MAX-PLANCK-GESELLSCHAFT

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Strong-field classical and quantum electrodynamics in intense laser fields Part II: Advanced concepts

Antonino Di Piazza

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Outline

- Introduction
- QED processes in a strong background electromagnetic field
- Strong-field QED in a strong plane-wave field
- Vacuum polarization effects
- Electromagnetic cascades
- Conclusions

Introduction

- QED was formulated by Schwinger, Feynman and Tomonaga (and many others…)
- The simple Lagrangian density

$$\mathscr{L}_{QED} = \bar{\psi}[\gamma^{\mu}(i\partial_{\mu} - eA_{\mu}) - m]\psi - \frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}$$

  accounts for an extremely large number of processes spanning from plasma physics, to atomic and molecular physics
- QED in vacuum is the most successful physical theory we have:

$$a_e(\text{experimental}) = 0.001159652180.73(0.28)$$
$$a_e(\text{theoretical}) = 0.001159652181.13(0.11)(0.37)(0.02)(0.77)$$

  where $a_e = (g - 2)/2$ is half the anomalous magnetic moment of the electron (Aoyama et al. 2012). Equivalent to measure the distance Earth-Moon with an accuracy of the order of the width of a single human hair!
- Proton-radius puzzle: r_p(experimental)= 0.84087(39) fm seven σ smaller than r_p(theoretical)=0.8775(51) fm (Pohl et al. 2013)

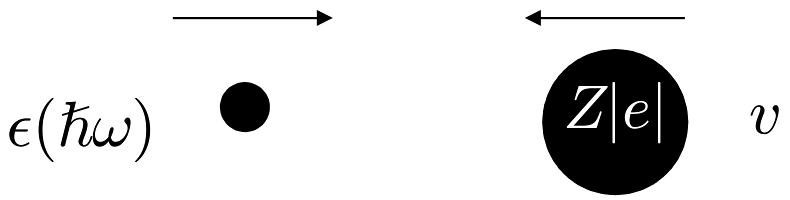
- The experimental tests of QED in the presence of background electromagnetic fields are not comparably numerous and accurate
- The reasons are:
  1. on the experimental side, the effective background field has to be of the order of the critical field of QED

     $$F_{cr} = \frac{\text{QED energy scale}}{(\text{positron charge}) \times (\text{QED length scale})} = \frac{mc^2}{|e| \times (\hbar/mc)}$$

     thus either very intense fields or high-energy particles are required to make in general the effects “measurable”
  2. on the theoretical side, calculations are feasible only for a few “special” background electromagnetic fields:
     - Constant and uniform electric/magnetic fields (relevant in astrophysics)
     - Coulomb field (relevant for atomic physics)
     - Plane-wave fields (relevant for laser physics)

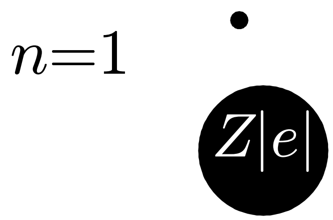
Strong-field QED in a strong atomic field

Highly-charged ions are sources of very strong fields and strong-field QED in the presence of strong atomic fields has been investigated since a long time (Bethe and Heitler 1934, Bethe and Maximon 1954). A bare nucleus with chrage number Z and with velocity v collides with a particle (e^-, e^+ or γ) with energy ϵ for an electron or a positron ($\hbar\omega$ for a photon)



High-order QED effects (Coulomb corrections) only depend on the Lorentz- and gauge-invariant parameter $Z\alpha$ ($\alpha=1/137$)

All the properties of the energies of a hydrogen-like atom depend only on $Z\alpha$ (Beresteskii et al., 1982)



$$d = \frac{\lambda_C}{Z\alpha}$$

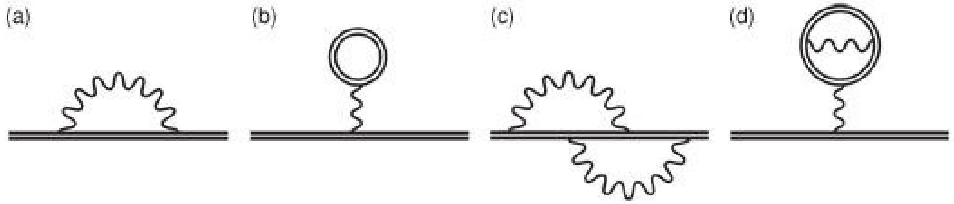
$$E = \frac{Z|e|}{d^2} = (Z\alpha)^3 E_{cr}$$

$$\lambda_C = \frac{\hbar}{mc} \qquad E_{cr} = \frac{m^2c^3}{\hbar|e|}$$

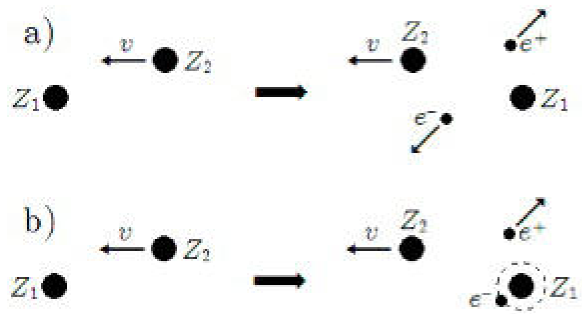
For Uranium 91^+ it is $Z\alpha\approx 0.66$

Experiments with highly-charged ions

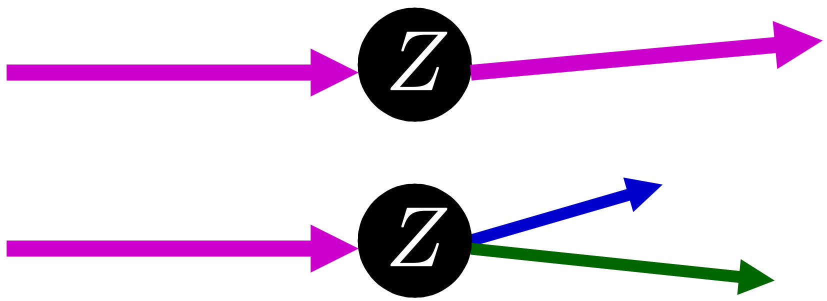
- Tests on bound-state QED (Lamb shift, Stoehlker et al. 2003)



- Tests via peripheral ultra-relativistic collisions of highly-charged ions (free and bound-free electron-positron pair production, Baur 2002)

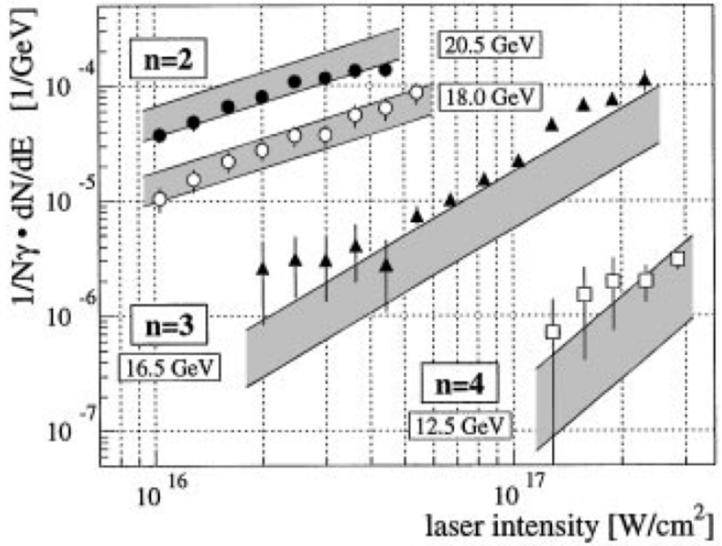


- Delbrueck scattering (Akhmadaliev et al. 1998) and photon splitting (Akhmadaliev et al. 2002) in bismuth (Z=83)



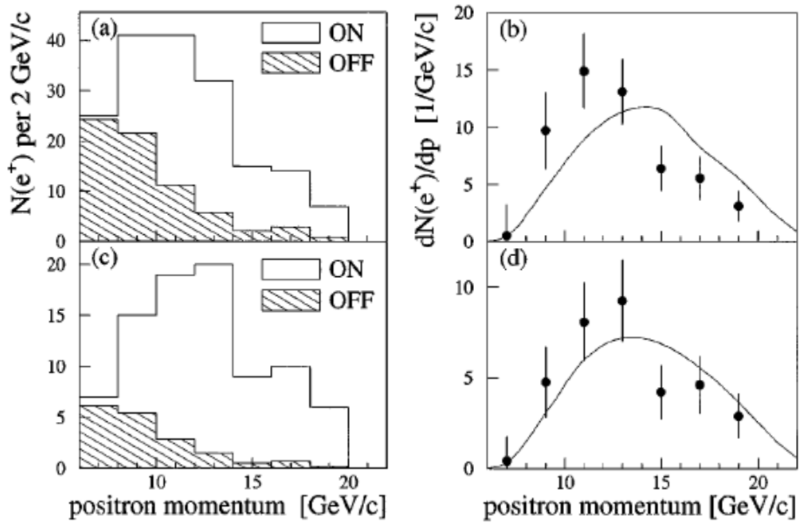
Experiment with intense laser fields
(SLAC, I_L=10^{17}-10^{19} W/cm^2, $\mathcal{E}$=50 GeV)

- Nonlinear Compton scattering:
  $e^- + n\omega_L \rightarrow e^- + \omega$
  (Bula et al. 1996)



- Electron-positron pair creation (two steps):
  1. $e^- + n\omega_L \rightarrow e^- + \omega$
  2. $\omega + l\omega_L \rightarrow e^- e^+$

  (Bula et al. 1997)



Electromagnetic field as classical field

- Following Bohr, an electromagnetic field can be treated as a classical field if the occupation numbers $n_{k,\lambda}$ corresponding to the number operators $N_{k,\lambda}=c^{\dagger}{}_{k,\lambda}c_{k,\lambda}$ are large. However, if all $n_{k,\lambda}$ are large, the energy of the field would be infinite (Landau and Lifshitz 1982)
- If the field is measured during a time Δt, angular frequencies larger than $\omega_0=1/\Delta t$ cannot be resolved
- Require that $n_{k,\lambda}\gg 1$ for $\omega=|\boldsymbol{k}|<\omega_0$
- Typical occupation number n in terms of the fields (E, B)

$$n \sim \frac{\text{Total field energy}}{\text{Typical number of states} \times \text{Typical states' energy}}$$

$$\sim \frac{\frac{1}{8\pi}(E^2+B^2)V}{\frac{V}{(2\pi\hbar)^3}\frac{4}{3}\pi\left(\frac{\hbar\omega_0}{c}\right)^3 \times (\hbar\omega_0)} = \frac{1}{2.3\times 10^4}\frac{I[\mathrm{W/cm^2}]}{(\hbar\omega_0[\mathrm{eV}])^4}$$

- Condition $n\gg 1$ easy to fulfill for available optical lasers
- The field should not be depleted during the interaction

QED in a strong background field

Lagrangian density of QED in the presence of a background field $A_{B,\mu}(x)$ (Furry, Phys. Rev. 1951)

Units with $\hbar$=c=1

$$\mathscr{L}_{QED} = \mathscr{L}_e + \mathscr{L}_\gamma + \mathscr{L}_{\rm int}$$

$$\mathscr{L}_e = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \qquad \longrightarrow \qquad \mathscr{L}_e = \bar{\psi}[\gamma^\mu(i\partial_\mu - eA_{B,\mu}) - m]\psi$$

$$\mathscr{L}_\gamma = -\frac{1}{16\pi} F_{\mu\nu}F^{\mu\nu} \qquad \longrightarrow \qquad \mathscr{L}_\gamma = -\frac{1}{16\pi} F_{\mu\nu}F^{\mu\nu}$$

$$\mathscr{L}_{\rm int} = -e\bar{\psi}\gamma^\mu\psi(A_\mu + A_{B,\mu}) \qquad \longrightarrow \qquad \mathscr{L}_{\rm int} = -e\bar{\psi}\gamma^\mu\psi A_\mu$$

- Only the interaction between the spinor and the radiation field is treated perturbatively:
  1. Solve the Dirac equation $[\gamma^\mu(i\partial_\mu - eA_{B,\mu}) - m]\psi=0$
     - find the "dressed" one-particle in- and out-electron states and the "dressed" electron propagator
  2. Write the Feynman diagrams of the process at hand
  3. Calculate the amplitude and then the cross sections (or the rates) using "dressed" states and propagators

QED in a strong laser field

- The laser field is approximated by a plane-wave field: $A_{B,\mu}(x)=A_{L,\mu}(\phi)$, $\phi=(k_L x)=\omega_L t-\boldsymbol{k}_L\cdot\boldsymbol{r}$ and $\omega_L=|\boldsymbol{k}_L|$
- One-particle states: Volkov states (Volkov, Z. Phys. 1936)

$$\psi_{p,\sigma}=\left[1+\frac{e}{2(k_Lp)}\hat{k}_L\hat{A}_L\right]\frac{u_{p,\sigma}}{\sqrt{2p_0}}\exp\Bigg\{\underbrace{-i(px)-i\int_{-\infty}^{\phi}d\phi'\left[\frac{e(pA_L)}{(k_Lp)}-\frac{e^2A_L^2}{2(k_Lp)}\right]}_{i(\text{Classical action})}\Bigg\}$$

Spin term

Electron momentum and spin at $t\to-\infty$

Free constant bi-spinor

- Technical notes:
  - Volkov states are quasiclassical
  - In- and out-states differ only by a constant phase
  - No tadpole diagrams
  - The vacuum in the presence of a plane wave is stable
- Study of QED radiative corrections in an intense laser field at $\chi\gg1$ (Ritus 1970, Narozhny 1979-1981, Fedotov 2016)

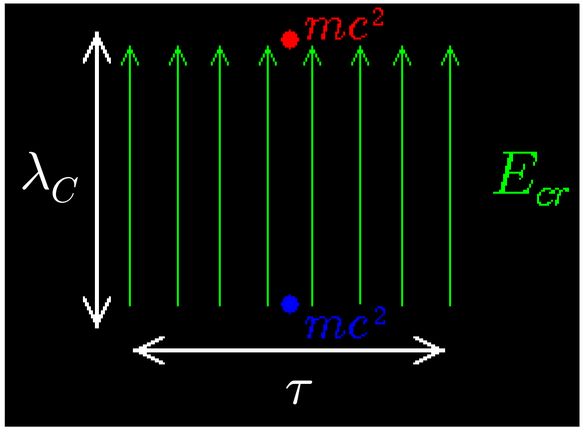
Physics of the QED vacuum

- Quantum vacuum is a region of space-time which contains no real particles (electrons, positrons, photons etc...)
  - Virtual particles are present
  - They live for a very short time and cover a very short distance ($\tau = \hbar/mc^2$ and $\lambda_C = \hbar/mc$, respectively). For electrons and positrons: $\lambda_C \sim 10^{-11}$ cm and $\tau \sim 10^{-21}$ s

- Physical meaning of the critical fields:

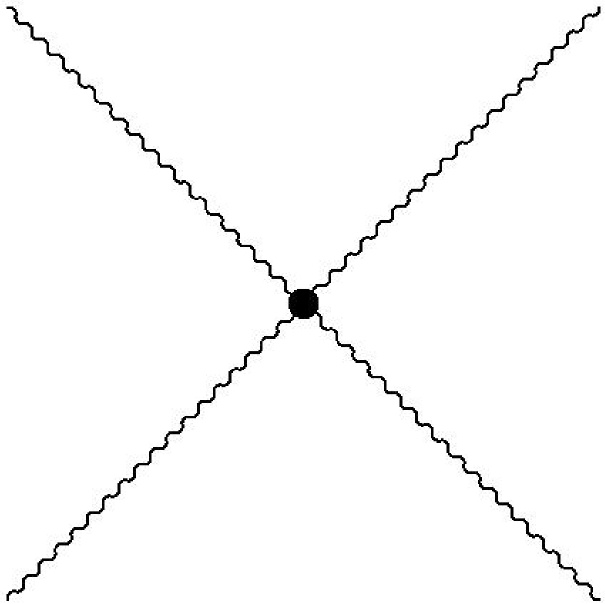
$$|e| E_{cr} \times \frac{\hbar}{mc} = mc^2$$

$$\frac{|e|\hbar}{mc} \times B_{cr} = mc^2$$



Probing the quantum vacuum: effective Lagrangian approach

- At a quantum level photons interact among them and with external electromagnetic fields:
  - The interaction is mediated by virtual electrons and positrons
  - The interaction is non local, the typical interaction distance being the Compton length $\lambda_C = \hbar / mc$
  - If the interacting photons and the external fields vary not much in a Compton length, the local approximation can be used (effective Lagrangian approach)



- Approach valid if $\hbar\omega \ll mc^2$ with ω the typical angular frequency of the photon and of the external fields

Methods to obtain the effective Lagrangian L_{eff}

1. The Lagrangian density L_{eff} is such that the Maxwell equations in vacuum are given by (Schwinger 1951)

$$\partial_\mu F^{\mu\nu} = 4\pi \langle 0(A)|j^\nu(x;A)|0(A)\rangle$$
$$= -4\pi i e \langle 0(A)|\mathrm{Tr}[\gamma^\nu G(x,x';A)]_{x'\to x}|0(A)\rangle$$

with $|0(A)\rangle$ and $G(x,x';A)$ being the vacuum state and the electron propagator, respectively, in the presence of the field electromagnetic $F^{\mu\nu}(x)=\partial^\mu A^\nu(x) - \partial^\nu A^\mu(x)$

2. The Lagrangian density L_{eff} is such that

$$\Delta L = L_{\text{eff}} - L_M = -[\mathcal{E}_{\text{vac}}(A) - \mathcal{E}_{\text{vac}}(0)]$$

with L_M the classical Maxwell Lagrangian density and $\mathcal{E}_{\text{vac}}(0/A)$ the vacuum energy in absence (in the presence) of the external field

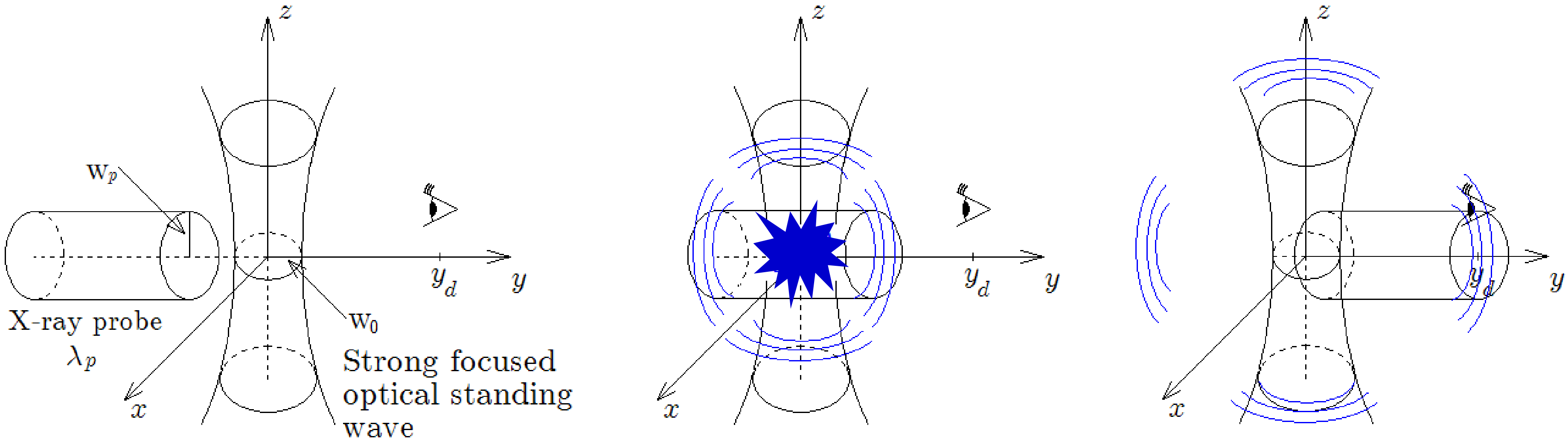
3. More formal methods through Feynman path integral (see, e.g., Peskin and Schroeder or any modern book on QFT)

- Euler and Heisenberg obtained the following effective Lagrangian for a constant and uniform electromagnetic field (E, B) described by the two Lorentz-invariant quantities $\mathcal{F}= (B^2-E^2)/2$ and $\mathcal{G}=E\cdot B$

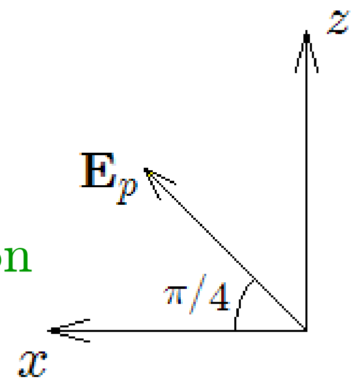
$$L_{\text{eff}} = -\frac{1}{4\pi}\mathcal{F}$$
$$-\frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3}e^{-m^2 s}\left\{(es)^2\mathcal{G}\frac{\operatorname{Re}\cosh\left[|e|s\sqrt{2(\mathcal{F}+i\mathcal{G})}\right]}{\operatorname{Im}\cosh\left[|e|s\sqrt{2(\mathcal{F}+i\mathcal{G})}\right]} - 1 - \frac{2}{3}(es)^2\mathcal{F}\right\}$$

- The quantum part of the Lagrangian density depends on the ratios $\mathcal{F}/F_{cr}{}^2$ and $|\mathcal{G}|/F_{cr}{}^2$
- The imaginary part of L_{eff} is connected with the pair production probability per unit volume and unit time
- If the external field is purely magnetic L_{eff} is real then no pair production while for purely electric fields L_{eff} contains an imaginary part (Narozhny et al. 2004)
- A plane wave field with $\mathcal{F} = \mathcal{G} = 0$ cannot give rise to any nonlinear vacuum polarization effect

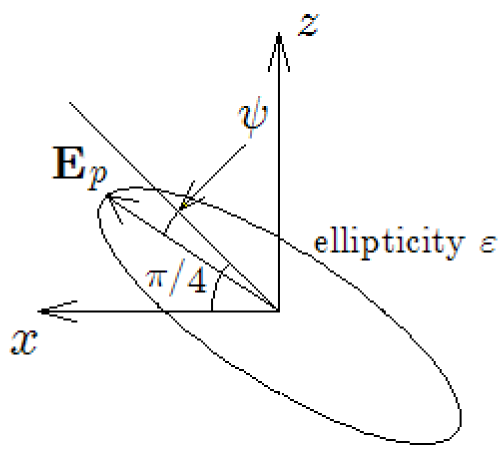
Light-by-light scattering



Probe polarization before the interaction



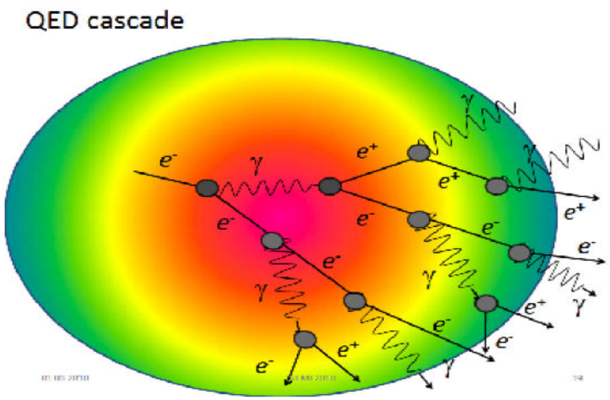
Probe polarization after the interaction



The scheme also works with a single strong traveling wave (electromagnetic cascade generation in a standing wave in the presence of residual electrons)

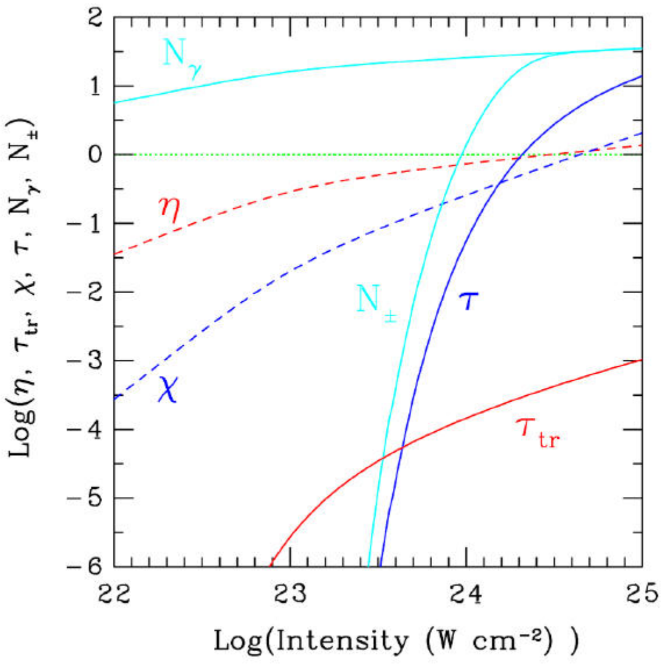
QED cascades

- In the E-144 experiment at SLAC only 100 positrons have been observed out of 22000 shots, each involving about 10^7 electrons
- Are there more efficient ways of producing positrons?
- By an avalanche or cascade process we mean here a process in which even a single electron in a field emits high-energy photons, which can interact with the field itself generating electron-positron pairs, which, in turn, emit photons again and so on (a cascade process may also be initiated by a photon rather than by an electron)



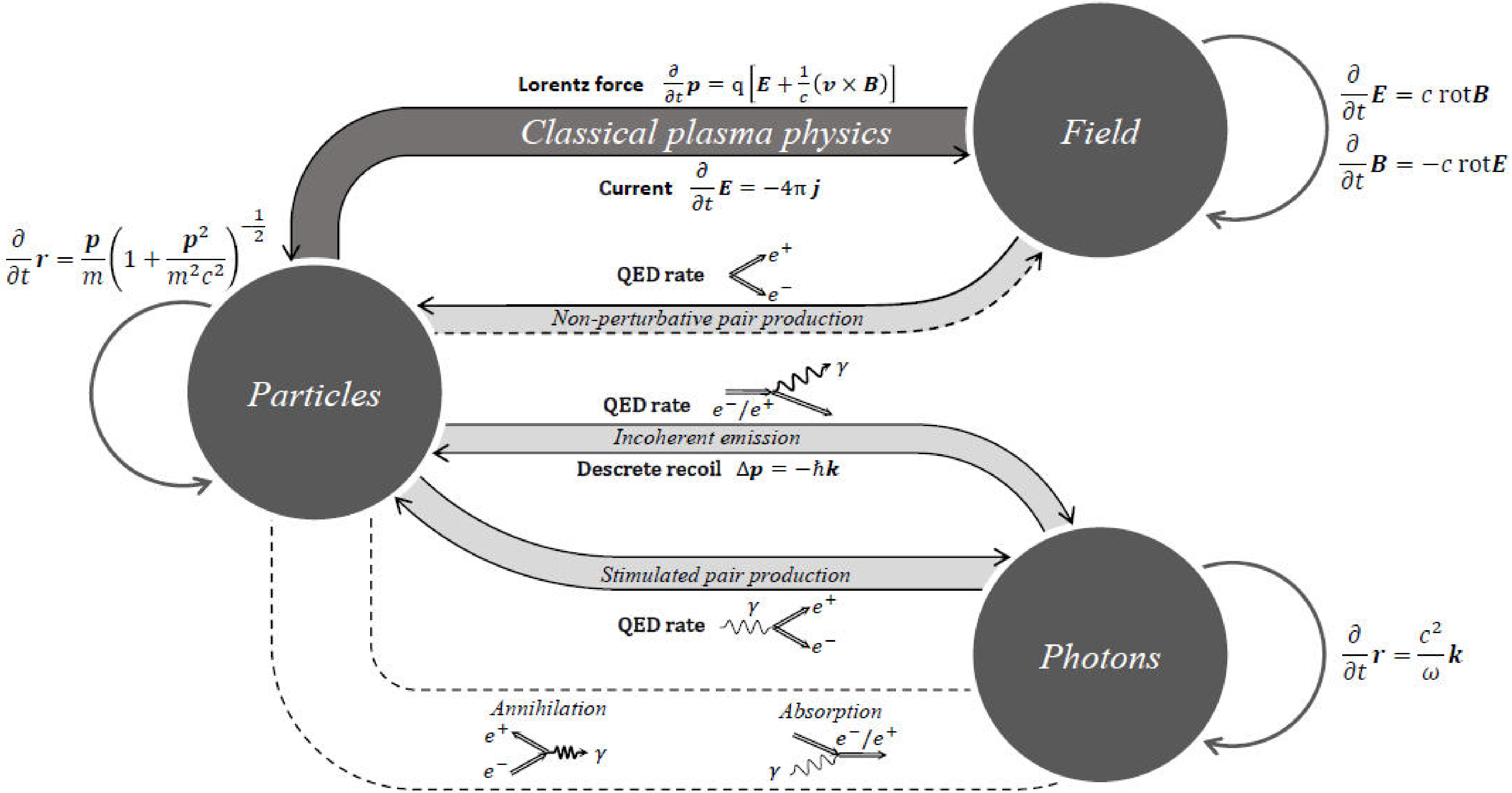
- Radiation-reaction effects prevent the development of a cascade in the collision of an electron/photon beam with a plane wave (Sokolov et al. PRL 2010)

- In (Kirk and Bell, PRL 2008) the first prediction of a cascade production was indicated if even a single electron is present in the focus of a standing wave with intensity larger than 10^{24} W/cm^2
- Idea: one of the laser beams acts as an accelerator for the electron that becomes ultrarelativistic and collides with the other beam



- This effect was exploited in (Fedotov et al., PRL 2010) to show that an intrinsic upper limit should exist for a laser field amplitude given by $\sim\alpha E_{cr}$ corresponding to an intensity $\sim 10^{25}$ W/cm^2
- This conclusion was questioned in (Bulanov et al., PRL 2010), where no upper limit is envisaged in the case of linear polarization, due to the reduced electromagnetic emission

- Implemented approach in most PIC codes like EPOCH and OSIRIS (see, e.g., Gonoskov et al. PRE 2015):



- Only the two basic processes (nonlinear Compton scattering and nonlinear Breit-Wheeler pair production) are implemented
- Some codes like OSIRIS also implement spontaneous pair production and vacuum-polarization effects

Conclusions Part II

- The Furry picture is a very powerful tool to investigate processes occurring in the presence of intense background fields, as it allows to take the field into account exactly from the beginning
- The Dirac equation in the corresponding background field has to be solvable analytically
- The quantum vacuum is an interesting and fascinating object to investigate
- Prolific production of electron-positron pairs can be achieved by colliding super intense electromagentic pulses in vacuum