

# X-ray Diagnostics of Warm Dense Matter

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Picture by courtesy of DLR Berlin



Bundesministerium  
für Bildung  
und Forschung

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Forschungsgemeinschaft



**INSTITUT FÜR PHYSIK**



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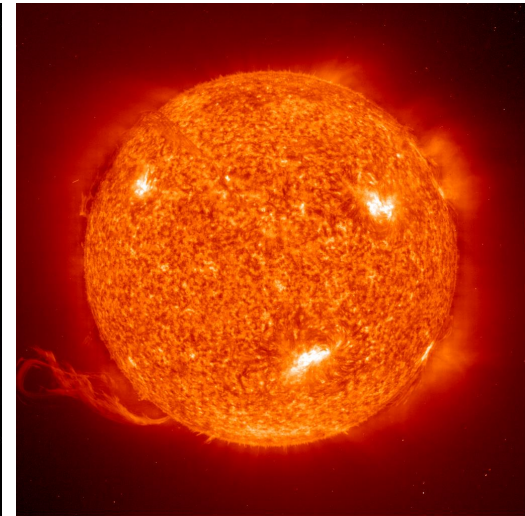
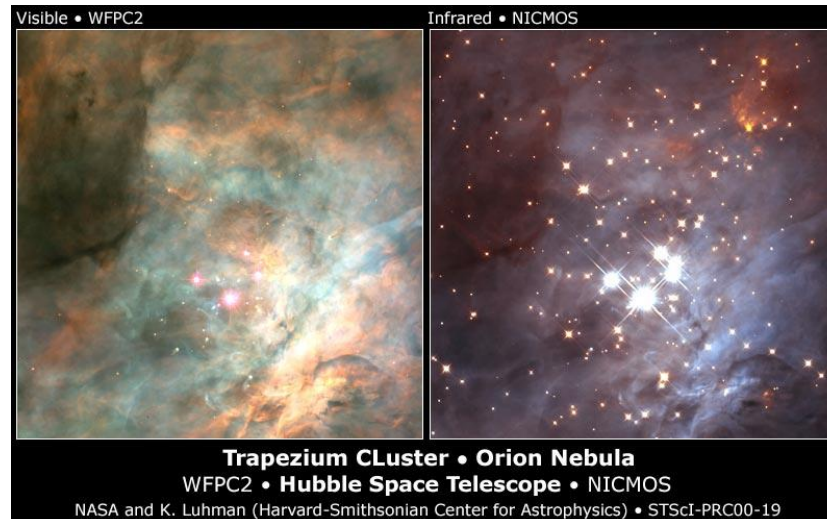
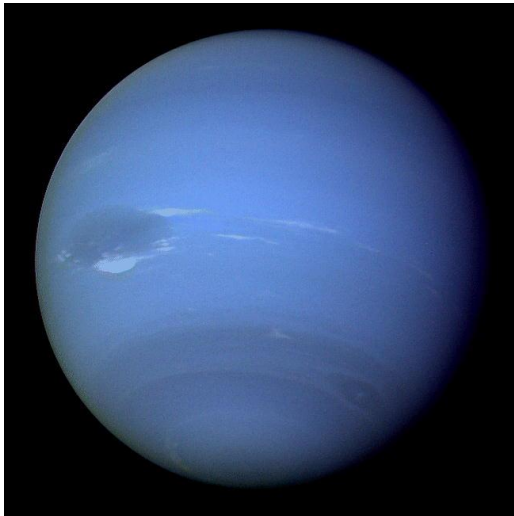
# Astrophysical objects

## Extreme states of matter

Giant Planets  $M < 13M_J$

Brown Dwarfs  $13M_J < M < 75M_J$

Stars  $M > 75M_J$



J, S, U, N and many  
extrasolar planets

... in the Orion Nebula  
seen in the IR spectrum

The Sun

Core (e.g. Jupiter):  
 $\approx 2 \times 10^4$  K & 40 Mbar  
**warm dense matter**

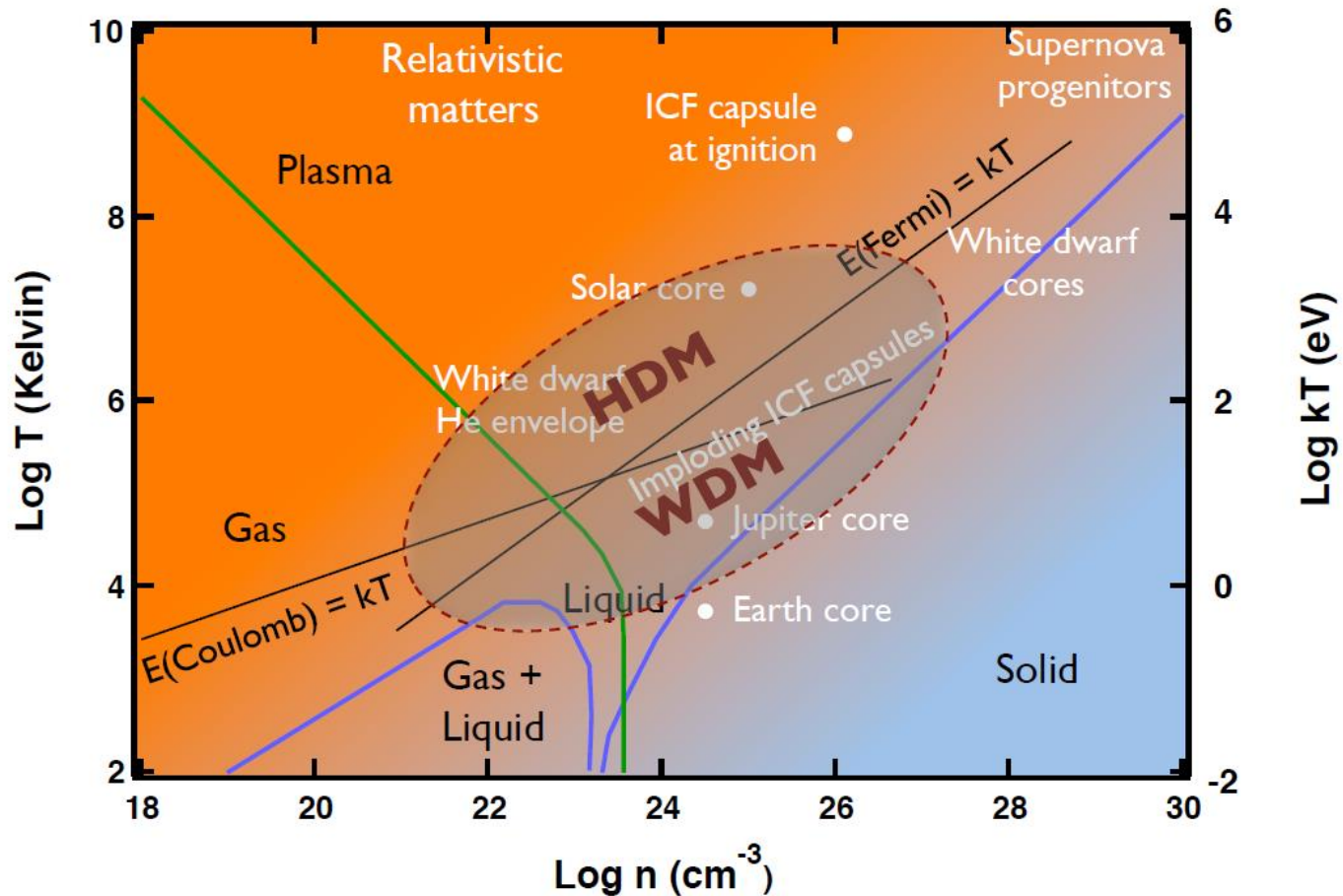
Cores (e.g. Gliese 229B):  
 $\approx 10^6$  K & 100 Gbar  
**degenerate matter**

Core of the Sun:  
 $\approx 15 \times 10^6$  K & 250 Gbar  
**hot dense matter**

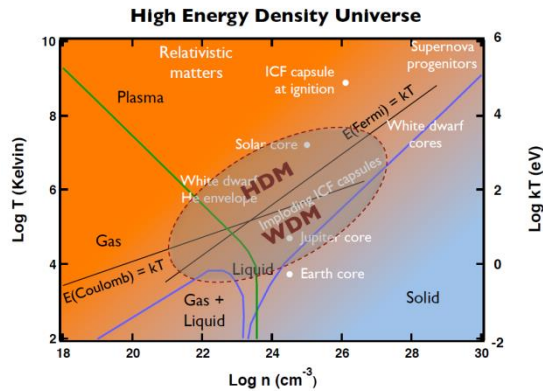
# Warm Dense Matter (WDM)

See: Basic Research Needs for High Energy Density Laboratory Physics (DOE Office of Science and NNSA, 2010)

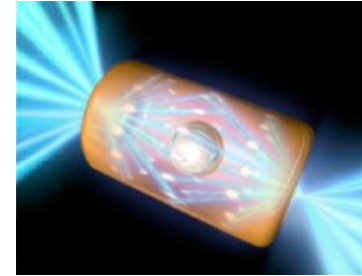
## High Energy Density Universe



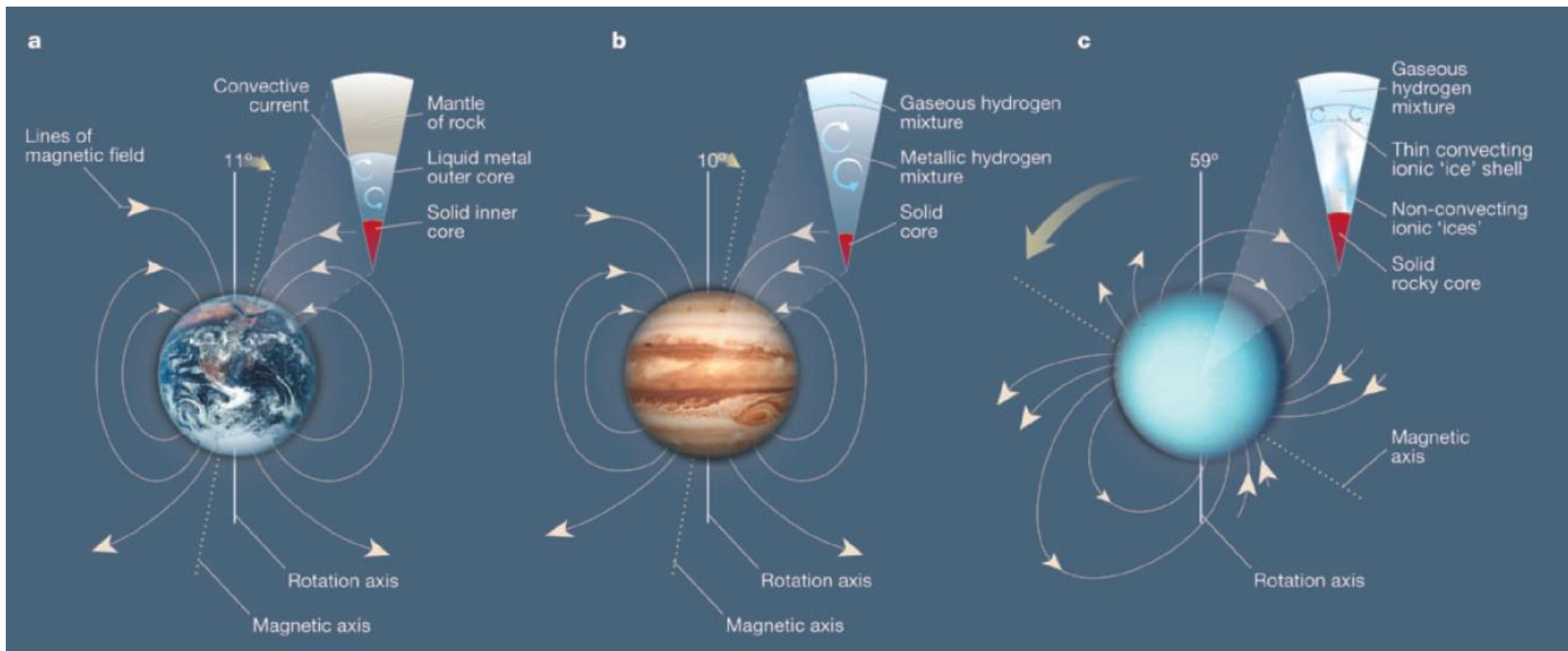
# Warm Dense Matter (WDM)



## Inertial Confinement Fusion (courtesy NIF)



$T > 10^8 \text{ K}$   
 $P > 100 \text{ Gbar}$

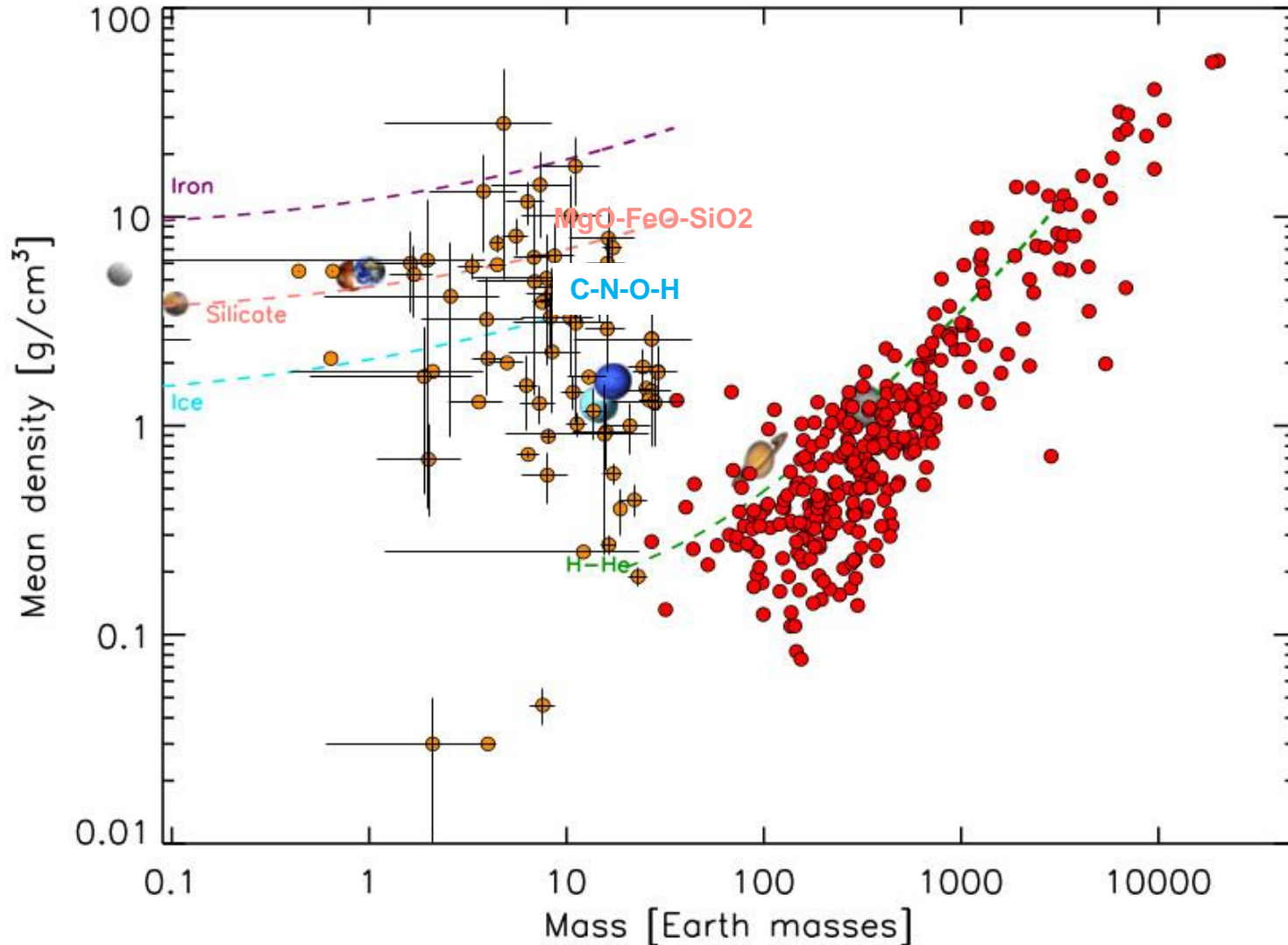


Interior and magnetic field of Solar System planets, see J. Aurnou, Nature 428, 134 (2004)

# Extrasolar transiting planets

## Mean density vs. mass

H. Rauer et al., *Exp. Astron.* **38**, 249 (2014)





# Basic equations for planetary modeling

mass conservation:

$$dm = 4\pi r^2 \rho(r) dr$$

hydrostatic equation of motion:

$$\frac{1}{\rho} \frac{dP}{dr} = \frac{dU}{dr}, \quad U = V + Q$$

gravitational potential:

$$V(\vec{r}) = -G \int_{V_0} d^3 r' \frac{\rho(r')}{|\vec{r} - \vec{r}'|}$$

expansion into Legendre polynomials:

$$V(r, \theta) = -\frac{GM}{r(\theta)} \left( 1 - \sum_{i=1}^{\infty} \left( \frac{R_{eq}}{r(\theta)} \right)^{2i} J_{2i} P_{2i}(\cos \theta) \right)$$

gravitational moments:

$$J_{2i} = -\frac{1}{MR_{eq}^{2i}} \int d^3 r' \rho(r'(\theta')) r'^{2i} P_{2i}(\cos \theta')$$

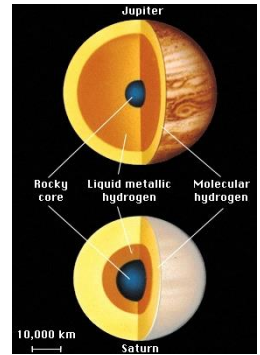
Calculations via theory of figures (Zharkov & Trubitsyn) with boundary conditions  $M_p(R_p)$ ,  $Y_1$ ,  $\bar{Y}$ ,  $P$  and  $T$  at 1 bar.

**Mass distribution along (piecewise) isentropes/isotherms according to EOS data for WDM – most important input!**



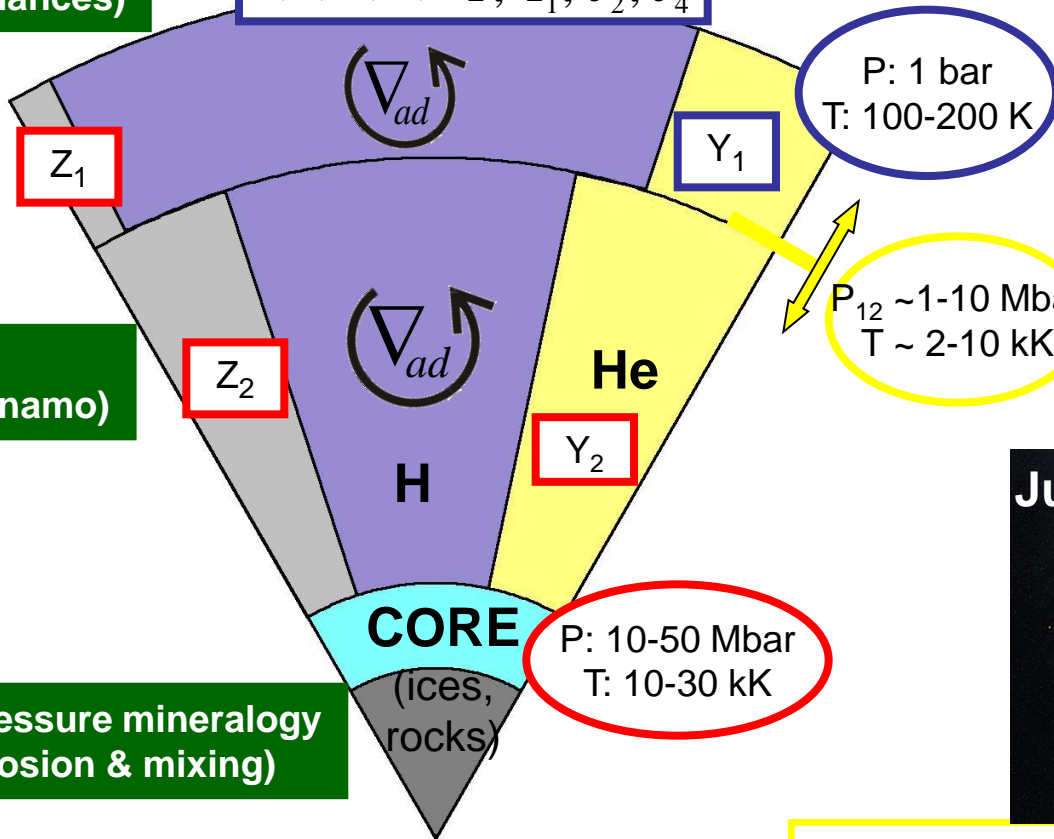
# Interior of Gas Giants: H-He

Three-layer model, input and constraints



Atmosphere models  
(luminosity, abundances)

$L, T, M, R, \bar{Y}, Y_1, J_2, J_4$



Physical origin and location of the layer boundary:  
 → MIT (PPT)  
 → H-He demixing

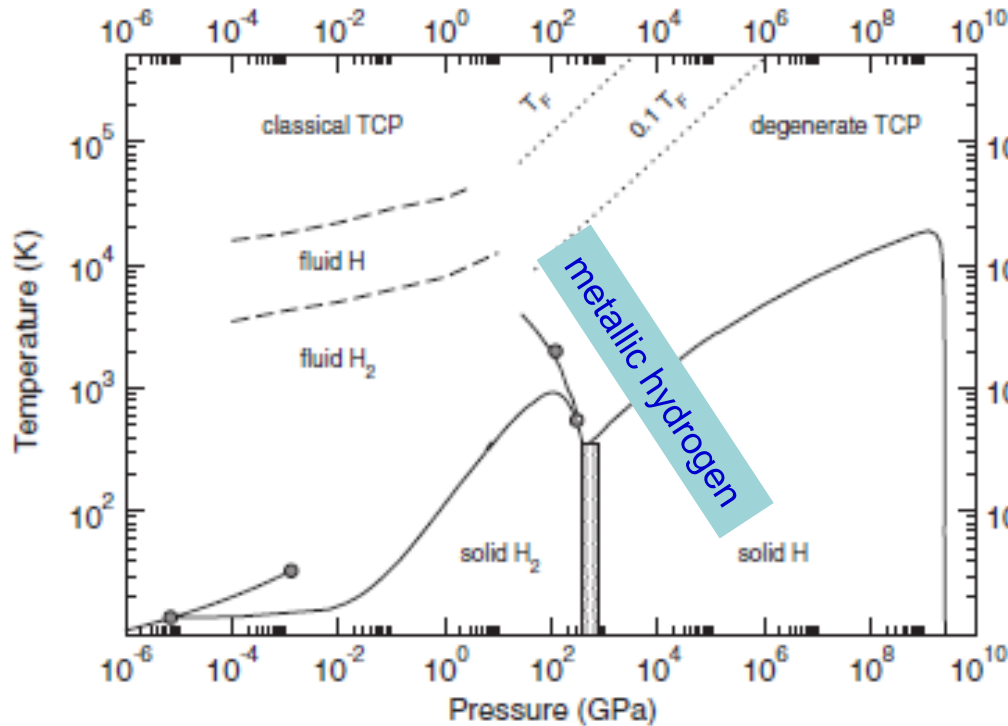


**Matter under extreme conditions (WDM):**

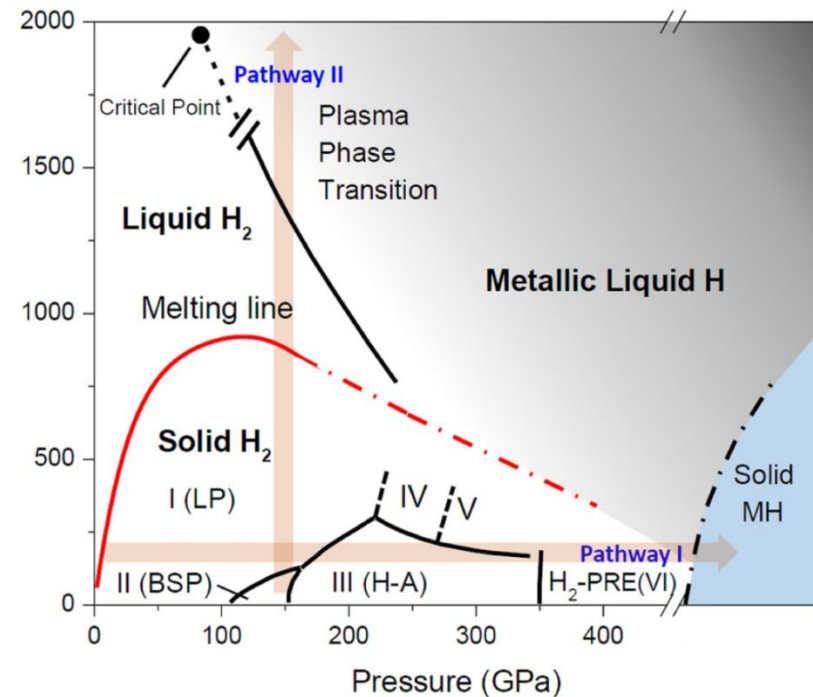
- High-pressure H-He phase diagram
- EOS of complex mixtures
- Electrical & thermal conductivity
- Diffusion & viscosity

# High-pressure phase diagram of H

Jupiter, Saturn, hot Jupiters  
Metalization & Plasma Phase Transition  
H-He demixing



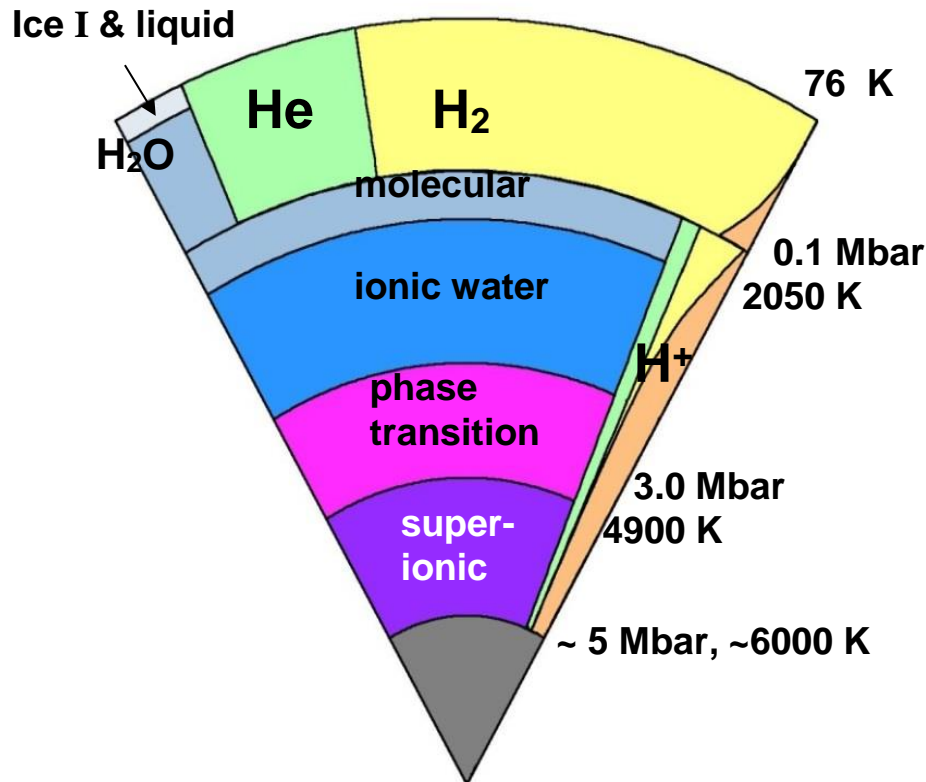
J.M. McMahon et al., RMP **84**, 1607 (2012)



R.P. Dias, I.F. Silvera, Science (2017)  
For recent DAC studies, see also:  
P. Dalladay-Simpson et al., Nature (2016)  
R.S. McWilliams et al., PRL (2016)

# Interior of Ice Giants: C-N-O-H mixtures

## Multi-layer models

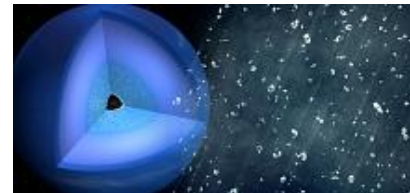


U & N

Neptune-like exos  
mini-Neptunes

Physical origin and location of layer boundaries:

- ice phase diagram
- superionic phase?
- carbon rain?
- solubility of rock material?
- inhomogeneous zone from formation: thermal boundary layer?



D. Kraus et al.,  
Nat. Astron.  
1, 606 (2017)

Interior structure models of this type are not uniquely defined.

**Accurate EOS data for warm dense C-N-O-H-He mixtures are needed and information on the high-P phase diagram.**

# High-pressure phase diagram of H<sub>2</sub>O

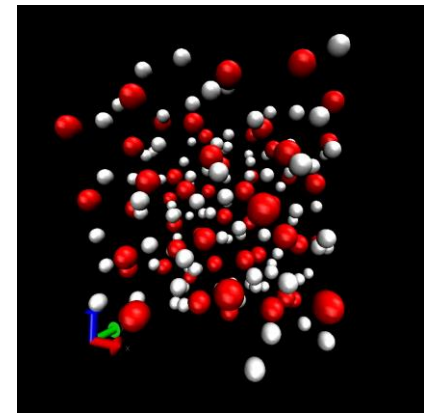
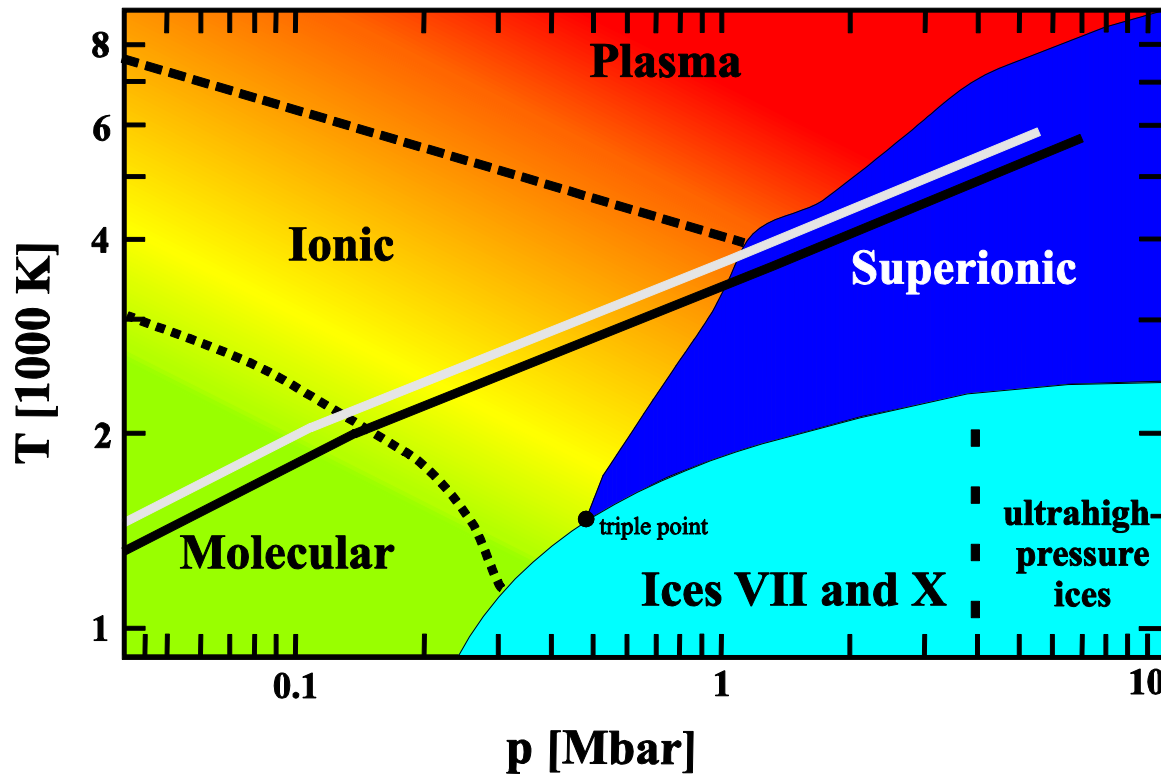
H<sub>2</sub>O: see RR et al., *Icarus* **211**, 798 (2011)

NH<sub>3</sub>: M. Bethkenhagen, M. French, RR, *JCP* **138**, 234504 (2013)

NH<sub>3</sub>-H<sub>2</sub>O: M. Bethkenhagen et al. *JPCA* **119**, 10582 (2015)

C-N-O-H: M. Bethkenhagen et al. *ApJ* **848**, 67 (2017)

Uranus (white), Neptune (black), ice giants, mini-Neptunes



C. Cavazzoni et al.,  
*Science* **283**, 44 (1999)  
T.R. Mattsson, M.P. Desjarlais,  
*PRL* **97**, 017801 (2006)  
E. Schwegler et al.,  
*PNAS* **105**, 14779 (2008)  
H.F. Wilson et al.,  
*PRL* **110**, 151102 (2013)

EOS and phase diagram:

M. French et al., *PRB* **79**, 054107 (2009), *PRE* **93**, 022140 (2016)

Transport properties (diffusion, conductivity):

M. French et al., *PRB* **82**, 174108 (2010), *PoP* **24**, 092306 (2017)



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# Diamond Anvil Cells (DACs)

Conventional DAC technique is limited to static pressures of few Mbar and low T using resistive or pulsed laser heating.

Dynamic dDAC (for molecular solids) > 2 Mbar

**Evans et al. 2007**

Double-stage dsDAC – potential to reach 10 Mbar

**Dubrovinsky et al. 2012: Re >6 Mbar**

**Dubrovinsky et al. 2015: Os ~8 Mbar**

X-ray diagnostics at 3rd generation synchrotrons:

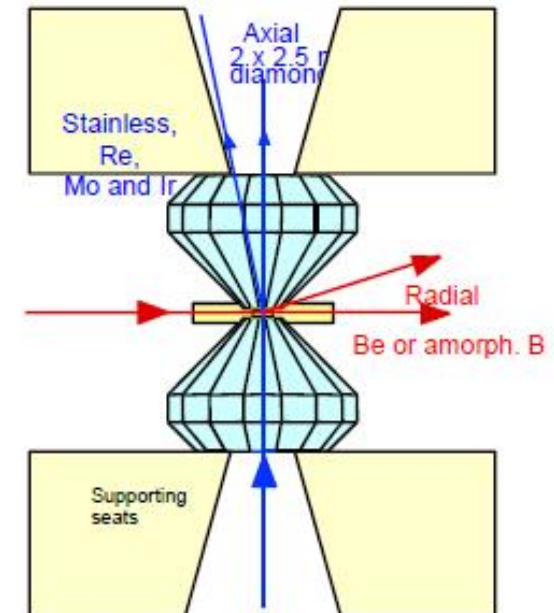
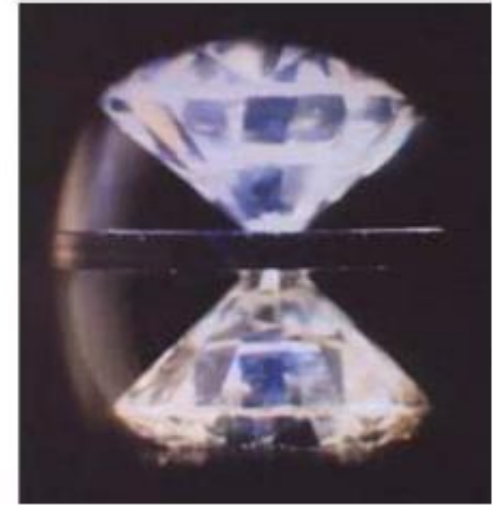
- ESRF, ECB@PETRA III, APS, Diamond ...
- Structure, phase transitions, EOS, reflectivity ...

Laser-driven shocks: NIF, Omega, Nike ... Orion, Vulcan, LMJ, LULI, PeTAL, Phelix ...

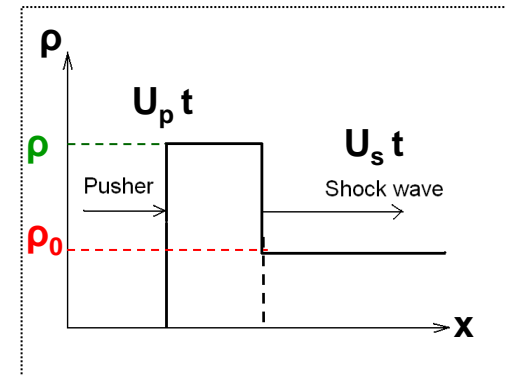
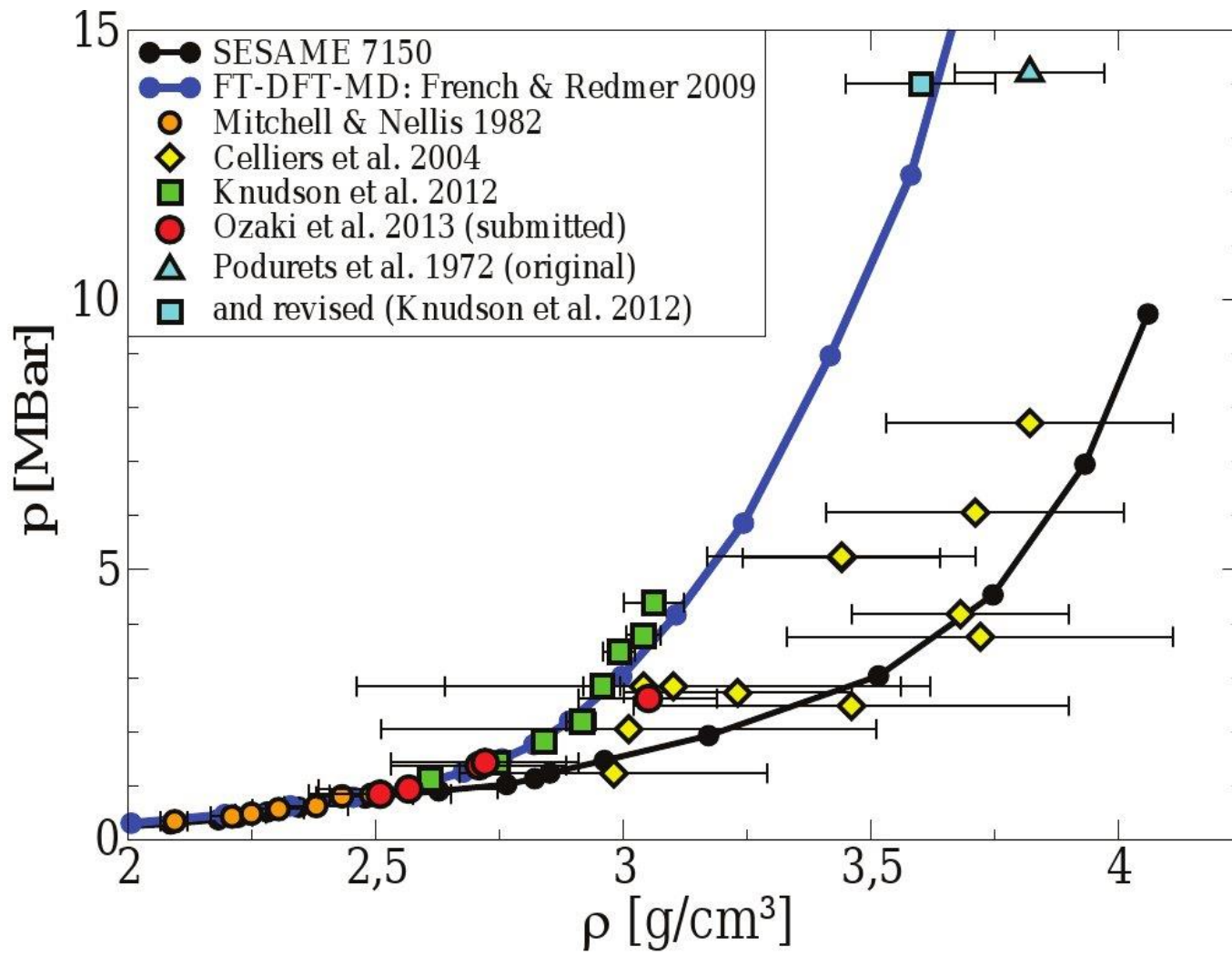
Combination of pre-compressed samples (DACs) and shock waves (lasers)

**Jeanloz et al. 2007, Eggert et al. 2008, Loubeyre et al. 2012, Torchio et al. 2016**

By courtesy of H.-P. Liermann (DESY)



# Shock waves: Hugoniot curve for H<sub>2</sub>O



**Data:**

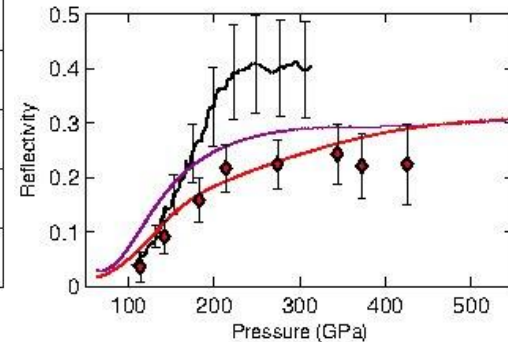
Red  $\diamond$   $\square$  : Sandia Z

Open  $\square$  : Laser shocks

**Theory: FT-DFT-MD**

Red: HSE

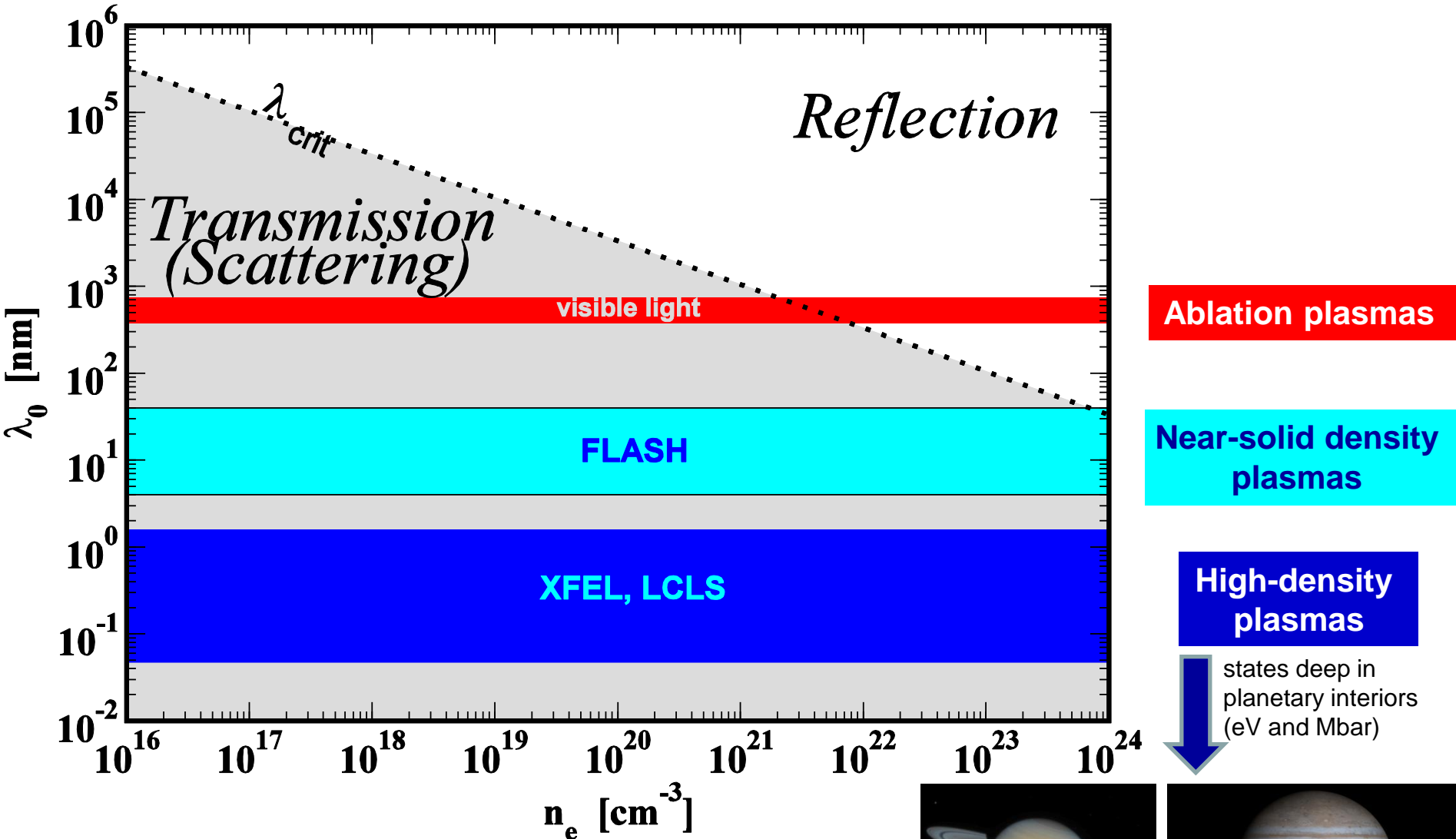
Magenta: PBE



# X-ray scattering:

$$\lambda_{crit} = \frac{2\pi c}{\omega_{pl}}$$

$$\omega_{pl} = \sqrt{\frac{e^2 n_e}{\epsilon_0 m_e}}$$

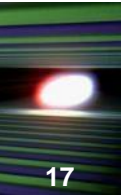


MEC @ LCLS, HED @ XFEL + HIBEF,  
HERMES @ SACLA; ESRF, PETRA III, APS

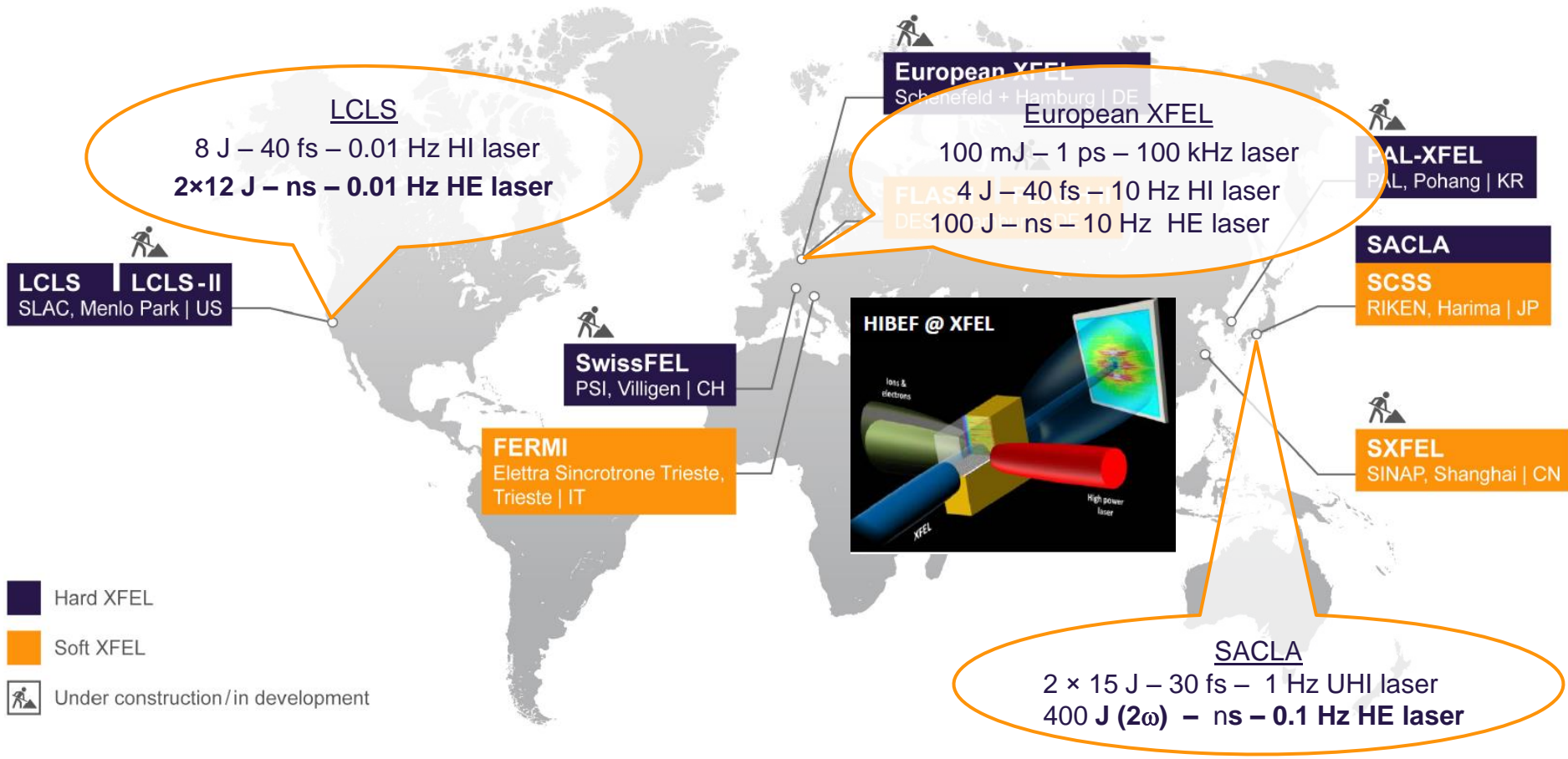




# X-ray free-electron lasers worldwide **with big OLs**



The European XFEL will put Europe in the lead among industrialized nations in a highly competitive scientific and technical environment.

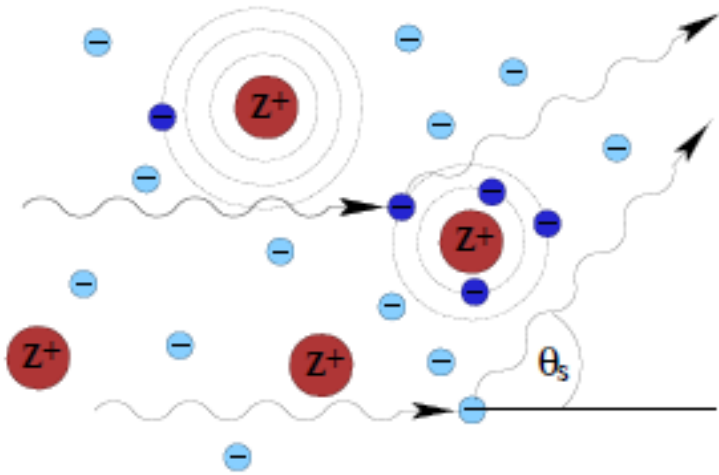


# X-ray diagnostics

See M. McMahon, U. Zastra:  
CDR on HED experiments, Feb 2017

- **X-ray diffraction (XRD):** EOS, phase diagram
- **X-ray phase contrast imaging (PCI):** dynamics
- **X-ray absorption spectroscopy (XAS):** e-structure
- **X-ray near-edge absorption spectroscopy (XANES):** K and L edges
- **X-ray emission spectroscopy (XES):**  
Auger & radiative decays
- **X-ray Thomson scattering (XRTS)**  
non-collective (particles cattering)  
collective (plasmon scattering)
- **High-resolution inelastic X-ray scattering (hrIXS):**  
dynamic properties of solids and liquids

# X-ray Thomson scattering (XRTS) in WDM



- free and bound electrons
- screening and correlation effects
- collisions between plasma particles

**Scattering on density fluctuations: scattering parameter  $\alpha$**

$$\alpha = \frac{1}{k\lambda_D} \begin{cases} < 1 & \text{thermal electron motion (non-collective)} \\ > 1 & \text{collective effects} \end{cases}$$

$\rightsquigarrow$  with  $\lambda_D = \sqrt{\epsilon_0 k_B T_e / n_e e^2}$  and  $k = 4\pi \sin(\theta_S/2) / \lambda_0$

S.H. Glenzer, RR, Rev. Mod. Phys. **81**, 1625 (2009)

E.E. Salpeter, Phys. Rev. **120**, 1528 (1960)

# Theory for the XRTS spectrum

## Dynamic structure factor $S_{ee}(\vec{k}, \omega)$

Scattered power  $P_s$  into frequency interval  $\omega \rightarrow \omega + d\omega$  and solid angle  $d\Omega$  per time, measured by a detector located at  $\vec{R}$

$$P_s(\vec{R}, \omega) d\Omega d\omega = \frac{P_i r_0^2 d\Omega}{2\pi A} \left| \hat{\vec{k}}_f \times (\hat{\vec{k}}_f \times \hat{\vec{E}}_{0i}) \right|^2 N S_{ee}(\vec{k}, \omega) d\omega$$

- $r_0 = \frac{e^2}{m_e c^2}$ : classical electron radius
- $N$ : number of scattering centers,  $A$ : irradiated plasma spot size
- $\left| \hat{\vec{k}}_f \times (\hat{\vec{k}}_f \times \hat{\vec{E}}_{0i}) \right|^2$ : geometrical factor due to laser polarization
- $S_{ee}(\vec{k}, \omega)$ : dynamic structure factor (DSF), spectral function of the (electron) density fluctuations in the plasma, accessible via
  - X-ray Thomson scattering experiments
  - Theory for the dielectric function  $\epsilon(\vec{k}, \omega)$ : fluctuation-dissipation theorem
  - Integral equation methods: HNC and classical-map HNC
  - *Ab initio* simulations: DFT-MD, WPMD



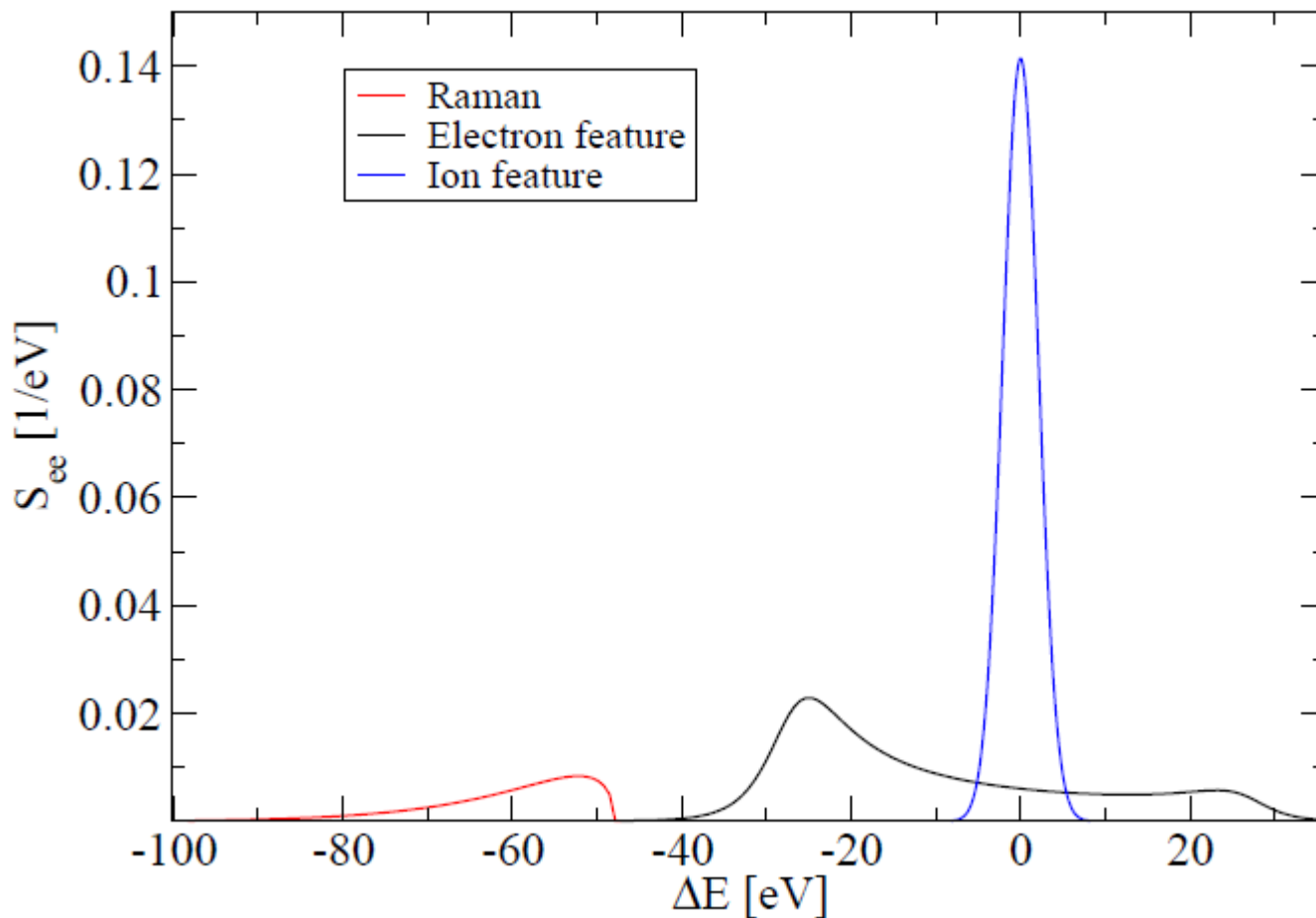
# Chihara formula

$$S_{ee}(k, \omega) = Z_f S_{ee}^0(k, \omega) + |f_i(k) + q(k)|^2 S_{ii}(k, \omega) + Z_b \int_{-\infty}^{\infty} d\omega' S_c(k, \omega - \omega') S_s(k, \omega')$$

J. Chihara, J. Phys.: Cond. Matt. **12**, 231 (2000)

G. Gregori et al., Phys. Rev. E **67**, 026412 (2003)

A. Höll et al., HEDP **3**, 120 (2007)



# Born-Mermin approximation (BMA) [1,2]

- Fluctuation-dissipation theorem [3]: 
$$S_{ee}^0(k, \omega) = -\frac{\epsilon_0 \hbar k^2}{\pi e^2 n_e} \frac{\text{Im} \epsilon^{-1}(k, \omega)}{1 - \exp(-\hbar\omega/k_B T_e)}$$
- Collisions via Mermin ansatz [4]: 
$$\epsilon^M(k, \omega) - 1 = \frac{\left(1 + i \frac{\nu(\omega)}{\omega}\right) [\epsilon^{\text{RPA}}(k, \omega + i\nu(\omega)) - 1]}{1 + i \frac{\nu(\omega)}{\omega} \frac{\epsilon^{\text{RPA}}(k, \omega + i\nu(\omega)) - 1}{\epsilon^{\text{RPA}}(k, 0) - 1}}$$
- RPA given by Lindhard [5]: 
$$\epsilon^{\text{RPA}}(\vec{k}, \omega) = 1 - \frac{1}{\Omega_0 \epsilon_0 k^2} \sum_{p,c} e_c^2 \frac{f_{p+(k/2)}^c - f_{p-(k/2)}^c}{\Delta E_{p,k}^c - \hbar(\omega + i\eta)}$$
- Dynamic collision frequency in Born approximation [6]:

$$\nu^{\text{Born}}(\omega) = -i \frac{\epsilon_0 n_i \Omega_0^2}{6\pi^2 e^2 n_e m_e} \int_0^\infty dq q^6 V_D^2(q) S_{ii}(q) \frac{1}{\omega} [\epsilon_{\text{RPA}}(q, \omega) - \epsilon_{\text{RPA}}(q, 0)]$$

[1] R. Redmer, H. Reinholz, G. Röpke, R. Thiele, A. Höll, IEEE Trans. Plasma Sci. **33**, 77 (2005)

[2] C. Fortmann et al., Laser Part. Beams **27**, 311 (2009)

[3] G.D. Mahan, *Many-Particle Physics* (Plenum Publishers, New York, 2000)

[4] N.D. Mermin, Phys. Rev. B **1**, 2362 (1970)

[5] J. Lindhard, Kgl. Dan. Videns. Selk. Mat. Fys. Medd. **28**, 8 (1954)

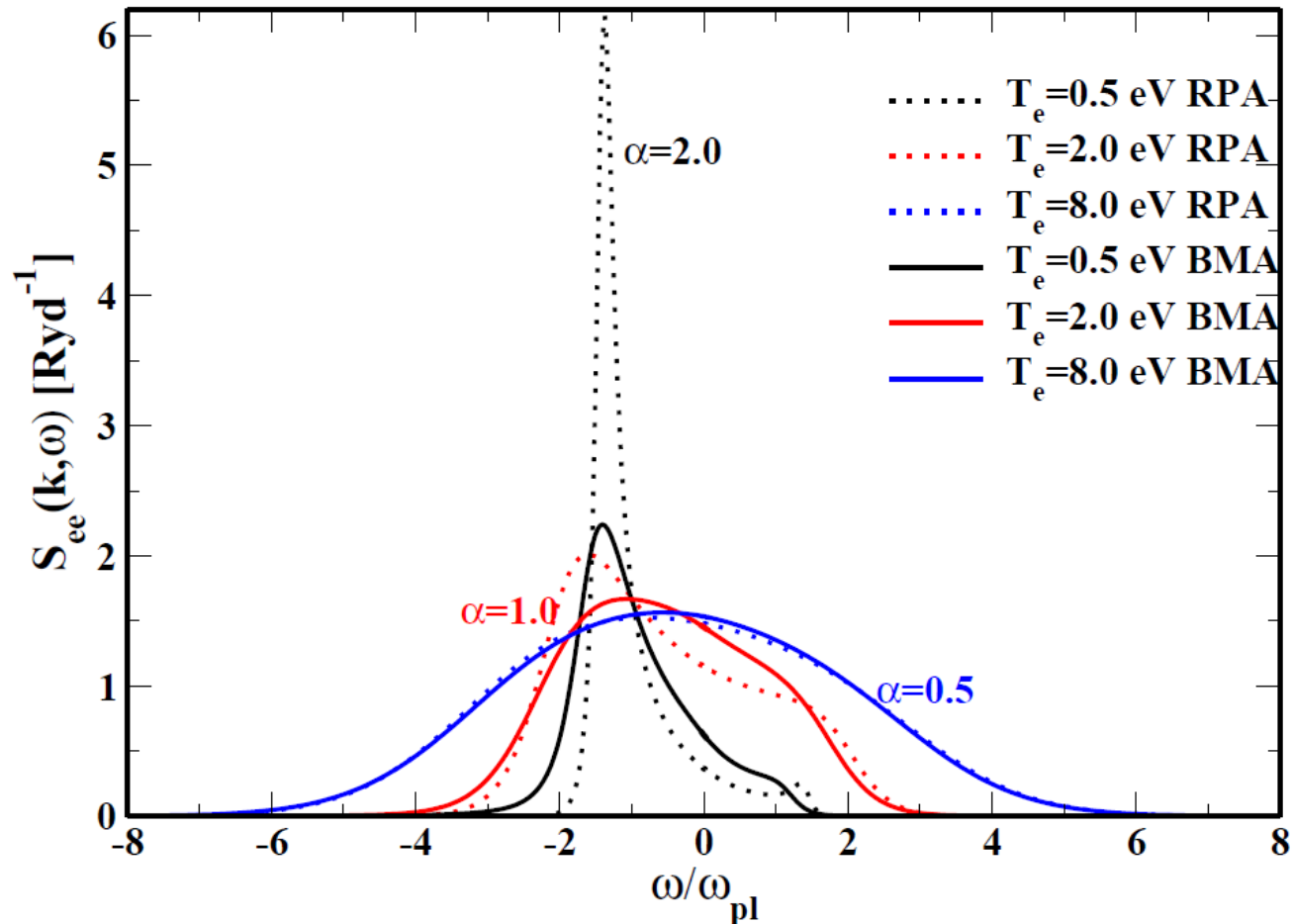
[6] H. Reinholz, R. Redmer, G. Röpke, A. Wierling, Phys. Rev. E **62**, 5648 (2000)

→ collisionality

→ conductivity

# Electron DSF $S_{ee}^0(k, \omega)$

Hydrogen plasma:  $n_e = 10^{21} \text{ cm}^{-3}$ ,  $\lambda_0 = 4.13 \text{ nm}$ ,  $Z=1$



G. Gregori et al., Phys. Rev. E **67**, 026412 (2003)

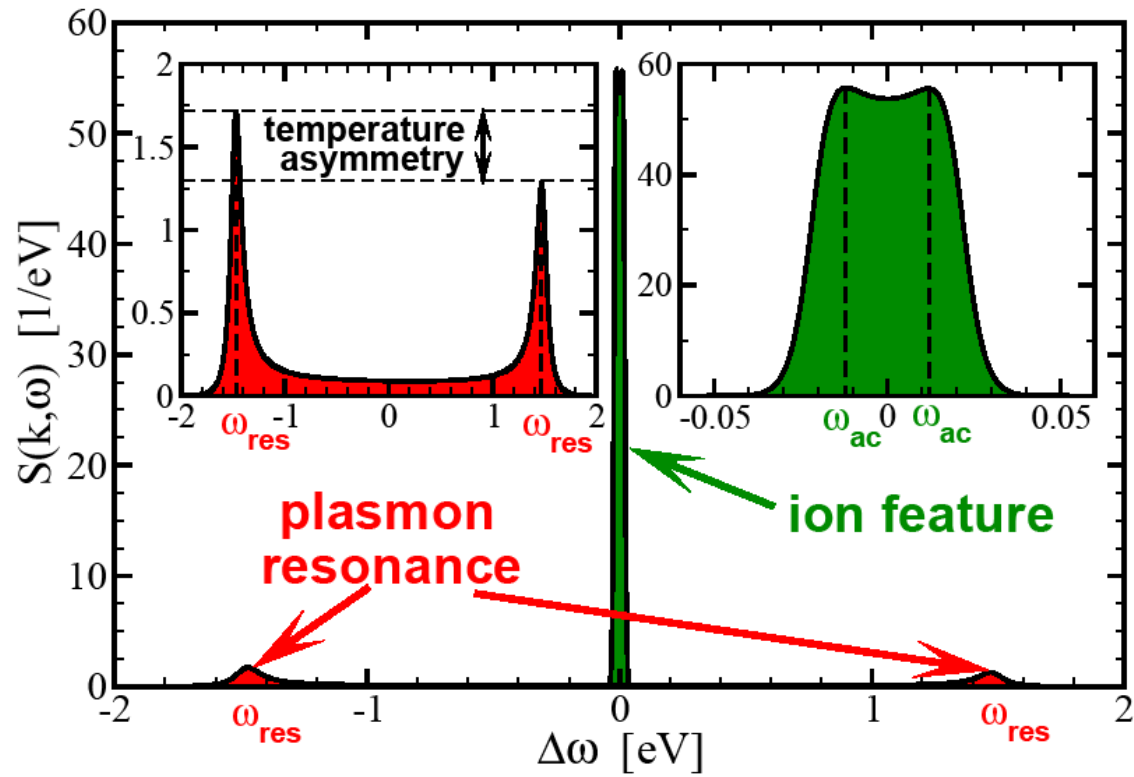
A. Höll et al., High Energy Dens. Phys. **3**, 120 (2007)

R. Thiele et al., Phys. Rev. E **78**, 026411 (2008)

# Collective XRTS: plasmons

$T_e$  via detailed balance relation:  $Y(\mathbf{k}, \omega) = \frac{S_{ee}^0(-\mathbf{k}, -\omega)}{S_{ee}^0(\mathbf{k}, \omega)} = \exp\left(-\frac{\hbar\omega}{k_B T_e}\right)$

Peak positions give  $n_e$ :  $\omega_{GB}^2(k) = \omega_{pl}^2 + \frac{3k_B T_e}{m_e} k^2$  Gross-Bohm (1949)



A. Höll et al., High Energy Dens. Phys. **3**, 120 (2007)

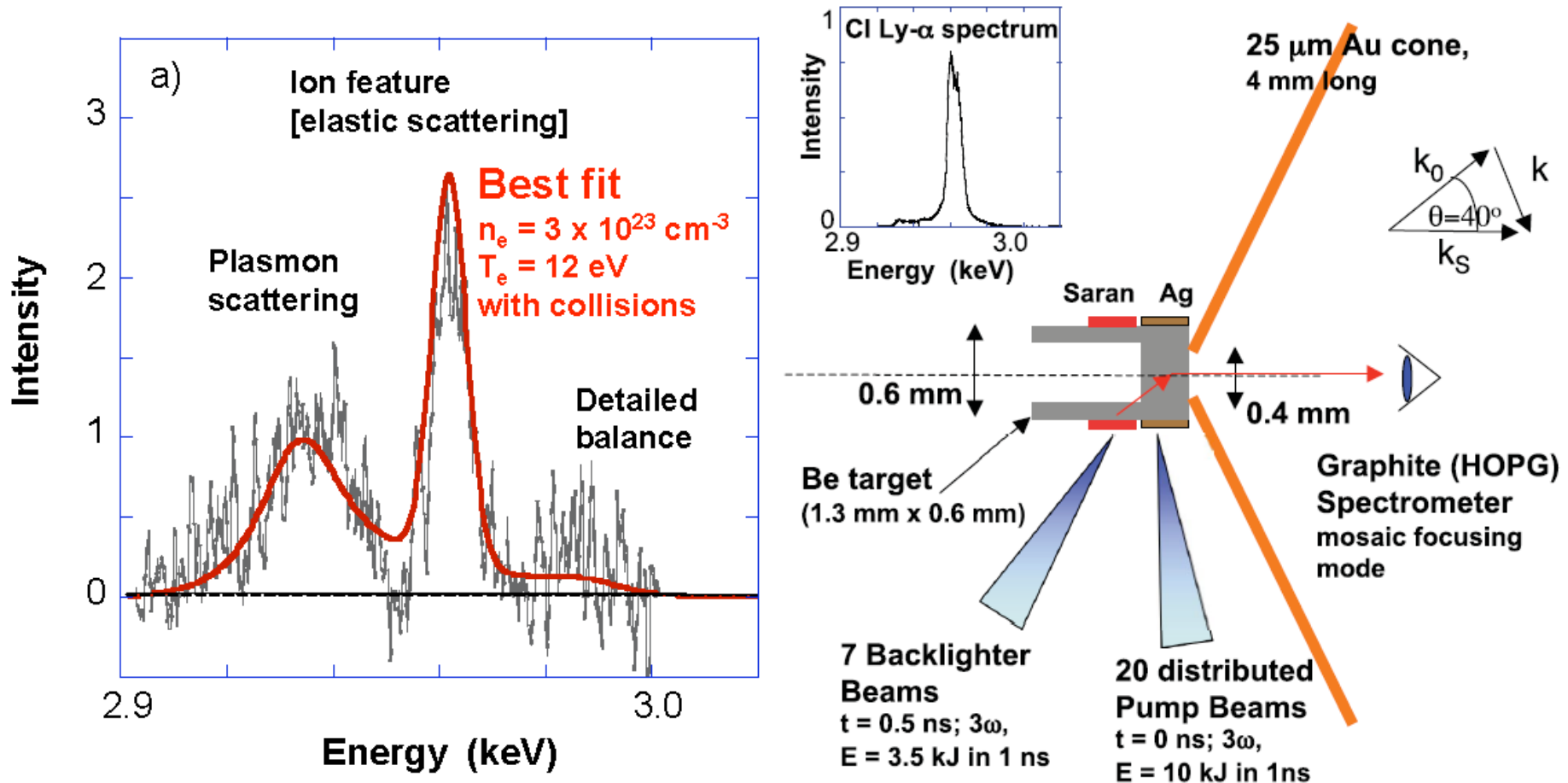
R. Thiele et al., Phys. Rev. E **78**, 026411 (2008)

S.H. Glenzer, RR, Rev. Mod. Phys. **81**, 1625 (2009)



# Collective XRTS: Be

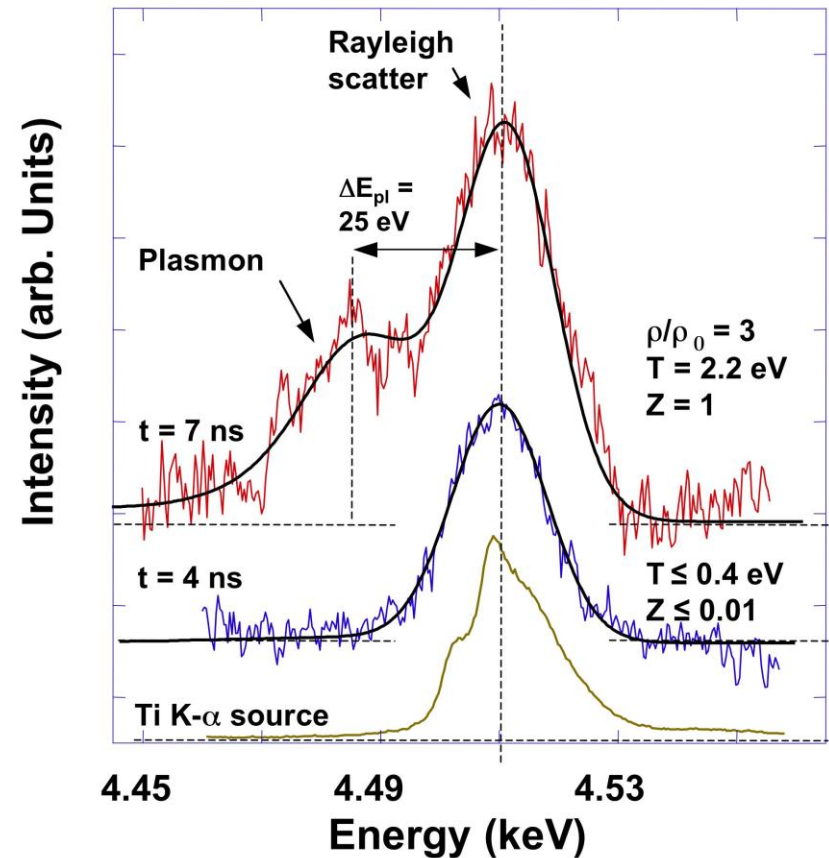
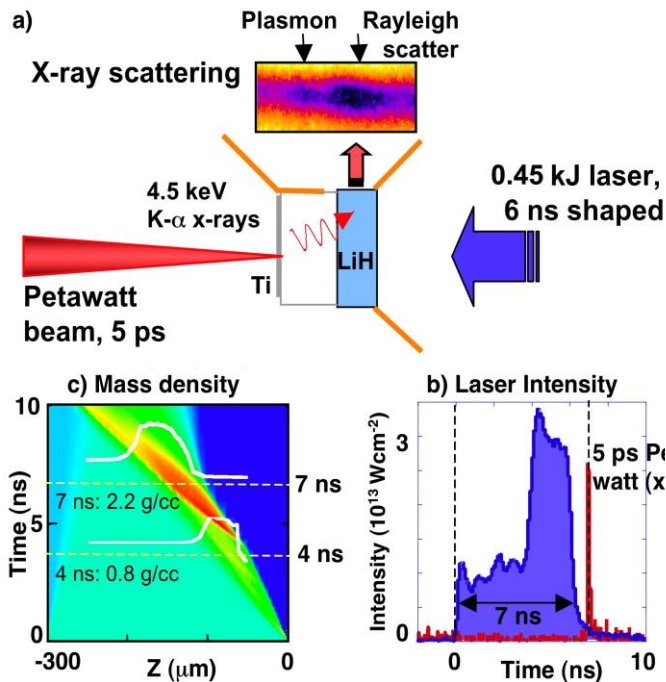
## Plasmon feature



Experiment at the Omega laser facility in Rochester:  
S.H. Glenzer et al., Phys. Rev. Lett. **98**, 065002 (2007)

# Pump-probe XRTS experiment: LiH

ns time resolution, nonmetal-to-metal transition

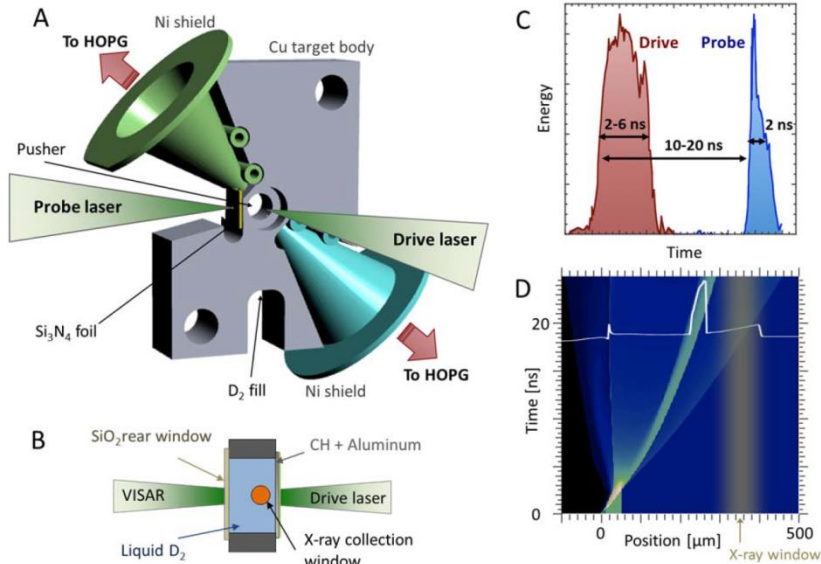


Experiment at the Titan laser facility at LLNL:

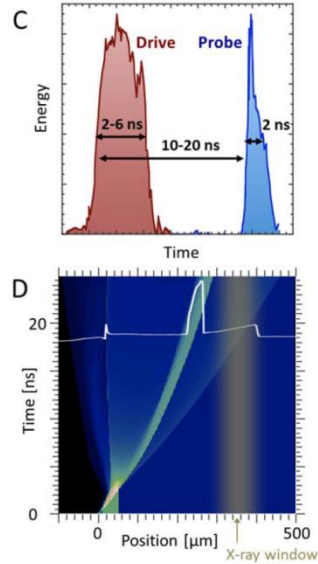
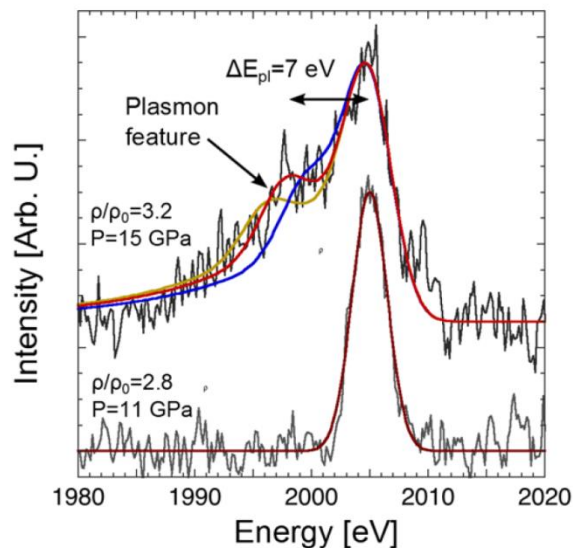
A. Kritcher et al., Science **322**, 69 (2008); PoP **16**, 056308 (2009).

# Pump-probe XRTS experiment: H

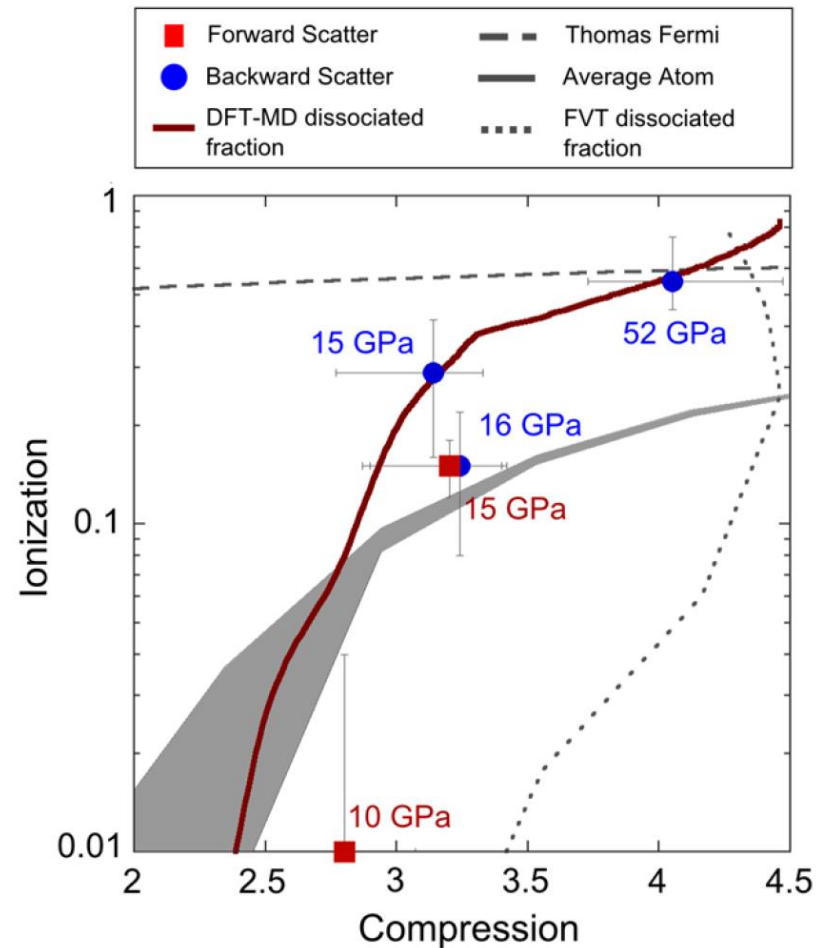
## Probes onset of dissociation in Jupiter



Probe: 2005 eV Si Lyman- $\alpha$  line



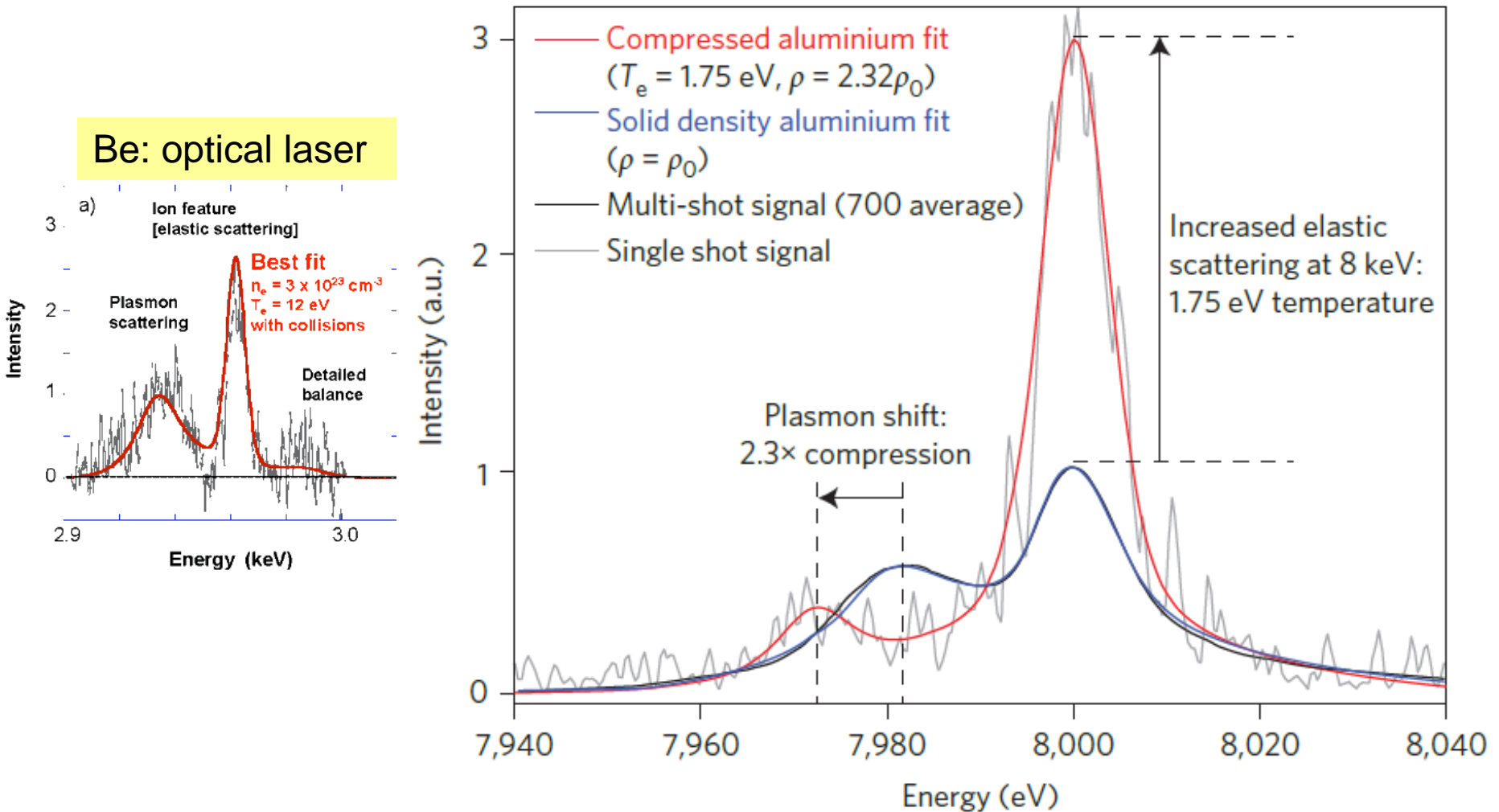
P. Davis et al., Nature Commun. 7, 11189 (2016)  
 Experiment at the Janus laser facility at LLNL  
 DFT-MD: A. Becker (U Rostock)





# Pump-probe XRTS experiment: Al

High spectral resolution at LCLS (seeded beam mode)



L.B. Fletcher et al., Nature Phot. **9**, 274 (2015)

Ion feature: DFT-MD  $\rightarrow$  Yukawa+SRR  $S(k)$

Plasmon feature: BMA and BMA+LFC



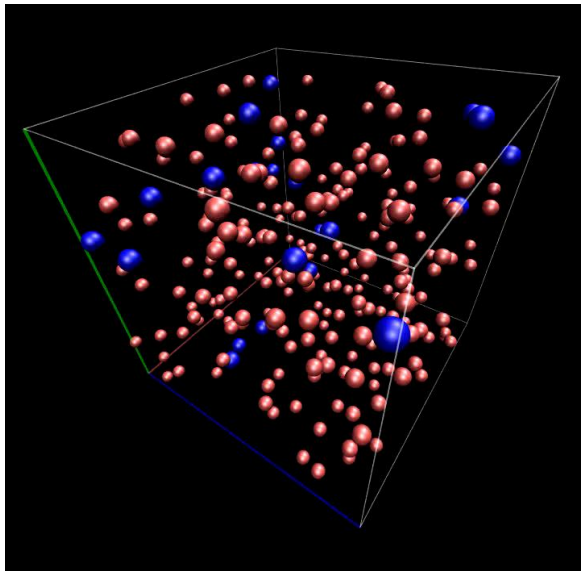
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# DFT-MD Simulations for WDM

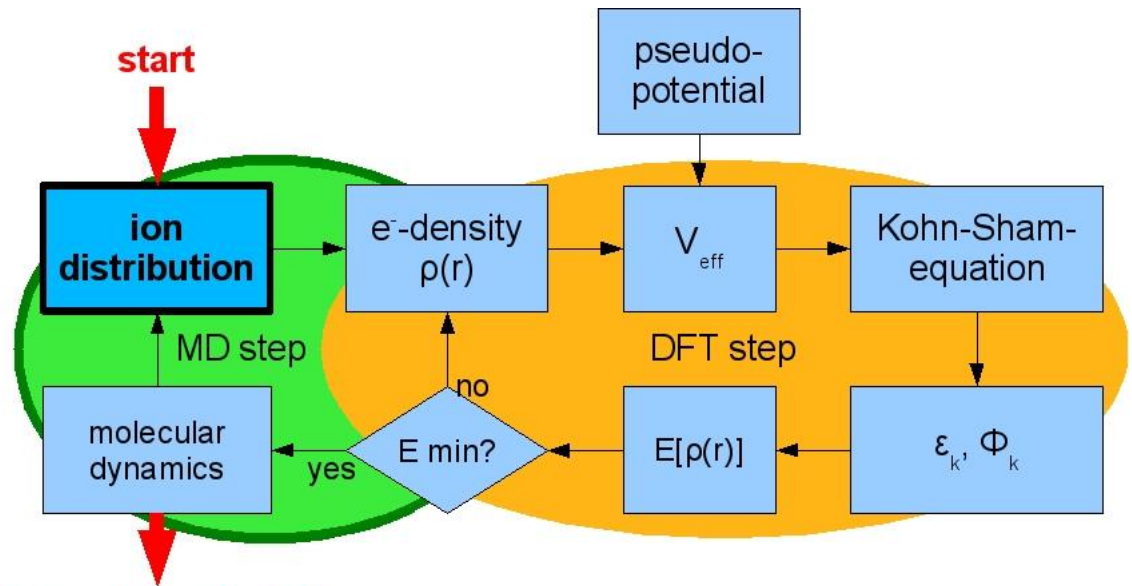
Born-Oppenheimer approximation: combination of (quantum) DFT and (classical) MD  
Warm Dense Matter: finite-temperature DFT-MD simulations based on  
N.D. Mermin, Phys. Rev. **137**, A1441 (1965)

Codes: **Vienna Ab-initio Simulation Package** (VASP) or Abinit, Quantum Espresso ...  
G. Kresse and J. Hafner, PRB **47**, 558 (1993), *ibid.* **49**, 14251 (1994)  
G. Kresse and J. Furthmüller, Comput. Mat. Sci. **6**, 15 (1996), PRB **54**, 11169 (1996)



H-He (8.6%) @ 1 Mbar, 4000 K

box length  $\sim 10^{-9}$  m



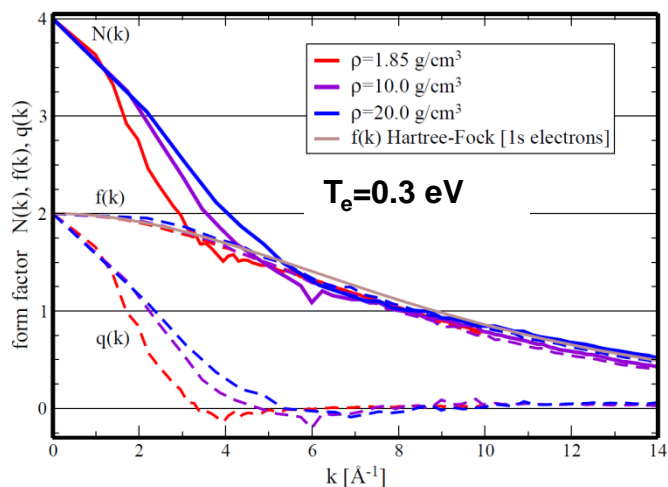
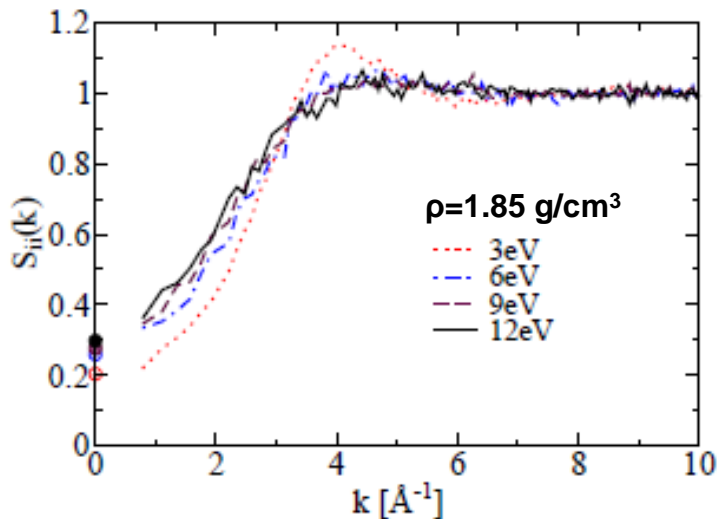
thermodynamic data  
high-pressure phase diagram  
pair correlation functions  
electrical & thermal conductivity  
diffusion coefficient  
viscosity, opacity



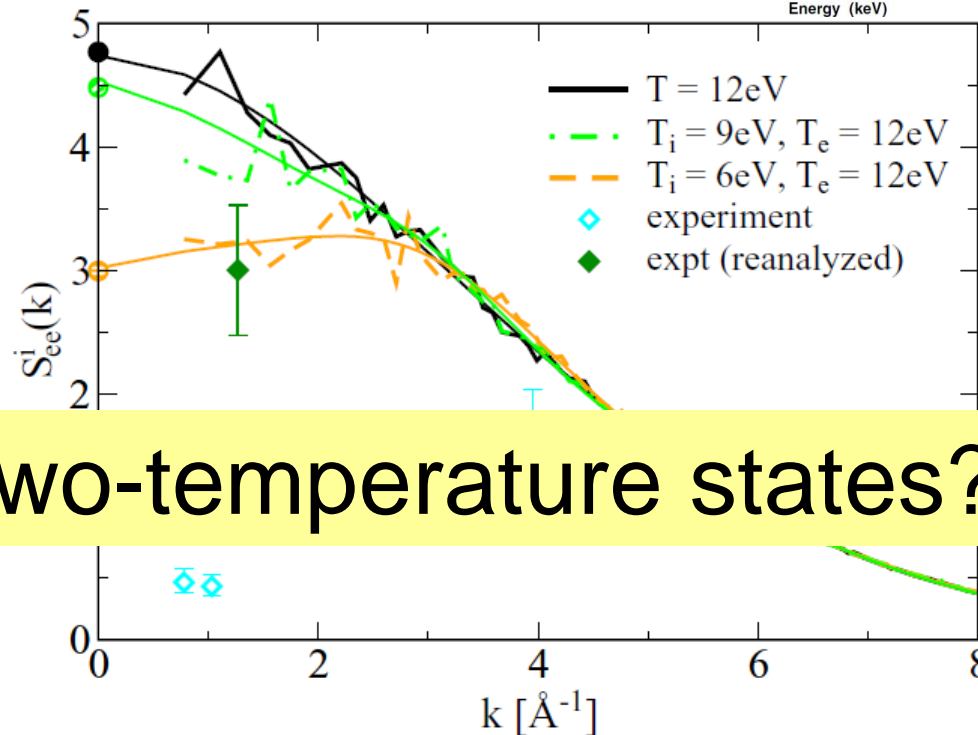
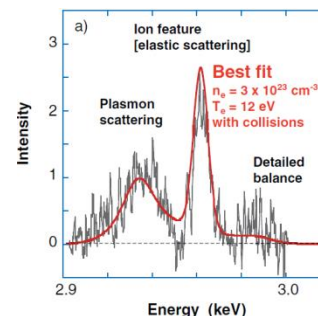
GP size  $\sim 10^8$  m

# Ion feature in warm dense Be

$$S_{ee}^i(\vec{k}) = |f(\vec{k}) + q(\vec{k})|^2 S_{ii}(\vec{k}) \equiv |N(\vec{k})|^2 S_{ii}(\vec{k}).$$



Warm dense Be:  
12 eV, 1.85 g/cm<sup>3</sup>

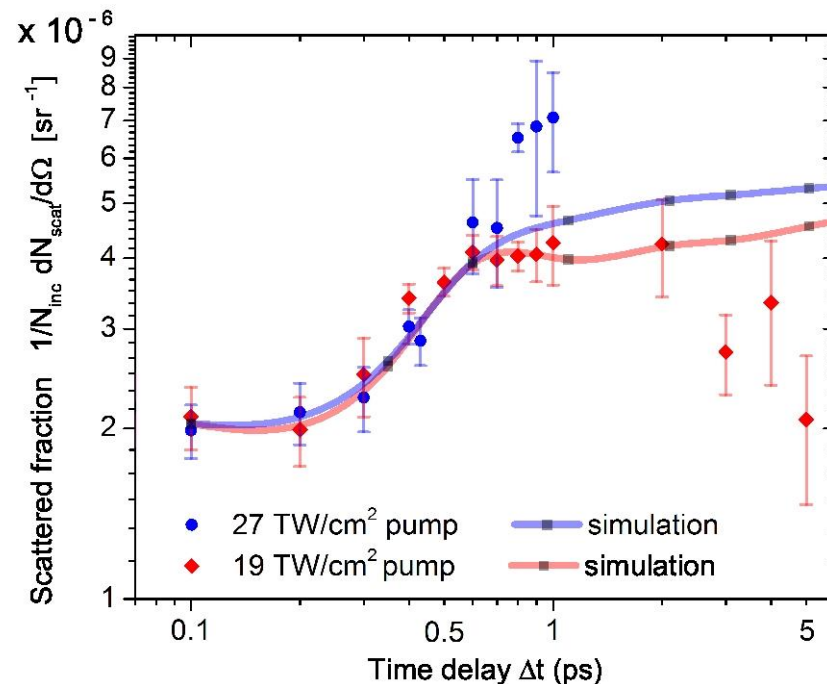
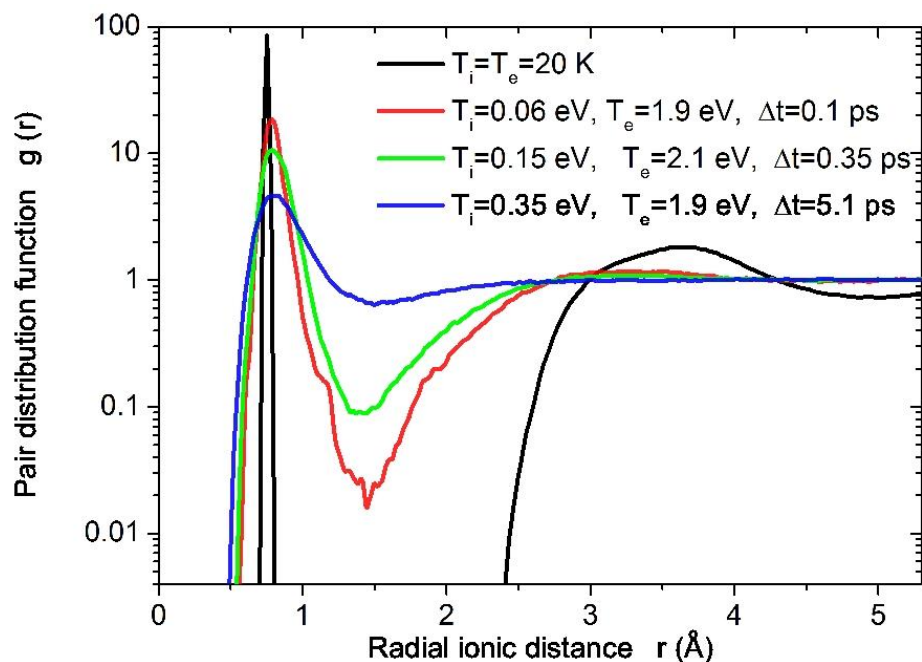
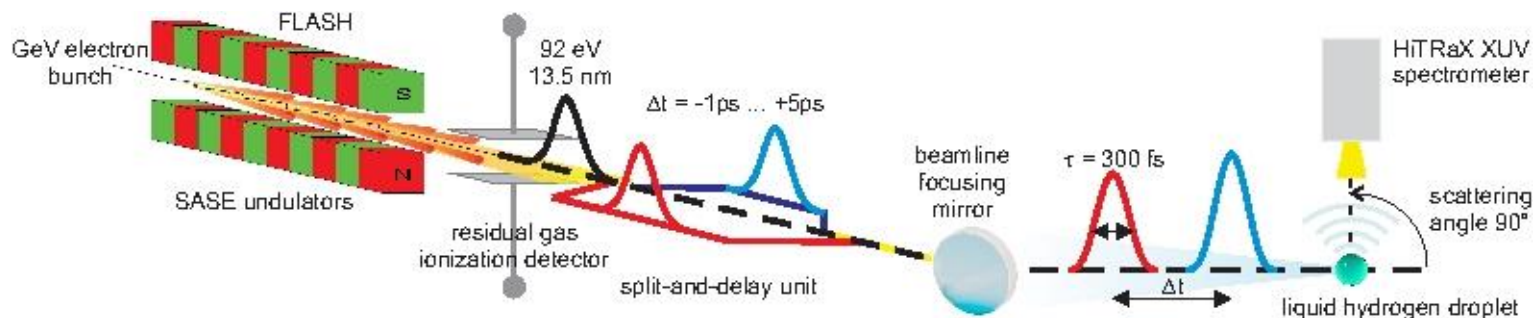


Two-temperature states?

Experiments: see Glenzer, RR, RMP **81**, 1625 (2009)  
 Reanalyzed spectrum from Glenzer et al., PRL **98**, 065002 (2007)  
 DFT-MD simulations: Plagemann et al., PRE **92**, 013103 (2015)

# Pump-probe experiments at FLASH

Temporal resolution of ultrafast heating in liquid hydrogen on ps time scales.  
 2T-DFT-MD simulations for  $S(k \rightarrow 0)$  based on rad-hydro target evolution.



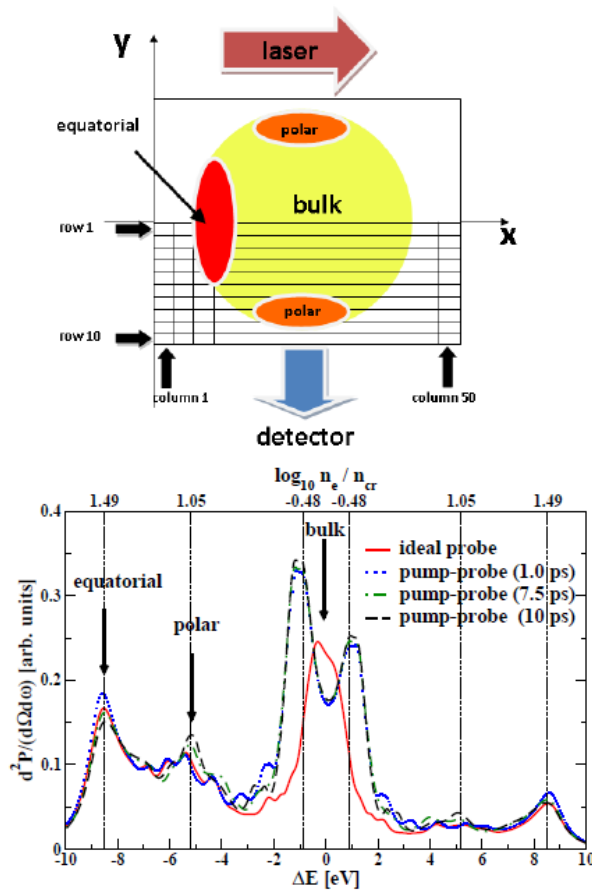
# XRTS for inhomogeneous plasmas

State-of-the-art experiments combine optical lasers (pump) and XFELs (probe).

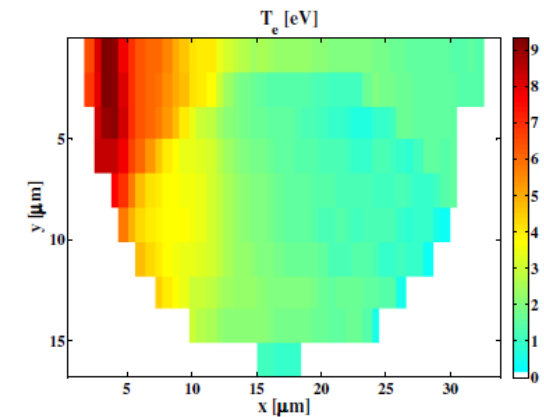
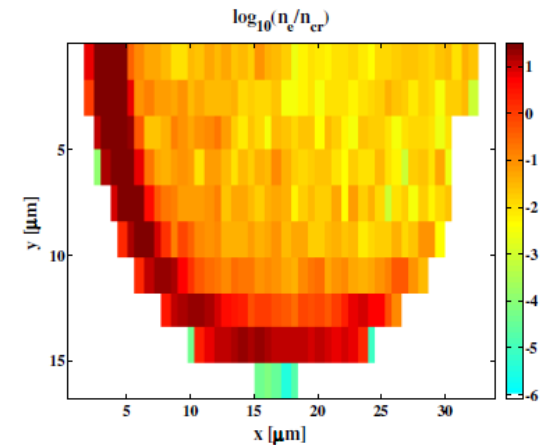
Interpret the XRTS signal of a strongly inhomogeneous plasma?

Perform PIC (VLPL3D) and hydrodynamic simulations (HELIOS) for laser-matter interaction.

Study the ultra-fast dynamics and relaxation in inhomogeneous plasmas!

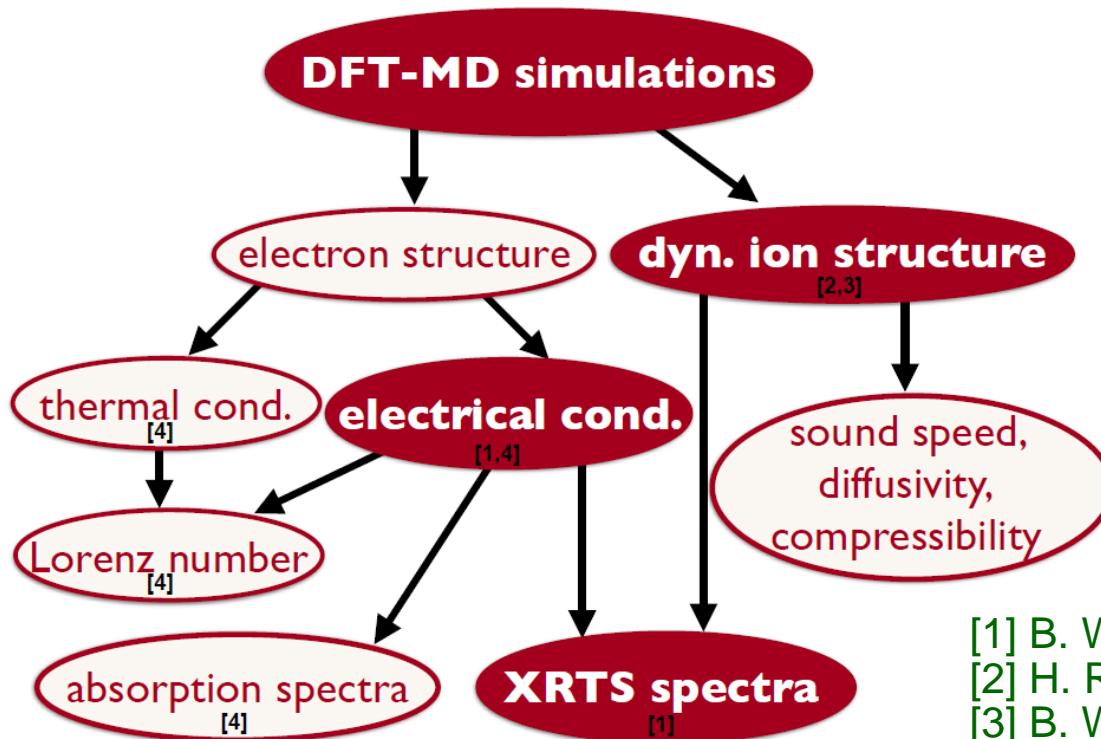


**OL:**  
 800 nm  
 150 fs  
 1.6 mJ  
**XFEL:**  
 13.5 nm  
 30 fs  
 0.05 mJ  
**Target:**  
 liquid H  
 15  $\mu\text{m}$





# Next step: derive all relevant quantities from DFT-MD



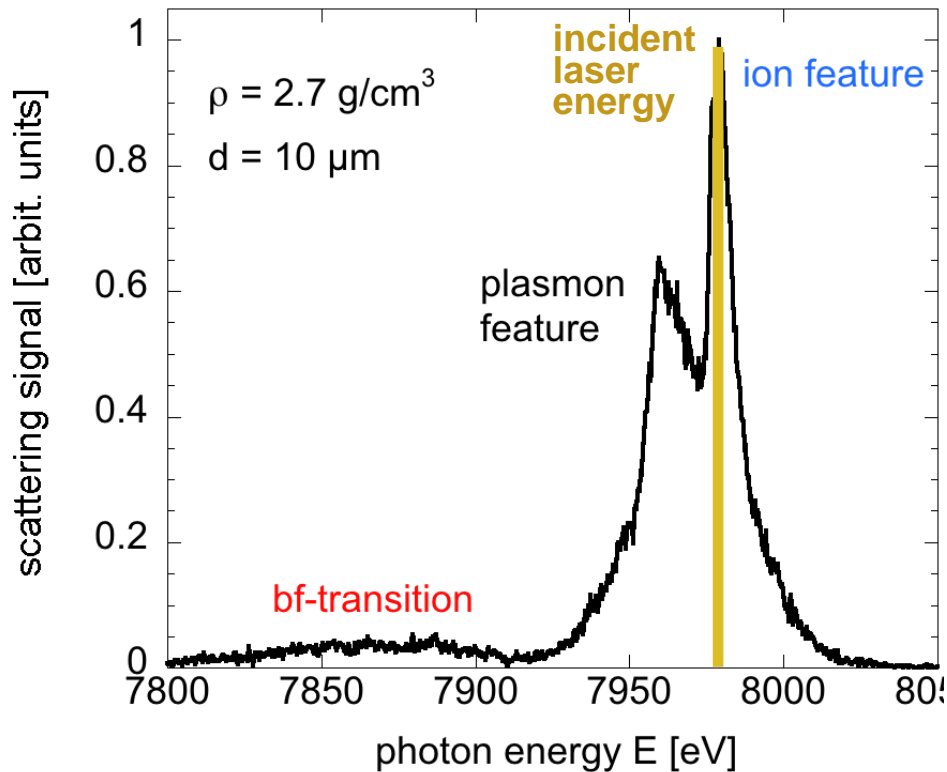
- [1] B. Witte et al., PRL **118**, 225001 (2017)  
 [2] H. Rüter, RR, PRL **112**, 145007 (2014)  
 [3] B. Witte et al., PRB **95**, 144105 (2017)  
 [4] B. Witte et al., PoP **25**, 056901 (2018)

## DSF & XRTS spectrum exclusively from DFT-MD

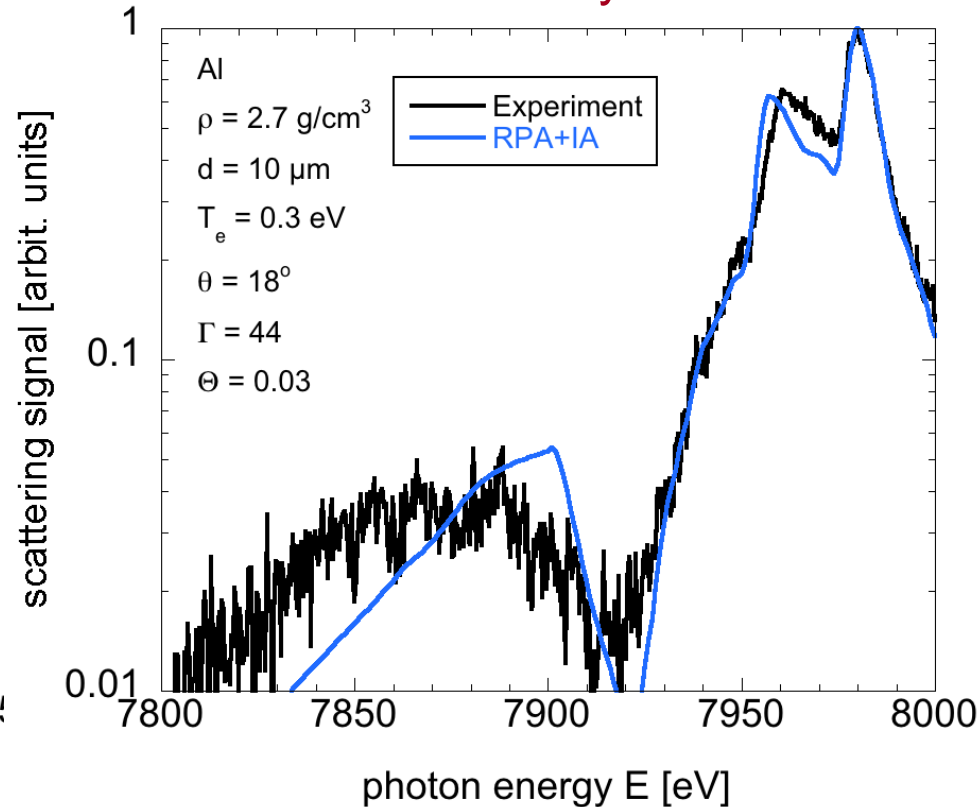
$$S_{ee}(k, \omega) = \underbrace{|f_I(k) + q(k)|^2}_{|\Psi_k|^2 \text{ [3]}} \underbrace{S_{ii}(k, \omega)}_{\langle n_k(0)n_{-k}(t) \rangle \text{ [2]}} + \underbrace{S_{et}(k, \omega)}_{|\langle \Psi_k | \hat{p} | \Psi_k \rangle|^2 \text{ [1]}}$$

# XRTS in isochorically heated Al

measured XRTS spectrum



standard theory fails



**Highly resolved XRTS spectrum of 50 μm thick Al foils averaged over  $10^3$ - $10^4$  shots at LCLS → challenges theory for the DSF and the dielectric function**

P. Sperling et al., PRL **115**, 115001 (2015)

B. Witte et al., PRL **118**, 225001 (2017)

# XRTS in isochorically heated Al

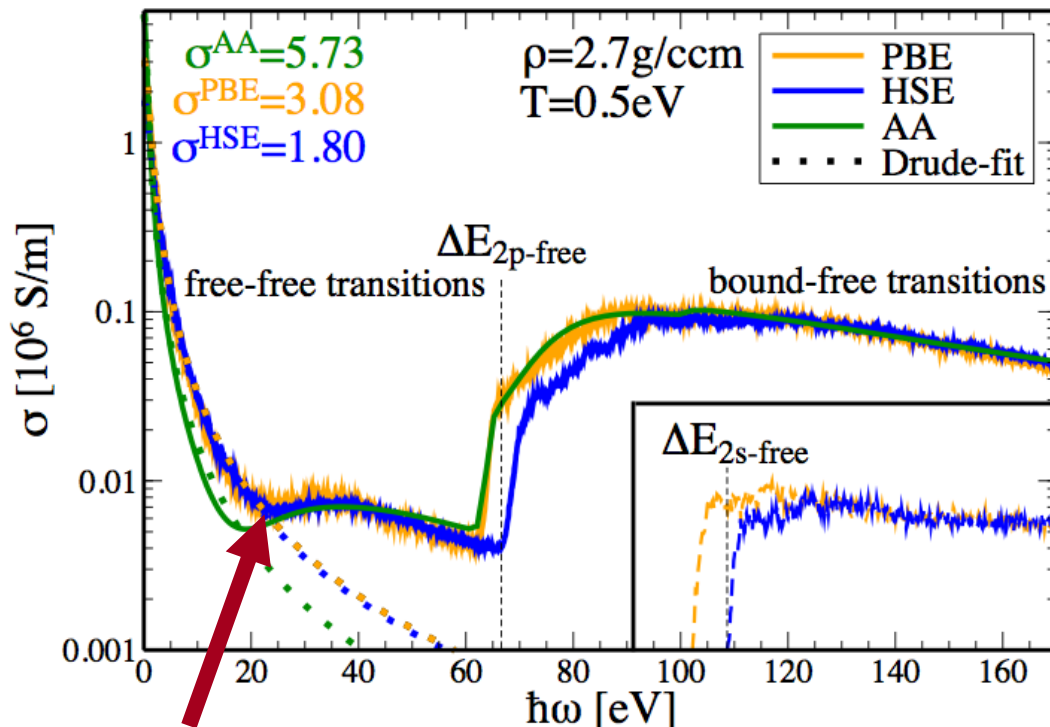
$$\sigma_e(\omega) \propto \sum_{\mathbf{k}\mu\nu} (f_{\mathbf{k}\mu} - f_{\mathbf{k}\nu}) |\langle \Psi_{\mathbf{k}\nu} | \hat{\mathbf{p}} | \Psi_{\mathbf{k}\mu} \rangle|^2 \delta(\Delta\epsilon_{\mathbf{k}\nu\mu} - \hbar\omega)$$

See B. Holst, M. French, RR, PRB 83 (2011)

Simple Drude model:

$$\sigma(\omega) = \frac{\epsilon_0 \nu \omega_{pl}^2}{\omega^2 + \nu^2}$$

XC functional matters: PBE, HSE



Cooper minimum: non-Drude behavior

DSF  $\rightarrow$  XRTS spectrum:

$$\text{Re } \epsilon(\omega) = 1 - \frac{1}{\epsilon_0 \omega} \text{Im } \sigma(\omega),$$

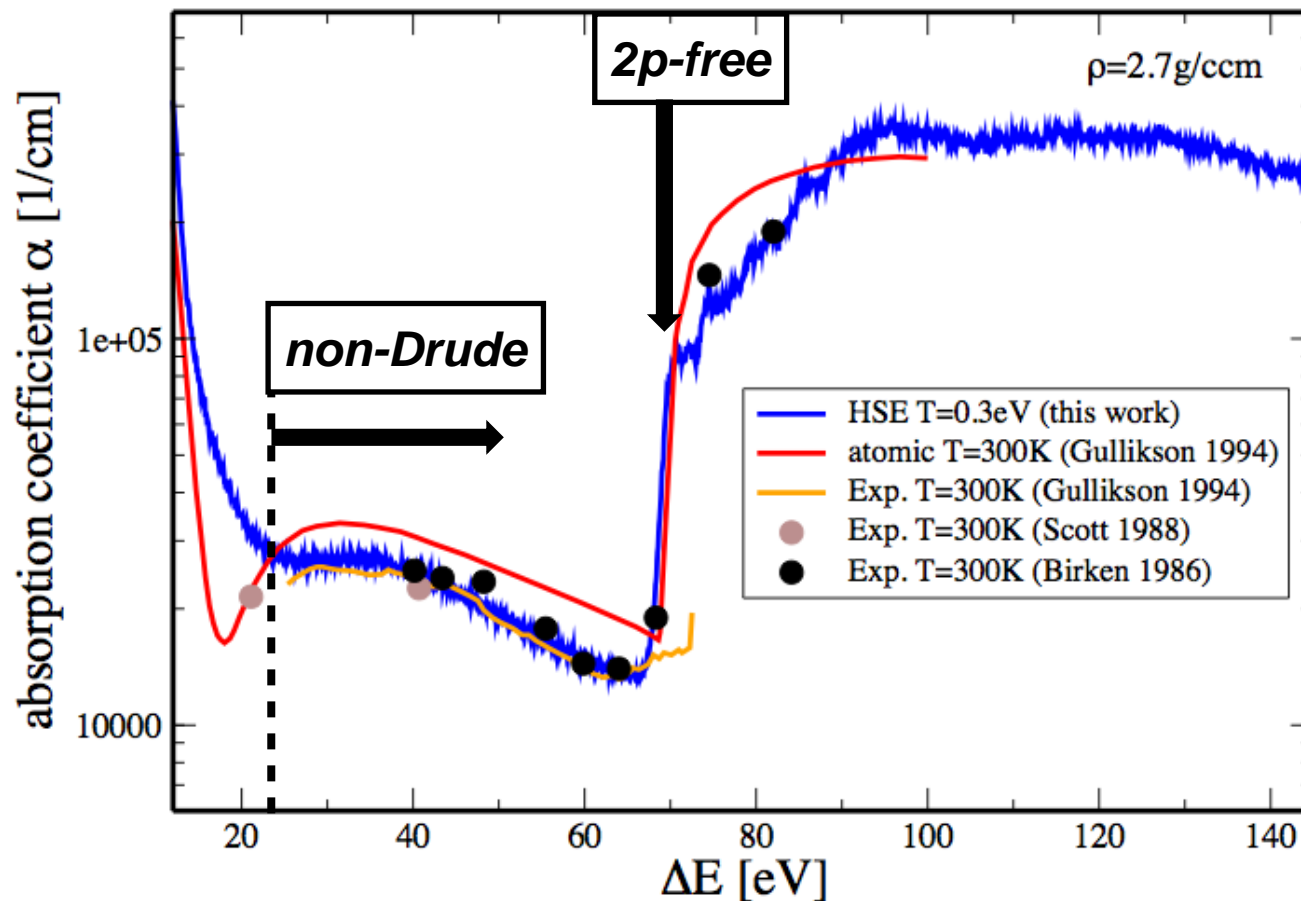
$$\text{Im } \epsilon(\omega) = \frac{1}{\epsilon_0 \omega} \text{Re } \sigma(\omega).$$

$$S_{ee}^0(k, \omega) = -\frac{\epsilon_0 \hbar k^2}{\pi e^2 n_e} \frac{\text{Im } \epsilon^{-1}(k, \omega)}{1 - \exp(-\frac{\hbar\omega}{k_B T_e})}$$

B. Witte et al., PRL 118, 225001 (2017)

# XRTS in isochorically heated Al

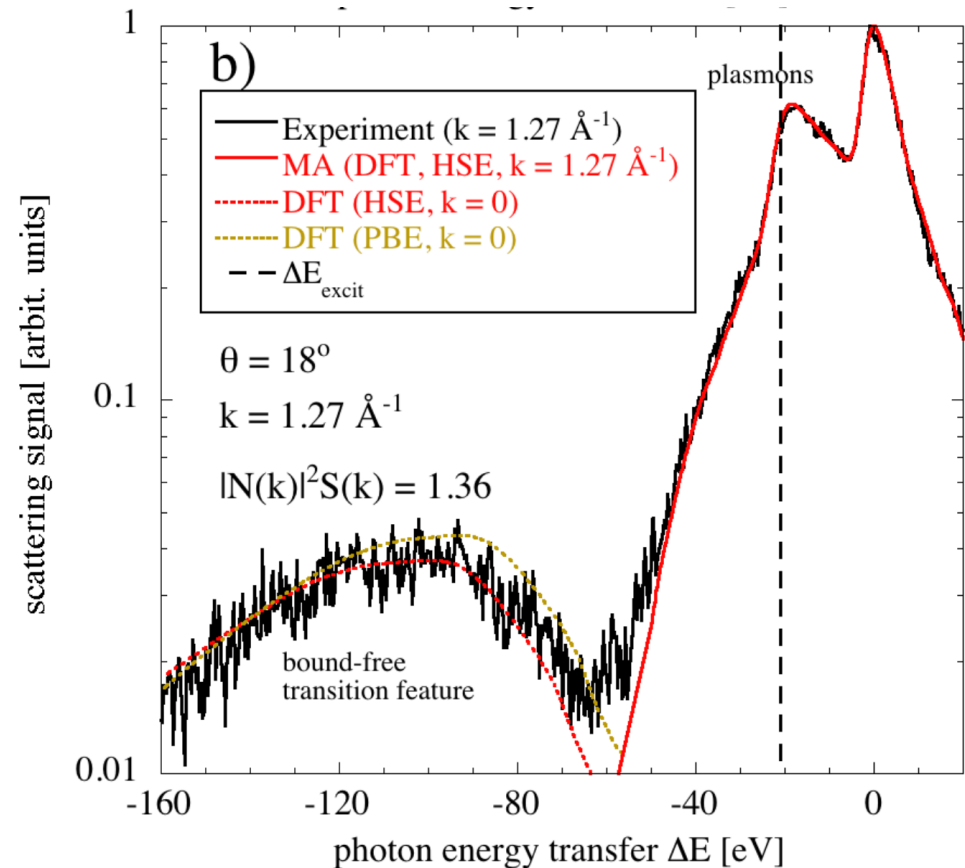
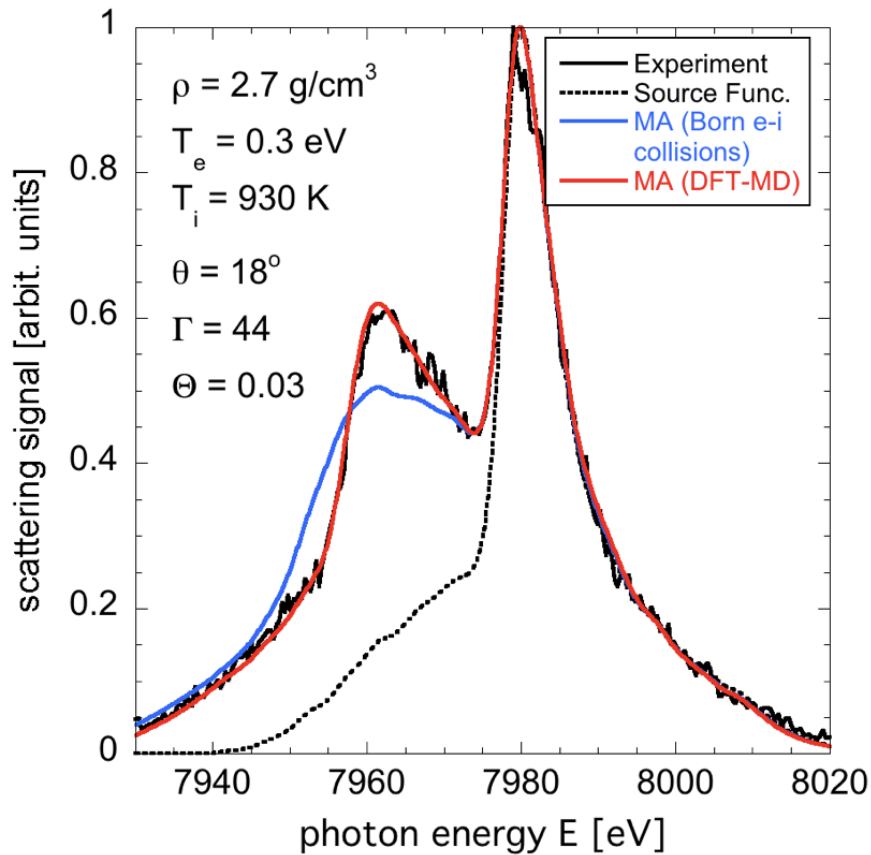
$$\alpha(\omega) = \frac{4\pi\sqrt{2}}{\sqrt{|\epsilon(\omega)|^2 + \text{Re}[\epsilon(\omega)]}} \text{Re}[\sigma(\omega)]$$



# XRTS in isochorically heated Al

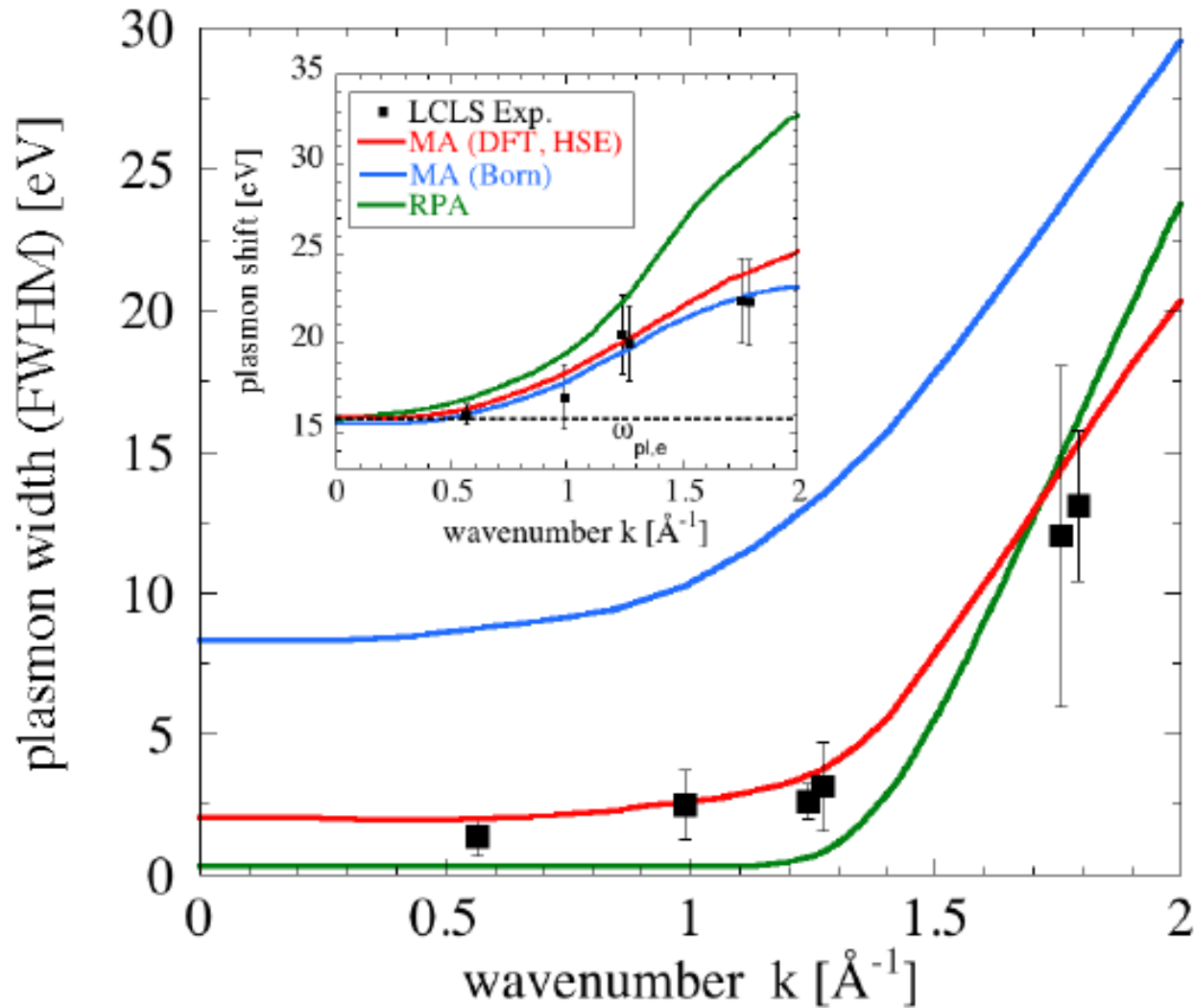
$$S_{et}(k \rightarrow 0, \omega) = - \lim_{k \rightarrow 0} \frac{\epsilon_0 \hbar k^2}{\pi e^2 n_e} \frac{\text{Im} [\epsilon^{-1}(k, \omega)]}{1 - \exp[\frac{-\hbar\omega}{k_B T_e}]}$$

Mermin approach for k-dependent plasmon



# XRTS in isochorically heated Al

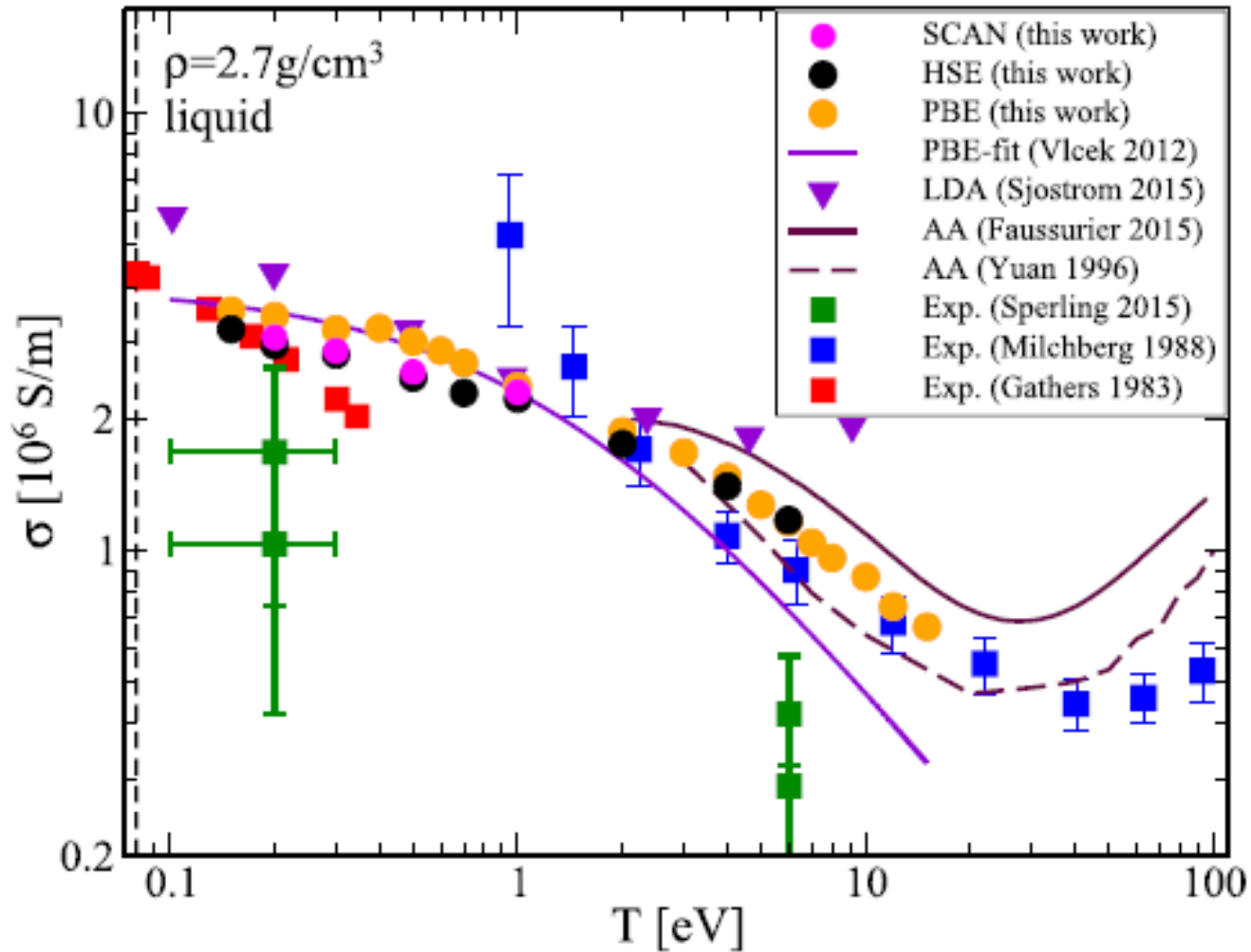
## Plasmon width and dispersion





# XRTS in isochorically heated Al

Electrical dc conductivity



# Contents

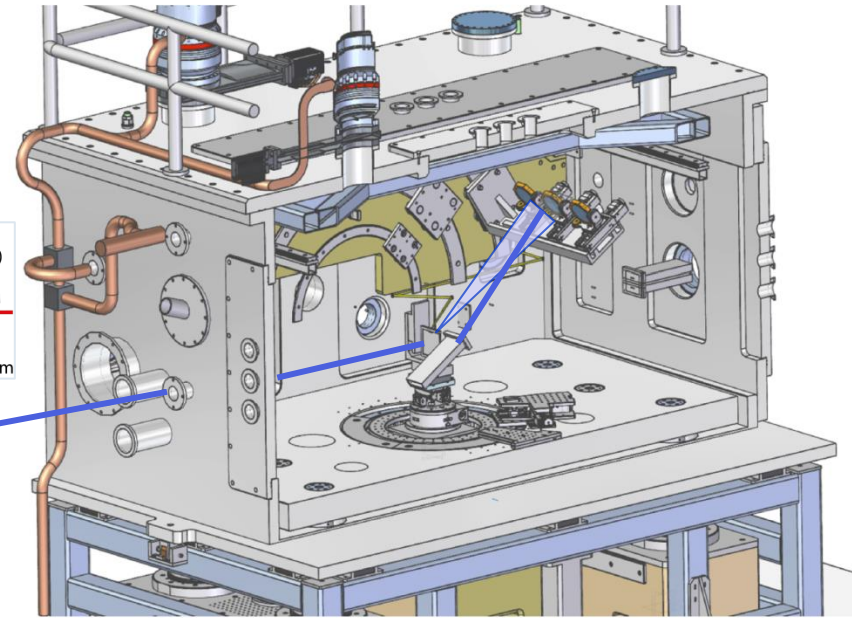
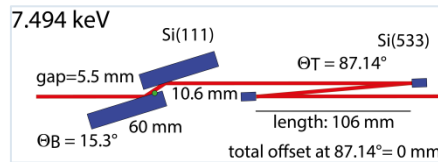
- 1. Introduction: Warm Dense Matter**
  - Planets, Exoplanets
  - Planetary Interiors & Phase Diagrams
- 2. High-Pressure Experiments**
  - DACs & Shock Waves
  - X-ray Thomson Scattering
- 3. DFT-MD Simulations**
  - Ion Feature
  - Plasmon Feature
- 4. Dynamic Ion-Ion Structure Factor**
  - Results of DFT-MD Simulations
  - Hydrodynamic Model
- 5. Summary**

# High-resolution inelastic X-ray scattering

## Implementation at HED@XFEL

### Monochromator with different bandwidths:

- $\Delta E/E = 10^{-3}$ : SASE
- $\Delta E/E = 10^{-4}$ : Si<sub>111</sub> monochromator
- $\Delta E/E = 10^{-5}$ : seeded
- $\Delta E/E = 10^{-6}$ : Si<sub>533</sub> at 7.5 keV



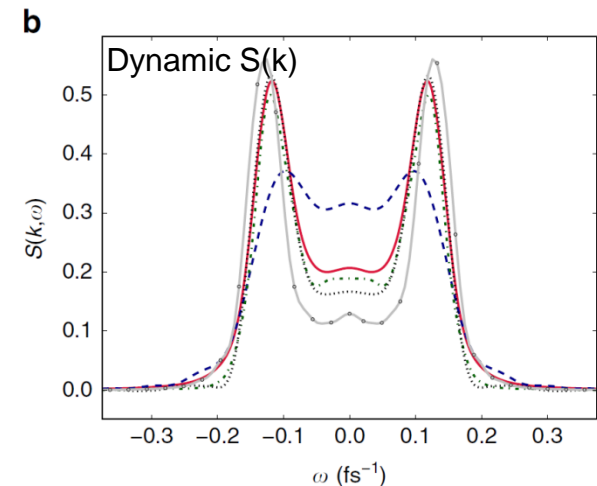
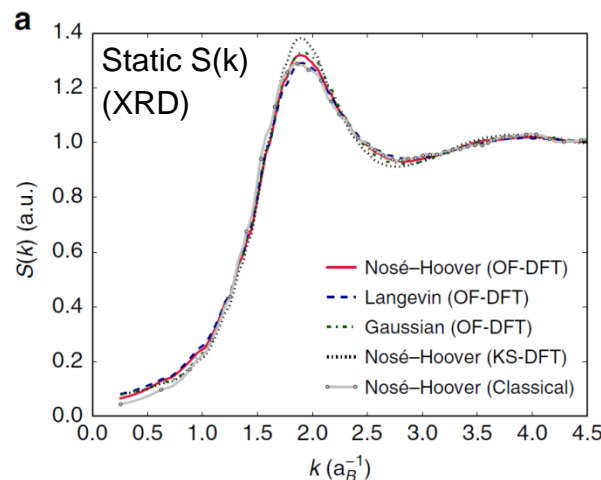
### 4 diced analyzer crystals from Si533 ( $\Delta E=25$ meV)

- 3 in forward direction,  $\rightarrow$  collective modes
- 1 in backward scattering  $\rightarrow$  Doppler broadening  $\rightarrow$  ion temperature  $T_i$

### Dynamic ion structure factor

allows accessing

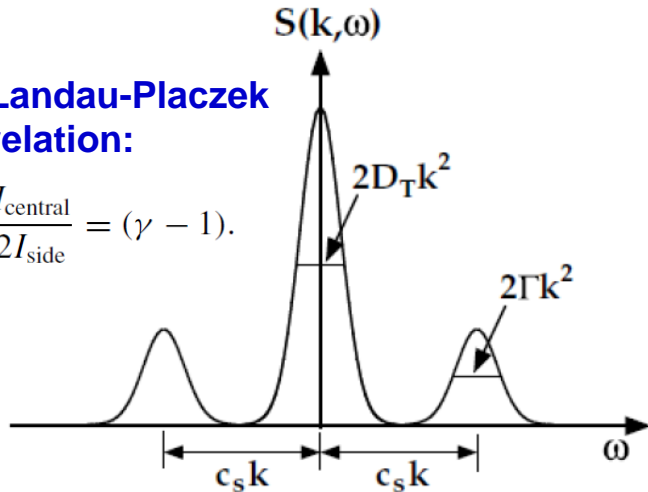
- dissipative processes
- viscosity
- thermal conductivity
- diffusive modes at  $\Delta k=0$



# Ion dynamics - hydrodynamic model

Landau-Placzek relation:

$$\frac{I_{\text{central}}}{2I_{\text{side}}} = (\gamma - 1).$$



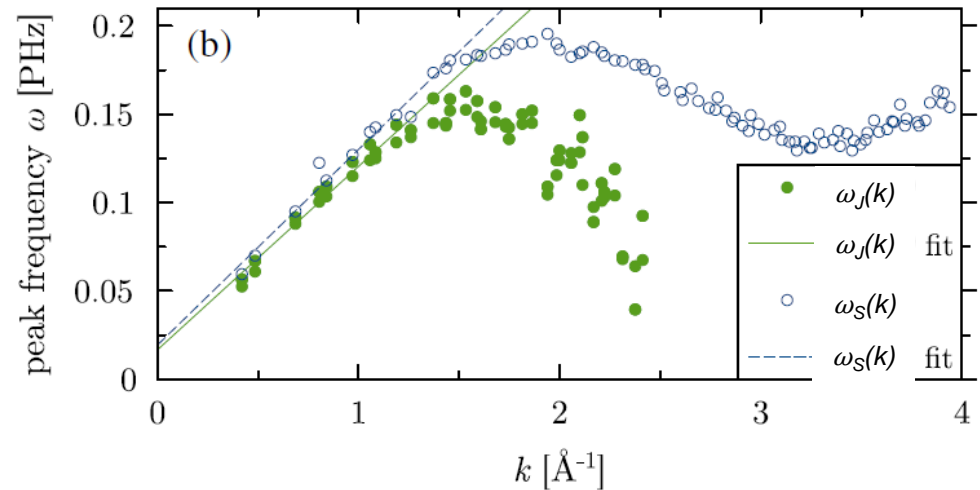
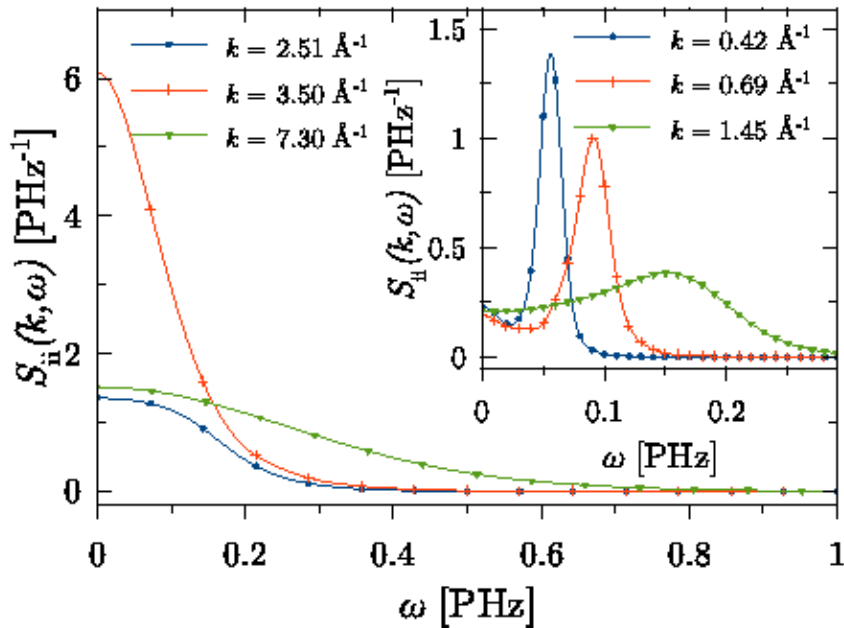
$$2\pi \frac{S_{ii}(k, \omega)}{S_{ii}(k)} = \left( \frac{\gamma - 1}{\gamma} \right) \frac{2D_T k^2}{\omega^2 + (D_T k^2)^2} + \frac{1}{\gamma} \left( \frac{\Gamma k^2}{(\omega + c_s k)^2 + (\Gamma k^2)^2} + \frac{\Gamma k^2}{(\omega - c_s k)^2 + (\Gamma k^2)^2} \right)$$

- Liquid: **Rayleigh line** centered at  $\omega = 0 \rightsquigarrow$  diffusive thermal mode
- Width given by **thermal diffusivity**  $D_T = \frac{a}{\gamma} = \frac{\lambda}{\rho c_p}$  with  $\gamma = \frac{c_p}{c_v}$  and  $a = \frac{\lambda}{\rho c_v}$
- Two **Brillouin lines** centered at  $\omega = \pm c_s k$ : ion acoustic modes with speed of sound  $c_s$
- Width given by sound attenuation coefficient  $\Gamma = \frac{a(\gamma-1)}{2\gamma} + \frac{b}{2}$
- Kinematic longitudinal **viscosity**  $b = (\frac{4}{3}\eta + \zeta)/(\rho m)$  with  $\eta$  shear and  $\zeta$  bulk viscosity
- Intensity ratio of **Rayleigh** versus **Brillouin** peaks:  $(\gamma - 1)\Gamma/D_T$

# $S_{ii}(k, \omega)$ for warm dense Al

$T=3.5$  eV,  $\rho=5.2$  g/cm<sup>3</sup>

typical for recent XRTS experiments (spectrum),  
see Ma et al. (2013), Fletcher et al. (2015)



Ion acoustic modes:  $\omega_s$

Long. current correlation function:  $\omega_l$

H.R. Rüter, RR, PRL **112**, 145007 (2014)  
Based on full Kohn-Sham DFT-MD

TABLE I. Adiabatic and apparent sound velocity for liquid (1000 K, 2.3565 g/cm<sup>3</sup>) and warm dense aluminum (40 600 K, 5.2 g/cm<sup>3</sup>).

$T$ (K)	$\rho$ (g/cm <sup>3</sup> )	$c_s$ (m/s)	$c_l$ (m/s)
1000	2.3565	4860	5010
40 600	5.2	10 380	11 070

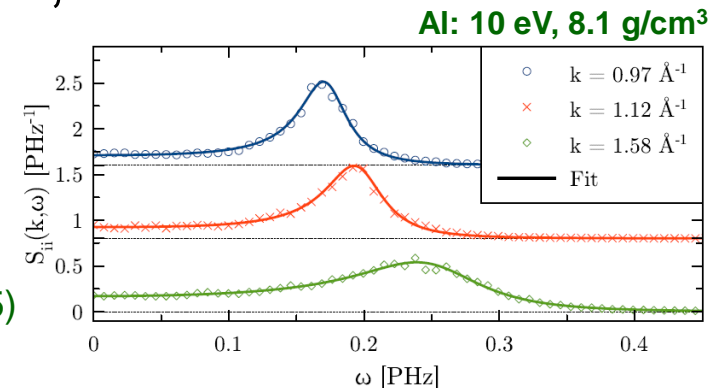
# Ion dynamics - generalized hydrodynamic model (GHM)

$$S_{ii}^{\text{GHD}}(\vec{k}, \omega) = \frac{1}{2\pi} \left( \frac{2A\alpha}{\alpha^2 + \omega^2} + \frac{B\beta}{\beta^2 + (\omega_0 + \omega)^2} + \frac{B\beta}{\beta^2 + (\omega_0 - \omega)^2} + \frac{C(\omega_0 + \omega)}{\beta^2 + (\omega_0 + \omega)^2} + \frac{C(\omega_0 - \omega)}{\beta^2 + (\omega_0 - \omega)^2} \right)$$

**A, B, C** (mode contributions: A diffusive, B: collective, C: shape),  
 **$\alpha, \beta$**  (decay coefficients),  **$\omega_0$**  (position of side peaks) depend on **k**.  
A, B, C determined via the 0th, 1st and 2nd frequency moments,  
 $\alpha, \beta, \omega_0$  are subject of a fitting procedure, see:

T. Bryk, J.F. Wax, J. Chem. Phys. **135**, 154510 (2011),  
J.F. Wax, T. Bryk, J. Phys.: CM **25**, 325104 (2013).

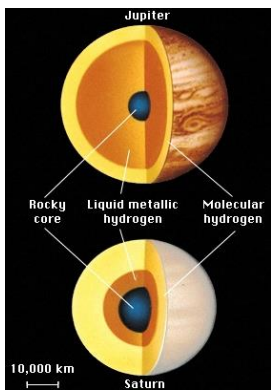
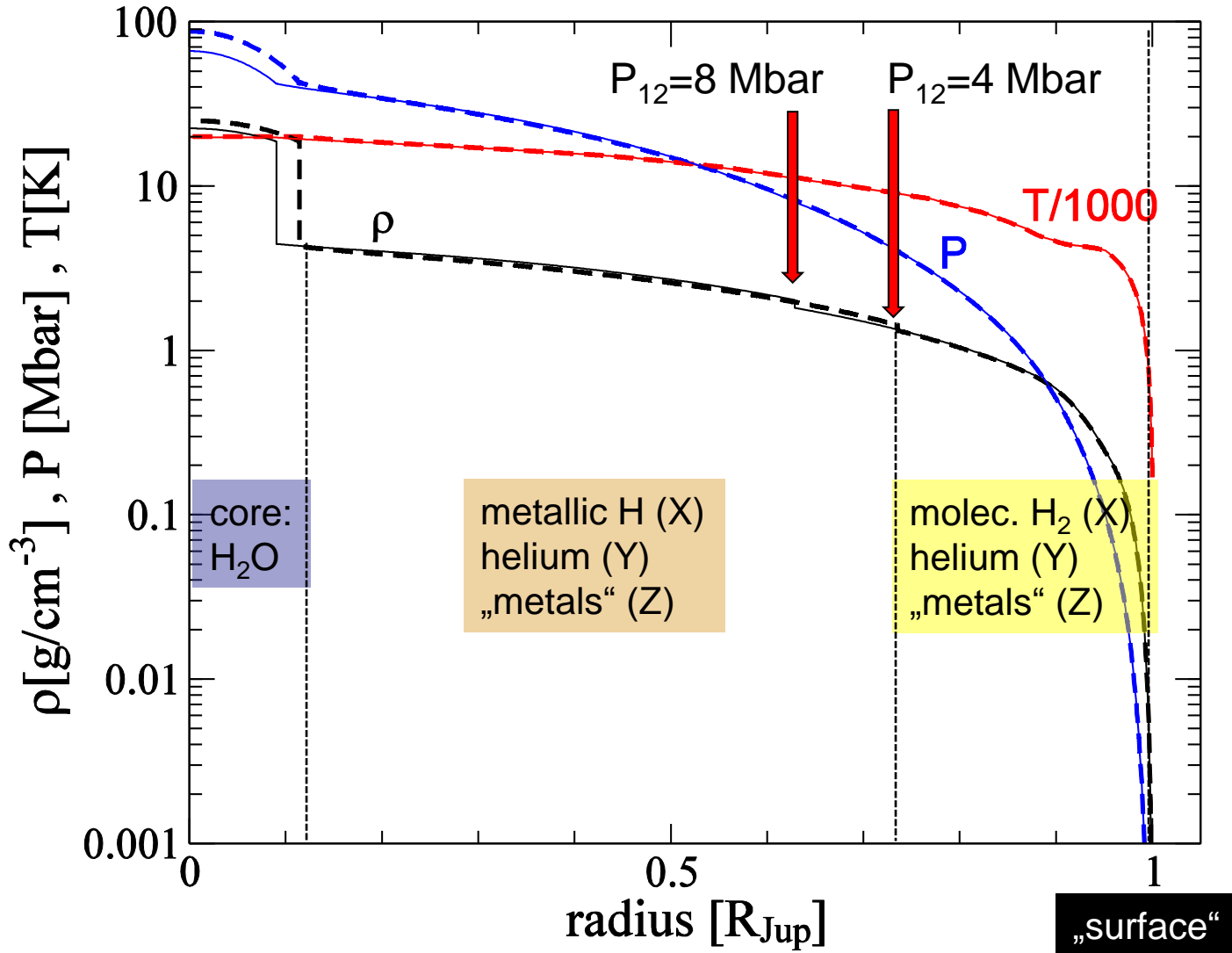
H.R. Rüter (2015)





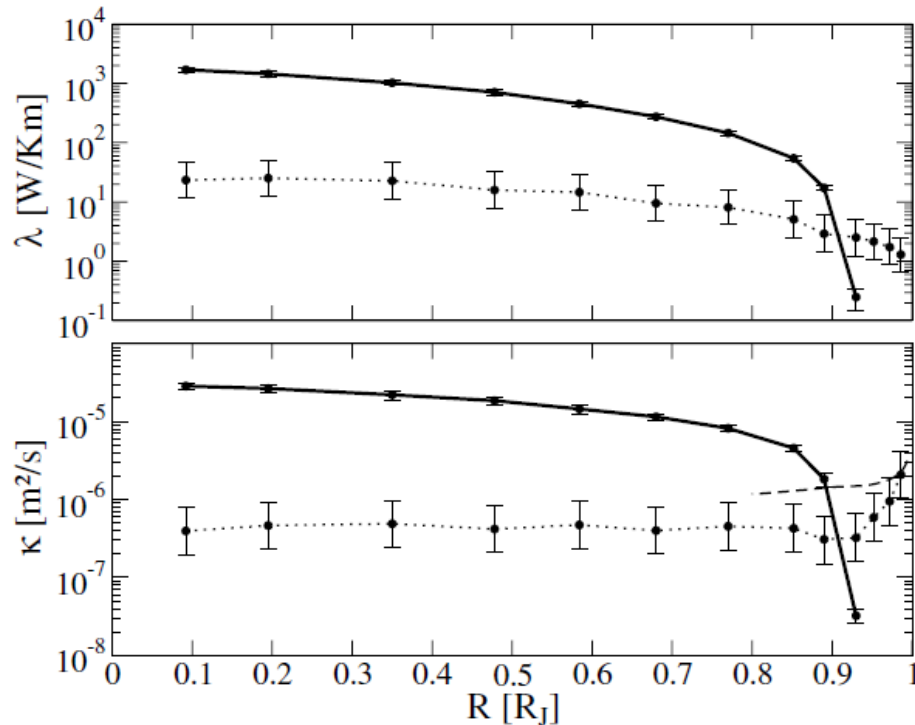
# Jupiter`s Interior with LM-REOS (H-He-H<sub>2</sub>O)

Assuming a three-layer structure



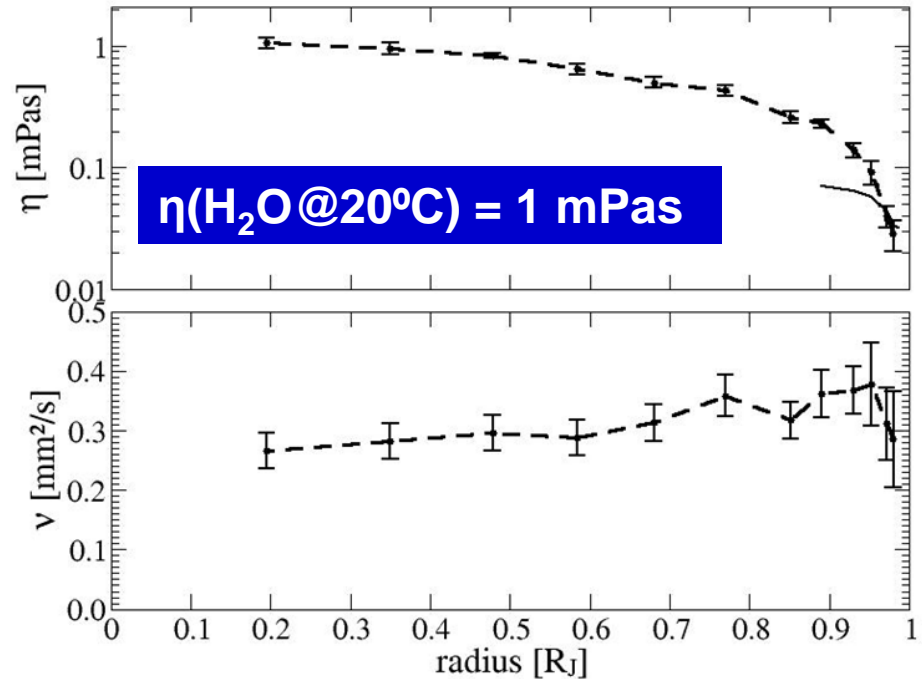
# Material Properties along Jupiter's Isentrope

M. French et al., ApJS **202**, 5 (2012): self-consistent EOS and material data from DFT-MD.  
Used for planetary modeling (interior, dynamo, evolution)



$$\kappa = \lambda / (\rho c_p)$$

Thermal conductivity  $\lambda$  and thermal diffusivity  $\kappa$  along Jupiter's isentrope.

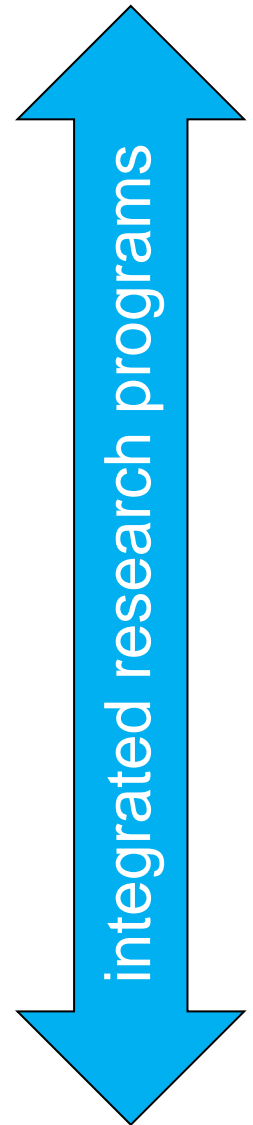


Dynamic ( $\eta$ ) and kinematic ( $\nu = \eta / \rho$ ) viscosity along Jupiter's isentrope.

$$\eta = \frac{\Omega}{3k_B T} \int_0^{\infty} dt \sum_{ij=\{xy, yz, zx\}} \langle p_{ij}(0) p_{ij}(t) \rangle$$

# Summary & Outlook

- **Fundamental properties of WDM**
  - ab initio simulations are an essential tool for
    - **EOS data and high-pressure phase diagram**
    - **Dynamic structure factor: conductivities, viscosity**
    - **Relaxation times**
- **Plasma diagnostics: light-matter interaction**
  - rad-hydro and hydro simulations
  - analyze DAC and shock wave experiments
  - analyze XRTS experiments at FELs: DSF, DF
- **Application: planetary physics - understand**
  - diversity of solar/extrasolar planets
  - interior, evolution, magnetic field (dynamo)



# Upcoming Workshops & Conferences

**16th Int. Conference on the Physics of Nonideal Plasmas (PNP-16)**

**September 24-28, 2018, Saint Malo**

**7th Joint Workshop on High Pressure, Planetary, and Plasma Physics (7HP4)**

**October 10-12, 2018, DLR Berlin**

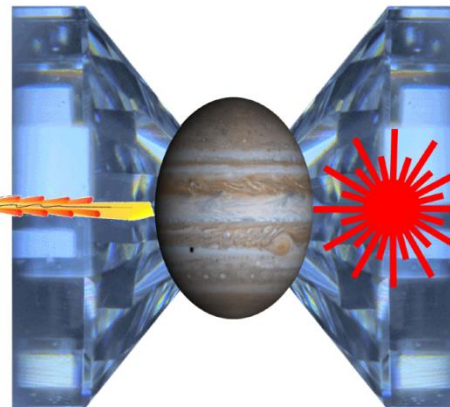
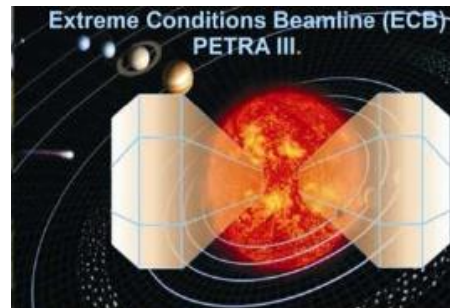
**18th Int. Workshop on Radiative Properties of Hot Dense Matter (RPHDM 2018)**

**October 21-26, 2018, DESY Hamburg**

**12th Int. Workshop on Planetary Formation and Evolution (PFE-12)**

**Feb 27 – March 1, 2019, U Rostock**

**FOR 2440 „Matter  
Under Planetary  
Interior Conditions“**



**U Rostock  
BGI Bayreuth  
DESY  
DLR Berlin  
European XFEL**