Crazy Sequential Representations: Without Subtraction and/or Division

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Introduction

Others have attempted to write the natural numbers from 1 to 11111 in terms of 1 to 9 (in increasing and decreasing order) by using the operations of addition, subtraction, multiplication, division and/or potentiation (and optionally parentheses).

For example:

Number	Increasing	Decreasing
10957	(1+2)^(3+4)*5–67+89	(9+8*7*65+4)*3–2*1
10958		(9+8*7*65+4)*3–2+1
10959	12+3+456*(7+8+9)	9+(8*76*(5+4)+3)*2*1
10960	12+(3^4+5+6)*7*(8+9)	9+(8*76*(5+4)+3)*2+1
10961	(1+2+34)*(5*6+7)*8+9	(9+8*7*65+4)*3+2*1
10962	12*3^4*5+678*9	9876+543*2*1

Generally these expressions are referred to as crazy sequential representations (CSR). Interestingly, only one CSR remains to be identified, the increasing CSR for 10958.

Previously

Authors validated the CSR as published by Inder Taneja [1,2,3,4,5] and provided eighteen corrections for the latest version of his work [5] (as publicly available on arXiv):

Increasing				
Number	**CSR by Inder Taneja**	**Error**	**Shortest CSR**	**CSR without subtraction/division**
292	1+2*3+4+56*7+89	Evaluates to 492	12+34+5*67-89	1^2+3*45+67+89
312	12+34*5+6+7+8+9	Evaluates to 212	-12-345+678-9	1^23+4*56+78+9
1548	1+2*34*(5+6)+789	Evaluates to 1538	12*3^4+56/7*8*9	12*(34+5*6+7*8+9)
2443	(9+8)*7*6+54*32+1	Not increasing	12+3*4*5*6*7-89	1+2*(3*(4+56*7+8)+9)
4498	(1+2+3^4+7*(5*6))*(8+9)	Evaluates to 4998	1^2*3+4567-8*9	-
9055	1+2*345*(6+7)+8*9	Evaluates to 9043	12/3*4*567-8-9	1+2*((3+4)*5+6*78)*9
9940	1+2*3*4*5*(6+7*8+9)	Evaluates to 8521	-12*-3*(45*6+7-8/9)	-
10637	-9*8+765*(4*3+2)-1	Not increasing	(12^3+45)*6+(7-8)^-9	-

Decreasing				
Number	**CSR by Inder Taneja**	**Error**	**Shortest Alternative**	**CSR without subtraction/division**
289	98+7+65*4+3+21	Evaluates to 389	98-76-54+321	9+87+65+4*32*1
6704	1–(2–34)*5*6*7–(8+9)	Not decreasing	-987+6^5-43*2+1	-
7683	(9*8*7+6)*5+4*3+21	Evaluates to 2583	9-8-7+6^5-43*2-1	-
8580	9+8*7+65*43*(2+1)	Evaluates to 8450	9*87+6^5^(4-3)+21	(9+8*7)*(65+4+3*21)
8989	9–8+7(6+5)*4*321	Invalid	98*76*5/4-321	-
9069	9*8*7*6(5+4)*3–2–1	Invalid	98/7*6^5/4/3-2-1	-
10498	1+23*456–(7–8)*9	Not decreasing	9876+5^4-3/(2-1)	-
10535	9+87*(6+5)(4*3)2–1	Invalid	98^(7-6)*5*43/2^1	-
10576	9+8*(7+(654+3)*2*1)	Evaluates to 10577	-9*8+(76-54)^3/(2-1)	-
10966	(1*2+3)*((4+5–6)^7+8)–9	Not decreasing	9-8+765*43/(2+1)	-

Authors provided 432 shorter CSR without subtraction and/or division, for example:

Increasing		
	CSR by Inder Taneja	**Shorter**
4306	(1^2+3)^4+5*6*(7+8)*9	1+2*(345*6+78)+9
4351	(1+2*3)^4+5*6*(7*8+9)	1+2^3*456+78*9
4402	1*(2+3*4*5)*(6+7*8+9)	1+2*3*4+56*78+9
4421	1+(2+3*4*5+6)*(7*8+9)	12^3+4+5*67*8+9
4423	1+2+34*(5+6+7*(8+9))	12+34+56*78+9
4437	1*2*34*5*(6+7)+8+9	1*2^3*456+789
4438	1+2*34*5*(6+7)+8+9	1+2^3*456+789
6320	(1+2)*(345*6+7)+89	12+345+67*89
7530	1*2*(3*(4+5)+6*7*89)	(1234+5)*6+7+89
8645	(12+3+4)*(5+(6*7+8)*9)	12+(34+56+7)*89
4306	(1^2+3)^4+5*6*(7+8)*9	1+2*(345*6+78)+9
4351	(1+2*3)^4+5*6*(7*8+9)	1+2^3*456+78*9

Decreasing		
	by Inder Taneja	**Shorter**
3119	9+8+(7*6+5)*(4^3+2)*1	9+8+7*(6+5+432)+1
3163	(98+7)*6*5+4+3^2*1	98+765*4+3+2*1
4890	9*87+6+5+4^(3+2+1)	9*8*7+65+4321
4944	(9+8+7)*(6+5*4*(3^2+1))	(987+654)*3+21
4963	(9*87+(6+5)*4)*3*2+1	9+8*(76+543)+2^1
4985	(9+8*(7*(6+5)*4+3)*2*1)	9+8*(76+543+2+1)
5027	9*(8*7+6)*(5+4)+3+2*1	(9+8+76)*54+3+2*1
5768	9*(87*6+5)+4^(3+2)+1	98+(7+65*4+3)*21
7610	(9*(8+76)+5)*(4+3+2+1)	98*7*(6+5)+43+21
8981	(98*(7+6)+5+4)*(3*2+1)	98+(76*5+43)*21
10556	(9+8)*76*5+4^(3*2)*1	98+(7*65+43)*21
10916	(9+8)*(7*6*5+4)*3+2*1	9+8+(7*65+4^3)*21

Authors provided up to 10 distinct CSR for the numbers from 0 up to 11111, example:

Increasing					
	CSR by Inder Taneja	**Shortest Overall**	**Shortest Without Division**	**Shortest Without Potentiation**	**Shortest Without Concatenation**
1531	1+(234+5)*6+7+89	12^3/4*5-6-7*89	12-3+4^5-6+7*8*9	-12*3+4*56*7+8-9	1+2^3+4^5-6+7*8*9
3650	12+34*(5+6+7+89)	1-234-5+6^7/8/9	-12-3^4+5+6*7*89	-1-23*4+5+6*7*89	(-1+2-3*4)*(5-6*7*8)+9
5263	(1+2)^3+4^5+6*78*9	1^2-3+45/6*78*9	12+(3*4+5+6*7)*89	-1+2-3+45/6*78*9	1+2+3-(-4^5*(6-7/8)-9)
7891	(1+(2*3)^4+5)*6+7+8*9	1^2+3*4*5/6*789	1+(-2-3+4+5+6)*789	-1+2+3*4*5/6*789	1*2-(-3-4^5+6*7)*8+9

Decreasing					
	CSR by Inder Taneja	**Shortest Overall**	**Shortest Without Division**	**Shortest Without Potentiation**	**Shortest Without Concatenation**
1531	98*(7+6)+5+4*3*21	9+8*765/4-3^2+1	9-8+76*5*4+3^2+1	9+8+76*5*4-3*2/1	9*8+7-6+(5+4)^3*2^1
3650	98*(7+6*5)+4*3*2*1	98/7*65*4+3^2+1	9+8*7*65-4+3+2^1	9+8*7*65-4+3+2/1	9*8*7*6+5^4+3-2/1
5263	(9*8+7)*65+4^3*2*1	987/6^(5-4)*32-1	987*6^(-5+4)*32-1	(9*8+7)*65+4*32*1	9*8+7+6^5*4/3/2*1
7891	9*8+7+6^5+4+32*1	9*876+54/3^2+1	9*876-5+4+3^2-1	9*876+5-4+3*2/1	9*8+7+6^5+4*3^2*1

Authors provided up to one CSR for numbers from 11111 up to 2147483647, for example:

Increasing	
2147483641	1+2^(-3+4+5*6)-7+8-9
2147483642	1/2^(3-4-5*6)-7-8+9
2147483643	1+2^(-3+4+5*6)-7-8+9
2147483644	-12/3-4*(-5-6+7)*8^9
2147483645	(-1+2^34+56-7)/8-9
2147483646	-1+2^34/(-5+6+7)+8-9
2147483647	-1+2^(34*5-67-8*9)

Decreasing	
2147483641	(9-8+7)^(6+5)/4-3*2-1
2147483642	(9-8+7)^(6+5)/4-3*2^1
2147483643	(9-8+7)^(6+5)/4-3-2^1
2147483644	(9-8+7)^(6+5)/4-3-2+1
2147483645	(9-8+7)^(6+5)/4-3^(2-1)
2147483646	(9-8+7)^(6+5)/4-3+2-1
2147483647	(9+8-7+6)^(5+4)/32-1

In order to prevent any ambiguity, authors defined negative counterpart definitions: Negative Crazy Sequential Representations (NCSR).

Authors provided up to 8 distinct NCSR for the numbers from 0 down to -11111, for example:

	Shortest Overall	Shortest Without Division	Shortest Without Potentiation	Shortest Without Concatenation
-3897	12*3-45-6^7/8/9	1-2^3*4*5-6*7*89	(-123-4)*5*6-78-9	1+2-3-4-5-6^7/8/9
-8182	1^23+4^5/(6-7)*8+9	1-2^3*4^5+(6-7)^8*9	-1+(-23-4)*(56*7-89)	1-2^(3+4*5-6-7)*8+9
-8650	1-2^3/4^-5-6*78+9	(-12^3+4)*5+6*7-8*9	-1*-2*(34-(56*78-9))	-1+(-2+3-4^5+6+7*8)*9
-3897	12*3-45-6^7/8/9	1-2^3*4*5-6*7*89	(-123-4)*5*6-78-9	1+2-3-4-5-6^7/8/9

	Shortest Overall	Shortest Without Division	Shortest Without Potentiation	Shortest Without Concatenation
-3897	9*(87-65/4*32)^1	-987+6-54^(3-2+1)	9*87-65*-4*(3-21)	-9+8-7-6^5/(4-3)/2-1
-8182	1^23+4^5/(6-7)*8+9	1-2^3*4^5+(6-7)^8*9	-1+(-23-4)*(56*7-89)	1-2^(3+4*5-6-7)*8+9
-8650	1-2^3/4^-5-6*78+9	(-12^3+4)*5+6*7-8*9	-1*-2*(34-(56*78-9))	-1+(-2+3-4^5+6+7*8)*9
-3897	9*(87-65/4*32)^1	-987+6-54^(3-2+1)	9*87-65*-4*(3-21)	-9+8-7-6^5/(4-3)/2-1

Authors provided up to one NCSR for numbers from -11111 down to -2147483647, for example:

Increasing	
-2147483638	1+(23-45+6)^7*8+9
-2147483639	(12*3+4-56)^7*8+9
-2147483640	-1+(23-45+6)^7*8+9
-2147483642	-1/2^(3-4-5*6)+7+8-9
-2147483643	-1-2^(-3+4+5*6)+7+8-9
-2147483645	1+2+(-3+4*5+6-7)*-8^9

Decreasing	
-2147483638	9-8^7*(6+5-43)^2+1
-2147483639	9-8^(7+6-5)*4*32^1
-2147483640	9-8^7*(6+5-43)^2-1
-2147483642	(-9+8-7)^(6+5)/4+3*2^1
-2147483643	(-9+8-7)^(6+5)/4+3+2*1
-2147483645	(-9+8-7)^(6+5)/4+3/(2-1)

Authors distinguished between genuine ₍N₎CSR (as defined by Inder Taneja) and pseudo ₍N₎CSR (less strict definition, allowing implicit multiplication by minus one). For example:

	Genuine CSR	Pseudo CSR	Pseudo CSR Expansion
388	1^23+456-78+9	-(1+2*3-4^5+6+7*89)	(-1)*(1+2*3-4^5+6+7*89)
1614	1*2*3*45*6-7-8+9	1234+5*-(6+7-89)	1234+5*(-1)*(6+7-89)
9911	((1234+5)-6+7)*8-9	12^3+4^5^-(6-7)*8-9	12^3+4^5^((-1)*(6-7))*8-9
9929	12^3-4^5/(-6+7)*-8+9	12^3+4^5/-(6-7)*8+9	12^3+4^5/(-1)*(6-7)*8+9
9733	9876-(5+4+3)^2+1	(-(-9+(-8-7)*6*54)-3)*2+1	((-1)*(-9+(-8-7)*6*54)-3)*2+1

₍N₎CSR were selected based on type (genuine versus pseudo) and length (absolute length). Genuine ₍N₎CSR were preferred over pseudo ₍N₎CSR, thus in case any genuine ₍N₎CSR was found, the shortest genuine ₍N₎CSR was chosen, also in case a shorter pseudo ₍N₎CSR was available. Only in case no genuine ₍N₎CSR was available, the shortest pseudo ₍N₎CSR was chosen.

Availability for the numbers from 0 up to 11111:

Increasing	Genuine CSR Available	Genuine CSR Unavailable	Pseudo CSR Available*	No CSR Available
Shortest Overall	11111	1	0	1
Without Division	11110	2	0	2
Without Potentiation	11053	59	50	9
Without Concatenation	10569	543	340	203

** Pseudo CSR available in case genuine CSR unavailable*

Decreasing	Genuine CSR Available	Genuine CSR Unavailable	Pseudo CSR Available*	No CSR Available
Shortest Overall	11112	0	0	0
Without Division	11108	4	2	2
Without Potentiation	11079	33	28	5
Without Concatenation	10891	221	173	48

** Pseudo CSR available in case genuine CSR unavailable*

Availability for the numbers from 11111 up to 2147483647:

	Genuine CSR Available	Pseudo CSR Available *
Increasing	544312	284380
Decreasing	767467	385935

** Pseudo CSR available in case genuine CSR unavailable*

Availability for the numbers from 0 down to -11111:

Increasing	Genuine NCSR Available	Genuine NCSR Unavailable	Pseudo NCSR Available*	No NCSR Available
Shortest Overall	11099	13	12	1
Without Division	11074	38	36	2
Without Potentiation	11029	83	74	9
Without Concatenation	10503	609	406	203

** Pseudo NCSR available in case genuine CSR unavailable*

Decreasing	Genuine NCSR Available	Genuine NCSR Unavailable	Pseudo NCSR Available*	No NCSR Available
Shortest Overall	11107	5	5	0
Without Division	11098	14	12	2
Without Potentiation	11058	54	49	5
Without Concatenation	10868	244	196	48

** Pseudo NCSR available in case genuine CSR unavailable*

Availability for the numbers from -11111 down to -2147483647:

	Genuine NCSR Available	Pseudo NCSR Available *
Increasing	535536	293157
Decreasing	681731	471671

** Pseudo NCSR available in case genuine CSR unavailable*

For various numbers, genuine CSR of equal length were identified, for example:

Increasing				Decreasing		
Result 161	**Result 174**	**Result 185**	**Result 191**	**Result 123**	**Result 124**	**Result 125**
12*3*4-5-67+89	12*3+45+6+78+9	12+3*4+5+67+89	12+34+5+67-8+9	9+8+76+54-3-21	98+76-54+3+2-1	98+76-5-43-2+1
1+23+4*56-78-9	12-3+4+5+67+89	123-4+56-7+8+9	123-45+6*7+8-9	98+76-5*4-32+1	98/7+65+43+2/1	98+76-54+3+2^1
1-23+45*6-78-9	123-4+5+67-8-9	123+4-5-6+78-9	1-234+5*67+8+9	9*8*7-6-54-321	98/7+65+43+2*1	98/7-6+54+3*21
12*3/4*56/7+89	123*4-5*67+8+9	1+234-56+7+8-9	1*23*4+5-67+89	9*87-654-3*2/1	9*87-654-3-2^1	98-7-6*5+43+21
12-3*4+5+67+89	1+2*3-456+7*89	123-4-5*6+7+89	1+23*4-56-7+89	9-87-6*5*4+321	98-7-6+54/3+21	98+7-65+43*2-1
123+4*5*6+7-89	1-234+5*67+8*9	123+4+56*7/8+9	12+34*5+6-78+9	98+7*6+5-43+21	98+76-5-43-2/1	98+76-54+3*2-1
1+234-5/6*78-9	123+4*5*6-78+9	12-3+4*5+67+89	1+23+45+67-8-9	98+76-5-43-2-1	9+87+6+54-32^1	98+76-54+3+2/1
123-4-5-6*7+89	123+4-5+6*78/9	12-3+45+6*7+89	123+4+56/7/8-9	9+87+6+54-32-1	98+76-5-43-2^1	98+7-65+4^3+21

For various numbers, genuine CSR of consecutive length were identified, for example:

	Increasing				
	Length 12	**Length 13**	**Length 14**	**Length 15**	**Length 16**
9	12-34-56+78+9	1+2-34-56+7+89	1^(23*4+5678)*9	1^(2-3*456*78)*9	1^(2-3/4*567*8)*9
11	123*4-56*7-89	1*23*4+56/7-89	1*2*34+5*6-78-9	12+3+4*(5-6)^789	1^2-3-4+5+6+7+8-9
15	123-45+6-78+9	12*3-4+5+67-89	1*2^3*4+5+67-89	1*2*3*4-56+7*8-9	12+3+(4-5-6+7)^89
18	12-34-56+7+89	1-23+45+67-8*9	12*3/4/56*7*8+9	1^2-3^4+5+6+78+9	1^2+3^4-5+6+7-8*9
25	1^23456+7+8+9	1-2+3+45+67-89	1^2+3+4-5-67+89	1*2^3/4-56+7+8*9	1^2+3-4-5+6+7+8+9
30	123+4-56/7-89	1^23*4-56-7+89	1^2^345*6+7+8+9	1^2+3^4+5*6+7-89	(1-2)^345*6*7+8*9
31	123-45+6*7-89	12*3+45-67+8+9	1*23*4+5-67-8+9	1-2*3+4/56*7*8*9	1*2*3*4+5-6+7-8+9
35	1+2345/67+8-9	1^23*4-56+78+9	1*2345*67^(8-9)	1*2-3+4/56*7*8*9	1^2+3*4+5*6-7+8-9

	Decreasing				
	Length 12	**Length 13**	**Length 14**	**Length 15**	**Length 16**
9	9+87-65-43+21	9+8-7-65+43+21	9*(8-7)^65432/1	9-8*7-6*5+43*2*1	9+8-7+6*5-4^3/2+1
11	98/7-6*54+321	9+8-7+65-43-21	98/7-65+4^3-2*1	9+8+7*6-54+3*2^1	9*8+7-6-5*4*3-2*1
15	98/7+65-43-21	98+76-54*3+2+1	98/7-65+4^3+2^1	98-7*6*5+4^3*2-1	9-8*7-6+5+4^3-2+1
18	98+7-65-43+21	98+7-65-4+3-21	9-87+6+5+4^3+21	9*8+7-65-4+3^2-1	9-8*7-6+5+4^3+2/1
25	98-76-54/3+21	9+8+7+65-43-21	9+87-6*5-43+2^1	9-8*76+5^4-3+2*1	9+8*7-6*5-4*3+2/1
30	98-7+65*4-321	98+7-6-5-43-21	98-7-65-4+3^2-1	98-7+6-5-4^3+2^1	9+8*7-6*5-4-3+2/1
31	9-876-5+43*21	98-76-5-4-3+21	98-76-5+4*3+2^1	9-8*76+5^4+3+2^1	9*8-7-6-5-4*3*2+1
35	98-76*5-4+321	98-76-5*4+32+1	9+87-65-4+3^2-1	9+8-7*6+54+3*2^1	9-8+7+6+5*4+3-2/1

For various numbers, sets of genuine CSR of equal length with specific operations at consecutive indexes were identified, for example:

Result 3 - Length 19				
Addition	**Subtraction**	**Multiplication**	**Division**	**Potentiation**
1+2-(-3+45-6*7)*89	1-(-23+4+5)*6+7-89	1*2+3-(-4-56+78)/9	1/2+3-4/56/7^(8-9)	1^234+5+6-(-7+8)*9
12+3-4*5+(-6+78)/9	12-(-3+4+56-7*8)*9	12*34-(-5-6+7*8)*9	12/(3+4+5)-6+7-8+9	12^(-3+4)-56/7+8-9
(-1+2-3)*45+6+78+9	1-(-23+4+5)*6+7-89	1-2*(-34-5)+6+7-89	1+2/(-3*4-56+78-9)	(-1^23+4*5-67)/8+9
12-3+4-5/(6+7-8)-9	12+3-4*5+(-6+78)/9	1-23*4+5+(-6+7)*89	(-12/-3+45*6-7)/89	12-3^-4*(56-7*8)-9
12/(3+4+5)-6+7-8+9	(-1+2-3)*45+6+78+9	1^(-2*345)-6+7-8+9	1+2-3/(4-5*6)*78-9	1+2-3^4*(5+67-8*9)
1-(-23+4+5)*6+7-89	1-2*(3-4)+5+67-8*9	12+3-4*5+(-6+78)/9	1+2*-3/(45+6-78)*9	1+2*34^(-5-67+8*9)
12/(3+4+5)-6+7-8+9	1*2+3-(-4-56+78)/9	12-3^-4*(56-7*8)-9	(-1-23)/4-56/7+8+9	1*2+345^(-6+7+8-9)
1-(-23+4+5)*6+7-89	1-2*(-34-5)+6+7-89	(-1+2-3)*45+6+78+9	1-2+3+45/(6+7-8)/9	(-1+2-3)^4+56-78+9
123+4+5+6+(-7-8)*9	(-1-23)/4-56/7+8+9	1-2*(34+5*6-7*8-9)	1-2*(3-45/6+7-8)-9	1*2*34-56^(-7+8)-9
(-1+2+3)*4+56-78+9	12/(3+4+5)-6+7-8+9	1+2+3+45-6*(7-8+9)	1^23^4+5-6/(7-8)-9	1^2+(-3+4)^567-8+9
(-1+2-3)*45+6+78+9	1^234+5+6-(-7+8)*9	1-(-23+4+5)*6+7-89	12-(-3+4-56/7+8)*9	12-3^4+5+67^(-8+9)
12/(3+4+5)-6+7-8+9	(-12/-3+45*6-7)/89	1+2-(-3+45-6*7)*89	(-1-23)/4-56/7+8+9	1-2*3-(-4+5)^678+9
1-(-23+4+5)*6+7-89	123+4+5+6+(-7-8)*9	12-(-3+4+56-7*8)*9	12+3-4-5/(-67/8+9)	1+23*4-(-5+6)^7-89
(-1-23)/4-56/7+8+9	12/(3+4+5)-6+7-8+9	(-1-23)/4+56-7*8+9	(-1^23+4*5-67)/8+9	1+2*3-4/(-5+6)^789
1+2+3+45-6*(7-8+9)	1-(-23+4+5)*6+7-89	1+2-(-3+45-6*7)*89	(-12/-3+45*6-7)/89	1+2*(-3-4+56/7)^89
12/(3+4+5)-6+7-8+9	1+2-3/(4-5*6)*78-9	1^234+5+6-(-7+8)*9	12+3-4*5+(-6+78)/9	1+2+34*(-56/7+8)^9

For more details, please refer to our manuscripts [6,7] (publicly available).

Background

Authors focused on CSR without subtraction and/or division because Inder Taneja preferred to publish CSR without subtraction and/or division (thus Inder Taneja solely published CSR with subtraction and/or division in case no CSR without subtraction and/or division was available). Within the latest version of his work [5] as publicly available on arXiv, CSR with subtraction and/or division were shown in italics, for example:

	Increasing CSR by Inder Taneja	Decreasing CSR by Inder Taneja
10803	*1 + 2 + 3 × (456 − 7 × 8) × 9.*	9 × 8 + 7 × (6 × 5 + 43) × 21.
10804	(1 + 2^3 × 4 × 5) × 67 + 8 + 9.	*9 − 8 × 76 + 543 × 21.*
10805	*1 × 2 + 3 × ((456 − 7) × 8 + 9).*	*(9 × 8 × 76 − 5 − 4^3) × 2 − 1.*
10806	*(1 + 2)$^{(3+4)}$ × 5 + 6 − (7 + 8) × 9.*	*(9 × 8 × 76 − 5 − 4^3) × 2 × 1.*
10807	*−12^3 + 4 × 56 × 7 × 8 − 9.*	*98 + 765 × (4 + 3) × 2 − 1.*
10808	*−1 + 2^3 × 4 × 5 × 67 + 89.*	98 + 765 × (4 + 3) × 2 × 1.
10809	1 × 2^3 × 4 × 5 × 67 + 89.	98 + 765 × (4 + 3) × 2 + 1.

Without Subtraction and/or Division

Authors assume Inder Taneja solely considered CSR without minus, subtraction and/or division characters (thus "/" and "-") to be CSR without subtraction and/or division.

Various CSR were considered CSR with subtraction and/or division by Inder Taneja, and shown in italics in the latest version of his work [5], while they do actually not contain any subtraction and/or division operations (only negative numbers).

For example, increasing CSR with subtraction and/or division (according to Inder Taneja):

Result	CSR by Inder Taneja
3980	-98+7+6*5^4+321
4487	-98+7*654+3*2+1
4954	-9+8+7*6+(5+4*3)^(2+1)
6032	-9+8*76+5432+1
6718	-9+8*7*6*5*4+3*2+1
7177	-98*7+6^5+43*2+1
7198	-98+76*(5+43)*2*1
7219	-98*7+6^5+4*32+1
7222	-987+6^5+432+1
7341	-98*7+6+(5*4)^3+21
7522	-98*7+6^5+432*1
8683	-9*8+7+6*(5+4)^3*2*1
8692	-9+876*5+4321
9022	-98+76*5*4*3*2*1
9023	-98+76*5*4*3*2+1
9056	-9*(8+7+6)+5*43^2*1
9065	-9+8*7*6*(5+4)*3+2*1
9066	-9+8+7+6^5+4*321

Result	CSR by Inder Taneja
9111	-9*8+765*4*3+2+1
9119	-9+8+76*5*4*3*2*1
9209	-9+8*(7+6+5)*4^3+2*1
9327	-9+(8+7)*6+5*43^2+1
9382	-987+6*54*32+1
9400	-98+(7+6)*(5+4)^3+21
9427	-98*76+5^4*3^(2+1)
9436	-(9+8)*7+65*(4+3)*21
9683	-98*7+6*54*32+1
10078	-987*6+(5*4)^3*2*1
10079	-987*6+(5*4)^3*2+1
10084	-9+87*(6*5+43*2)+1
10108	-98+7*6*(5+4)*3^(2+1)
10124	-9876+5^4*32*1
10138	-987+6*(5+43^2)+1
10172	-9+8*7+(6+5+4)^3*(2+1)
10228	-98*7+(6*5+4)*321
10247	-9+8+7*(6+(5+4)^3*2*1)

Result	CSR by Inder Taneja
10277	-98+7+6*54*32*1
10309	-9*8+7+6*(54*32+1)
10340	-9*8+76*(5+4*(32+1))
10367	-9+8+(76+5)*4*32*1
10390	-98+76*(5+4^3)*2*1
10411	-9+8+76*(5+4*(32+1))
10554	-9+(8+7)*(6+5)*4^3+2+1
10567	-98*7+6*5^4*3+2+1
10568	-9*8+76*5*4*(3*2+1)
10755	-9*8*7+(6*5^4+3)*(2+1)
10779	-9*(8+7)+(6*5+4)*321
10823	-98+7*65*4*3*2+1
10919	-9+8+7*65*4*3*2*1
11006	-(9+8)*7+6*(5+43^2)+1
11031	-9+8*((7*65+4)*3+2+1)
11039	-98+7+6*(5+43^2+1)
11087	-9+8+7*(6+5)*(4*3)^2*1
11110	-987+6^5+4321

For example, decreasing CSR with subtraction and/or division (according to Inder Taneja):

Result	CSR by Inder Taneja
4430	-1+2*34*56+7*89
5021	-1+2*3^4*5+6*78*9
5323	-1*2+3^4*56+789
5368	-1+2345+6*7*8*9
6374	-1^2*34+(5+67)*89
6686	-123+4*5+6789
7340	-1+(2*3+4^5+6)*7+89
7358	-1+(23*45+6)*7+8*9
7415	-1+2*3456+7*8*9
7448	-1+2*3*4*5*(6+7*8)+9
7468	-1*23*4+56*(7+8)*9
7492	-1*2*34+56*(7+8)*9
7598	-1+2*3^4*5+6789
7613	-1+2*3456+78*9
7622	-1^2+(3+4+56*(7+8))*9
7717	-12+3+4+(5+6)*78*9
7915	-12^3+4+567*(8+9)
7953	-12+3*(4+5*6)*78+9
8293	-1+2^3*4^5+6+7+89
8326	-1+2^3*(4^5+6)+78+9
8408	-1+2^(3+4)*5*(6+7)+89
8482	-1*23+45*(6+7+8)*9
8486	-1+23*(4*5+6+7+8)*9
8536	-1+2^3*4^5+6*7*8+9
8554	-1*2+3*4*(5+6+78*9)
8638	-1*2+3*4*5*6*(7+8+9)
8674	-12+3+4+(5+6)*789
8701	-1+2^3*4^5+6+7*8*9
8878	-1+2^3*4^5+678+9
8899	-1+2^3*4^5+6+78*9
8923	-1+23*(4+5*(67+8)+9)
8941	-1+2^3*(4^5+6)+78*9
8951	-1*2+(3^4+5)*(6+7)*8+9
9069	-12+3*(4+5)*6*7*8+9
9094	-1*2+3*4*(56+78*9)
9206	-1+2*3^4*56+(7+8)*9
9322	-1*23+(4+5+6)*7*89
9382	-1*23+(4^5+6+7+8)*9
9517	-1*23+4*5*(6*78+9)
9524	-1+(23*4+5*6)*78+9
9773	-1+2*3^4*56+78*9
9822	-123+45*(6+7)*(8+9)
9859	-1*2+3*4^5+6789
9932	-12*3+(45+67)*89
9933	-12+3*(45+6)*(7*8+9)
9939	-1*2*3+45*(6+7)*(8+9)
9967	-1+23+45*(6+7)*(8+9)
10036	-1+23*4*(5+(6+7)*8)+9
10087	-1*2+3*4*56*(7+8)+9
10096	-1+(23*4+5)*(6+7)*8+9
10129	-1*23+(4^5+(6+7)*8)*9
10228	-1*23+(4+5)*67*(8+9)
10273	-1+23+(4+5)*67*(8+9)
10277	-1+2*3^4*(56+7)+8*9
10312	-1+23*4*(56+7*8)+9
10348	-1*2+345*(6+7+8+9)
10390	-1*23+(4+5)*(6+7)*89
10460	-1+(2+3*4*(5+6))*78+9
10471	-1+(2^3*4+56)*7*(8+9)
10475	-1+2*(3^4*56+78*9)
10510	-1+23*(4*(56+7*8)+9)
10546	-1+2*3^4*5*(6+7)+8+9
10583	-1+23*456+7+89
10606	-123+4*5*67*8+9
10616	-1+2*(3*4+56)*78+9
10622	-1+23*456+(7+8)*9
10678	-1+23+4*(5*6+7)*8*9
10691	-1+(2*3)^4*5+6*78*9
10748	-1*2^3+4*(5*67*8+9)
10786	-1+23*(4+5*(6+78+9))
10808	-1+2^3*4*5*67+89
10877	-1+2*3*4^5+6*789
10928	-1+(2+(3+4*5)*6)*78+9
10966	-1+23+456*(7+8+9)
10995	-12+(3*45+6)*78+9
11014	-1+234*(5+6*7)+8+9
11047	-1+2^3*(4*(5*67+8)+9)
11069	-1+234*(5+6*7)+8*9
11082	-1+(2+3)*4^5+67*89
11086	-1+234*(5+6*7)+89
11108	-1+23*(4+5+6*(7+8*9))

During previous evaluation [6,7] authors limited the CSR without subtraction and/or division category to CSR without minus, subtraction and/or division characters, like Inder Taneja.

Authors were only interested in CSR without subtraction and/or division that were also shorter than the CSR without subtraction and/or division as published by Inder Taneja.

Unfortunately authors only searched for “shorter CSR without subtraction and/or division” within the “shortest overall” category, thereby missing various CSR. For example:

	CSR by Inder Taneja	Shortest Overall	Shortest Without Division	Shortest Without Potentiation	Shortest Without Concatenation
59	1*2*3*4+5+6+7+8+9	12+34-56+78-9	12*3+45+67-89	12-34-56/7+89	1+2+3+4-5*6+7+8*9
75	12*3+4+5+6+7+8+9	123+45-6-78-9	1^2345*6+78-9	12-34+56/7+89	1-2+3-4+5*6+7*8-9
97	9+8*7+6+5*4+3+2+1	9+87+65-43-21	9*87-654-32^1	9*87-654-32/1	9+8+7+6+5+4^3-2*1
108	9+8+76+5+4+3+2+1	98-76+54+32*1	98-76+54+32^1	98-76+54+32/1	9*8-7+6*5+4+3^2*1

For these numbers the “shortest overall” category does not contain any CSR without subtraction and/or division, however “shorter CSR without subtraction and/or division” do exist (just being longer than the “shortest overall” expression), for example:

	CSR by Inder Taneja	Shortest Overall	Shorter Without Subtraction/Division
59	1*2*3*4+5+6+7+8+9	12+34-56+78-9	1^2345*6*7+8+9
75	12*3+4+5+6+7+8+9	123+45-6-78-9	1^23*45+6+7+8+9
97	9+8*7+6+5*4+3+2+1	9+87+65-43-21	9+8+7*6+5+4*3+21
108	9+8+76+5+4+3+2+1	98-76+54+32*1	9+8+7+6+54+3+21

Existing Definitions

Default Notation

Notation as used by most programming languages, restricted to following characters:

1	2	3	4	5	6	7	8	9	+	-	*	/	^	(	)

Potential CSR / NCSR

Valid mathematical expression, thus well-formed interpretable syntactic construct, matching against either of the following regular expressions (using @ delimiter):

@^ [-+*/^()]* **1** [-+*/^()]* **2** [-+*/^()]* **3** [-+*/^()]* **4** [-+*/^()]* **5** [-+*/^()]* **6** [-+*/^()]* **7** [-+*/^()]* **8** [-+*/^()]* **9** [-+*/^()]* **$**
@^ [-+*/^()]* **9** [-+*/^()]* **8** [-+*/^()]* **7** [-+*/^()]* **6** [-+*/^()]* **5** [-+*/^()]* **4** [-+*/^()]* **3** [-+*/^()]* **2** [-+*/^()]* **1** [-+*/^()]* **$**

Ignoring evaluation result (natural, integer, real, rational, indeterminate, etc.).

Genuine CSR

Natural number (or zero) in terms of 1 to 9 (in increasing or decreasing order) by using the operations of addition, subtraction, multiplication, division and/or potentiation (and optionally parentheses).

Genuine NCSR

Negative integer (or zero) in terms of 1 to 9 (in increasing or decreasing order) by using the operations of addition, subtraction, multiplication, division and/or potentiation (and optionally parentheses).

Pseudo CSR

Potential non-genuine CSR evaluating to natural number (or zero).
For example, expressions with implicit multiplication by minus one.

Pseudo NCSR

Potential non-genuine NCSR evaluating to negative integer (or zero).
For example, expressions with implicit multiplication by minus one.

In terms of 1 to 9

Digits 1 to 9 occur once and in order, either in increasing or decreasing order.
Digits can be used as individual numbers (thus 1, 2, 3, 4, 5, 6, 7, 8 and 9).
Digits can be concatenated into larger numbers (for example 123, 4, 5, 6 and 789).
Negative counterparts of numbers may be used as well (also used by Inder Taneja).

For more details (and examples), please refer to our manuscripts [6,7] (publicly available).

New Definitions

CSR without subtraction and/or division

Genuine CSR without subtraction, minus and/or division characters,
thus without “/” or “-” characters for CSR in default notation.

Length

Absolute string length (based on CSR in default notation).

Aim

Using the latest version by Inder Taneja [5] as reference, optimize for length,
while respecting the preference for CSR without subtraction and/or division.

For invalid reference CSR

- Ideally identify the shortest genuine CSR without subtraction and/or division.
- Otherwise identify the shortest genuine CSR with subtraction and/or division.

For valid reference CSR with subtraction and/or division

- Ideally identify the shortest genuine CSR without subtraction and/or division.
- Otherwise identify any shorter genuine CSR with subtraction and/or division.

For valid reference CSR without subtraction and/or division

- Identify any shorter genuine CSR without subtraction and/or division

Newly identified genuine CSR will be referred to as enhanced CSR.

Final Notes

Authors consider following CSR to be proof-of-work,
as identification of CSR is computationally expensive,
while verification of CSR is computationally inexpensive.

Authors do not guaranty:

- Published NCSR are the shortest NCSR in existence.
- Published NCSR are in their simplest form.
- Unavailable NCSR do not exists.

Results

Authors counted the number of CSR without subtraction and/or division and the number of CSR with subtraction and/or division, as published by Inder Taneja [5].

Order	CSR without subtraction and/or division	CSR with subtraction and/or division	CSR unavailable	CSR invalid
Increasing	10492	611	1	8
Decreasing	10463	639	0	10

Various CSR as published by Inder Taneja [5] proofed invalid during previous [6] validation. Authors previously [6] provided seven alternatives without subtraction and/or division and eleven alternatives with subtraction and/or division.

Order	Number	Shortest Genuine CSR	Genuine CSR without subtraction/division
Increasing	292		1^2+3*45+67+89
Increasing	312		1^23+4*56+78+9
Increasing	1548		12*(34+5*6+7*8+9)
Increasing	2443		1+2*(3*(4+56*7+8)+9)
Increasing	4498	1^2*3+4567-8*9	
Increasing	9055		1+2*((3+4)*5+6*78)*9
Increasing	9940	-12*-3*(45*6+7-8/9)	
Increasing	10637	(12^3+45)*6+(7-8)^-9	
Decreasing	289		9+87+65+4*32*1
Decreasing	6704	-987+6^5-43*2+1	
Decreasing	7683	9-8-7+6^5-43*2-1	
Decreasing	8580		(9+8*7)*(65+4+3*21)
Decreasing	8989	98*76*5/4-321	
Decreasing	9069	98/7*6^5/4/3-2-1	
Decreasing	10498	9876+5^4-3/(2-1)	
Decreasing	10535	98^(7-6)*5*43/2^1	
Decreasing	10576	-9*8+(76-54)^3/(2-1)	
Decreasing	10966	9-8+765*43/(2+1)	

Authors identified one previously unknown CSR without subtraction and/or division.

	Increasing CSR by Inder Taneja	Enhanced CSR
6704	1-(2-34)*5*6*7-(8+9)	(1+2^3)^4+56+78+9

Authors identified 2577 enhanced CSR for the numbers from 1 up to 11111.

	Increasing	Decreasing
Shorter genuine CSR without subtraction and/or division	1099	1099
Shorter genuine CSR without subtraction and/or division	183	196

Authors identified 2198 shorter genuine CSR without subtraction and/or division for the numbers from 0 up to 11111 (1099 increasing and 1099 decreasing), for example:

	CSR by Inder Taneja	Enhanced CSR	CSR by Inder Taneja	Enhanced CSR
66	1*2^3+4+5*6+7+8+9	1^23456+7*8+9	9+8+7+6+(5+4+3)*(2+1)	9+8+7+6+5+4+3^(2+1)
74	1+2+3+4+5+6*7+8+9	1+23+4*5+6+7+8+9	9+8+7*6+5+4+3+2+1	9+8+7+6+5*4+3+21
87	1+2*3+4+5+6+7*8+9	1^23456*78+9	9+8*7+6+5+4+3*2+1	9+8+7+6*5+4*3+21
89	1*2+3+4+56+7+8+9	1^234567*89	9+8+7*6+5+4*3*2+1	9+8+7+6+54+3+2*1
93	1+2+3*4*5+6+7+8+9	1^2345*6+78+9	9+8+7+6+5*4*3+2+1	9+8+7+6+54+3^2*1
97	1+2+3+4*5+6+7*8+9	1^23456+7+89	9+8*7+6+5*4+3+2+1	9+8+7*6+5+4*3+21
99	1+2+3+4+5+67+8+9	1^234+5+6+78+9	9+8+7+65+4+3+2+1	9+8+7+6+5+43+21
100	1+2+3+4+5+6+7+8*9	12+34+5*6+7+8+9	9*8+7+6+5+4+3+2+1	9+8+7+6*5+43+2+1
102	12+3*4*5+6+7+8+9	1^2345*6+7+89	9+8+7+6+5+4^3+2+1	9+8+7+65+4+3^2*1
108	1+2+3+4+5+6+78+9	1^234+5+6+7+89	9+8+76+5+4+3+2+1	9+8+7+6+54+3+21
112	1*2+3*4+5+6+78+9	1^23+4+5+6+7+89	9*8+7+6+5*4+3*2+1	9+8+76+5+4*3+2^1
115	1+23+4*5+6+7*8+9	1^2345+6*7+8*9	9*8+7+6+5+4*3*2+1	9+8*7+6+5*4+3+21
118	1+23+4+5+6+7+8*9	1^234+5*6+78+9	9+87+6+5+4+3*2+1	9+8+7+6*5+43+21
119	1+2+3+4*5+6+78+9	12+3+45+6*7+8+9	9*8+7+6*5+4+3+2+1	9+8+76+5*4+3+2+1
121	1*2+3*4+5+6+7+89	1^2^34*56+7*8+9	9+8+7*6+5*4*3+2*1	9+8+7+6+5+43*2^1
122	1+2+3*4+5+6+7+89	1^234+56+7*8+9	9+87+6+5+4*3+2+1	9+8+7+65+4*3+21
124	1+2+3*4+5*6+7+8*9	1^2+34+5+67+8+9	9*8+7+6*5+4*3+2+1	9+8*7+6+5*4+32+1
128	1+2+3+4*5+6+7+89	1+23+45+6*7+8+9	9+87+6+5*4+3+2+1	9+8+7*6+5+43+21
129	12*3+4+5+67+8+9	1^23*45+67+8+9	9*8+7*6+5+4+3+2+1	98+7+6+5+4+3^2*1
130	1*2+3+4+56+7*8+9	1^23+45+67+8+9	9*8+7*6+5+4+3*2+1	98+7+6+5+4*3+2^1
131	1+2+3+4+56+7*8+9	1^2345*6*7+89	98+7+6+5+4*3+2+1	9+8+76+5+4*3+21

Authors identified 379 shorter genuine CSR with subtraction and/or division for the numbers from 0 up to 11111 (183 increasing and 196 decreasing), for example:

	CSR by Inder Taneja	Enhanced CSR	CSR by Inder Taneja	Enhanced CSR
0	12+34-56-7+8+9	1^234567+8-9	98-7-6-54-32+1	98-76-54+32*1
7	1+23-4+56-78+9	1^23456+7+8-9	98-7-6-54-3-21	98/7*65-43*21
11	1+23+4+5+67-89	1^23456-7+8+9	9+8-7+65-43-21	98/7-6*54+321
13	1-23+4-56+78+9	1-234+5*67-89	98-7-6-54+3-21	98/7-65+43+21
14	12-3-45+67-8-9	123*4-567+89	98+7-6-54-32+1	98*7-654+3-21
23	1+2-3+45+67-89	1^2345-67+89	9+87-65-4-3-2+1	-9-876+5+43*21
41	12-34-5+67-8+9	1^234-56+7+89	98-76+5-4-3+21	9*8*76-5432+1
5048	1-2+3*4*5*(6+78)+9	-123-4-(-567-8)*9	9+(8+76)*5*4*3-2+1	-9*87+6^5/4*3-2+1
10079	(1+2*(34+56)*7)*8-9	-1+2*3*45*6*7*8/9	-987*6+(5*4)^3*2+1	98/7*6*5*4*3*2-1
10670	(-1+23)*(4+56*7+89)	(12+3)^4/5+67*8+9	(98-76)*(54*3^2-1)	9-8+(76-54)^3+21
10678	-1+23+4*(5*6+7)*8*9	1^2+(3^4+56)*78-9	(9+876+5)*4*3-2*1	-987+6^5/4*3*2+1
10806	(1+2)^(3+4)*5+6-(7+8)*9	1+2-3*((-456+7)*8-9)	(9*8*76-5-4^3)*2*1	-9*(87-6*5*43)-21
10919	(1+2)^(3+4)*5-6+7-8-9	-1+234*-5*(-6-78)/9	-9+8+7*65*4*3*2*1	98/7*65*4*3-2+1
10928	-1+(2+(3+4*5)*6)*78+9	-1+234*5/6*7*8+9	-9-8+76*(5+4+3)^2+1	98/7-(6*-5-4)*321
11078	-1+2*3*(4*56+7)*8-9	1-2-3*(45+6*7*-89)	9-8*7+6*(5+43^2)+1	98/7-6*(5-43^2^1)

CSR by Inder Taneja and enhanced CSR were tabulated side-by-side (see supplement 1).

Authors identified 35493 increasing and 51948 decreasing genuine CSR without subtraction and/or division for the numbers from 11111 to 2147483647, for example:

Result	Increasing NCSR
2113929252	12*3+4^(5+6)*7*8*9
2113929339	123+4^(5+6)*7*8*9
2113930944	12^3+4^(5+6)*7*8*9
2135539721	(1+2*3)*((4+5+6)^7+8^9)
2140798983	12^(3+4)*56+7+8^9
2142770112	1*2^3*(4+5)^6*7*8*9
2142770113	1+2^3*(4+5)^6*7*8*9
2144505981	(12+3*4*5*6)*7^8+9
2144505982	1+(2+3*4*5)*6*7^8+9
2144506531	1+(2+3*4*5)*(6*7^8+9)
2147186334	((1+2)^3+4)^5*(67+8)+9

Result	Decreasing NCSR
2127482687	(9^8+(7+6)^5)*(4+3)^2+1
2128664522	(987+6*5+4)^3*2*1
2128664523	(987+6*5+4)^3*2+1
2134590003	9*(8+(76+543)^(2+1))
2138798832	9+(8+7*65+4)^3*21
2138799012	(9+(8+7*65+4)^3)*21
2140946490	((9+8)^7+65^4)*(3+2^1)
2140946491	((9+8)^7+65^4)*(3+2)+1
2143550952	(9+876+5+4)^3*(2+1)
2143588900	(9+(8*7+65)^4)*(3^2+1)
2144772911	((9+8+7*6)^5+4)*3+2^1

See supplement 2 for the increasing series and supplement 3 for the decreasing series.

Discussion

Authors noticed that five CSR as published by Inder Taneja [5] were pseudo CSR, namely:

	CSR by Inder Taneja	Expansion
9070	-(1+2^3)^4+5^6+7+8-9	(-1)*(1+2^3)^4+5^6+7+8-9
9436	-(9+8)*7+65*(4+3)*21	(-1)*(9+8)*7+65*(4+3)*21
9647	-(9-87+6+5)*(4*3)^2-1	(-1)*(9-87+6+5)*(4*3)^2-1
11006	-(9+8)*7+6*(5+43^2)+1	(-1)*(9+8)*7+6*(5+43^2)+1
11013	-(9*8-7)*6+543*21	(-1)*(9*8-7)*6+543*21

Allowing pseudo CSR to be included in the CSR with subtraction and/or division category, numerous shorter CSR with subtraction and/or division can be identified. For example:

Result	Shorter Pseudo CSR
5044	-12*-(345+678/9)
7454	-12*-(3-4-5/6+7*89)
7539	1-2^3*-(4^5+6)-78*9
8420	-12*-(3-4*5/6+78*9)
8890	1-2*-(3^4+56*78)-9
8950	-1*-2*-(3-4567+89)
9013	1^2-3*-(45*67-8)-9
9020	1-23*-(4+56*7)-89
9053	1-2*-(3-4+567*8-9)

Result	Shorter Pseudo CSR
9886	-12*-(34+5/6+789)
9886	-12*-(34+5/6+789)
9926	-1-23*-4*-(5*-6-78)-9
10096	-1-23*-(-4*5+6*78-9)
10163	1-2*-(3-(4^5-678*9))
10174	1^2-3*-(4+5*678)-9
10394	(-1-2)*-(3456+78/9)
10706	98/7+6*-54*-(32+1)
10768	9+8-7*-(-6+54)*32-1

Result	Shorter Pseudo CSR
7181	-(-123-4)*56+78-9
7364	-(-1234+5)*6+7-(8+9)
8411	-(-1234+5*6)*7-8-9
8615	-(-12^3+4)*5+67-8*9
8734	-(-1234+5-6)*7+89
9238	-(98/7-6-5*43^2-1)
9427	-(-98-76)*54+32-1

Result	Shorter Pseudo CSR
9904	-(1-(234*-5-67)*-8)+9
9904	-(1-(234*-5-67)*-8)+9
10292	-(-12^3+4-5)*6+7-89
10394	-((-1-2)*(3456+78/9))
11075	-(1-234*(56-78/9))
11108	-(1^2-34*(5*67-8))-9
11108	-(1^2-34*(5*67-8)+9)

However, as previously indicated, authors did restrict the optimization to genuine CSR.

Inder Taneja considered CSR starting with a negative number to be CSR with subtraction and/or division (as previously shown), even though they do not contain any subtraction and/or division operations (the minus character just represents a negative number).

Considering genuine CSR starting a negative number, but without any subtraction and/or division operations and without any other negative numbers, to be CSR without subtraction and/or division, various new CSR without subtraction and/or division can be identified.

	CSR by Inder Taneja	CSR starting with negative number	CSR by Inder Taneja	CSR starting with negative number
3	123-45-6-78+9	-12*3+4+5+6+7+8+9	98-76-5+4+3-21	-98+7+6*5+43+21
4	12-34-56-7+89	-12^3+4^5+6+78*9	98-7-65-43+21	-98+76+5*4+3*2^1
5	12-34+5-67+89	-1^2345*67+8*9	98-76+5-43+21	-98+7+6+5+4^3+21
6	12+34+56-7-89	-123+45+67+8+9	98-7-65+4-3-21	-987+6*54*3+21
7	1+23-4+56-78+9	-123+4+5*6+7+89	98-7-6-54-3-21	-98+7+65+4*3+21
12	123+45-67-89	-1*2*34+56+7+8+9	987-654-321	-9*8+7*6+5+4+32+1
13	1-23+4-56+78+9	-1*(23+4+56)+7+89	98-7-6-54+3-21	-98+7*6+5+43+21
14	12-3-45+67-8-9	-1^23*45+6*7+8+9	98+7-6-54-32+1	-9*8+7+65+4*3+2^1
15	123-45+6-78+9	-123+45+6+78+9	98-76-5-4+3-2+1	-98*(7+6)+5+4*321
16	1-2+34+5+67-89	-1^234567+8+9	98-7-6-5-43-21	-98+76+5+4*3+21
20	12+3-45+67-8-9	-1*(2^3+45+6)+7+8*9	98+7-65+4-3-21	-98+76+5+4+32+1
21	1-23-45+6-7+89	-123+4*5*6+7+8+9	9+87-6-5-43-21	-9*(87+6+5)+43*21
22	1-23+4-56+7+89	-1^2345*67+89	9-87+65+4+32-1	-98+76+5*4+3+21
23	1+2-3+45+67-89	-123*4+5+6+7*8*9	9+87-65-4-3-2+1	-9*8+76+5+4*3+2^1
27	12-3-45-6+78-9	-1^23*(4+56)+78+9	9-87+65+43-2-1	-9*8+7+6+54+32^1
28	12+3-4-5-67+89	-123+4*5+6*7+89	98-7+6-5-43-21	-98+76+5+43+2^1
29	12+34+5+67-89	-1^2345+6+7+8+9	9-87+65+43-2+1	-98+76+5+43+2+1
33	12+34+56-78+9	-123+4+56+7+89	98+7+6-54-3-21	-9*8+7+65+4*3+21
34	123+4-5-6+7-89	-1*23*4+5*6+7+89	9+8+76+5-43-21	-98+7*6+5+4^3+21
38	12+3+45+67-89	-1^23+4+5+6+7+8+9	98-7-6-5-43+2-1	-98+7+65+43+21

Considering genuine CSR with one or more negative numbers, but without any subtraction and/or division operations, to be CSR without subtraction and/or division, various new CSR without subtraction and/or division can be identified, for example:

	CSR by Inder Taneja	Enhanced CSR	CSR with multiple negative numbers
3585	(9+8)*7*6*5+4*3+2+1	(9+87)*(6*5+4)+321	(-98+76)*-54*3+21
5822	(9+8*7*6)*5+4^(3*2)+1	?	((9*-8+7)*-6+5432)*1
7129	9*(8*7*6+5*4*3)*2+1	9*8+(7+65+4*3)^2+1	(-98+76)*-54*3*2+1
7344	9*8*(76+5*4+3+2+1)	?	(-98+76+5)*-432^1
9201	9*(8+7*(6+5))*4*3+21	?	(-9+8)*-765*4*3+21
10656	(9*8)*(76+5+4+3*21)	?	(-987+654)*-32*1

Authors identified 16049 increasing genuine CSR and 20419 decreasing genuine CSR starting with a negative number, see supplement 4 and 5 respectively.

Authors identified 107127 increasing genuine CSR and 151611 decreasing genuine CSR with one or more negative numbers, see supplement 6 and 7 respectively.

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