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Strong-field classical and quantum electrodynamics in intense laser fields
Part I: Radiation reaction

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Outline

- Introduction on classical electrodynamics (CED) and quantum electrodynamics (QED)
- Nonlinear Thomson and Compton scattering
- Radiation reaction in CED and in QED
- Conclusions

Contact

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  http://www.mpi-hd.mpg.de/keitel/dipiazza/

Suggested material

- For more information on the subjects of my lecture see the reviews:

1. V. I. Ritus, J. Sov. Laser Res. **6**, 497 (1985)
2. F. Ehlotzky et al., Rep. Prog. Phys. **72**, 046401 (2009)
3. A. Di Piazza et al., Rev. Mod. Phys. 84, 1177 (2012)
4. B. King et al., High Power Laser Sci. Eng. 4, e5 (2016)

and the books:

1. V. B. Berestetskii et al., Quantum Electrodynamics (Elsevier Butterworth-Heinemann, Oxford, 1982)
2. W. Dittrich et al., Effective Lagrangians in Quantum Electrodynamics (Springer, Berlin, 1985)
3. E. S. Fradkin et al., Quantum Electrodynamics with Unstable Vacuum (Springer, Berlin, 1991)
4. V. N. Baier et al., Electromagnetic Processes at High Energies in Oriented Single Crystals (World Scientific, Singapore, 1998)

Electromagnetic interaction

- The electromagnetic interaction is one of the four "fundamental" interactions in Nature. It is the interaction among electric charged particles (e.g., electrons and positrons) and it is mediated by the electromagnetic field (photons, in the "quantum" language)
- Both classically and quantum mechanically it is described theoretically by a Lagrangian/Hamiltonian which depends on two parameters:
  - Electron mass m=9.1×10^{-28} g
  - Electron charge e, with $|e|$=4.8×10^{-10} statcoulomb
- The typical scales of classical electrodynamics (CED) are determined by the parameters m and e and by the speed of light c, whereas those of quantum electrodynamics (QED) are also determined by the (reduced) Planck constant $\hbar$

Typical scales of CED and QED

	CED	QED
Energy	Electron's rest energy: $\varepsilon_0 = mc^2 = 0.5$ MeV	
Momentum	$p_0 = \varepsilon_0 / c = 0.5$ MeV$/c$	
Length	Classical electron's radius: $r_0 = e^2/mc^2 = 2.8\times10^{-13}$ cm (from the Thomson cross section)	Compton wavelength: $\lambda_C = \hbar/p_0 = 3.9\times10^{-11}$ cm (from Heisenberg uncertainty principle)
Time	$r_0/c = 1.0\times10^{-23}$ s	$\lambda_C/c = 1.3\times10^{-21}$ s

$r_0 = \alpha\lambda_C$, where $\alpha = e^2/\hbar c \approx 1/137$ is the fine-structure constant

Field scales of QED (critical or Schwinger field)

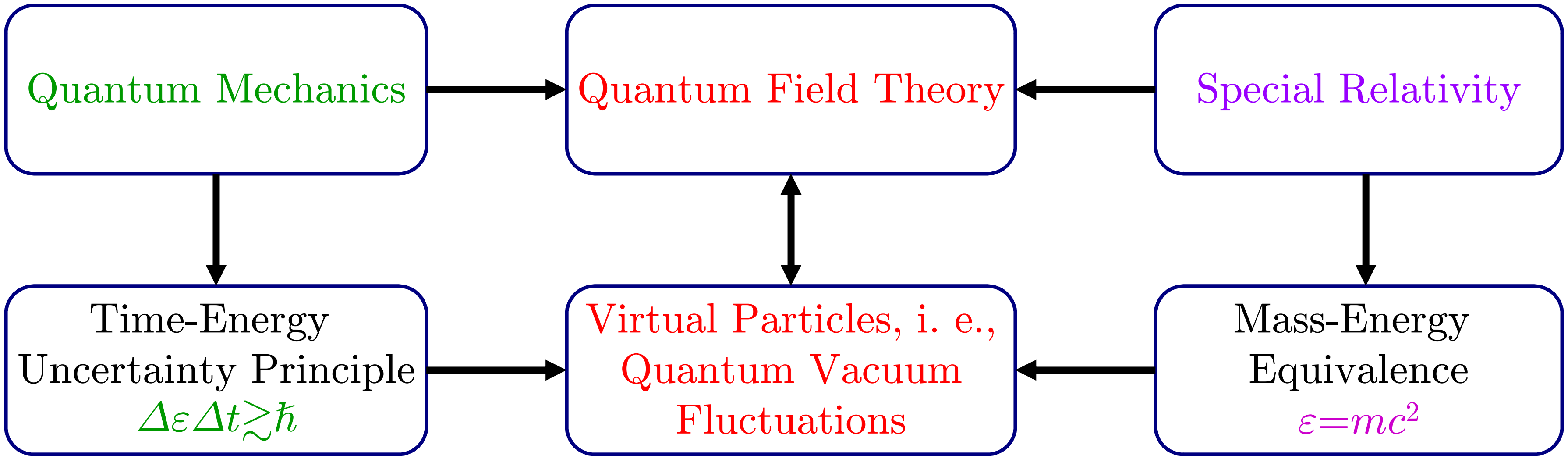
$$E_{cr} = \frac{m^2c^3}{\hbar|e|} = 1.3\times10^{16}\ \mathrm{V/cm}$$

$$B_{cr} = \frac{m^2c^3}{\hbar|e|} = 4.4\times10^{13}\ \mathrm{G}$$

$$\Rightarrow \quad I_{cr} = \frac{cE_{cr}^2}{4\pi} = 4.6\times10^{29}\ \mathrm{W/cm^2}$$

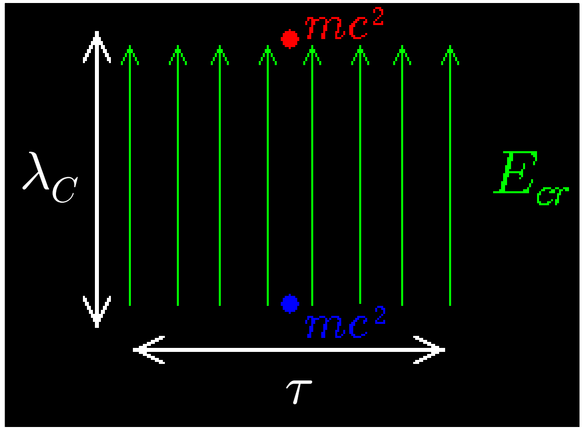
QED critical fields and vacuum physics

- In classical physics the vacuum is a region of space-time where neither particles nor fields are present
- In quantum field theory the vacuum is the lowest-energy state where no real particles (electrons, positrons, photons etc...) are present
  - Virtual particles are present
  - They "live" for a very short time and cover a very short distance (for electrons and positrons $\tau = \hbar/mc^2 \sim 10^{-21}$ s and $\lambda_C = \hbar/mc \sim 10^{-11}$ cm, respectively)



- Physical meaning of the critical fields:

$$|e|E_{cr} \times \frac{\hbar}{mc} = mc^2$$



- Vacuum instability and electromagnetic cascades (Bell et al., PRL 2008, Bulanov et al., PRL 2010, Fedotov et al., PRL 2010)
- The interaction energy of a Bohr magneton with a magnetic field of the order of B_{cr} is of the order of the electron rest energy
- In the presence of background electromagnetic fields of the order of the critical ones a new regime of QED, the strong-field QED regime, opens:

1. where the properties of the vacuum are substantially altered by the fields
2. where a tight interplay unavoidably exists between collective (plasma-like) and quantum effects
3. which is inaccessible to conventional accelerators because it requires coherent fields

- QED in vacuum is considered to be the most successful physical theory in terms of agreement with experiments
- Experiments on QED in the presence of intense background electromagnetic fields
  - ✓ aim at testing the theory in a sector complementary to the conventional high-energy/short-distance sector explored by means of accelerator facilities (interaction among many particles at the same time)
  - ✓ are not comparably numerous and accurate as those in vacuum
- The reasons are:
  1. on the experimental side, the critical electromagnetic field of QED is very "large"
  2. on the theoretical side, exact analytical calculations are feasible only for a few background electromagnetic fields: constant and uniform electric/magnetic field, Coulomb field, plane-wave field

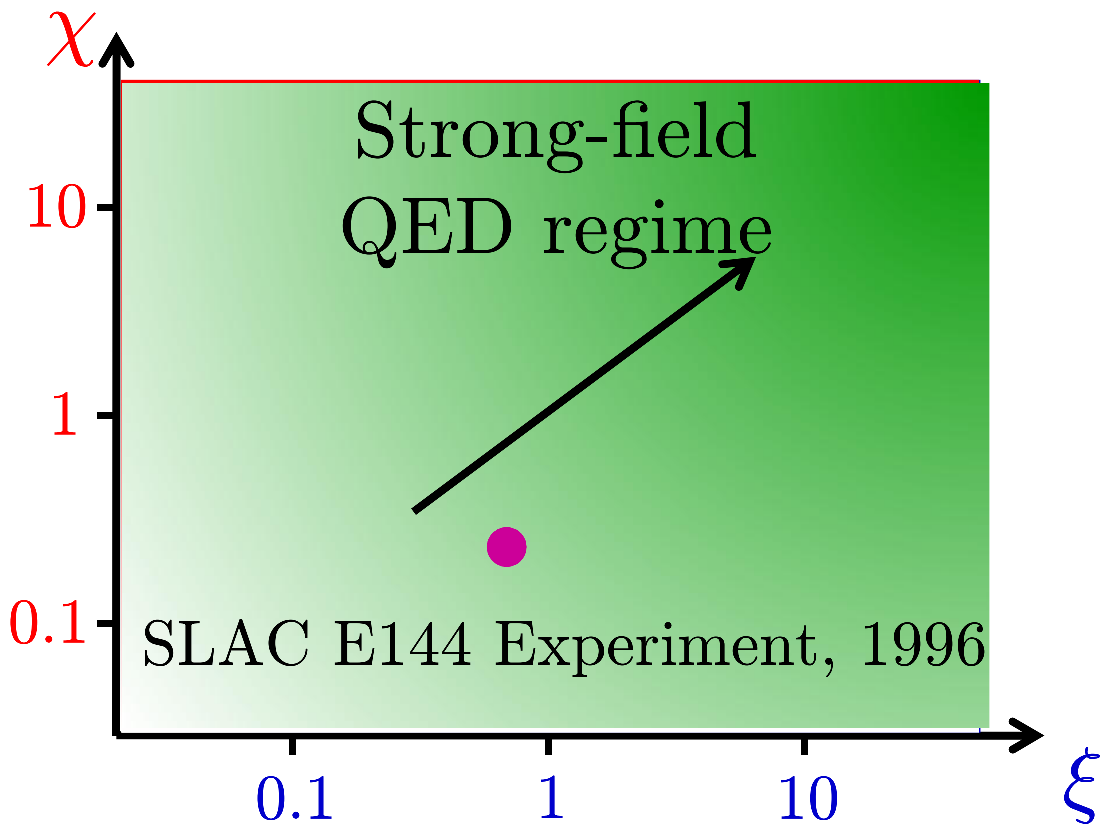
Regimes of QED in a strong laser field

A particle (e^-, e^+ or γ) with energy $\mathcal{E}$ for an electron or a positron ($\hbar\omega$ for a photon) collides head on with a plane wave with amplitude E_L and angular frequency ω_L (wavelength λ_L)

Relevant Lorentz- and gauge-invariant parameters (Ritus 1985):

$$\xi = \frac{1}{2\pi}\frac{|e|E_L\lambda_L}{mc^2} = \frac{|e|E_L\lambda_C}{\hbar\omega_L}$$

$$\chi = 2\frac{\hbar\omega}{mc^2}\frac{E_L}{E_{cr}} = \left.\frac{E_L}{E_{cr}}\right|_{\text{r.f.}}$$



Optical laser technology

Optical laser technology ($\hbar\omega_L$=1 eV, λ_L=1 μm)	Energy (J)	Pulse duration (fs)	Spot radius (μm)	Intensity (W/cm^2)
State-of-art (Yanovsky et al., Opt. Express 2008)	10	30	1	2×10^{22}
Soon (APOLLON, ELI-NP, ELI Beamlines etc...)	10÷100	10÷100	1	$10^{22}\div10^{23}$
Near future (ELI 4th pillar, XCELS)	10^4	10	1	$10^{25}\div10^{26}$

Electron accelerator technology

Electron accelerator technology	Energy (GeV)	Beam duration (fs)	Spot radius (μm)	Number of electrons
Conventional accelerators (PDG)	10÷50	$10^3\div10^4$	10÷100	$10^{10}\div10^{11}$
Laser-plasma accelerators (Leemans et al., Phys. Rev. Lett. 2014)	4.2	40	50	8×10^8

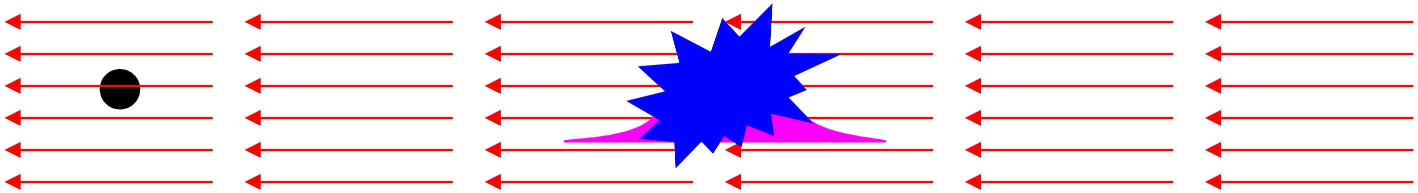
$$\xi = 6.0\sqrt{I_L[10^{20}\ \mathrm{W/cm^2}]}\lambda_L[\mu\mathrm{m}]$$

$$\chi = 5.9\times10^{-2}\epsilon[\mathrm{GeV}]\sqrt{I_L[10^{20}\ \mathrm{W/cm^2}]}$$

Present technology allows in principle the experimental investigation of strong-field QED

Nonlinear Thomson and Compton scattering

- Nonlinear Thomson and Compton scattering are among the most fundamental processes in electrodynamics

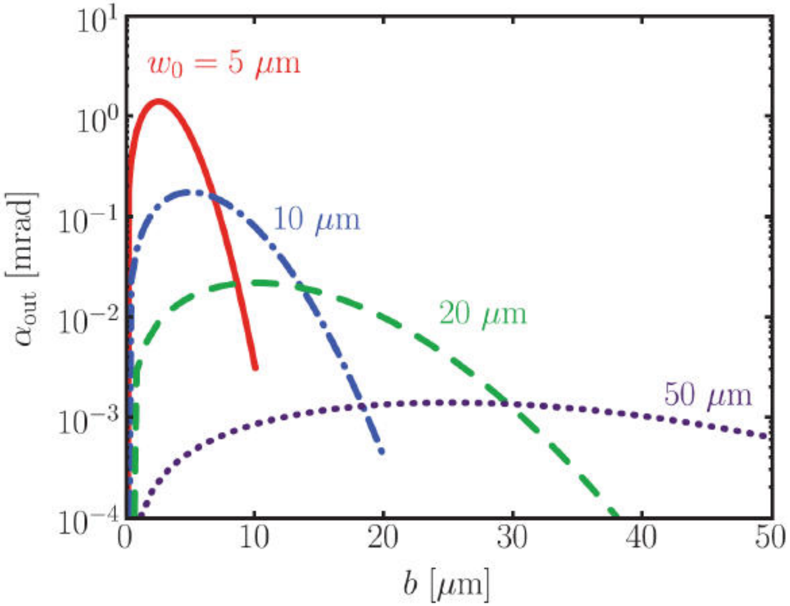


1. Classically: the electron is accelerated by the laser field and emits electromagnetic radiation (nonlinear Thomson scattering)
2. Quantum mechanically: the electron exchanges many photons with the laser field and emits a high-energy photon (nonlinear Compton scattering)

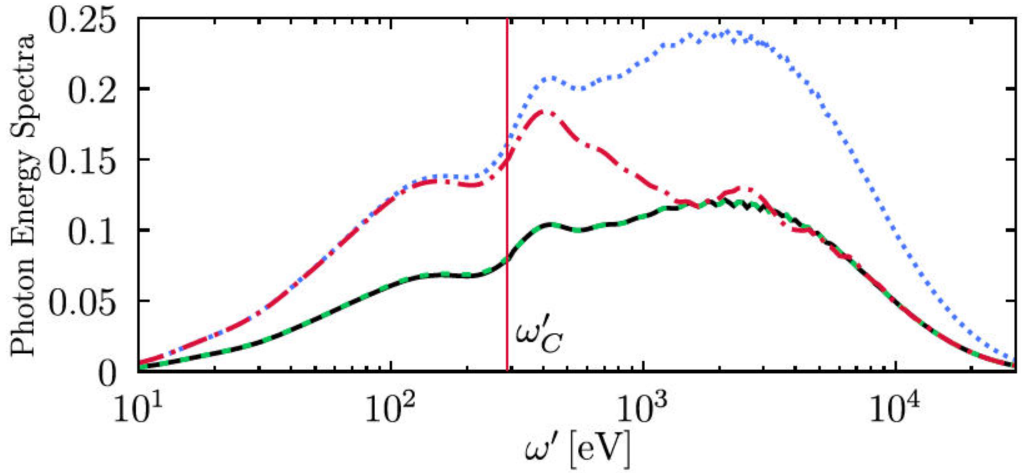
- The main quantum effect is the photon recoil: the energy-momentum carried away by the photon from the electron. This energy is typically χ times the electron energy $\mathcal{E}$ at $\chi \lesssim 1$ and the quantum photon-energy spectrum reduces to the classical one at $\chi \ll 1$
- For a monochromatic plane-wave: Brown et al. 1964, Nikishov et al. 1964, Narozhny et al. 1965

- Results in the monochromatic case have confirmed some classical prediction:
  1. The electron moves inside the linearly polarized laser field as having an effective (square) "dressed" mass
  $$m^{*2} = m^2\left(1+\frac{\xi^2}{2}\right)$$
  2. At large values of ξ, about ξ^3 laser photons are absorbed by the electron from the laser field (nonlinearity orders of about 10^6 at a laser intensity of 10^{22} W/cm^2)
  3. In the ultra-relativistic regime, the electron mainly emits along its velocity within a cone of aperture $\sim 1/\gamma$ (Landau and Lifshitz 1947)
- In the so-far unique experiment on multiphoton Compton scattering performed at SLAC (Bula et al. 1996) it was ξ=0.6 and only the mass-dressing effect was observed as the position of the emission lines depended on the laser intensity
- Classically the dependence of the emission lines on the laser intensity is a consequence of the Doppler effect and the mass dressing is due to the electron's quivering motion in the wave

- Recent investigations on nonlinear Thomson and Compton scattering focus on alterations induced in the photon spectra by the finite extension of the laser pulse (Boca et al. 2009, Heinzl et al., 2010). Main effects: emission line broadening and appearance of subpeaks



- Nonlinear Compton scattering by many electrons: coherence effects in strong-field QED
- Quantum recoil effects can limit the emission of coherent radiation if the recoil is comparable with the width of the electrons' wave packets in momentum space (Antioi and Di Piazza, Phys. Rev. Lett., in press)



Radiation reaction in CED

- What is the equation of motion of an electron in an external, given electromagnetic field $F^{\mu\nu}(x)$?
- The Lorentz equation

Units with $\hbar = c = 1$

$$m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu$$

does not take into account that while being accelerated the electron generates an electromagnetic radiation field and it loses energy and momentum

- One has to solve self consistently the coupled Lorentz and Maxwell equations (Barut 1980)

$$m_0\frac{du^\mu}{ds} = eF_T^{\mu\nu}u_\nu$$
$$\partial_\mu F_T^{\mu\nu} = e\int ds\delta(x - x(s))u^\nu$$

Lorenz gauge →

$$m_0\frac{du^\mu}{ds} = e(\partial^\mu A_T^\nu - \partial^\nu A_T^\mu)u_\nu$$
$$\Box A_T^\nu = e\int ds\delta(x - x(x))u^\nu$$

where now m_0 is the electron's bare mass and $F_{T,\mu\nu}=\partial_\mu A_{T,\nu}-\partial_\nu A_{T,\mu}$ is the total electromagnetic field (external field plus the one generated by the electron)

- One first solves the inhomogeneous wave equation exactly with the Green's-function method

$$\Box A_T^{\nu} = e \int ds\delta(x - x(x))u^{\nu} = j^{\nu}(x) \longrightarrow A_T^{\nu}(x) = A^{\nu}(x) + \int dx'\mathcal{D}(x - x')j^{\nu}(x')$$

and then re-substitute the solution into the Lorentz equation:

$$(m_0 + \delta m)\frac{du^{\mu}}{ds} = eF^{\mu\nu}u_{\nu} + \frac{2}{3}e^2\left(\frac{d^2u^{\mu}}{ds^2} + \frac{du^{\nu}}{ds}\frac{du_{\nu}}{ds}u^{\mu}\right)$$

where, for a uniform sphere with radius a in the initial rest frame, it is $\delta m = e^2/2a$ which diverges in the limit $a \to 0$

- After "mass renormalization" one obtains the Lorentz-Abraham-Dirac (LAD) equation

$$m\frac{du^{\mu}}{ds} = eF^{\mu\nu}u_{\nu} + \frac{2}{3}e^2\left(\frac{d^2u^{\mu}}{ds^2} + \frac{du^{\nu}}{ds}\frac{du_{\nu}}{ds}u^{\mu}\right)$$

- The second term in the RR force is the Larmor term and it directly accounts for the energy-momentum loss
- The first term in the RR force is the so-called Schott term and is responsible of all inconsistencies of the LAD equation (it must be there for the on-shell condition $u^2=1$ to be fulfilled at all times)

- The LAD equation is plagued by serious inconsistencies: runaway solutions. Consider its non-relativistic limit

$$m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2}{3}e^2\left(\frac{d^2u^\mu}{ds^2} + \frac{du^\nu}{ds}\frac{du_\nu}{ds}u^\mu\right) \longrightarrow \quad m\frac{d\boldsymbol{v}}{dt} = e(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}) + \frac{2}{3}e^2\frac{d^2\boldsymbol{v}}{dt^2}$$

- In the free case E=B=0, it admits the solution $a(t)$=a_0e^{t/τ_0}, where τ_0=$(2/3)e^2/m$. Note: the solution is non-perturbative in e.
- Avoiding the runaways: integro-differential LAD equation (Rohrlich 1961)

$$m\frac{du^\mu}{ds} = \frac{e^{s/\tau_0}}{\tau_0}\int_s^\infty ds' e^{-s'/\tau_0}\left[eF^{\mu\nu}u_\nu + \frac{2}{3}e^2\frac{du^\nu}{ds'}\frac{du_\nu}{ds'}u^\mu\right]$$

- Problem: preacceleration at time scales of the order of τ_0
- If $\chi \ll 1/\alpha \sim 10^2$ (always true in CED, as $\chi \ll 1$ is required to neglect QED effects), a reduction of order can be applied to transforms the LAD equation into the so-called Landau-Lifshitz equation (Landau and Lifshitz 1947), which is safe from inconsistencies

$$m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2}{3}e^2\left[\frac{e}{m}(\partial_\alpha F^{\mu\nu})u^\alpha u_\nu - \frac{e^2}{m^2}F^{\mu\nu}F_{\alpha\nu}u^\alpha + \frac{e^2}{m^2}(F^{\alpha\nu}u_\nu)(F_{\alpha\lambda}u^\lambda)u^\mu\right]$$

- Recent experiments on classical (quantum?) radiation reaction in laser fields: Cole et al., PRX 2018, Poder et al., PRX, in press

- The LAD equation is "too exact" (but in a wrong way):

$$m\frac{du^\mu}{ds} = a_1^\mu e + a_2^\mu e^2 + a_3^\mu e^3 + a_4^\mu e^4 + \cdots$$

- In the LAD equation the series of classical terms in e is "summed" exactly (essential non-perturbative effects in e are predicted) but lower-order quantum terms are much larger than higher-order classical terms
- In the ultrarelativistic case radiation-reaction effects
    1. are mainly due to the "Larmor" damping term
    2. scale with the parameter $R_C\Phi$, where Φ is the total phase of the laser pulse and $R_C=\alpha\xi\chi$
- The condition $R_C \approx 1$ means that the energy emitted by the electron in one laser period is of the order of the initial energy (classical radiation dominated regime) (Koga et al. 2005)

Radiation reaction in QED

- We introduced the problem of radiation reaction in CED by saying that the Lorentz equation has to be modified as it does account for the energy-momentum loss of the accelerating and then emitting electron
- Thus one could be tempted to say that radiation reaction is automatically taken into account in QED already in the "basic" emission process (nonlinear Compton scattering, Ritus 1985)

  because photon recoil, i.e., the energy-momentum subtracted by the photon to the electron is automatically included
- However, this cannot be the case because

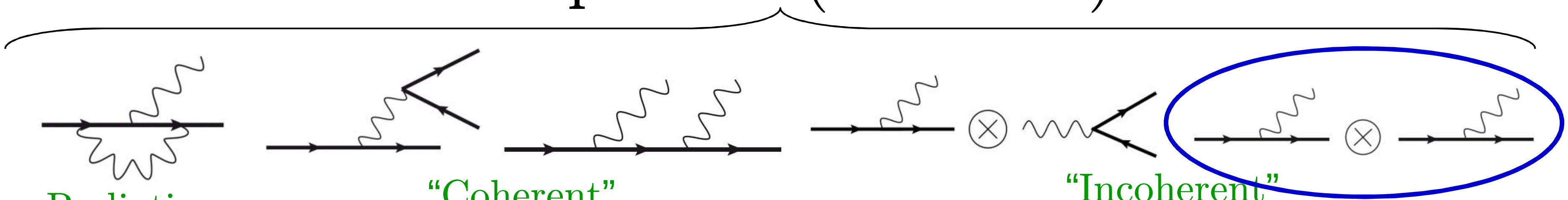
1. in the classical limit $\chi \ll 1$, the spectrum of nonlinear Compton scattering goes into the classical spectrum calculated via the Lorentz equation, i.e., without radiation reaction
2. the photon recoil $\hbar\omega$ is proportional to $\hbar$ and it does not have a classical analogue
3. radiation reaction would always be a small correction classically, which is not the case in the classical radiation dominated regime

- To determine the dynamics of the electron via the Lorentz-Abraham-Dirac equation amounts to solve self-consistently Maxwell and Lorentz equations

$$m_0\frac{du^\mu}{ds} = eF_T^{\mu\nu}u_\nu \qquad \partial_\mu F_T^{\mu\nu} = e\int ds\delta(x-x(s))u^\nu \quad \longleftrightarrow \quad m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2}{3}e^2\left(\frac{d^2u^\mu}{ds^2}+\frac{du^\nu}{ds}\frac{du_\nu}{ds}u^\mu\right)$$

- This corresponds in QED to determine the evolution of a single-electron state in background field+radiation field generated by the electron

$$|t=-\infty\rangle = |e^-\rangle \longrightarrow \text{Complete evolution operator } (S\text{-matrix}) \longrightarrow |t=+\infty\rangle$$



Radiative corrections

"Coherent" high-order processes

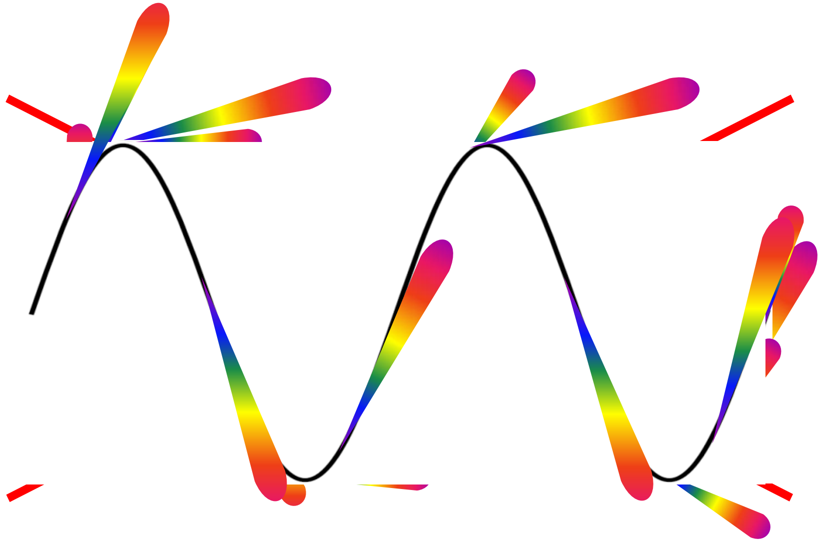
"Incoherent" high-order processes

- In QED radiation reaction includes all possible QED processes (relation with the QED cascades)
- At $\xi \gg 1$ and $\chi \sim 1$ the multiple incoherent emission gives the main contribution (Di Piazza et al. 2010), which starts playing a role if the total probability $P_1 \sim \alpha\xi\Phi$ of emitting one photon in a laser pulse exceeds unity and it has to be interpreted as the average number of photons emitted

- Quantum analogous of each term in the LAD equation

LAD equation	$m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2}{3}e^2\left(\frac{d^2u^\mu}{ds^2} + \frac{du^\nu}{ds}\frac{du_\nu}{ds}u^\mu\right)$

- The Larmor term corresponds to the cascade emission of many photons (Elkina et al. 2011)
- No multiple coherent emission
- Classical limit: the electron emits a large number of photons ($N \sim \alpha\xi\Phi \to \infty$) but all with a small recoil ($\omega \sim \chi\mathcal{E}_0 \to 0$), such that the average energy emitted ($N\omega \sim (\alpha\xi\Phi) \times (\chi\mathcal{E}_0) = R_C\mathcal{E}_0\Phi$) is finite



- Quantum radiation dominated regime (Di Piazza et al. 2010): multiple photon emission already in one laser period ($P_1 \sim \alpha\xi \gtrsim 1$) with a large recoil ($\chi \sim 1$)
- The Schott term is related to the effects of the near field (radiative corrections)

When does radiation reaction become important?

	CED	QED
Physical condition	When the total energy emitted is of the same order of the initial electron energy	When the total probability $P_1 \sim \alpha\xi\Phi$ of emitting one photon exceeds unity and has to be interpreted as the average number of photons emitted (it indicates that incoherent multiphoton emission occurs)
Mathematical condition	$\alpha\chi\xi\Phi \gtrsim 1$	$\alpha\xi\Phi \gtrsim 1$

Classical and quantum radiation dominated regime

	CED	QED
Radiation reaction parameter	$R_C = \alpha\chi\xi$	$R_Q = \alpha\xi$
Physical meaning	Energy emitted in one laser period in units of the initial electron energy	Average number of photons emitted incoherently in one laser period
Radiation dominated regime	$\chi \ll 1$ and $R_C = \alpha\chi\xi \gtrsim 1$	$\chi \sim 1$ and $R_Q = \alpha\xi \gtrsim 1$

Conclusions Part I

- Present and next-generation lasers can offer a unique possibility of accessing new extreme regimes of interaction, where the effective strength of the electromagnetic fields becomes close to the critical fields of QED
- In these regimes new effects are predicted to occur by classical and quantum electrodynamics and, in particular, the electron dynamics is strongly dominated by radiation-reaction and quantum effects
- Strong laser facilities can help clarifying the fundamental and still unsolved problem of radiation reaction and its quantum origin