

Crazy Sequential Representations: Negative Integers

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DOI

10.5281/zenodo.1288892

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Introduction

Others have attempted to write the natural numbers from 1 to 11111 in terms of 1 to 9 (in increasing and decreasing order) by using the operations of addition, subtraction, multiplication, division and/or potentiation (and optionally parentheses).

For example:

Number	Increasing	Decreasing
10957	$(1+2)^{(3+4)} * 5 - 67 + 89$	$(9+8*7*65+4)*3-2*1$
10958		$(9+8*7*65+4)*3-2+1$
10959	$12+3+456*(7+8+9)$	$9+(8*76*(5+4)+3)*2*1$
10960	$12+(3^4+5+6)*7*(8+9)$	$9+(8*76*(5+4)+3)*2+1$
10961	$(1+2+34)*(5*6+7)*8+9$	$(9+8*7*65+4)*3+2*1$
10962	$12*3^4*5+678*9$	$9876+543*2*1$

Generally these expressions are referred to as crazy sequential representations (CSR). Interestingly, only one CSR remains to be identified, the increasing CSR for 10958.

Previously

Authors validated the CSR as published by Inder Taneja ^{1,2,3,4,5} and provided eighteen corrections for the latest version of his work ⁵ (as publicly available on arXiv):

Increasing				
Number	CSR by Inder Taneja	Error	Shortest CSR	CSR without subtraction/division
292	$1+2*3+4+5+6+7+8+9$	Evaluates to 492	$12+3+4+5+6+7+8+9$	$1^2+3*4+5+6+7+8+9$
312	$12+3+4+5+6+7+8+9$	Evaluates to 212	$-12-3+4+5+6+7+8+9$	$1^2+3+4*5+6+7+8+9$
1548	$1+2*3+4*(5+6)+7+8+9$	Evaluates to 1538	$12*3+4+5+6+7+8+9$	$12*(3+4+5+6+7+8+9)$
2443	$(9+8)*7+6+5+4*3+2+1$	Not increasing	$12+3*4+5*6+7+8+9$	$1+2*(3*(4+5+6+7+8)+9)$
4498	$(1+2+3+4+7*(5*6))*(8+9)$	Evaluates to 4998	$1^2*3+4+5+6+7+8+9$	-
9055	$1+2*3+4*5*(6+7)+8+9$	Evaluates to 9043	$12/3+4*5+6+7+8+9$	$1+2*((3+4)*5+6+7+8)*9$
9940	$1+2*3*4*5*(6+7+8+9)$	Evaluates to 8521	$-12*-3*(4+5+6+7+8+9)$	-
10637	$-9*8+7+6*(4*3+2)-1$	Not increasing	$(12^3+4+5)*6+(7+8)^{-9}$	-

Decreasing				
Number	CSR by Inder Taneja	Error	Shortest Alternative	CSR without subtraction/division
289	$9+8+7+6+5+4+3+2+1$	Evaluates to 389	$98-76-54+3+2+1$	$9+8+7+6+5+4+3+2+1$
6704	$1-(2-3+4)*5+6*7-(8+9)$	Not decreasing	$-98+7+6+5-4+3+2+1$	-
7683	$(9*8*7+6)*5+4*3+2+1$	Evaluates to 2583	$9-8-7+6+5-4+3+2+1$	-
8580	$9+8*7+6+5*4+3*(2+1)$	Evaluates to 8450	$9*8+7+6+5*(4+3)+2+1$	$(9+8*7)*(6+5+4+3+2+1)$
8989	$9-8+7(6+5)*4+3+2+1$	Invalid	$98*7+6+5+4+3+2+1$	-
9069	$9*8*7*6(5+4)*3+2+1$	Invalid	$98/7+6+5+4+3+2+1$	-
10498	$1+2+3+4+5+6-(7+8)*9$	Not decreasing	$987+6+5+4+3+2+1$	-
10535	$9+8*7*(6+5)(4*3)+2+1$	Invalid	$98^8(7+6)*5+4+3+2+1$	-
10576	$9+8*(7+(6+5+4+3)*2+1)$	Evaluates to 10577	$-9*8+(76-54)^3/(2+1)$	-
10966	$(1*2+3)*((4+5+6)^7+8)-9$	Not decreasing	$9-8+7+6+5+4+3+2+1$	-

Authors identified 432 shorter CSR without subtraction and/or division, for example:

Genuine			Decreasing		
	CSR by Inder Taneja	Shorter		by Inder Taneja	Shorter
4306	$(1^2+3)^4+5*6*(7+8)*9$	$1+2*(3+4+5+6+7+8)+9$	3119	$9+8+(7*6+5)*(4+3+2)*1$	$9+8+7*(6+5+4+3+2+1)$
4351	$(1+2*3)^4+5*6*(7*8+9)$	$1+2^3*4+5+6+7+8+9$	3163	$(9+8+7)*6+5+4+3+2+1$	$98+7+6+5+4+3+2+1$
4402	$1*(2+3*4*5)*(6+7+8+9)$	$1+2*3*4+5+6+7+8+9$	4890	$9*8+7+6+5+4*(3+2+1)$	$9*8*7+6+5+4+3+2+1$
4421	$1+(2+3*4*5+6)*(7*8+9)$	$12^3+4+5+6+7+8+9$	4944	$(9+8+7)*(6+5+4*(3+2+1))$	$(98+7+6+5+4+3+2+1)$
4423	$1+2+3+4*(5+6+7*(8+9))$	$12+3+4+5+6+7+8+9$	4963	$(9*8+7+(6+5)*4)*3+2+1$	$9+8*(7+6+5+4+3+2+1)$
4437	$1*2*3+4*5*(6+7)+8+9$	$1*2^3+4+5+6+7+8+9$	4985	$(9+8*(7*(6+5)*4+3)*2+1)$	$9+8*(7+6+5+4+3+2+1)$
4438	$1+2*3+4*5*(6+7)+8+9$	$1+2^3*4+5+6+7+8+9$	5027	$9*(8*7+6)*(5+4)+3+2+1$	$(9+8+7)*5+4+3+2+1$
6320	$(1+2)*(3+4+5+6+7)+8+9$	$12+3+4+5+6+7+8+9$	5768	$9*(8*7+6+5)+4*(3+2)+1$	$98+(7+6+5+4+3+2+1)*2+1$
7530	$1*2*(3*(4+5)+6*7+8+9)$	$(12+3+4+5)*6+7+8+9$	7610	$(9*(8+7+6)+5)*(4+3+2+1)$	$98*7*(6+5)+4+3+2+1$
8645	$(12+3+4)*(5+(6*7+8)*9)$	$12+(3+4+5+6+7)*8+9$	8981	$(98*(7+6)+5+4)*(3+2+1)$	$98+(7+6+5+4+3+2+1)*2+1$
4306	$(1^2+3)^4+5*6*(7+8)*9$	$1+2*(3+4+5+6+7+8)+9$	10556	$(9+8)*7+6+5+4*(3+2)*1$	$98+(7*6+5+4+3+2+1)*2+1$
4351	$(1+2*3)^4+5*6*(7*8+9)$	$1+2^3*4+5+6+7+8+9$	10916	$(9+8)*(7*6+5+4)*3+2+1$	$9+8+(7*6+5+4+3+2+1)*2+1$

Authors provided up to 10 distinct CSR for the numbers from 0 up to 11111, for example:

Increasing					
	CSR by Inder Taneja	Shortest Overall	Shortest Without Division	Shortest Without Potentiation	Shortest Without Concatenation
1531	$1+(2+3+4+5)*6+7+8+9$	$12^3/4+5-6-7*8+9$	$12-3+4^5-6+7*8+9$	$-12*3+4*5+6+7+8+9$	$1+2^3+4^5-6+7*8+9$
3650	$12+3+4*(5+6+7+8+9)$	$1-2+3-4+5+6+7+8+9$	$-12-3+4+5+6+7*8+9$	$-1-2+3+4+5+6+7*8+9$	$(-1+2-3+4)*(5-6+7*8)+9$
5263	$(1+2)^3+4+5+6+7+8+9$	$1^2-3+4+5+6+7+8+9$	$12+(3+4+5+6+7)*8+9$	$-1+2-3+4+5+6+7+8+9$	$1+2+3-(-4+5*(6-7+8)-9)$
7891	$(1+(2*3)^4+5)*6+7+8+9$	$1^2+3*4+5+6+7+8+9$	$1+(-2-3+4+5+6)*7+8+9$	$-1+2+3*4+5+6+7+8+9$	$1*2-(-3-4+5+6*7)*8+9$

Decreasing					
	CSR by Inder Taneja	Shortest Overall	Shortest Without Division	Shortest Without Potentiation	Shortest Without Concatenation
1531	$98*(7+6)+5+4*3+2+1$	$9+8*7+6+5+4+3+2+1$	$9-8+7+6+5+4+3+2+1$	$9+8+7+6+5+4-3+2+1$	$9*8+7-6+(5+4)^3+2+1$
3650	$98*(7+6+5)+4*3+2+1$	$98/7+6+5+4+3+2+1$	$9+8*7+6+5-4+3+2+1$	$9+8*7+6+5-4+3+2+1$	$9*8*7+6+5+4+3+2+1$
5263	$(9*8+7)*6+5+4+3+2+1$	$987/6+(5-4)*3+2+1$	$987*6^-(5-4)*3+2+1$	$(9*8+7)*6+5+4+3+2+1$	$9*8+7+6+5+4+3+2+1$
7891	$9*8+7+6+5+4+3+2+1$	$9*8+7+6+5+4+3+2+1$	$9*8+7+6+5+4+3+2+1$	$9*8+7+6+5+4+3+2+1$	$9*8+7+6+5+4+3+2+1$

Authors provided up to one CSR for numbers from 11111 up to 2147483647, for example:

Increasing		Decreasing	
2147483638	$-1-(23-45+6)^7*8-9$	2147483638	$-9+8^7*(6+5-43)^2-1$
2147483639	$(12+34-5*6)^7*8-9$	2147483639	$-9+8^7*(6+5-43)^2/1$
2147483640	$1-(23-45+6)^7*8-9$	2147483640	$-9+8^7*(6+5-43)^2+1$
2147483641	$1+2^{\wedge}(-3+4+5*6)-7+8-9$	2147483641	$(9-8+7)^{\wedge}(6+5)/4-3*2-1$
2147483642	$1/2^{\wedge}(3-4-5*6)-7-8+9$	2147483642	$(9-8+7)^{\wedge}(6+5)/4-3*2^{\wedge}1$
2147483643	$1+2^{\wedge}(-3+4+5*6)-7-8+9$	2147483643	$(9-8+7)^{\wedge}(6+5)/4-3-2^{\wedge}1$
2147483644	$-12/3-4*(-5-6+7)*8^9$	2147483644	$(9-8+7)^{\wedge}(6+5)/4-3-2+1$
2147483645	$(-1+2^{\wedge}34+56-7)/8-9$	2147483645	$(9-8+7)^{\wedge}(6+5)/4-3^{\wedge}(2-1)$
2147483646	$-1+2^{\wedge}34/(-5+6+7)+8-9$	2147483646	$(9-8+7)^{\wedge}(6+5)/4-3+2-1$
2147483647	$-1+2^{\wedge}(34*5-67-8*9)$	2147483647	$(9+8-7+6)^{\wedge}(5+4)/32-1$

Authors distinguished between genuine CSR (as defined by Inder Taneja) and pseudo CSR (less strict definition, allowing implicit multiplication by minus one). For example:

	Genuine CSR	Pseudo CSR	Pseudo CSR Expansion
388	$1^{\wedge}23+456-78+9$	$-(1+2*3-4^{\wedge}5+6+7*89)$	$(-1)*(1+2*3-4^{\wedge}5+6+7*89)$
1614	$1*2*3*45*6-7-8+9$	$1234+5*(-6+7-89)$	$1234+5*(-1)*(6+7-89)$
9911	$((1234+5)-6+7)*8-9$	$12^{\wedge}3+4^{\wedge}5^{\wedge}-(6-7)*8-9$	$12^{\wedge}3+4^{\wedge}5^{\wedge}((-1)*(6-7))*8-9$
9929	$12^{\wedge}3-4^{\wedge}5/(-6+7)*-8+9$	$12^{\wedge}3+4^{\wedge}5/-(6-7)*8+9$	$12^{\wedge}3+4^{\wedge}5/(-1)*(6-7)*8+9$
9733	$9876-(5+4+3)^2+1$	$(-(-9+(-8-7)*6*54)-3)*2+1$	$((-1)*(-9+(-8-7)*6*54)-3)*2+1$

Authors preferred genuine CSR over pseudo CSR, thus pseudo CSR were only published in case no genuine CSR was available. Availability for the numbers from 0 up to 11111:

Increasing	Genuine CSR Available	Genuine CSR Unavailable	Pseudo CSR Available*	No CSR Available
Shortest Overall	11111	1	0	1
Without Division	11110	2	0	2
Without Potentiation	11053	59	50	9
Without Concatenation	10569	543	340	203

* Pseudo CSR available in case genuine CSR unavailable

Decreasing	Genuine CSR Available	Genuine CSR Unavailable	Pseudo CSR Available*	No CSR Available
Shortest Overall	11112	0	0	0
Without Division	11108	4	2	2
Without Potentiation	11079	33	28	5
Without Concatenation	10891	221	173	48

* Pseudo CSR available in case genuine CSR unavailable

Availability for the numbers from 11111 up to 2147483647:

	Genuine Available	Pseudo Available *
Increasing	544312	284380
Decreasing	767467	385935

* Pseudo CSR available in case genuine CSR unavailable

Authors provided up to one genuine CSR with reversible digits and up to one genuine CSR with reversible characters for the numbers from 0 up to 11111. For example:

Reversible Digit CSR				Reversible Character CSR			
25	$98-76-54/3+21$	$12-34-56/7+89$	59	25	$1^{\wedge}23456+7+8+9$	$9+8+7+65432^{\wedge}1$	65456
26	$12+34*5-67-89$	$98+76*5-43-21$	414	26	$12*3/4-5-67+89$	$98+76-5-4/3*21$	141
27	$12/3+45+67-89$	$98/7+65+43-21$	101	27	$12-3*4+5-67+89$	$98+76-5+4*3-21$	160
28	$123-45-67+8+9$	$987-65-43+2+1$	882	28	$1^{\wedge}234+5-67+89$	$98+76-5+432^{\wedge}1$	601
29	$12+34+5+67-89$	$98+76+5+43-21$	201	29	$123+45-67-8*9$	$9*8-76-54+321$	263
30	$123+4-56/7-89$	$987+6-54/3-21$	954	30	$9+87+6-54+3-21$	$12-3+45-6+78+9$	135
31	$123-45+6*7-89$	$987-65+4*3-21$	913	31	$9-87*6+543+2-1$	$1-2+345+6*78-9$	803
32	$12-3+45+67-89$	$98-7+65+43-21$	178	32	$12-3+45+67-89$	$98-76+54+3-21$	58
33	$12+34+56-78+9$	$98+76+54-32+1$	197	33	$98+76-54*3+21$	$12+3*45-67+89$	169
34	$12-34+5/6*78-9$	$98-76+5/4*32-1$	61	34	$1/2*34-5-67+89$	$98+76-5-43*2/1$	83
35	$98-76*5-4+321$	$12-34*5-6+789$	625	35	$12*3/4-56-7+89$	$98+7-65-4/3*21$	12

For various numbers, genuine CSR of equal length were identified, for example:

Increasing				Decreasing		
Result 161	Result 174	Result 185	Result 191	Result 123	Result 124	Result 125
$12*3*4-5-67+89$	$12*3+45+6+78+9$	$12+3*4+5+67+89$	$12+34+5+67-8+9$	$9+8+76+54-3-21$	$98+76-54+3+2-1$	$98+76-5-43-2+1$
$1+23+4*56-78-9$	$12-3+4+5+67+89$	$123-4+56-7+8+9$	$123-45+6*7+8-9$	$98+76-5*4-32+1$	$98/7+65+43+2/1$	$98/7-6+54+3*2^1$
$1-23+45*6-78-9$	$123-4+5+67-8-9$	$123+4-5-6+78-9$	$1-234+5*67+8+9$	$9*8*7-6-54-321$	$98/7+65+43+2*1$	$98/7-6+54+3*21$
$12*3/4*56/7+89$	$123*4-5*67+8+9$	$1+234-56+7+8-9$	$1*23*4+5-67+89$	$9*87-654-3*2/1$	$9*87-654-3-2^1$	$98-7-6*5+43+21$
$12-3*4+5+67+89$	$1+2*3-456+7*89$	$123-4-5*6+7+89$	$1+23*4-56-7+89$	$9-87-6*5*4+321$	$98-7-6+54/3+21$	$98+7-65+43*2-1$
$123+4*5*6+7-89$	$1-234+5*67+8*9$	$123+4+56*7/8+9$	$12+34*5+6-78+9$	$98+7*6+5-43+21$	$98+76-5-43-2/1$	$98+76-54+3*2-1$
$1+234-5/6*78-9$	$123+4*5*6-78+9$	$12-3+4*5+67+89$	$1+23+45+67-8-9$	$98+76-5-43-2-1$	$9+87+6+54-32^1$	$98+76-54+3+2/1$
$123-4-5-6*7+89$	$123+4-5+6*78/9$	$12-3+45+6*7+89$	$123+4+56/7/8-9$	$9+87+6+54-32-1$	$98+76-5-43-2^1$	$98+7-65+4^3+21$

For various numbers, genuine CSR of consecutive length were identified, for example:

Increasing					
	Length 12	Length 13	Length 14	Length 15	Length 16
9	$12-34-56+78+9$	$1+2-34-56+7+89$	$1^(23*4+5678)*9$	$1^(2-3*456*78)*9$	$1^(2-3/4*567*8)*9$
11	$123*4-56*7-89$	$1*23*4+56/7-89$	$1*2*34+5*6-78-9$	$12+3+4*(5-6)^789$	$1^2-3-4+5+6+7+8-9$
15	$123-45+6-78+9$	$12*3-4+5+67-89$	$1*2^3*4+5+67-89$	$1*2*3*4-56+7*8-9$	$12+3+(4-5-6+7)^89$
18	$12-34-56+7+89$	$1-23+45+67-8*9$	$12*3/4/56*7*8+9$	$1^2-3^4+5+6+78+9$	$1^2+3^4-5+6+7-8*9$
25	$1^23456+7+8+9$	$1-2+3+45+67-89$	$1^2+3+4-5-67+89$	$1*2^3/4-56+7+8*9$	$1^2+3-4-5+6+7+8+9$
30	$123+4-56/7-89$	$1^23*4-56-7+89$	$1^2^345*6+7+8+9$	$1^2+3^4+5*6+7-89$	$(1-2)^345*6*7+8*9$
31	$123-45+6*7-89$	$12*3+45-67+8+9$	$1*23*4+5-67-8+9$	$1-2*3+4/56*7*8*9$	$1*2*3*4+5-6+7-8+9$
35	$1+2345/67+8-9$	$1^23*4-56+78+9$	$1*2345*67^8(8-9)$	$1*2-3+4/56*7*8*9$	$1^2+3*4+5*6-7+8-9$

Decreasing					
	Length 12	Length 13	Length 14	Length 15	Length 16
9	$9+87-65-43+21$	$9+8-7-65+43+21$	$9*(8-7)^65432/1$	$9-8*7-6*5+43*2*1$	$9+8-7+6*5-4^3/2+1$
11	$98/7-6*54+321$	$9+8-7+65-43-21$	$98/7-65+4^3-2*1$	$9+8+7*6-54+3*2^1$	$9*8+7-6-5*4*3-2*1$
15	$98/7+65-43-21$	$98+76-54*3+2+1$	$98/7-65+4^3+2^1$	$98-7*6*5+4^3*2-1$	$9-8*7-6+5+4^3-2+1$
18	$98+7-65-43+21$	$98+7-65-4+3-21$	$9-87+6+5+4^3+21$	$9*8+7-65-4+3^2-1$	$9-8*7-6+5+4^3+2/1$
25	$98-76-54/3+21$	$9+8+7+65-43-21$	$9+87-6*5-43+2^1$	$9-8*76+5^4-3+2*1$	$9+8*7-6*5-4*3+2/1$
30	$98-7+65*4-321$	$98+7-6-5-43-21$	$98-7-65-4+3^2-1$	$98-7+6-5-4^3+2^1$	$9+8*7-6*5-4-3+2/1$
31	$9-876-5+43*21$	$98-76-5-4-3+21$	$98-76-5+4*3+2^1$	$9-8*76+5^4+3+2^1$	$9*8-7-6-5-4*3*2+1$
35	$98-76*5-4+321$	$98-76-5*4+32+1$	$9+87-65-4+3^2-1$	$9+8-7*6+54+3*2^1$	$9-8+7+6+5*4+3-2/1$

For various numbers, sets of genuine CSR of equal length with specific operations at consecutive indexes were identified, for example:

Result 3 - Length 19				
Addition	Subtraction	Multiplication	Division	Potentialiation
$1+2-(-3+45-6*7)*89$	$1-(-23+4+5)*6+7-89$	$1*2+3-(-4-56+78)/9$	$1/2+3-4/56/7^8(8-9)$	$1^234+5+6-(-7+8)*9$
$12+3-4*5+(-6+78)/9$	$12-(-3+4+56-7*8)*9$	$12*34-(-5-6+7*8)*9$	$12/(3+4+5)-6+7-8+9$	$12^2(-3+4)-56/7+8-9$
$(-1+2-3)*45+6+78+9$	$1-(-23+4+5)*6+7-89$	$1-2*(-34-5)+6+7-89$	$1+2/(-3*4-56+78-9)$	$(-1^23+4*5-67)/8+9$
$12-3+4-5/(6+7-8)-9$	$12+3-4*5+(-6+78)/9$	$1-23*4+5+(-6+7)*89$	$(-12/3+45*6-7)/89$	$12-3-4*(56-7*8)-9$
$12/(3+4+5)-6+7-8+9$	$(-1+2-3)*45+6+78+9$	$1^(-2*345)-6+7-8+9$	$1+2-3/(4-5*6)*78-9$	$1+2-3^4*(5+67-8*9)$
$1-(-23+4+5)*6+7-89$	$1-2*(3-4)+5+67-8*9$	$12+3-4*5+(-6+78)/9$	$1+2*3/(45+6-78)*9$	$1+2*34^(-5-67+8*9)$
$12/(3+4+5)-6+7-8+9$	$1*2+3-(-4-56+78)/9$	$12-3^4*(56-7*8)-9$	$(-1-23)/4-56/7+8+9$	$1*2+345^(-6+7+8-9)$
$1-(-23+4+5)*6+7-89$	$1-2*(-34-5)+6+7-89$	$(-1+2-3)*45+6+78+9$	$1-2+3+45/(6+7-8)/9$	$(-1+2-3)^4+56-78+9$
$123+4+5+6+(-7-8)*9$	$(-1-23)/4-56/7+8+9$	$1-2*(34+5)*6-7*8-9$	$1-2*(3-45/6+7-8)-9$	$1*2*34-56^(-7+8)-9$
$(-1+2+3)*4+56-78+9$	$12/(3+4+5)-6+7-8+9$	$1+2+3+45-6*(7-8+9)$	$1^23^4+5-6/(7-8)-9$	$1^2+(-3+4)^567-8+9$
$(-1+2-3)*45+6+78+9$	$1^234+5+6-(-7+8)*9$	$1-(-23+4+5)*6+7-89$	$12-(-3+4-56/7+8)*9$	$12-3^4+5+67^(-8+9)$
$12/(3+4+5)-6+7-8+9$	$(-12/-3+45*6-7)/89$	$1+2-(-3+45-6*7)*89$	$(-1-23)/4-56/7+8+9$	$1-2*3-(-4+5)^678+9$
$1-(-23+4+5)*6+7-89$	$123+4+5+6+(-7-8)*9$	$12-(-3+4+56-7*8)*9$	$12+3-4-5/(-67/8+9)$	$1+23*4-(-5+6)^7-89$
$(-1-23)/4-56/7+8+9$	$12/(3+4+5)-6+7-8+9$	$(-1-23)/4+56-7*8+9$	$(-1^23+4*5-67)/8+9$	$1+2*3-4/(-5+6)^789$
$1+2+3+45-6*(7-8+9)$	$1-(-23+4+5)*6+7-89$	$1+2-(-3+45-6*7)*89$	$(-12/-3+45*6-7)/89$	$1+2*(-3-4+56/7)^89$
$12/(3+4+5)-6+7-8+9$	$1+2-3/(4-5*6)*78-9$	$1^234+5+6-(-7+8)*9$	$12+3-4*5+(-6+78)/9$	$1+2+34*(-56/7+8)^9$

For more details, please refer to our previous CSR manuscript ⁶ (publicly available).

Existing Definitions

Default Notation

Notation as used by most programming languages, restricted to following characters:

1 2 3 4 5 6 7 8 9 + - * / ^ ()

Potential CSR

Valid mathematical expression, thus well-formed interpretable syntactic construct, matching against either of the following regular expressions (using @ delimiter):

@^[+-*/^]*1[-+*/^]*2[-+*/^]*3[-+*/^]*4[-+*/^]*5[-+*/^]*6[-+*/^]*7[-+*/^]*8[-+*/^]*9[-+*/^]*\$

@^[+-*/^]*9[-+*/^]*8[-+*/^]*7[-+*/^]*6[-+*/^]*5[-+*/^]*4[-+*/^]*3[-+*/^]*2[-+*/^]*1[-+*/^]*\$

Ignoring evaluation result (natural, integer, real, rational, indeterminate, etc.).

Genuine CSR

Natural number (or zero) in terms of 1 to 9 (in increasing or decreasing order) by using the operations of addition, subtraction, multiplication, division and/or potentiation (and optionally parentheses).

Pseudo CSR

Potential non-genuine CSR evaluating to **natural number** (or zero).
For example, expressions with implicit multiplication by minus one.

In terms of 1 to 9

Digits 1 to 9 occur once and in order, either in increasing or decreasing order. Digits can be used as individual numbers (thus 1, 2, 3, 4, 5, 6, 7, 8 and 9). Digits can be concatenated into larger numbers (for example 123, 4, 5, 6 and 789). Negative counterparts of numbers may be used as well (also used by Inder Taneja).

Strictness

In order to prevent any ambiguity, authors introduced negative counterpart definitions: Negative Crazy Sequential Representations (NCSR).

New Definitions

Potential NCSR

Valid mathematical expression, thus well-formed interpretable syntactic construct, matching against either of the following regular expressions (using @ delimiter):

$$\begin{aligned} & @^{\wedge}[-+*/^{\wedge}]*1[-+*/^{\wedge}]*2[-+*/^{\wedge}]*3[-+*/^{\wedge}]*4[-+*/^{\wedge}]*5[-+*/^{\wedge}]*6[-+*/^{\wedge}]*7[-+*/^{\wedge}]*8[-+*/^{\wedge}]*9[-+*/^{\wedge}]*\$ \\ & @^{\wedge}[-+*/^{\wedge}]*9[-+*/^{\wedge}]*8[-+*/^{\wedge}]*7[-+*/^{\wedge}]*6[-+*/^{\wedge}]*5[-+*/^{\wedge}]*4[-+*/^{\wedge}]*3[-+*/^{\wedge}]*2[-+*/^{\wedge}]*1[-+*/^{\wedge}]*\$ \end{aligned}$$

Ignoring evaluation result (natural, integer, real, rational, indeterminate, etc.).

Genuine NCSR

Negative integer (or zero) in terms of 1 to 9 (in increasing or decreasing order) by using the operations of addition, subtraction, multiplication, division and/or potentiation (and optionally parentheses).

Pseudo NCSR

Potential non-genuine CSR evaluating to **negative integer** (or zero). For example, expressions with implicit multiplication by minus one.

Adapted Pseudo NCSR

Any genuine CSR and pseudo CSR can be adapted to a pseudo NCSR by introducing an implicit multiplication by minus one. Authors wanted to distinguish between newly identified pseudo NCSR and pseudo NCSR based on previously published CSR. CSR published within the first four supplements of our previous CSR manuscript⁶ were used as reference for being previously published. For example:

Result	Previously Published CSR	Result	Adapted Pseudo NCSR
4179	$1+2^{(3*4)}+5*(6+7)+8+9$	-4179	$-(1+2^{(3*4)}+5*(6+7)+8+9)$
4868	$1*2*3^4*5*6+7-8+9$	-4868	$-(1*2*3^4*5*6+7-8+9)$
5739	$1^2*3*((4+5*6)*7*8+9)$	-5739	$-(1^2*3*((4+5*6)*7*8+9))$
5970	$(1+2+3)^4*5-6-7*8*9$	-5970	$-((1+2+3)^4*5-6-7*8*9)$
4179	$1+2^{(3*4)}+5*(6+7)+8+9$	-4179	$-(1+2^{(3*4)}+5*(6+7)+8+9)$
4868	$1*2*3^4*5*6+7-8+9$	-4868	$-(1*2*3^4*5*6+7-8+9)$

Distinction between pseudo NCSR and adapted pseudo NCSR was introduced to prove that authors did not “simply convert CSR for the number from 1 op to 2147483647 into pseudo NCSR for the numbers from -1 down to -2147483647”.

Aim

Identify NCSR for the numbers from -1 to -2147483647.

Selection

NCSR were selected based on type (genuine versus pseudo) and length (absolute length). Genuine NCSR were preferred over pseudo NCSR, thus in case any genuine NCSR was found, the shortest genuine NCSR was chosen, also in case a shorter pseudo NCSR was available. Only in case no genuine NCSR was available, the shortest pseudo NCSR was chosen.

Authors avoided adapted pseudo NCSR where possible, thus in case multiple qualifying pseudo NCSR (equal length pseudo NCSR) were found, the non-adapted NCSR was chosen.

For example:

Result	Previously Published CSR	Result	Newly Published NCSR
11000	$(-1-23+4)*(-567+8+9)$	-11000	$(-1+23)*(45-67*8-9)$
11001	$12+(3*45+6)*78-9$	-11001	$(1^2-3*456-7)*8-9$
11002	$(1+2)^(3+4)*5-67*(8-9)$	-11002	$((1+2)^(3+4)*5+67)/(8-9)$
11003	$12+(3*-456-7)*-8-9$	-11003	$-(1+2)^(3+4)*5-67+8-9$
11004	$1*2*3*(4^5+6*(7+8)*9)$	-11004	$1+2+(3*-45-6)*78-9$
11005	$-1*2-(3*-45-6)*78+9$	-11005	$(1/2-3*456-7)*8-9$
11006	$1-2-(3*-45-6)*78+9$	-11006	$1+2+(3*-456-7)*8-9$
11007	$1^2*(3*45+6)*78+9$	-11007	$(-12-3*45+6)*78-9$
11008	$1^2+(3*45+6)*78+9$	-11008	$1-2+(3*-45-6)*78-9$
11009	$1*2+(3*45+6)*78+9$	-11009	$((1-234+5)*6-7)*8-9$
11010	$1+2+(3*45+6)*78+9$	-11010	$12*3-(-4*-5-6)*789$
11011	$1*2+(3*456+7)*8+9$	-11011	$-1*2+(3*-456-7)*8-9$
11012	$1+2+(3*456+7)*8+9$	-11012	$-(1+2+(3*456+7)*8+9)$
11013	$(1+23)*456+78-9$	-11013	$(-1-23)*456-(78-9)$
11014	$-1+234*(5+6*7)+8+9$	-11014	$1*2+(-3-456)*(7+8+9)$
11015	$1*234*(5+6*7)+8+9$	-11015	$(-1-2-3*456-7)*8+9$

Genuine NCSR were available for 14 out of 16 negative integers.

For negative integer -11003 no genuine NCSR was identified and adaptation of previously published CSR resulted in a 21 character pseudo NCSR. Since another qualifying pseudo NCSR of equal length was available, this pseudo NCSR was published instead (light pink).

For negative integer -11012 no genuine NCSR was identified and adaptation of previously published CSR resulted in a 20 character pseudo NCSR. Since no other qualifying pseudo NCSR of equal/shorter length was found, the adapted pseudo NCSR was selected (dark pink).

NCSR for the negative integers from -1 to -11111 were categorized into four categories (shortest overall, shortest without division, shortest without potentiation and shortest without concatenation). These NCSR were also selected based on uniqueness (as compared to other categories) and validity (for other categories).

Final Notes

Authors consider following NCSR to be proof-of-work, as identification of NCSR is computationally expensive, while verification of NCSR is computationally inexpensive.

Authors do not guaranty:

- Published NCSR are the shortest NCSR in existence.
- Published NCSR are in their simplest form.
- Unavailable NCSR do not exists.

Results

No increasing NCSR was found for -10958.

Authors provide up to 8 distinct NCSR for the numbers from 0 down to -11111, for example:

	Shortest Overall	Shortest Without Division	Shortest Without Potentiation	Shortest Without Concatenation
-3897	$12*3-45-6^7/8/9$	$1-2^3*4^5-6*7*89$	$(-123-4)*5*6-78-9$	$1+2-3-4-5-6^7/8/9$
-7293	$-1-2-3*45/6^(7-8)*9$	$-1-2-3*45*6^(-7+8)*9$	$-1-2-3*-45*6/(7-8)*9$	$(-1-2*3)*(4^5-(6/7-(8+9)))$
-8182	$1^2+3+4^5/(6-7)*8+9$	$1-2^3*4^5+(6-7)^8*9$	$-1+(-23-4)*(56*7-89)$	$1-2^(3+4*5-6-7)*8+9$
-8650	$1-2^3/4^5-6*78+9$	$(-12^3+4)*5+6*7-8*9$	$-1*-2*(34-(56*78-9))$	$-1+(-2+3-4^5+6+7*8)*9$
-9423	$-123*45-6^7/8/9$	$(-1+2+3+4^5)*(6-7-8)-9$	$(-12-3)*(-4/5+6+7*89)$	$(-1+2-(3+4^5+6+7+8))*9$
-10279	$(1+2+3)^4*-56/7+89$	$12^3*((4-5)^6-7)+89$	$-12*-3*-4*(5+67)+89$	$-1-(2-3+4)^5*6*7-8*9$
-3897	$12*3-45-6^7/8/9$	$1-2^3*4^5-6*7*89$	$(-123-4)*5*6-78-9$	$1+2-3-4-5-6^7/8/9$

	Shortest Overall	Shortest Without Division	Shortest Without Potentiation	Shortest Without Concatenation
-3897	$9*(87-65/4*32)^1$	$-987+6-54^(3-2+1)$	$9*87-65*-4*(3-21)$	$-9+8-7-6^5/(4-3)/2-1$
-7293	$9*8*7-6^5/(4-3)-21$	$9*8*7-6^5(5^(4-3))-21$	$-9*-8*(7+6*-54/3)-21$	$-9*-8*7-6^5+(-4-3)*(2+1)$
-8182	$9-8*7*65/4*3^2-1$	$9-(8+7-6-5)^4*32+1$	$-98*(76+5+4-3/2)+1$	$9+8*(7-6-5-4)^3*2+1$
-8650	$98/7-6*(5-43)^2/1$	$(-987+6+5*4)*3^2-1$	$9+8+(-7-6*-5+4)*-321$	$-9*(8-7+6*-5*-4^3/2)-1$
-9423	$(-987-6-54)/3^2-2^1$	$((-987-6-54)*3^2)^1$	$-9*(87-6*(-54*3+2/1))$	$(-9+(-8-7*(6+5))*4)*3^2(2+1)$
-10279	$98-7-6^5*4/3-2^1$	$9-8*(7-65-(-4^3*21))$	$9-8*(7-6*(-5*43+2)+1)$	$-9*(((-8-7)*-6+5)*4*3+2)-1$
-3897	$9*(87-65/4*32)^1$	$-987+6-54^(3-2+1)$	$9*87-65*-4*(3-21)$	$-9+8-7-6^5/(4-3)/2-1$

See supplement 1 for the increasing series and supplement 2 for the decreasing series.

Number of increasing NCSR for the numbers from 0 down to -11111:

	Genuine Available	Genuine Unavailable	Pseudo* Available	Only** Adapted	None Available
Overall	11099	13	12	1	1
Without Division	11074	38	36	12	2
Without Potentiation	11029	83	74	12	9
Without Concatenation	10503	609	406	46	203

* Pseudo NCSR available in case genuine NCSR unavailable

** Pseudo NCSR was based on previously published CSR

For the following numbers, at least one adapted pseudo NCSR was used:

Adapted Pseudo NCSR
-4179, -4868, -5739, -5970, -6548, -6788, -6814, -6928, -6981, -6986, -7328, -7404, -7422, -7508, -7526, -7556, -8014, -8291, -8346, -8362, -8490, -8606, -8824, -8887, -8894, -9092, -9580, -9677, -9700, -9723, -9812, -9863, -9868, -9986, -10054, -10078, -10102, -10114, -10163, -10264, -10282, -10326, -10330, -10363, -10364, -10444, -10480, -10534, -10543, -10544, -10546, -10553, -10555, -10564, -10699, -10724, -10739, -10813, -10820, -10987, -11012, -11075, -11083, -11093, -11099,

Missing increasing NCSR:

	Only pseudo NCSR available	No NCSR available
Without Division	-7330, -7508, -7526, -7539, -7942, -7996, -8402, -9092, -9340, -9436, -9733, -9806, -9812, -9907, -9925, -9986, -10078, -10085, -10316, -10330, -10346, -10364, -10492, -10636, -10748, -10813, -10820, -11002, -11003, -11012, -11075, -11077, -11080, -11083, -11093, -11099	-10958, -11027
Without Potentiation	-6548, -7330, -7357, -7508, -7576, -7646, -7673, -7942, -7987, -7996, -8014, -8170, -8293, -8324, -8563, -8564, -8795, -8797, -8834, -8851, -8869, -8885, -8887, -9092, -9232, -9302, -9340, -9532, -9733, -9771, -9806, -10020, -10078, -10082, -10084, -10085, -10102, -10129, -10156, -10213, -10222, -10238, -10253, -10264, -10292, -10298, -10316, -10330, -10334, -10346, -10364, -10366, -10388, -10456, -10460, -10492, -10543, -10577, -10636, -10645, -10739, -10748, -10820, -10822, -10828, -10843, -10918, -10931, -10987, -11003, -11012, -11020, -11093, -11102	-8312, -9238, -9986, -10838, -10840, -10886, -10958, -11002, -11027
Without Concatenation	-3460, -3820, -4179, -4468, -4496, -4548, -4558, -4768, -4817, -4827, -4856, -4868, -4886, -4972, -5278, -5295, -5315, -5318, -5332, -5333, -5431, -5443, -5566, -5582, -5666, -5674, -5692, -5699, -5708, -5709, -5739, -5756, -5774, -5792, -5884, -5970, -6266, -6303, -6304, -6378, -6380, -6382, -6388, -6456, -6602, -6700, -6748, -6773, -6788, -6789, -6814, -6815, -6820, -6861, -6926, -6928, -6932, -6933, -6981, -6986, -6995, -6996, -7016, -7328, -7358, -7360, -7365, -7387, -7404, -7405, -7422, -7437, -7445, -7454, -7456, -7472, -7492, -7508, -7518, -7519, -7537, -7539, -7556, -7563, -7571, -7573, -7580, -7597, -7620, -7621, -7654, -7655, -7664, -7675, -7715, -7834, -7868, -7906, -7918, -7942, -7961, -7963, -7964, -7971, -7972, -7996, -8036, -8044, -8067, -8068, -8269, -8291, -8323, -8342, -8346, -8348, -8357, -8362, -8373, -8456, -8488, -8490, -8507, -8508, -8557, -8564, -8565, -8574, -8588, -8590, -8594, -8596, -8606, -8607, -8609, -8617, -8660, -8662, -8698, -8699, -8717, -8723, -8725, -8752, -8770, -8777, -8778, -8779, -8787, -8788, -8798, -8822, -8824, -8833, -8834, -8836, -8842, -8843, -8866, -8888, -8890, -8894, -8897, -8943, -8944, -8957, -8980, -8986, -8987, -8989, -9038, -9093, -9122, -9238, -9247, -9422, -9427, -9436, -9442, -9447, -9491, -9507, -9526, -9553, -9560, -9566, -9580, -9581, -9582, -9591, -9592, -9597, -9625, -9632, -9634, -9654, -9677, -9678, -9680, -9682, -9683, -9686, -9700, -9723, -9731, -9762, -9763, -9785, -9786, -9807, -9812, -9821, -9822, -9824, -9832, -9853, -9863, -9866, -9867, -9868, -9872, -9884, -9890, -9896, -9903, -9907, -9914, -9917, -9919, -9921, -9922, -9924, -9925, -9932, -9934, -9939, -9941, -9958, -9974, -9980, -9988, -10003, -10006, -10014, -10015, -10022, -10045, -10048, -10054, -10055, -10057, -10059, -10070, -10074, -10078, -10094, -10096, -10100, -10114, -10117, -10119, -10121, -10130, -10138, -10163, -10226, -10227, -10228, -10244, -10253, -10254, -10264, -10266, -10274, -10275, -10282, -10283, -10307, -10326, -10338, -10344, -10346, -10356, -10363, -10364, -10372, -10379, -10382, -10388, -10444, -10445, -10462, -10470, -10471, -10480, -10482, -10484, -10490, -10491, -10494, -10495, -10498, -10510, -10518, -10524, -10525, -10526, -10534, -10535, -10542, -10544, -10545, -10546, -10550, -10551, -10553, -10554, -10555, -10559, -10564, -10569, -10573, -10576, -10578, -10579, -10590, -10592, -10596, -10599, -10600, -10607, -10609, -10614, -10619, -10623, -10627, -10630, -10635, -10643, -10667, -10668, -10675, -10677, -10696, -10697, -10698, -10699, -10706, -10724, -10726, -10748, -10754, -10768, -10778, -10793, -10797, -10802, -10807, -10811, -10820, -10821, -10823, -10828, -10830, -10832, -10839, -10846, -10849, -10861, -10865, -10866, -10867, -10869, -10877, -10884, -10886, -10892, -10907, -10909, -10924, -10939, -10964, -10967, -10968, -10969, -10973, -10978, -10995, -11000, -11003, -11011, -11022, -11045, -11046, -11048, -11056, -11062, -11064, -11073, -11075, -11080, -11091, -11092, -11093, -11098, -11106	-4948, -5276, -5296, -5307, -5522, -5710, -5773, -5794, -5812, -6746, -6819, -6836, -6964, -6980, -6982, -7322, -7330, -7394, -7408, -7414, -7436, -7438, -7466, -7468, -7538, -7574, -7576, -7594, -7598, -7643, -7645, -7646, -7898, -7915, -7916, -7958, -7960, -7986, -8012, -8438, -8486, -8548, -8555, -8556, -8572, -8573, -8597, -8608, -8635, -8692, -8780, -8782, -8786, -8803, -8818, -8851, -8867, -8896, -8898, -8962, -8966, -9050, -9113, -9424, -9428, -9445, -9446, -9448, -9452, -9470, -9506, -9508, -9533, -9535, -9542, -9562, -9572, -9573, -9574, -9578, -9598, -9607, -9608, -9616, -9626, -9643, -9644, -9646, -9650, -9655, -9687, -9688, -9706, -9713, -9733, -9740, -9788, -9808, -9813, -9814, -9885, -9886, -9893, -9902, -9908, -9915, -9923, -9938, -9947, -9948, -9949, -9977, -9978, -9986, -10012, -10040, -10058, -10068, -10069, -10076, -10091, -10093, -10102, -10118, -10316, -10317, -10342, -10357, -10402, -10436, -10451, -10453, -10468, -10469, -10474, -10475, -10516, -10552, -10568, -10571, -10572, -10577, -10588, -10589, -10597, -10598, -10604, -10606, -10613, -10615, -10618, -10621, -10633, -10634, -10636, -10645, -10669, -10678, -10715, -10733, -10757, -10759, -10771, -10784, -10805, -10813, -10825, -10831, -10833, -10834, -10838, -10843, -10844, -10847, -10852, -10856, -10858, -10860, -10868, -10958, -10970, -10972, -10986, -10987, -10991, -10996, -11002, -11012, -11036, -11037, -11038, -11047, -11054, -11055, -11066, -11068, -11072, -11077, -11081, -11082, -11083, -11095, -11108

Number of decreasing NCSR for the numbers from 0 down to -11111:

	Genuine Available	Genuine Unavailable	Pseudo * Available	Only ** Adapted	None Available
Overall	11107	5	5	1	0
Without Division	11098	14	12	5	2
Without Potentiation	11058	54	49	8	5
Without Concatenation	10868	244	196	15	48

* Pseudo NCSR available in case genuine NCSR unavailable

** Pseudo NCSR was based on previously published CSR

For the following numbers, at least one adapted pseudo NCSR was used:

Adapted Pseudo NCSR
-5276, -6351, -6595, -6872, -8894, -9020, -9274, -9508, -9524, -9616, -9634, -9949, -10174, -10330, -10526, -10550, -10553, -10604, -10645, -10706, -10731, -10739, -10810, -10815, -10882, -11104,

Missing decreasing NCSR:

	Only pseudo NCSR available	No NCSR available
Without Division	-9508, -9524, -9644, -9767, -9949, -10068, -10526, -10706, -10732, -10802, -10832, -10988	-9668, -11038
Without Potentiation	-7082, -7108, -7171, -7220, -7222, -8122, -8123, -8467, -8539, -8894, -9412, -9508, -9524, -9596, -9616, -9634, -9644, -9650, -9670, -9733, -9767, -10068, -10076, -10085, -10156, -10280, -10309, -10317, -10334, -10411, -10484, -10492, -10526, -10546, -10550, -10553, -10558, -10604, -10636, -10645, -10670, -10732, -10768, -10832, -10882, -10972, -11003, -11038, -11056	-8258, -9668, -9970, -10316, -10498
Without Concatenation	-5276, -5278, -5377, -5387, -5963, -6277, -6285, -6286, -6314, -6351, -6366, -6378, -6388, -6402, -6412, -6415, -6419, -6422, -6437, -6467, -6593, -6595, -6610, -6676, -6872, -6926, -6938, -6944, -6980, -7044, -7045, -7052, -7076, -7077, -7082, -7093, -7413, -7466, -7541, -8309, -8310, -8364, -8428, -8444, -8446, -8482, -8490, -8518, -8542, -8552, -8553, -8562, -8563, -8564, -8573, -8696, -8921, -8933, -8957, -8962, -8974, -8979, -8980, -8988, -9020, -9022, -9031, -9032, -9037, -9067, -9104, -9105, -9114, -9115, -9122, -9123, -9131, -9133, -9148, -9155, -9170, -9172, -9176, -9274, -9355, -9357, -9364, -9406, -9415, -9416, -9428, -9436, -9445, -9493, -9497, -9515, -9518, -9524, -9527, -9543, -9562, -9572, -9573, -9574, -9616, -9628, -9629, -9637, -9646, -9651, -9663, -9669, -9692, -9704, -9733, -9746, -9813, -9820, -9821, -9824, -9853, -9876, -9886, -9915, -9922, -9931, -9971, -10054, -10068, -10084, -10142, -10168, -10172, -10174, -10177, -10222, -10244, -10245, -10253, -10267, -10309, -10310, -10312, -10317, -10330, -10424, -10459, -10461, -10479, -10491, -10492, -10493, -10514, -10515, -10520, -10542, -10555, -10561, -10565, -10588, -10590, -10604, -10605, -10689, -10714, -10731, -10734, -10739, -10754, -10756, -10762, -10776, -10798, -10810, -10811, -10815, -10833, -10857, -10859, -10860, -10861, -10868, -10910, -11003, -11021, -11028, -11029, -11030, -11055, -11056, -11057, -11058, -11092, -11095, -11104, -11110	-6404, -6523, -7034, -7078, -8363, -8492, -8572, -8944, -8989, -9508, -9542, -9556, -9578, -9634, -9643, -9644, -9678, -9885, -10052, -10069, -10460, -10474, -10480, -10516, -10526, -10544, -10564, -10597, -10671, -10715, -10732, -10733, -10741, -10742, -10751, -10753, -10757, -10758, -10775, -10784, -10814, -10831, -10832, -10858, -11006, -11044, -11066, -11068

Authors provide up to one NCSR for numbers from -11111 down to -2147483647. For example:

Result	Increasing NCSR	Result	Decreasing NCSR
-2147483638	$1+(23-45+6)^{7*8+9}$	-2147483638	$9-8^{7*(6+5-43)^{2+1}}$
-2147483639	$(12*3+4-56)^{7*8+9}$	-2147483639	$9-8^{(7+6-5)*4*32^1}$
-2147483640	$-1+(23-45+6)^{7*8+9}$	-2147483640	$9-8^{7*(6+5-43)^{2-1}}$
-2147483642	$-1/2^{(3-4-5*6)+7+8-9}$	-2147483642	$(-9+8-7)^{(6+5)/4+3*2^1}$
-2147483643	$-1-2^{(-3+4+5*6)+7+8-9}$	-2147483643	$(-9+8-7)^{(6+5)/4+3+2*1}$
-2147483645	$1+2+(-3+4*5+6-7)*-8^9$	-2147483645	$(-9+8-7)^{(6+5)/4+3/(2-1)}$
-2147483648	$-1*2^3/4*56/7*8^9$	-2147483647	$9-8-(7-6+5-4)^{(32-1)}$

See supplement 3 for the increasing series and supplement 4 for the decreasing series.

Number of NCSR for the numbers from -11111 down to -2147483647:

	Genuine Available	Pseudo Available *	Only Adapted **
Increasing	535536	293157	16365
Decreasing	681731	471671	48083

* Pseudo NCSR available in case genuine NCSR unavailable

** Pseudo NCSR was based on previously published CSR

Miscellaneous

During NCSR evaluation authors also identified 34 shorter genuine CSR, as compared to the genuine CSR published in our previous CSR manuscript.

Result	Increasing CSR	Result	Decreasing CSR
255449	$(-1-23)*4^5+6^7+89$	165912	$9+8+(7+6^5*4^3/(2+1))$
274728	$1+(-2-3)*4^5+6^7-89$	531578	$(9/(8-7))^{6+5+4*(32+1)}$
394730	$(1*2^3)^4+(5/(6-7))^8+9$	1134008	$9-8+7*((6*5)^4/(3+2)+1)$
397195	$(12-3)^4+(5/(6-7))^8+9$	5198105	$98*(7+6)^5/(4+3)+2+1$
781098	$1-2*(3^4-(5/(6-7))^8)+9$	7528297	$(-98/7)^6-(-5+4^3)*21$
781192	$1-2*(34-(5/(6-7))^8)+9$	15120017	$9+8*(7*(6*5)^4+3)/(2+1)$
836645	$-1*2-3*(4^5-6^7)-89$	33064584	$9^8-(-7+6^5*4)*321$
1562533	$1+23+4*(5/(6-7))^8+9$	36025140	$9^8-(-7+6^5*43)*21$
3125004	$1-2*(3-4*(5/(6-7))^8)+9$	41304001	$9^8-7*(6^5+4)*32*1$
5763010	$(-12-3)*-4*-5*6+7^8+9$	42525260	$9^8+(-7-6^5)*(4+3*21)$
8984293	$1-23*(4-(5/(6-7))^8)+9$	42884729	$9^8+7-(6*5)^4/(3+2)+1$
9437201	$(1+23)^{(4+5)}/6^7+8+9$	42982881	$9^8-7*(6^5+4^3*21)$
134193219	$(-1-23)*4^5+6^7+8^9$	43068331	$9^8+(7^6/(5+4/3^2)+1)$
134202435	$(-12-3)*4^5+6^7+8^9$	44180720	$9^8+(7*(6*5)^4/(3+2)-1)$
195471973	$-1*2-34*(5^6-7^8)-9$	56606412	$9*(8^7+6-5^4)*3+21$
298959324	$-12*(3+4^5-6^7*89)$	387538597	$((9/(8-7)-6)^{(5+4)+3})^{2+1}$
478473089	$1+(-2-3^4)*(56-(7^8-9))$	603801505	$9*(8^7+6-5^4)*32+1$

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