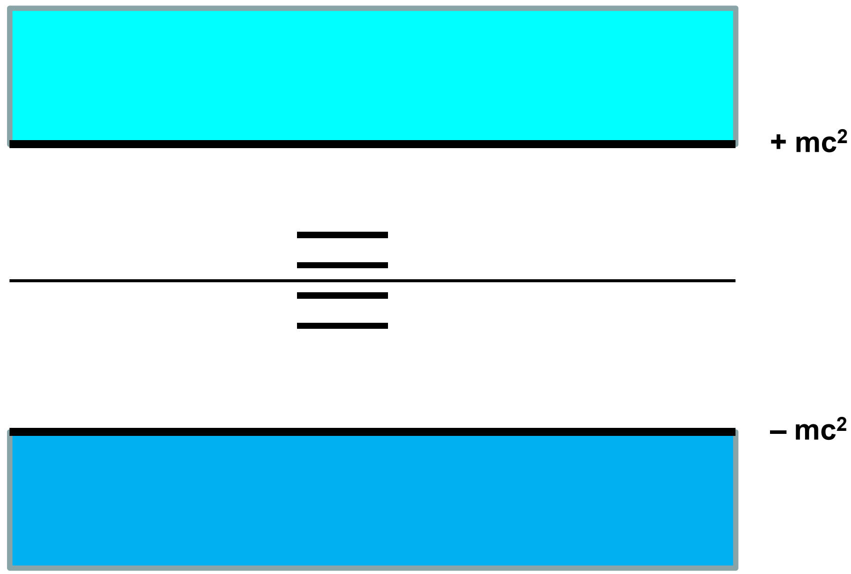
Interaction with an oscillating pure electric field [through Lorentz transformation, along the propagation direction]. Quasi – energy between $-mc^2$ and $+mc^2$; 'Gap states'.

$$y' = \frac{y - \beta ct}{\sqrt{1-\beta^2}}, \quad ct' = \frac{ct - \beta y}{\sqrt{1-\beta^2}}, \quad \beta = n_m \quad \boldsymbol{A}' = \boldsymbol{e}_x \frac{cF_0}{\omega_0}\cos\left(\frac{\omega_0}{\gamma} t'\right)$$

$$\varepsilon_m = 1 - \omega_p^2 / \omega_0^2 \quad \Rightarrow \gamma = \omega_0 / \omega_p \quad \Rightarrow E_n'^{(k)} = \pm \tfrac{1}{\sqrt{2}} \hbar \omega_p \sqrt{\eta_n^{(k)}}$$





S. V., *Laser Physics Letters* 10 095301 (2013). S. V. *ibid.* 11 (2014). S. V,. *Nucl. Instr. Meth. in Physics Res.* A 740 (2014) 280-283.

Connection with the Gordon–Volkov states. Adiabatic transition at the vacuum–plasma interface (in the presence of a laser field); $p_x = nhk_p$ is fixed, $n \to$ infinity, $\omega_p \to 0$.

$$S_p = p \cdot x + (1/2k \cdot p)\int [2\varepsilon p \cdot A(\xi) - \varepsilon^2 A^2(\xi)]d\xi$$

$$A = e_x (F_0 / k_0)\cos \xi, \quad \xi = \omega_0 (t - y/c)$$

$$e^{-iS_p} \propto \exp(-i\alpha \sin \xi - i\beta \sin 2\xi)$$

Jacobi–Anger formula:

$$e^{-i\alpha \sin \xi} = \sum_n J_n(\alpha) e^{-in\xi}$$

The usual way to obtain the harmonic components is to use the Jacobi–Anger formula (generating function of the Bessel functions). Instead, we can also expand the exponential expression as:

$$\exp(-i\alpha \sin \xi - 2i\beta \sin \xi \cos \xi) =$$

$$\sum_{n=0}^{\infty} (-2i\alpha)^n \sum_{k=0}^{n} \frac{(2\beta/\alpha)^k}{(n-k)!k!} (\cos \xi)^k (\sin \xi)^{n-k}$$

$$\varphi_n^k(\xi \mid a)$$

$$p_x = n\hbar k_p$$

This expansion of the usual Gordon–Volkov states is a superposition of the asymptotic solutions of the Ince equation. ['Barut – Girardello distribution' × Binomial (Poisson) distribution.]

Summary.

• Basics on the classic Gordon – Volkov solutions.

• Unusual properties of the new exact solutions:
exponentially high-contrast longitudinal charge density modulation,
half-integer harmonics,
[Lorentz transformation of the plasmon wave to an oscillating pure electric field.]
'gap states' between $-mc^2$ and $+mc^2$.

• Brief discussion of the boundary-value problem at the vacuum-plasma transition regions.

Acknowledgments.
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Appendices

Abstract.

Varró S, **Quantum description of relativistic charged particles interacting with a strong laser field in a plasma, represented by Lánczos-Proca vector bosons.**

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We analyse the new exact solutions of the relativistic wave equations of a charged particle propagating in a plasmon wave, which we represent by a Lanczos-Proca vector boson field. The nonlinearities associated to these solutions depend on a new intensity parameter, which is the work done by the laser field along the plasmon wavelength divided by the laser photon energy. These solutions describe a high-contrast periodic longitudinal density structure on the plasma length scale (a sort of 'quantum-bubble'), whose existence relies on the discrete absorption of momentum quanta along the wave's transverse electric field. For vanishing plasma density, certain 'coherent superpositions' of our solutions reproduce the Volkov states, which, by now, have been the only closed form exact solutions in a plane wave in vacuum..

[1] Varró S, *Las. Phys. Lett.* 10 (2013) 095301. [2] Varró S, *ibid.* 11 (2014) 016001.

[3] Varró S, *Nucl. Instr. Meth. Phys. Res.* A 740 (2014) 280-283.

[4] Volkov D M, *Zeitschrift für Physik* 94 (1935) 250-260.

$h\nu - A$

Multiphoton Compton scattering; first (relativistic) calculation of HHG [Alperin, 1944]. „To the theory of light scattering on free electrons"

ЖУРНАЛ
ЭКСПЕРИМЕНТАЛЬНОЙ И ТЕОРЕТИЧЕСКОЙ ФИЗИКИ
ТОМ 14 1944 ВЫП. 1—2

$$\Psi^*_f \Psi_i e^{-i\omega t} \sim e^{\frac{i}{\hbar}(E_f - E_i)t} e^{-i\omega' t}$$

$$A_{scatt} = \frac{1}{c} \int \frac{j_{p'p}(r, t - R/c)}{R} d^3r d^3p'$$

$$j_{p'p} = \overline{\Psi}_{p's'} \gamma \Psi_{ps}$$

К ТЕОРИИ РАССЕЯНИЯ СВЕТА НА СВОБОДНЫХ ЭЛЕКТРОНАХ

М. Альперин

В работе получено точное решение уравнений Дирака для электрона в поле плоской волны. Применение найденных решений дает возможность на основе принципа соответствия рассмотреть задачу о рассеянии света на свободных электронах. *Получена обобщенная формула Клейна — Нишины, распространенная на случай больших интенсивностей падающего излучения и учитывающая возможность одновременного поглощения нескольких квантов.*

Note: Alperin quotes Klein and Nishina (1929), Tamm (1930) and Heitler' book. [The unitary equivalence with the Volkov solution see: Bergou J, S. Varró: Wave functions of a free electron in an external field and their application in intense field interactions, II. Relativistic treatment. *J. Phys. A* 13, 2823-2837 (1980).]

$h\nu - A$

W. Becker's analysis on the 'strong-field photon-electron interaction' in a medium [1977]

Physica **87A** (1977) 601–613 © *North-Holland Publishing Co.*

$$n_m > 1$$

$$k^2 < 0$$

RELATIVISTIC CHARGED PARTICLES IN THE FIELD OF AN ELECTROMAGNETIC PLANE WAVE IN A MEDIUM

W. BECKER

Institut für Theoretische Physik, der Universität Tübingen, D-7400 Tübingen, West Germany

Received 1 November 1976

Solutions of the Klein–Gordon and Dirac equation for a charged particle in the field of an electromagnetic plane wave in a medium with a constant refractive index n are discussed. Generally, for $n^2 < 1$, spontaneous pair creation from the vacuum, and for $n^2 > 1$, energy bands are observed. The interplay of Compton and Cherenkov scattering is discussed. Some doubts are formulated as to the physical relevance of calculating pair creation in a homogeneous electric field as it is usually done.

W. Becker, Relativistic charged particles in the field of an electromagnetic plane wave in a medium. *Physica A* 87, 601-613 (1977)

$h\nu - A$

attosecond

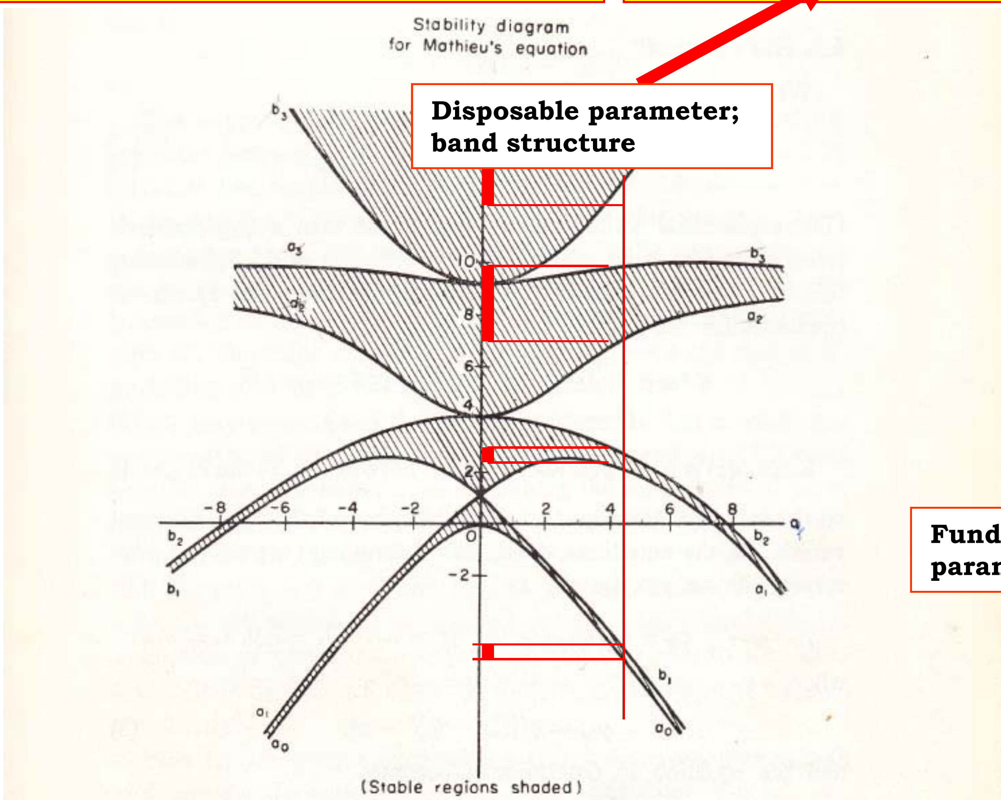
The 'strong-field photon-electron interaction' in a medium.

Mathieu, Hill eq.

$$\xi = k_\mu x^\mu = \omega(t - n_m y / c)$$

$$w'' + (\lambda - 2h^2 \cos 2z)w = 0$$

$$\lambda \propto k \cdot p, \quad h^2 \propto eA_0 p_\perp$$



Fundamental parameter.

MATHIEU EQUATION. In the standard notation: 2z = ξ. [Figure copied from Arscott F M, *Periodic differential equations* (Pergamon Press, Oxford, 1964) p.123.]. Nikishov & Ritus (1967), Nikishov (1970), Narozhny & Nikishov (1974) pair creation..., Becker (1977) Cherenkov..., Fedorov, McIver … FEL theories.

New exact, closed form solutions of the Dirac and Klein–Gordon equations in a linearly polarized plane e.m. wave propagating in a medium ($n_m < 1$). I.

$$\xi = k_\mu x^\mu = \omega(t - n_m y / c)$$

$$k^2 > 0$$

$$[(i\partial - \varepsilon A)^2 - \kappa^2 - \tfrac{1}{2}\varepsilon\sigma \cdot F]\Psi = 0$$

$$\Psi_p^{(\pm)} = \sum_{s=1}^{4} \Psi_{ps}^{(\pm)} u_s$$

$$\Psi_{ps}^{(\pm)} = e^{-ip\cdot x + i(k\cdot p/k^2)\xi} f e^{-a\cos\xi}$$

Hill equation. Narozhny and Nikishov (1974) for n_m=0.

Ince's transformation (SV, 2013).

$$a \equiv 4\varepsilon A_0 / k_p = 4eF_0\lambda_p / \hbar\omega_0$$

Ince type equation (SV, 2013). [Unexplored in mathematics.]

$$\frac{d^2 f}{dz^2} + a\sin 2z\left(\frac{df}{dz} + if\right) + (\eta - qa\cos 2z) f = 0$$

$$2z \equiv \xi$$

$$2p_x \equiv (q+1)k_p$$

$$k_p \equiv k_0\sqrt{1 - n_m^2}$$

$$2(k\cdot p)^2 / k_p^4 \equiv \eta$$

S. V., *Laser Physics Letters* 10 095301 (2013). S. V. *ibid.* 11 (2014). S. V,. *Nucl. Instr. Meth. in Physics Res.* A 740 (2014) 280-283.

$h\nu - A$

New exact, closed form solutions of the Dirac [and Klein–Gordon] equations in a linearly polarized plane e.m. wave propagating in a medium (n_m < 1). II.

$p_x = nk_p$ $(n = 1,2,\dots)$

$p_x = (n' + ½)k_p$ $(n' = 0,1,\dots)$

$$f = f_n^k(\xi, a \,|\, +) = \sum_{r=-n+1}^{n} D_r^{(k)}(a \,|\, 2n)\exp(-ir\xi)$$

$$\begin{bmatrix} 4(-n+1)^2 & (+1)a & 0 & 0 & 0 \\ (2n-1)a & 4(-n+2)^2 & \cdots & 0 & 0 \\ 0 & (2n-2)a & \cdots & (2n-2)a & 0 \\ 0 & 0 & \cdots & 4(n-1)^2 & (2n-1)a \\ 0 & 0 & 0 & (+1)a & 4n^2 \end{bmatrix} \cdot \begin{bmatrix} D_{-n+1} \\ D_{-n+2} \\ \vdots \\ D_{n-1} \\ D_n \end{bmatrix} = \eta_n^{(k)} \cdot \begin{bmatrix} D_{-n+1} \\ D_{-n+2} \\ \vdots \\ D_{n-1} \\ D_n \end{bmatrix}$$

S. V., New exact solutions of the Dirac equation of a charged particle interacting with an electromagnetic plane wave in a medium. *Laser Physics Letters* 10 095301 (2013).; [A new class of exact solutions of the Klein-Gordon equation of a charged particle interacting with an electromagnetic wave in a medium. In press *Laser Physics Letters* 11 (2014).] Varró S, New exact solutions of the Klein-Gordon and Dirac equations of a charged particle propagating in a strong laser field in an underdense plasma. *Nuclear Instruments and Methods in Physics Research* A 740 (2014) 280-283.

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attosecond

Recent related works on electron – positron plasmas (cascades) and QED in plasmas.

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PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **14**, 054401 (2011)

QED cascades induced by circularly polarized laser fields

N. V. Elkina,[1] A. M. Fedotov,[2] I. Yu. Kostyukov,[3] M. V. Legkov,[2] N. B. Narozhny,[2] E. N. Nerush,[3] and H. Ruhl[1]

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PHYSICS OF PLASMAS **20**, 056701 (2013)

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Dense electron-positron plasmas and bursts of gamma-rays from laser-generated quantum electrodynamic plasmas[a)]

C. P. Ridgers,[1,b)] C. S. Brady,[2] R. Duclous,[3] J. G. Kirk,[4] K. Bennett,[2] T. D. Arber,[2] and A. R. Bell[1]

PHYSICS OF PLASMAS **21**, 023109 (2014)

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Energy partition, γ-ray emission, and radiation reaction in the near-quantum electrodynamical regime of laser-plasma interaction

L. L. Ji,[1,2] A. Pukhov,[1,3] E. N. Nerush,[4,3] I. Yu. Kostyukov,[4,3] B. F. Shen,[2] and K. U. Akli[5]

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PHYSICS OF PLASMAS **21**, 053103 (2014)

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The Lagrangian formulation of strong-field quantum electrodynamics in a plasma

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The Lagrangian formulation of the scalar and spinor quantum electrodynamics in the presence of

Raicher E and Eliezer S, Analytical Solutions of the Dirac and the Klein-Gordon Equations in Plasma Induced by High Intensity Laser.Phys. Rev. A 88, 022113 (2013).