Bayesian modelling of the ionosphere

Long baseline workshop

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Motivation

There is a *gap* between the physical description of the ionosphere, and the methods we use to mitigate it.

… and a Bayesian perspective deals well with noise dominated data.

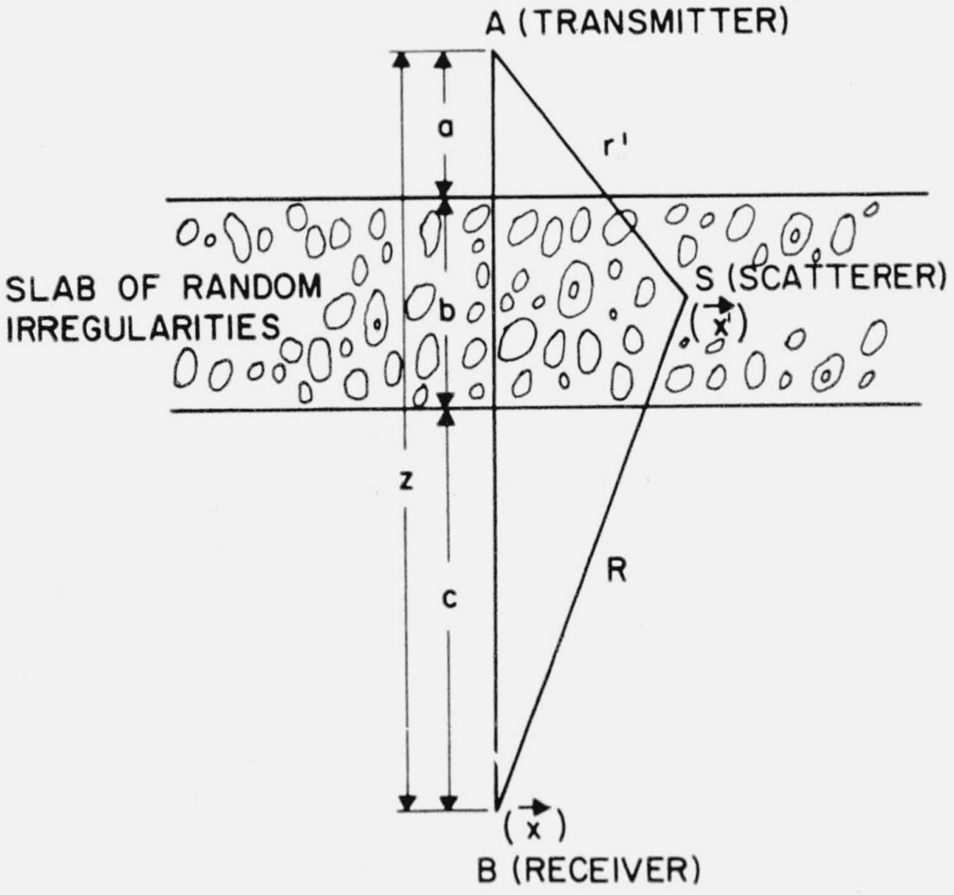
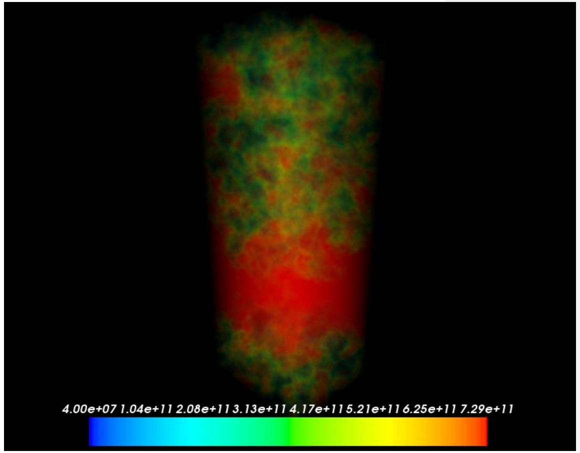


FIGURE 1. *Geometry of the problem.*

Description

A single scattering event produces asymptotic Gaussian correlations on the ground.

Is this approximation valid?
SPAM-like approaches rely on it and break down.



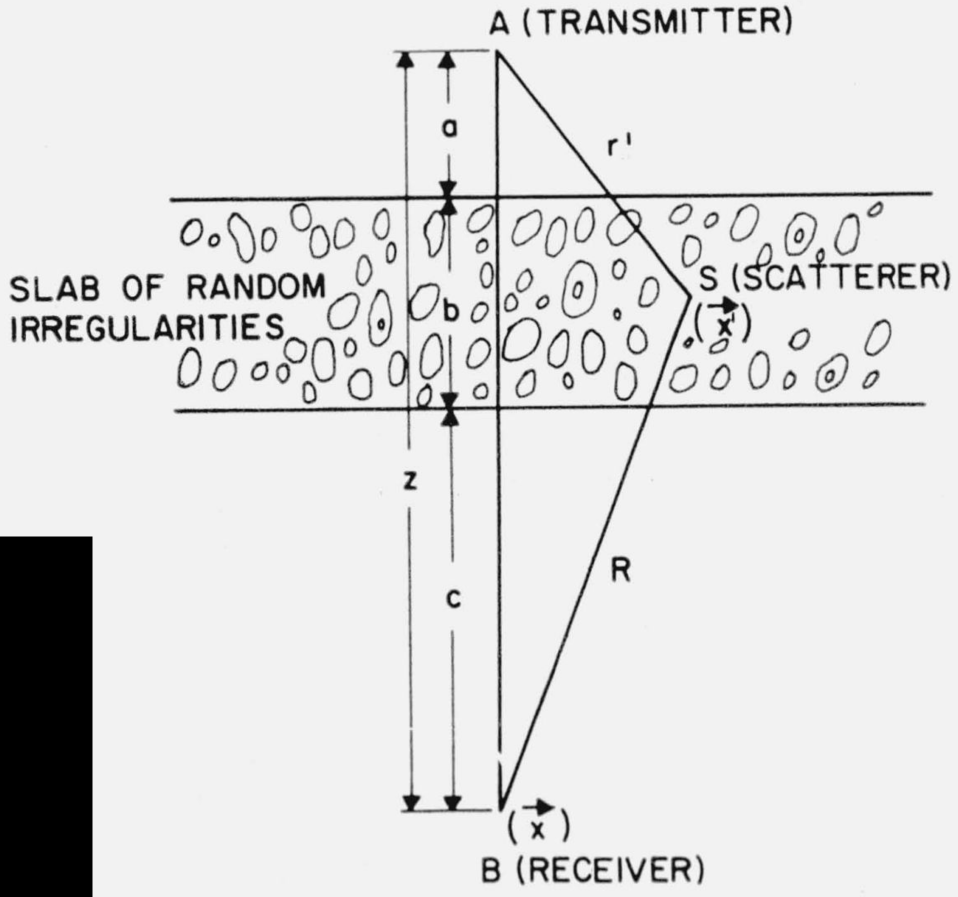


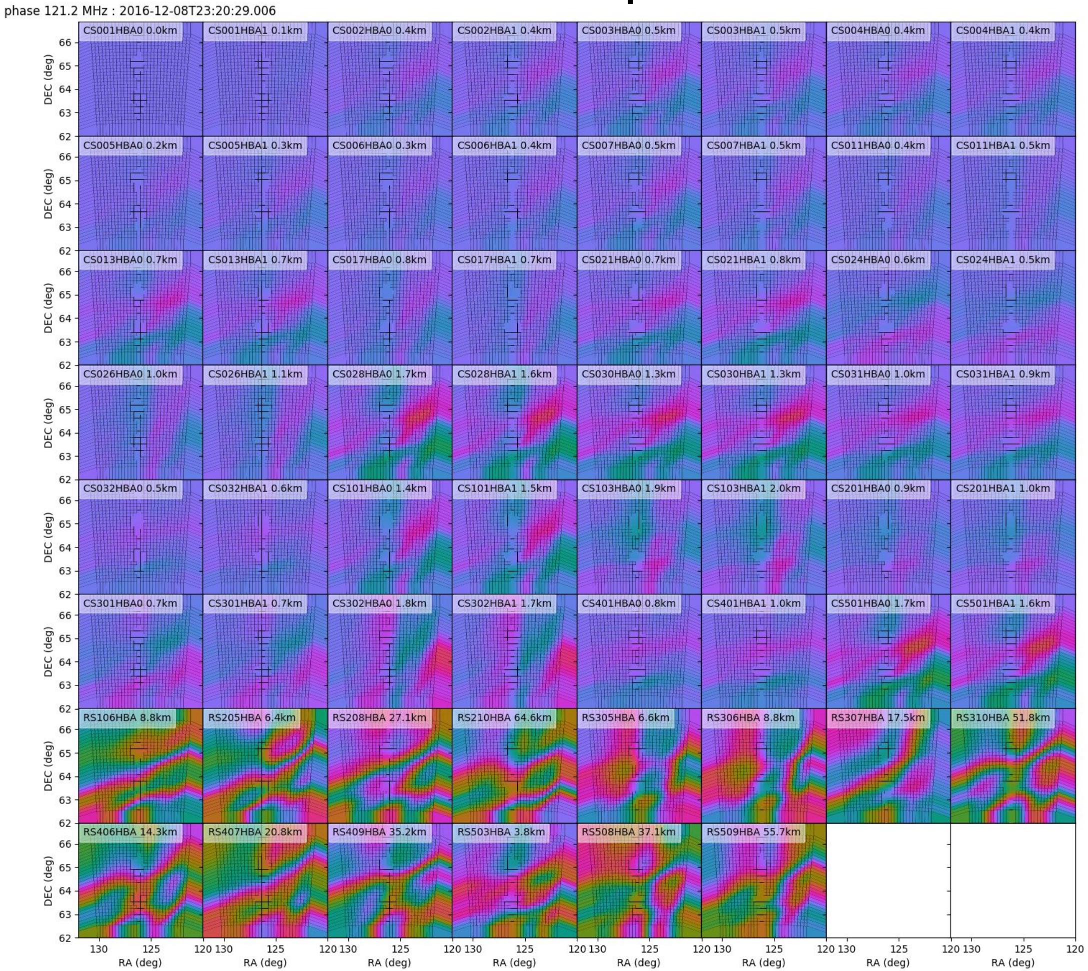
FIGURE 1. *Geometry of the problem.*

Phase screens

1. Continuous over the directional field
2. Absolute phase encodes geometric properties

Homogeneity occurs when the ionosphere behave similarly over all antennas.

Simulated phase screens



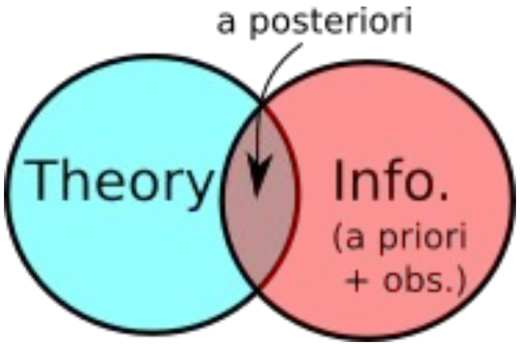
Bayesian phase screens

let $\Theta(\mathbf{V}|\phi, X)$ be the *forward equation* distribution

Assume that the RIME is a perfect theory

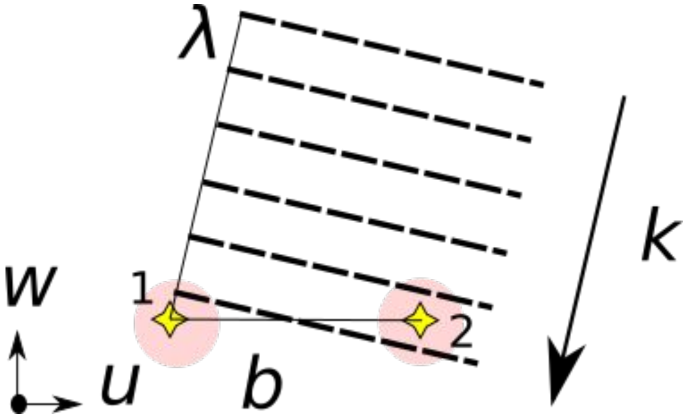
$$\Theta(\mathbf{V}|\phi, X) = \delta(\mathbf{V} - \mathbf{g}_V(\phi, X))$$

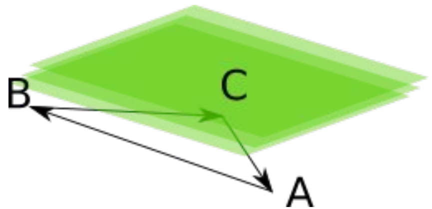
$$P_\phi(\phi^{\mathrm{inf}}|\mathbf{V}^{\mathrm{obs}}, X) \propto Q_\phi(\phi^{\mathrm{inf}}) \int \Theta(\mathbf{V}|\phi^{\mathrm{inf}}, X) \wedge P_V(\mathbf{V})\, d\mathbf{V}$$



Clock+TEC separation -> high variance estimate

Sky brightness is so large we are in a noise dominated regime.





Clock and Propagation terms

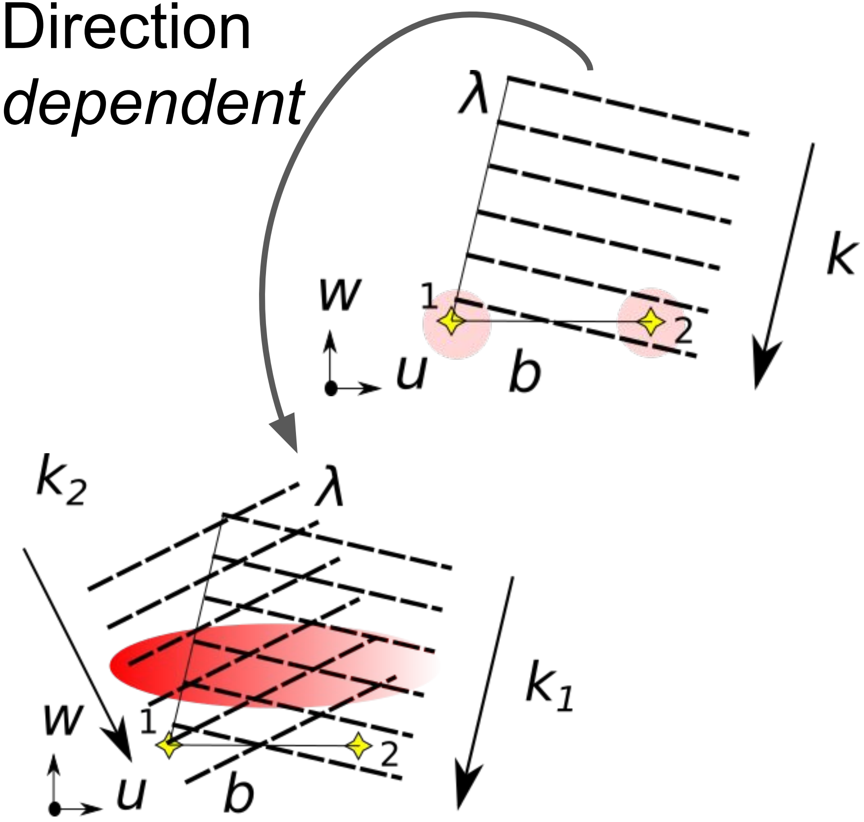
$$\phi_{A,\alpha} \approx 2\pi\nu\tau_A + \frac{C}{\nu}\mathrm{TEC}_{A,\alpha}$$

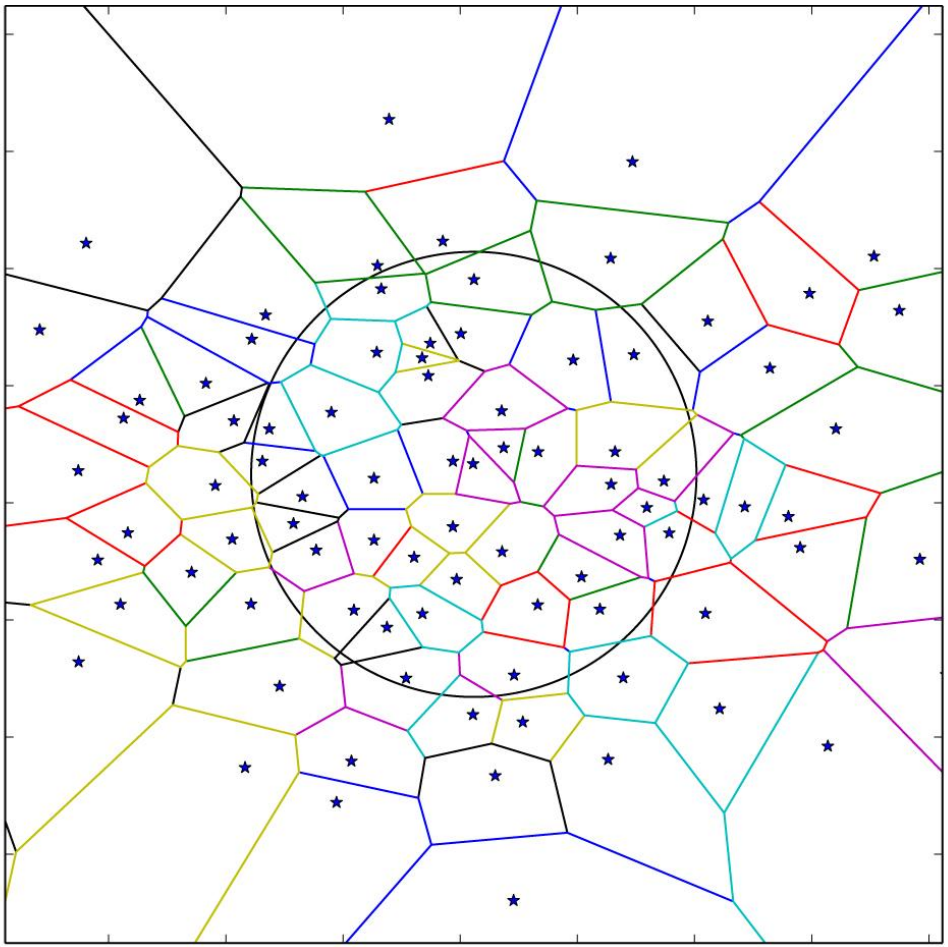
IFF there is **sufficient ν coverage**, and the path differences are less than the *coherence length*, then we can solve...

$$\Delta\mathrm{TEC}_{A,\alpha}$$

Differential electron content (dTEC)

Direction
dependent

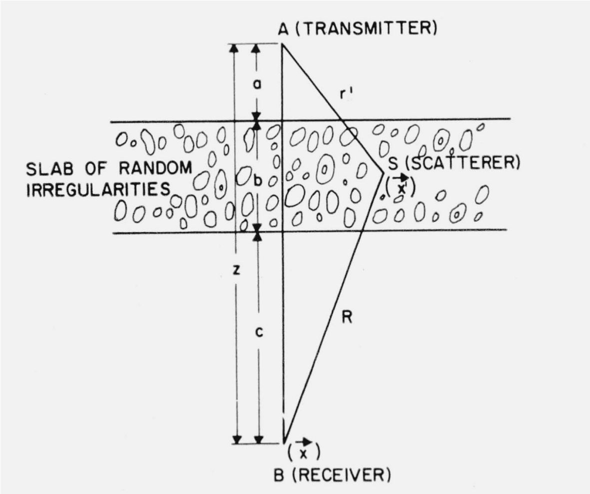




Bayesian modeling phase screens

- A slab produces quasi Gaussian correlated phase screens.
- So why not Gaussian processes!

$$Q_\phi(\phi^{\text{inf}}) = \mathcal{G}(\phi^{\text{inf}} \mid \mu(\vec{x}, \vec{k}, t), K(\vec{x}, \vec{k}, t))$$



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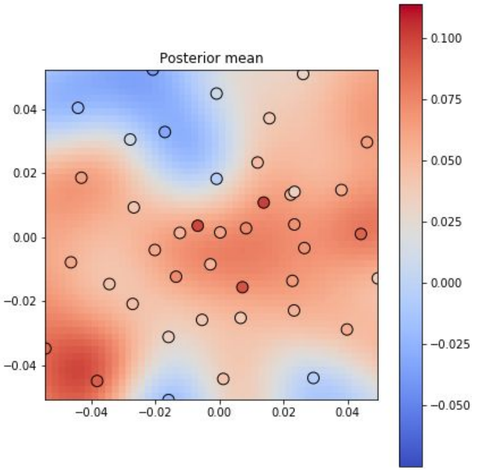
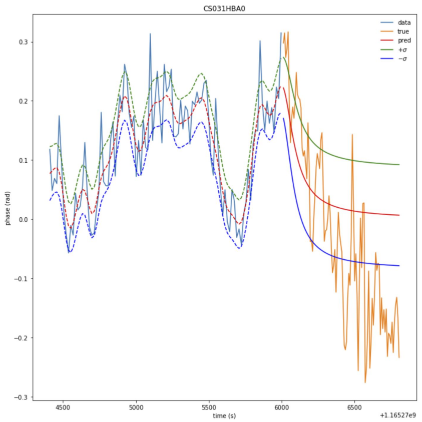




Bayesian modeling phase screens

$$Q_\phi(\phi^{\text{inf}}) = \mathcal{G}(\phi^{\text{inf}} \mid \mu(\vec{x}, \vec{k}, t), K(\vec{x}, \vec{k}, t))$$

$$\begin{aligned} K(t,x,y,\alpha,\beta) = &\ \mathcal{D}(t,x,y,\alpha,\beta) \\ &+ \exp - \frac{t^2}{2\tau_{slow}^2} - \frac{t^2}{2\tau_{quick}^2} \\ &+ \mathrm{RQ}_{1/6}(x,y;l_{inertia}) \exp - \frac{x^2+y^2}{2L_{outer}^2} \\ &+ \exp - \frac{\sin^2(\alpha)+\sin^2(\beta)}{2\gamma^2} \end{aligned}$$

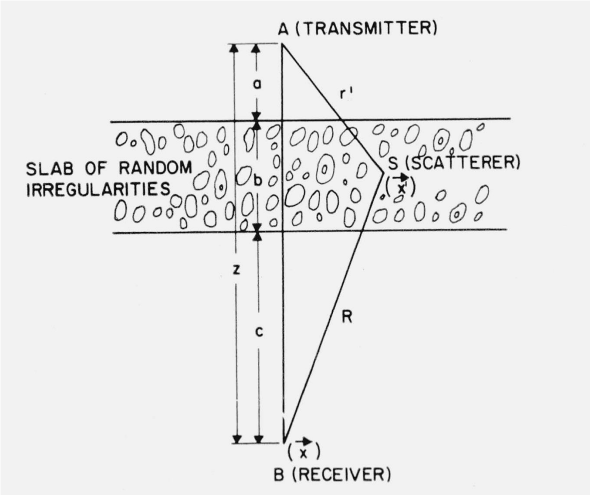
Example kernel

Uncorrelated noise

Slow and fast disturbances

Kolmogorov Turbulence

Angular coherency



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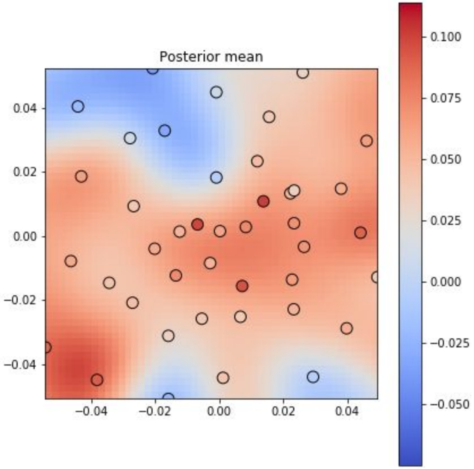
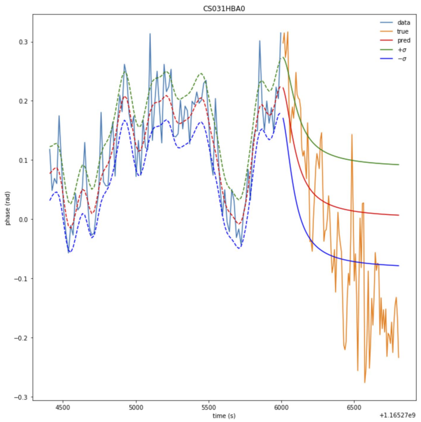
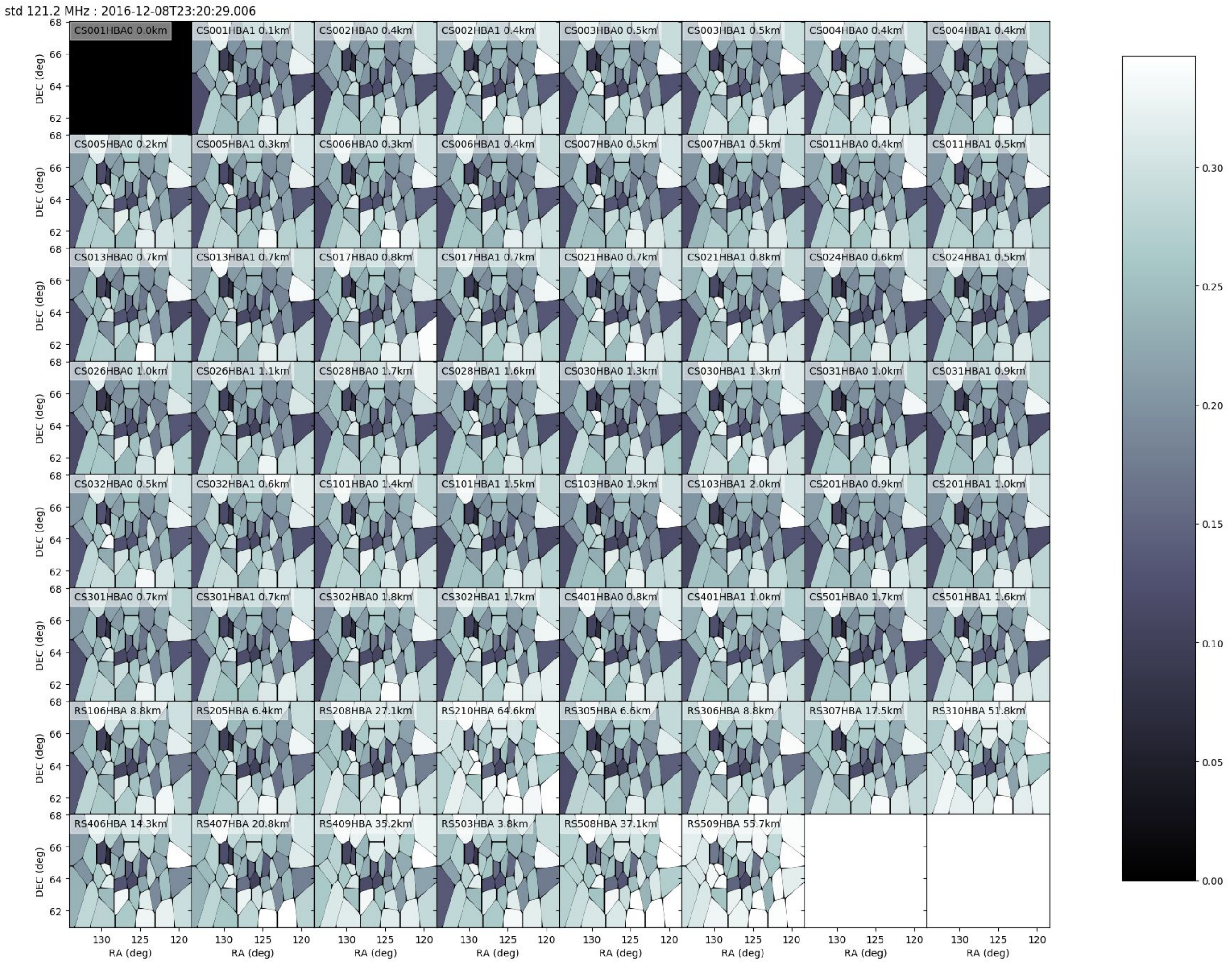




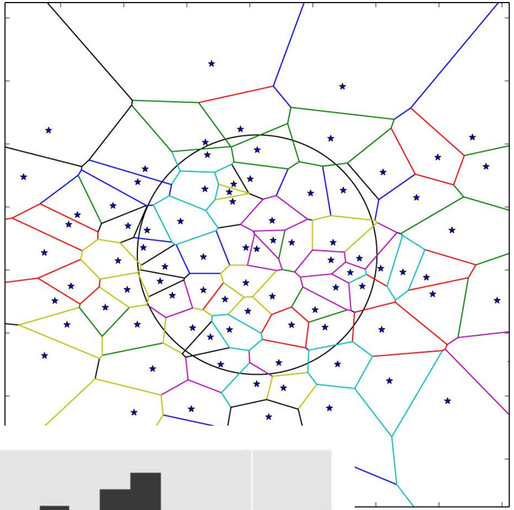
Least-square residuals from RIME after clock+TEC fitting

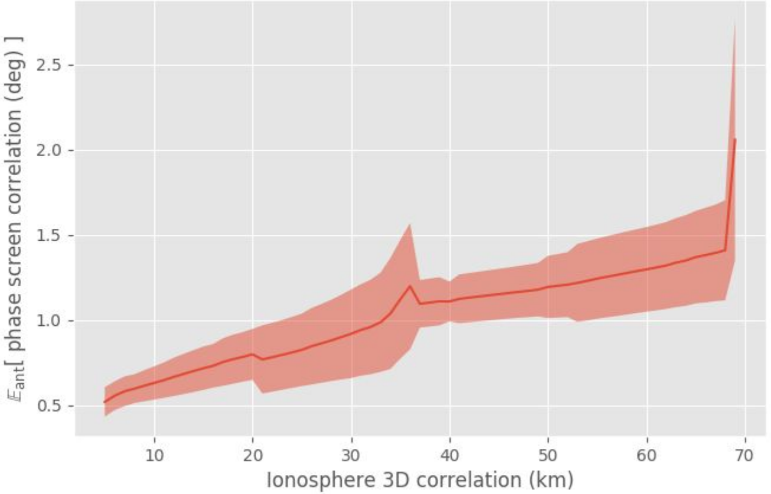


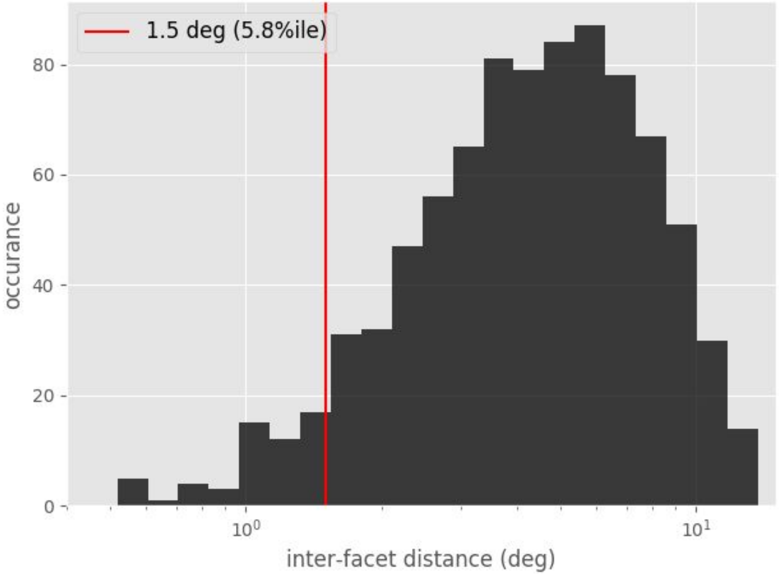
How much information does one need?

Neighbouring dimensions borrow information via correlation.

Furthermore, we can assume spatial homogeneity.



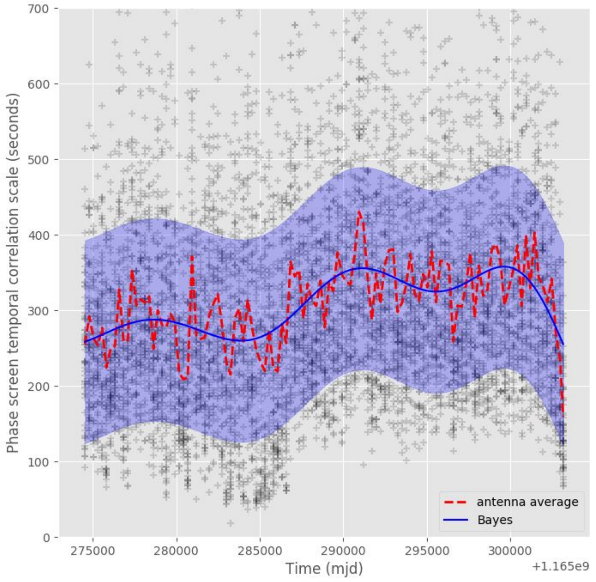




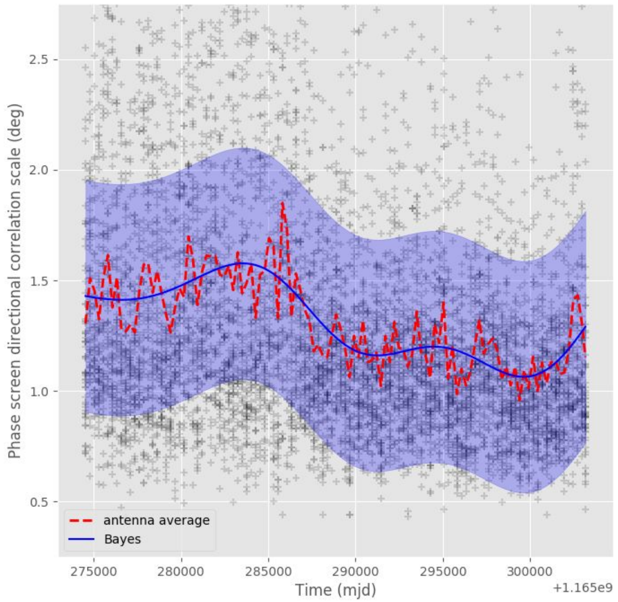
“Wilder” ionospheres are quicker but larger scale

Large facet approximations are more valid in “wilder” ionospheres.
I.e. you’re better able to transfer solutions larger separations.

Temporal corr. scale



Directional corr. scale

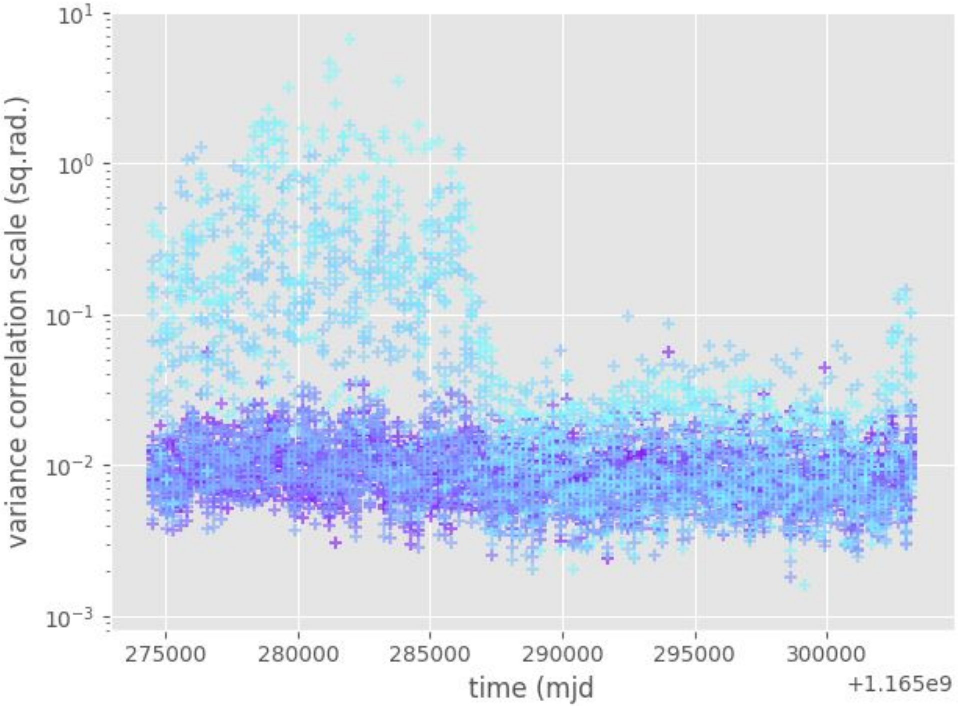


What does “wild” mean?

Are larger magnitude variations worse than smaller scale variations?

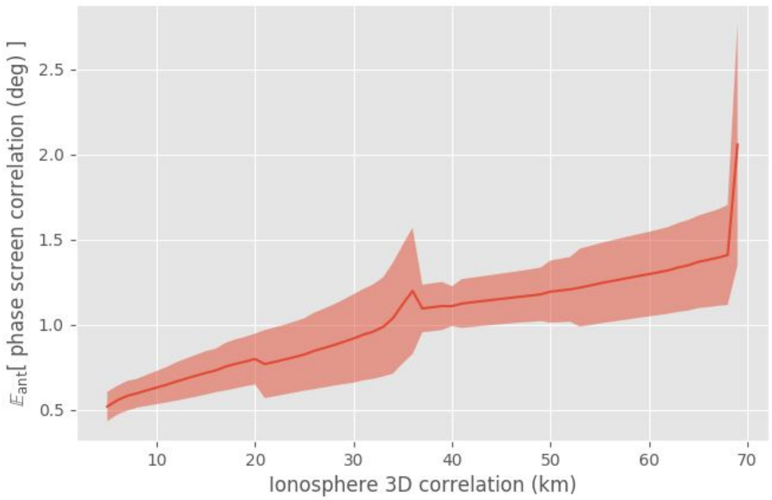
For long baseline, probably large directional scale is probably better...

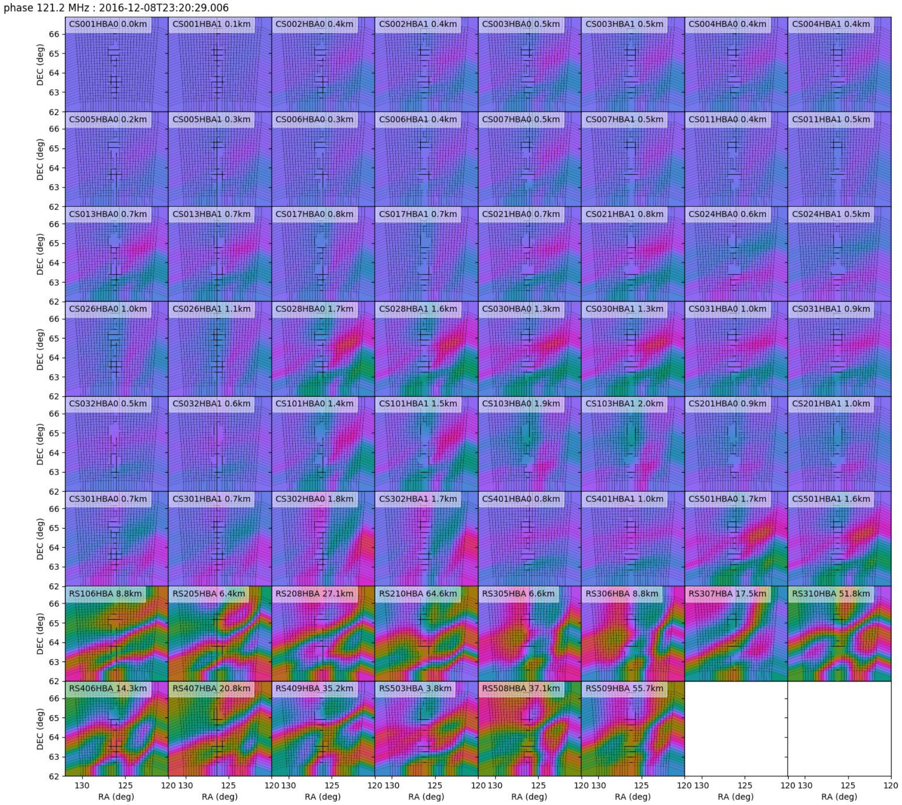
Variance corr. Scale (colour coded by antenna distance from ref.)



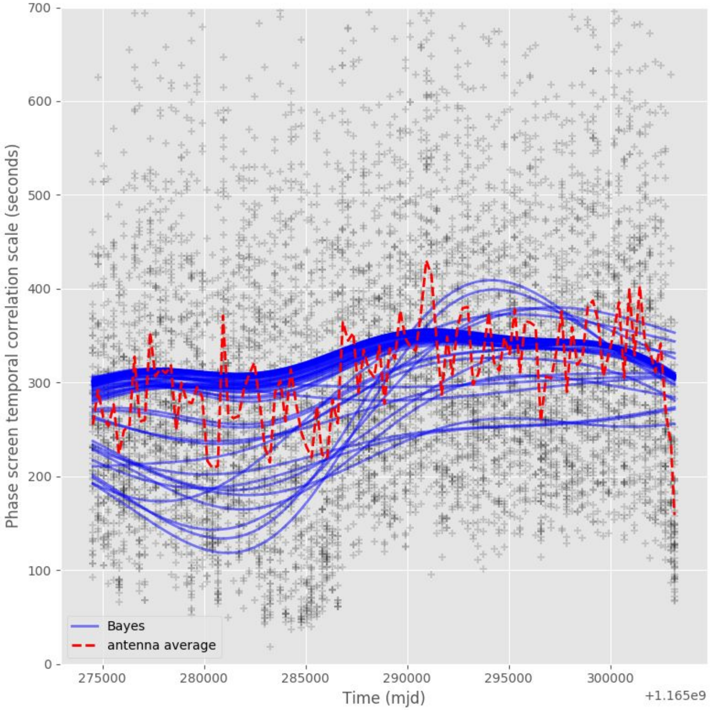
Of special interest to long baseliners

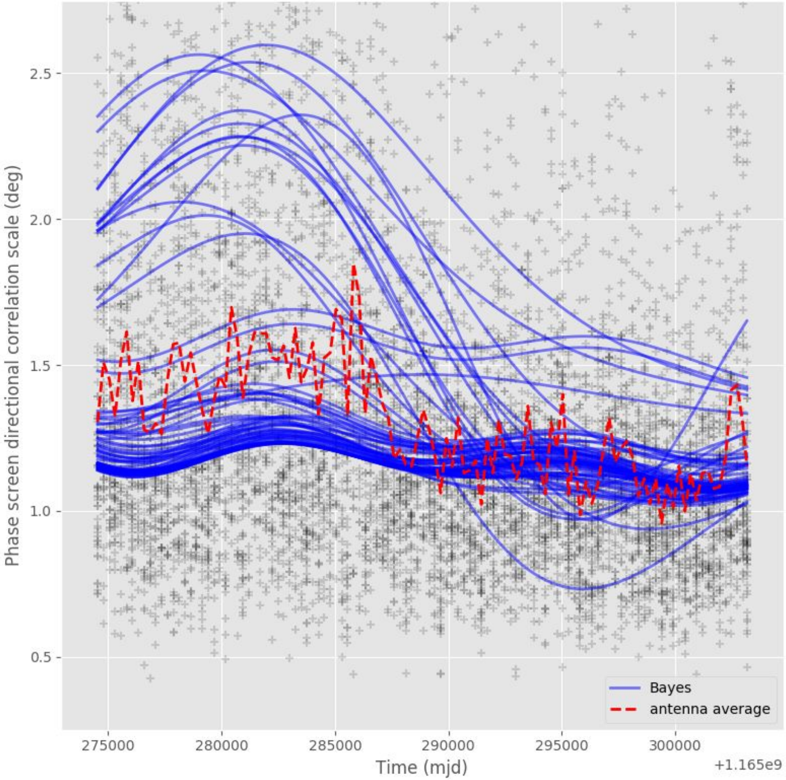
Homogeneity across antennas?
At what distance point does it break down?
Could IS leverage information from their neighbours?





Allowing spatial dependence is more realistic





Summary and outlook

- Phase screens seem to be well approximated by Gaussian processes.
  - But what about at scales below ~1deg where we lack directional info?
  - Images improve accordingly (not shown here today)
- “Wilder” means larger directional scales and shorter temporal scales.
- Could a scalable implementation of Gaussian processes aid Loop 2+3 in the pipeline?