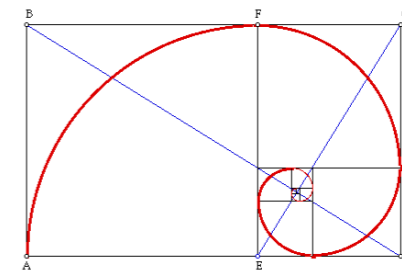
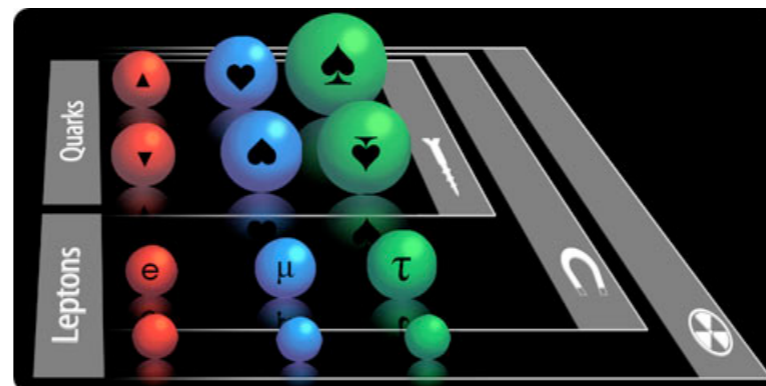
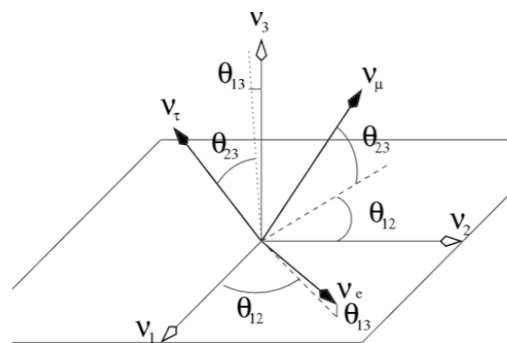


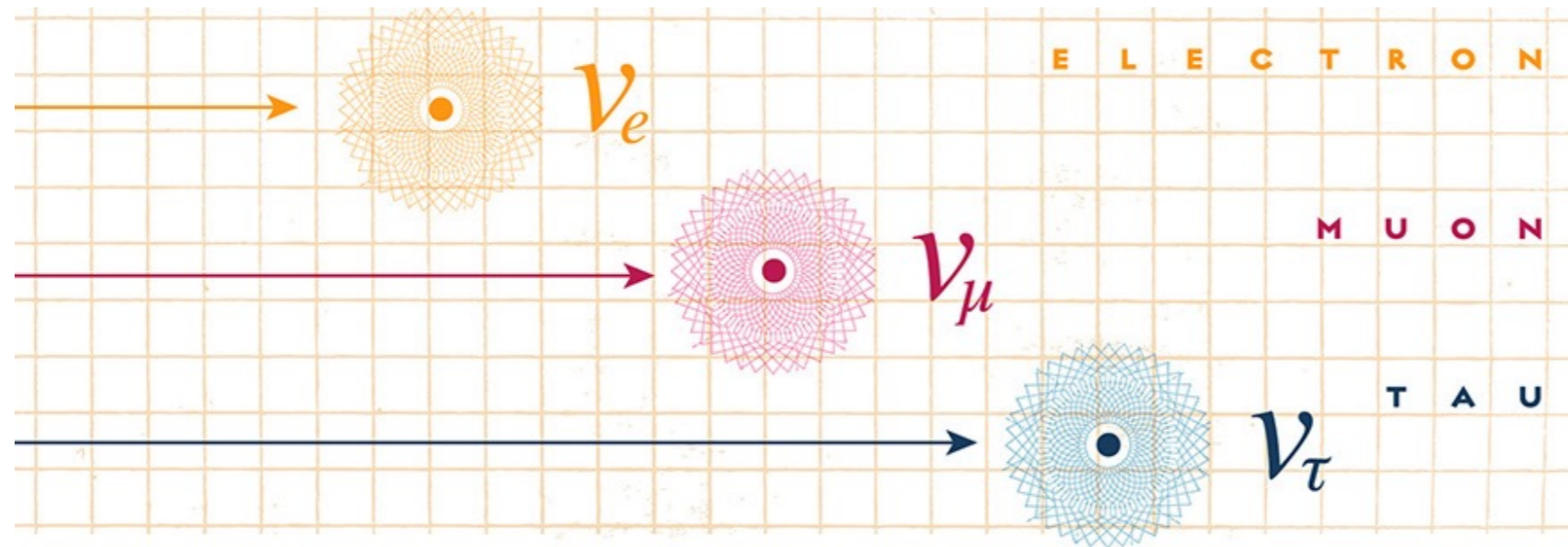
Theories of Neutrino Masses

Lisa L. Everett
University of Wisconsin-Madison

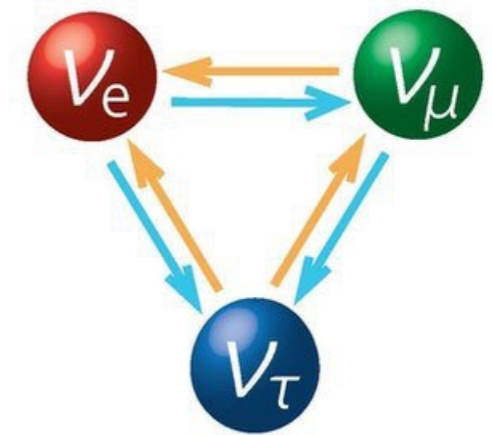
**Neutrino 2018: XXVIII International Conference
on Neutrino Physics and Astrophysics**



A wealth of discoveries in neutrino physics since 1998...



(image credit: C. Wiens)



Some highlights:

- 1998: atmospheric ν_μ disappearance (SK)
- 2002: solar ν_e disappearance (SK)
- 2002: solar ν_e appear as ν_μ, ν_τ (SNO)
- 2004: reactor $\bar{\nu}_e$ oscillations (KamLAND)
- 2004: accelerator ν_μ disappearance (K2K)
- 2006: accelerator ν_μ disappearance (MINOS)

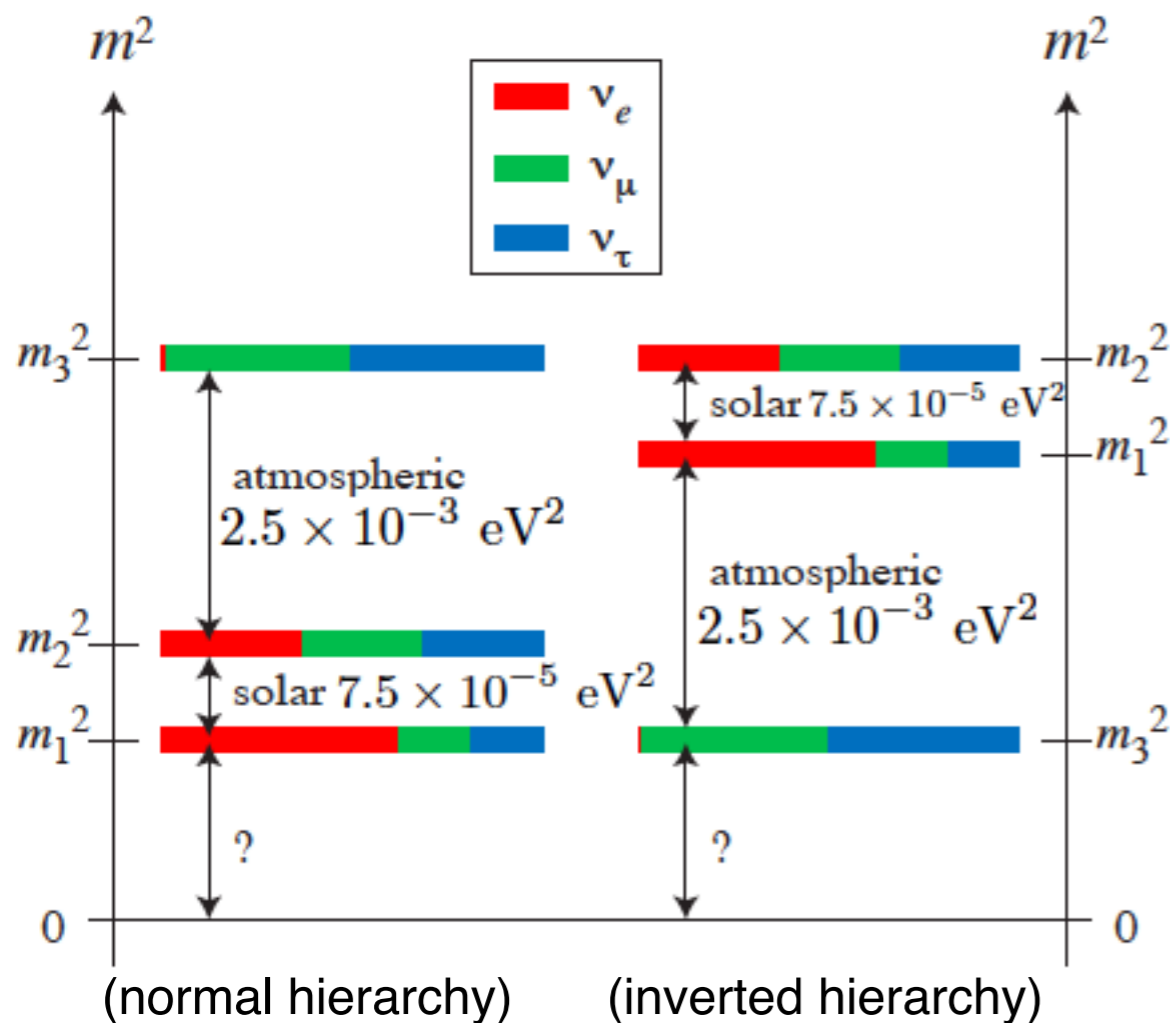
- 2011: accelerator ν_μ appear as ν_e (T2K, MINOS)
- 2012: reactor $\bar{\nu}_e$ disappear (Daya Bay, RENO)
reactor angle measured!
- 2014: hint for CP violation? (T2K)
- 2015: hints for normal hierarchy? (SK, T2K, NOvA)
- 2016: hint for non-maximal atm mixing? (NOvA)
- 2018: trivial Dirac phase disfavored at 2σ (T2K)

...

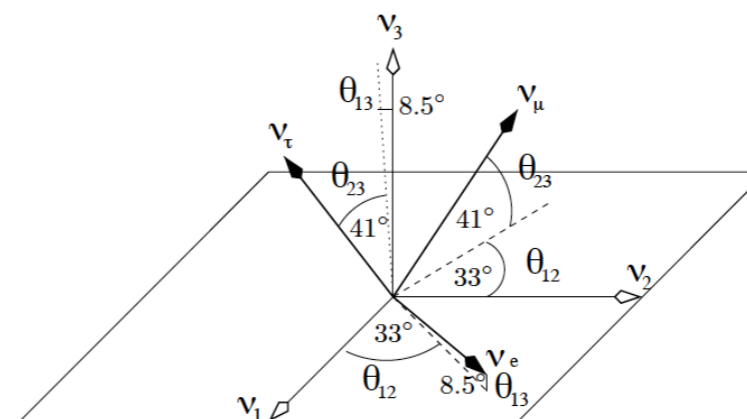
Signals physics beyond the Standard Model (SM)!

The emergent picture...

a (seemingly) robust 3-neutrino mixing scheme



(image credits: King, Luhn)



Global Fits:

Forero et al., '17 Capozzi et al., '18

Gonzalez-Garcia et al., (www.nu-fit.org)

NuFIT 3.2 (2018)

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 4.14$)		Any Ordering 3 σ range
	bfp $\pm 1\sigma$	3 σ range	bfp $\pm 1\sigma$	3 σ range	
$\sin^2 \theta_{12}$	$0.307^{+0.013}_{-0.012}$	0.272 \rightarrow 0.346	$0.307^{+0.013}_{-0.012}$	0.272 \rightarrow 0.346	0.272 \rightarrow 0.346
$\theta_{12}/^\circ$	$33.62^{+0.78}_{-0.76}$	31.42 \rightarrow 36.05	$33.62^{+0.78}_{-0.76}$	31.43 \rightarrow 36.06	31.42 \rightarrow 36.05
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$\theta_{13}/^\circ$	$8.54^{+0.15}_{-0.15}$	8.09 \rightarrow 8.98	$8.58^{+0.14}_{-0.14}$	8.14 \rightarrow 9.01	8.09 \rightarrow 8.98
$\delta_{CP}/^\circ$	234^{+43}_{-31}	144 \rightarrow 374	278^{+26}_{-29}	192 \rightarrow 354	144 \rightarrow 374
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.40^{+0.21}_{-0.20}$	6.80 \rightarrow 8.02	$7.40^{+0.21}_{-0.20}$	6.80 \rightarrow 8.02	6.80 \rightarrow 8.02
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.494^{+0.033}_{-0.031}$	+2.399 \rightarrow +2.593	$-2.465^{+0.032}_{-0.031}$	-2.562 \rightarrow -2.369	$[+2.399 \rightarrow +2.593]$ $[-2.536 \rightarrow -2.395]$

Caveat: sterile neutrino(s)?

(image credit: ParticleBites)

Anomalies in the data:

- 1995: $\bar{\nu}_e$ appearance (LSND)
- 2007: $\bar{\nu}_e$ appearance (MiniBooNE)
- 2012: ν_e appearance (MiniBooNE)
- 1995: ν_e disappearance (Gallium)
- 2011: ν_e disappearance (Reactor)

[lots of results, investigation in the interim...]

[well-documented tension between appearance and disappearance data]

See IPA 2017 Huber talk for “scorecard”

2018: [some highlights]

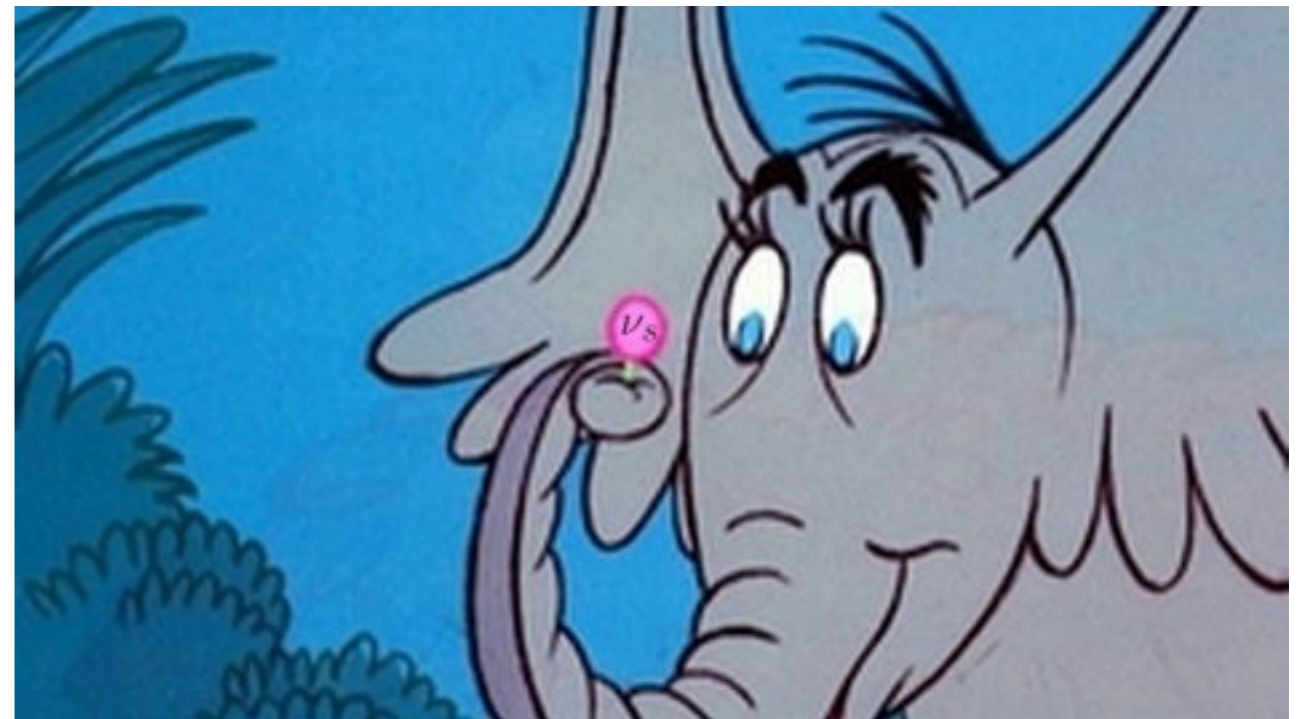
global analysis (March 2018)

Dentler et al. '18

?????

brand new MiniBooNE result (May 2018)

For this talk, focus on 3 active families only...



Mass Generation

Quarks, Charged Leptons

“natural” mass scale tied to electroweak scale

Dirac mass terms, parametrized by Yukawa couplings



$$Y_{ij} H \cdot \bar{\psi}_{Li} \psi_{Rj} \longrightarrow \mathcal{M}_u, \mathcal{M}_d, \mathcal{M}_e$$

top quark: O(1) Yukawa coupling

rest: suppression (flavor symmetry)

Neutrinos

Main question: origin of neutrino mass suppression

Options: **Dirac**

$$\Delta L = 0$$



Majorana

$$\Delta L = 2$$



Majorana first: $\Delta L = 2$

advantages: naturalness, leptogenesis, $0\nu\beta\beta$



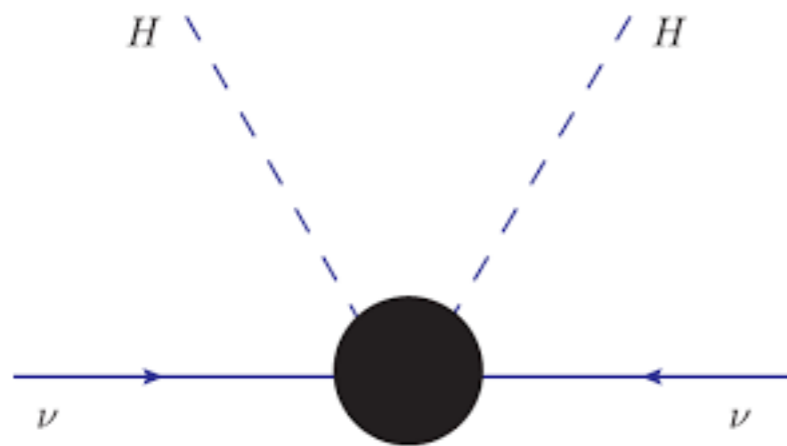
SM at NR level: Weinberg dimension 5 operator

$$\frac{\lambda_{ij}}{\Lambda} L_i H L_j H$$

if $\lambda \sim O(1)$ $\Lambda \gg m \sim O(100 \text{ GeV})$ (but wide range possible)

Underlying mechanism:

3 tree-level options



(image credit: Dinh et al.)

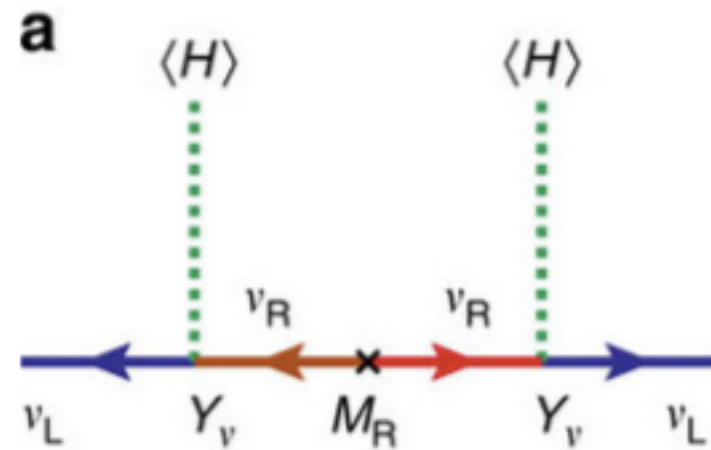
- a. Type I seesaw** ν_R (fermion singlet)
- b. Type II seesaw** Δ (scalar triplet)
- c. Type III seesaw** Σ (fermion triplet)

(prediction: superheavy particles)

Prototype: Type I seesaw

Type I: Minkowski; Yanagida; Gell-Mann, Ramond, Slansky; Mohapatra, Senjanovic;...

right-handed neutrinos:



(image credit: T. Ohlsson et al., Nat. Comm.)

$$Y_{ij} L_i \nu_{Rj} H + M_{Rij} \nu_{Ri} \nu_{Rj}^c$$

$$\mathcal{M}_\nu \sim \langle H \rangle^2 Y M_R^{-1} Y^T$$

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

$$m \sim \mathcal{O}(100 \text{ GeV})$$

$$M \gg m$$

$$m_1 \sim \frac{m^2}{M} \quad m_2 \sim M \gg m_1 \quad \nu_{1,2} \sim \nu_{L,R} + \frac{m}{M} \nu_{R,L}$$

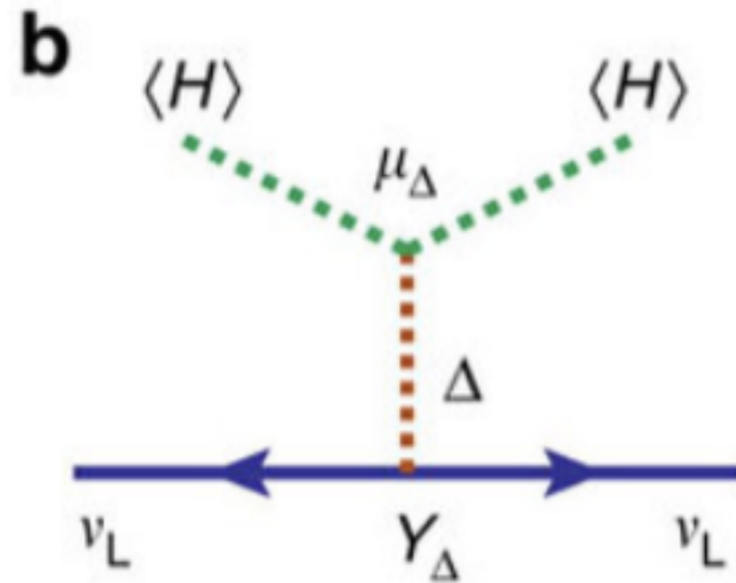
advantages: naturalness, connection to grand unification, leptogenesis,...

disadvantage: testability without model assumptions (even at low scales)

Other tree-level seesaws

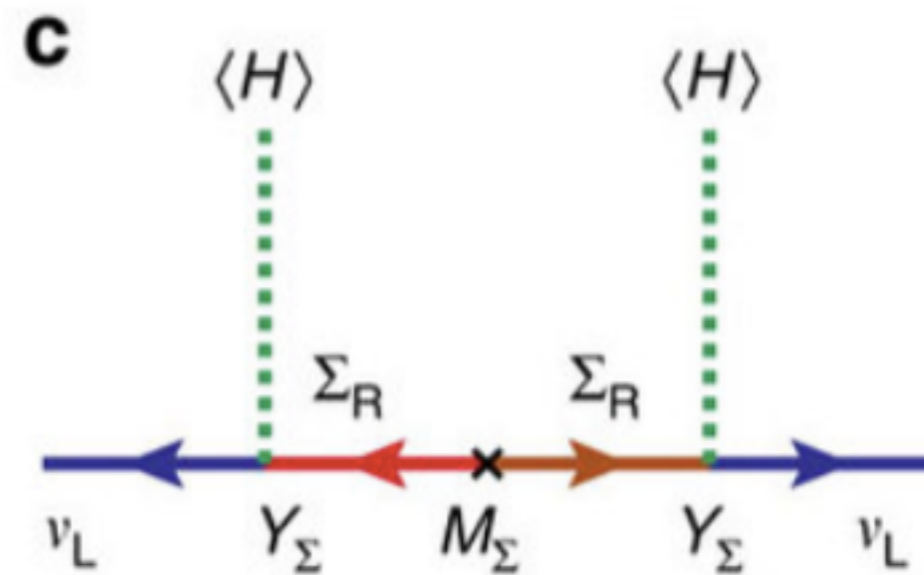
(image credits: T. Ohlsson et al., Nat. Comm.)

Type II



$$\mathcal{M}_\nu \sim \langle H \rangle^2 Y_\Delta \mu_\Delta / M_\Delta^2$$

Type III



$$\mathcal{M}_\nu \sim \langle H \rangle^2 Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T$$

advantage: testability (when completed)

usually accompanied by new EW charged states – visible at LHC?

disadvantages: naturalness, economy (subjective)

Type II: Konetchsy, Kummer; Cheng, Li; Lazarides, Shafi, Wetterich; Schecter, Valle; Mohapatra et al.; Ma;...

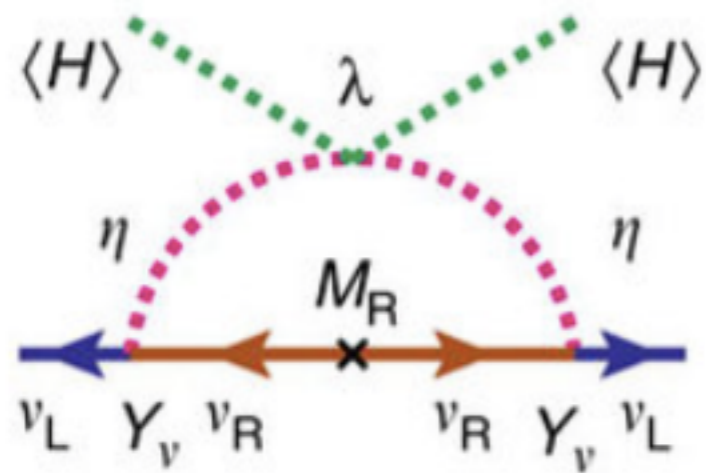
Type III: Foot, He, Joshi; Ma;...

Radiative neutrino mass generation:

complete Weinberg operator via loops

(loop suppression factor aids in overall mass suppression)

A canonical example: “scotogenic” model



(image credit: T. Ohlsson et al., Nat. Comm.)

introduce new electroweak doublet
and right-handed neutrinos

$$\mathcal{M}_\nu \sim \lambda \frac{\langle H \rangle^2}{16\pi^2} Y M_R^{-1} Y^T$$

(new states can be DM candidates)

Generic feature of radiative models:

superheavy states no longer required!

advantage: testability

Radiative neutrino mass generation:

(odd mass dimension $d > 5$)

can have other NR operators in SM with $\Delta L = 2$

Babu and Leung '01
de Gouvea and Jenkins '07

d=7

$$LLLe^c H$$

$$LLQd^c H$$

$$LL\bar{Q}\bar{u}^c H$$

$$L\bar{e}^c\bar{u}^c d^c H$$

d=9

$$LLLe^c Le^c \quad (\text{Zee, Babu})$$

$$LLQd^c Qd^c$$

+ many others...

NP scale can be accessible at LHC (subject to LFV bounds)

Connection between loop-induced mass generation and B-physics anomalies...

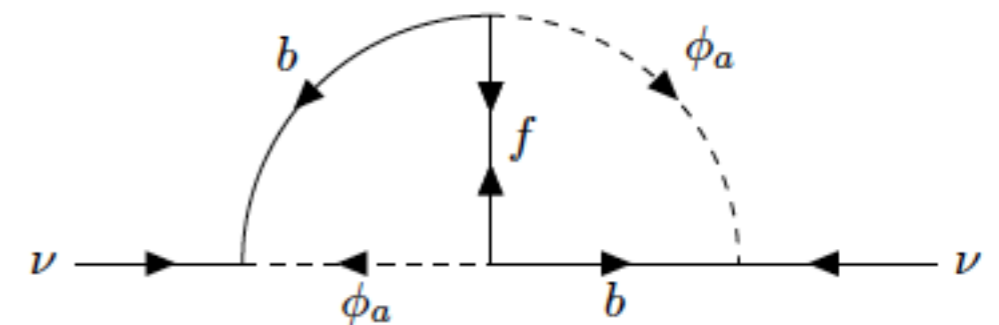
One way **leptoquarks** can manifest themselves!

Päs and Schumacher, '15

A two-loop example:

scalar leptoquark $\phi \sim (\mathbf{3}, \mathbf{1}, -1/3)$

+ octet fermion $f \sim (\mathbf{8}, \mathbf{1}, 0)$



Cai, Gargalones, Schmidt, Volkas '17



see Volkas talk at Neutrino 2018

Many other ideas for Majorana neutrino masses...



more seesaws (double, inverse,...),
SUSY with R-parity violation, RS models...

lepton number violation



Majorana ν masses

Now for Dirac neutrino masses:

Require strong suppression

$$Y_\nu \sim 10^{-14}$$

Less intuitive, but mechanisms exist...

extra dimensions, new gauge symmetries (non-singlet ν_R),
SUSY breaking effects, string instanton effects,...



General themes:

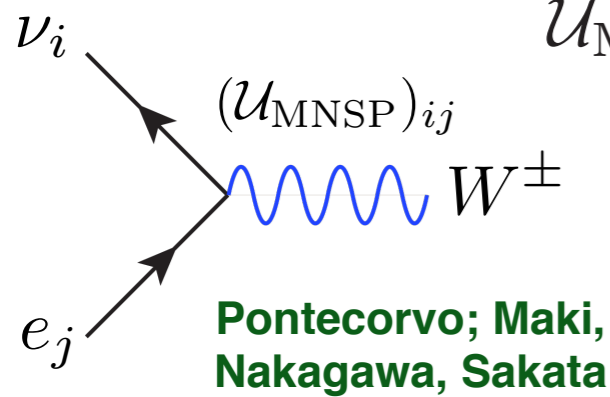
Much richer than quark and charged lepton sectors.

Trade-off between naturalness and testability.



see Dev talk at Neutrino 2018

Lepton mixings



$$U_{\text{MNSP}} = \mathcal{R}_1(\theta_{23})\mathcal{R}_2(\theta_{13}, \delta)\mathcal{R}_3(\theta_{12})\mathcal{P}$$

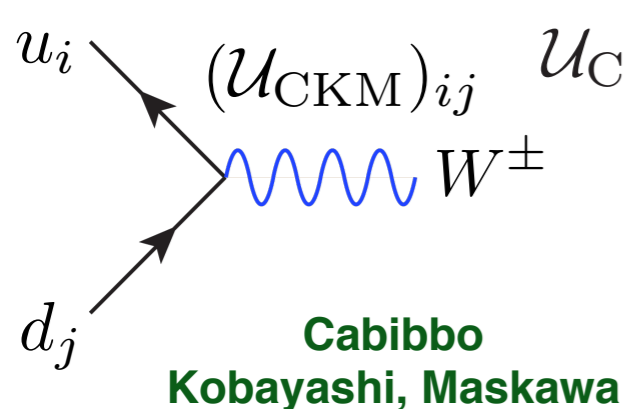
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix} \begin{pmatrix} \cos \theta_{13} & 0 & \sin \theta_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -\sin \theta_{13} e^{-i\delta} & 0 & \cos \theta_{13} \end{pmatrix} \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathcal{P}$$

diagonal phase matrix
(Majorana neutrinos)

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Compare quarks:



$$U_{\text{CKM}} = \mathcal{R}_1(\theta_{23}^{\text{CKM}})\mathcal{R}_2(\theta_{13}^{\text{CKM}}, \delta_{\text{CKM}})\mathcal{R}_3(\theta_{12}^{\text{CKM}})$$

$$\theta_{12}^{\text{CKM}} = 13.0^\circ \pm 0.1^\circ = \theta_C \quad (\text{Cabibbo angle})$$

$$\theta_{23}^{\text{CKM}} = 2.4^\circ \pm 0.1^\circ$$

$$\theta_{13}^{\text{CKM}} = 0.2^\circ \pm 0.1^\circ$$

$$\delta_{\text{CKM}} = 60^\circ \pm 14^\circ$$

3 “small” angles, 1 $O(1)$ phase

Lepton mixings

$$\mathcal{U}_{\text{MNSP}} = \mathcal{R}_1(\theta_{23})\mathcal{R}_2(\theta_{13}, \delta)\mathcal{R}_3(\theta_{12})\mathcal{P}$$

Certainly **two large mixing angles**: θ_{23}, θ_{12}

Dirac phase: too soon to say, but intriguing hints

Majorana phases: unlikely to know anytime soon**

A basic question: is θ_{13} “large” or “small”?

Large reactor angle:



the case for **anarchy**

vs.

Small reactor angle:



the case for **symmetry**

Neutrino anarchy

\mathcal{U}_ν from a random draw of unbiased distribution of 3x3 unitary matrices

statistical tests: lower bound on $|\mathcal{U}_{e3}|^2$

Post-reactor angle measurement: renewed focus

de Gouvea and Murayama '12 Altarelli et al. '12, Bai and Torroba '12 ,...

Some recent highlights:

RG analysis Brdar, Konig, Kopp '15

Model-building + quark sector Babu et al. '16,...

Fortin et al. '17

(anarchy also popular approach for NP flavor violation at scales ~ 10 TeV)

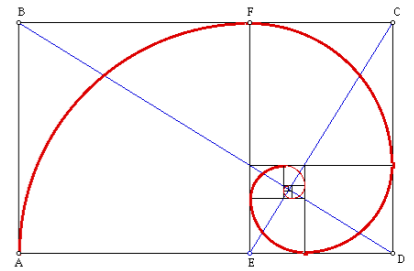
Baumgart et al. '15,...

Anarchy hypothesis alone does not provide information on Δm^2



(character: Watterson)

Family symmetries (structure)



Very different structure for leptons and quarks!

Quarks:

spontaneously broken family symmetry at scale M

Froggatt, Nielsen

$$Y_{ij} H \cdot \bar{\psi}_{Li} \psi_{Rj} \longrightarrow \left(\frac{\varphi}{M} \right)^{n_{ij}} H \cdot \bar{\psi}_{Li} \psi_{Rj}$$

φ = "flavon"

small mixings and hierarchical masses:

→ continuous family symmetry

both Abelian and non-Abelian: many examples!

$\mathcal{M}_u, \mathcal{M}_d$ approx diagonalized by same unitary transformation

(can choose basis where both close to diagonal)

$$\mathcal{U}_{\text{CKM}} = \mathcal{U}_u \mathcal{U}_d^\dagger \sim 1 + \mathcal{O}(\lambda) \quad \lambda \sim \frac{\varphi}{M}$$

Wolfenstein parametrization: $\lambda \equiv \sin \theta_c = 0.22$

suggests Cabibbo angle (or some power) as a flavor expansion parameter

Leptons:




For the **charged leptons**: hierarchical masses  similar strategy?

But now, in basis where \mathcal{M}_e is diagonal, \mathcal{M}_ν is not diagonal:


\mathcal{M}_ν diagonalization requires **1 small, 2 large mixing angles!**

Arguably the **most challenging*** pattern:

(* for three families)

3 small angles		\sim diagonal \mathcal{M}_ν	}
1 large, 2 small		\sim Rank $\mathcal{M}_\nu < 3$	
3 large angles		anarchical \mathcal{M}_ν	

relatively straightforward
at leading order

1 small, 2 large  **fine-tuning, non-Abelian**



A model-building opportunity!

Lepton mixings:

No unique theoretical starting point for the flavor expansion!

$$\mathcal{U}_{\text{MNSP}} \sim \mathcal{W} + O(\lambda')$$

↑
mixing angles $(\theta_{12}^\nu, \theta_{23}^\nu, \theta_{13}^\nu)$
(diagonal charged lepton basis)

← flavor expansion parameter

“Bare” mixing angles generically shift due to $O(\lambda')$ corrections

A priori, expansions in quark and lepton sectors unrelated.

Unification paradigm (broad sense): set $\lambda' = \lambda_C$

ideas of **quark-lepton complementarity** and “**Cabibbo haze**”

Raidal '04, Minakata+Smirnov '04, many others...
 (“haze” terminology from Datta, L.E., Ramond '05)

Pre-measurement, speculation that **reactor angle is a Cabibbo effect**

$$\theta_{13}^\nu = 0 \quad \theta_{13} \sim \frac{\lambda_C}{\sqrt{2}}$$

Vissiani '98, '01
Ramond '04

Possible starting points:

Most studied: maximal atmospheric, zero reactor $\theta_{23}^\nu = \frac{\pi}{4}$ $\theta_{13}^\nu = 0$



classify scenarios by bare solar angle

tri-bimaximal mixing: $\sin^2 \theta_{12}^\nu = 1/3$

Harrison, Perkins, Scott '02;
Xing '02; He, Zee '02; Ma '03...

bimaximal mixing: $\sin^2 \theta_{12}^\nu = 1/2$

Vissiani '97; Barger et al. '98; Baltz, A.
Goldhaber, M. Goldhaber '98;...

golden ratio (A) mixing: $\sin^2 \theta_{12}^\nu = 1/(2 + r) \sim 0.276$
 $r = (1 + \sqrt{5})/2$

Datta, Ling, Ramond '03;
Kajiyama, Raidal, Strumia '08;...

golden ratio (B) mixing: $\sin^2 \theta_{12}^\nu = (3 - r)/4 \sim 0.345$

Rodejohann '09,...

hexagonal mixing: $\sin^2 \theta_{12}^\nu = 1/4$

Albright, Duecht, Rodejohann
'10, Kimand, Seo '11,...

Also can study scenarios without $\theta_{13}^\nu = 0$

Lam '13; Holthausen et al. '12; Hagendorn...

many others...

All can be obtained via discrete non-Abelian family symmetries

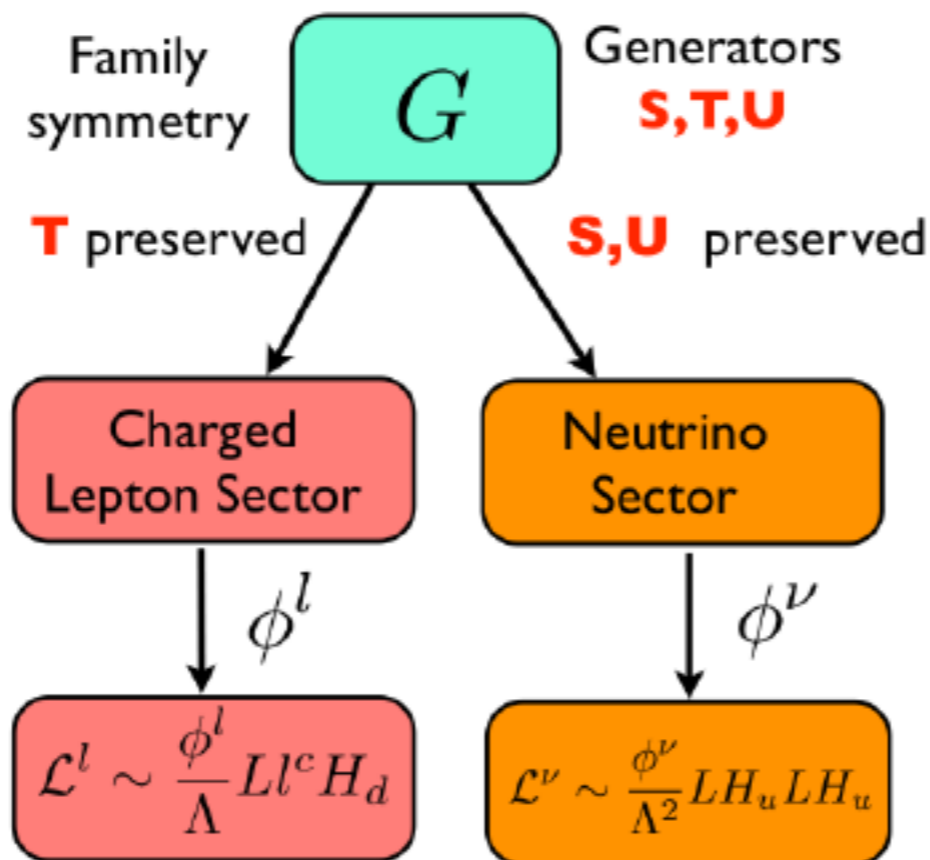
Model-building approach

Choose a discrete non-Abelian group for family symmetry

Options: $SU(3)$, $SO(3)$ subgroups:

$$A_4 \quad S_4 \quad A_5 \quad \Delta(3n^2) \quad \Delta(6n^2) \quad D_n \quad T' \quad \mathcal{I}' \quad \dots$$

(image credit: King, Luhn)



see e.g. reviews by
King, Luhn '13,
King '17

Example (Majorana ν):

Flavons:

$$\phi^l, \phi^\nu$$

Residual symmetries:

$$T \langle \phi^l \rangle \approx \langle \phi^l \rangle$$

$$S, U \langle \phi^\nu \rangle \approx \langle \phi^\nu \rangle$$

(or broken further, e.g.
only S or U unbroken)

corrections in flavor expansion: (i) NLO in flavons, (ii) “charged lepton”/kinetic/RG...

Many papers! Some authors (not comprehensive):

King, Ma, Ding, Feruglio, Lam, Rodejohann, Chen, Hagedorn, Luhn, Stuart, LE...

Example: tri-bimaximal mixing (TBM/HPS)

(Majorana neutrinos, Type I seesaw)

$$\mathcal{U}_{\text{MNSP}}^{(\text{HPS})} = \begin{pmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (\sim \text{Clebsch-Gordan coeffs!})$$

Meshkov, Zee...

Many models pre-dated reactor angle measurements

see e.g. Albright et al. '10



current data requires Cabibbo-sized corrections

Prototypical scenarios: \mathcal{A}_4 \mathcal{S}_4 \mathcal{T}' (typically SUSY/SUSY-GUT)

Many, many authors!!

Ma et al.; Altarelli, Feruglio; Carone et al.; Chen et al.; King et al.; Ding; Lam...



“minimal” flavor group
(contains S, T, U generators)

Lam; Ding et al;...

Residual symmetries: $\mathcal{Z}_3 \sim T$ $\mathcal{Z}_2 \times \mathcal{Z}_2 \sim S, U, SU$ (Klein symmetry)

Can further break down Klein symmetry:

1 column only of HPS matrix preserved: TM1, TM2 + corrections

see e.g. King '17 for review

Example: tri-bimaximal mixing (TBM/HPS)

Bottom-up approach: get needed corrections through “Cabibbo Haze”

Interesting (very) recent example:

asymmetric charged lepton corrections to TBM/HPS
(with a dash of grand unification)

Rahat, Ramond, Xu '18

SU(5), SO(10) GUT-inspired relations:

symmetric Yukawas \longrightarrow insufficient corrections to θ_{13}

Kile, Perez, Ramond, Zhang '14

asymmetric Yukawas \longrightarrow possible for specific $O(\lambda_C)$
corrections to Y_e (via $Y_{\bar{5}}$)

Notable feature:

phase required in $\mathcal{U}_\nu \sim \mathcal{U}^{(\text{HPS})}$ for consistency with mixing angle data

numerical example: $\delta \simeq \pm 1.3\pi$, $J \simeq \mp 0.03$

good agreement!

CP Violation

Consider case of spontaneous CP violation — calculable phases.

Idea of generalized CP: $X^T \mathcal{M}_\nu X = \mathcal{M}_\nu^*$ $Y^\dagger \mathcal{M}_e \mathcal{M}_e^\dagger Y = (\mathcal{M}_e \mathcal{M}_e^\dagger)^*$

“ordinary” CP has $X = Y = 1$

Branco, Lavoura, Rebelo '86...

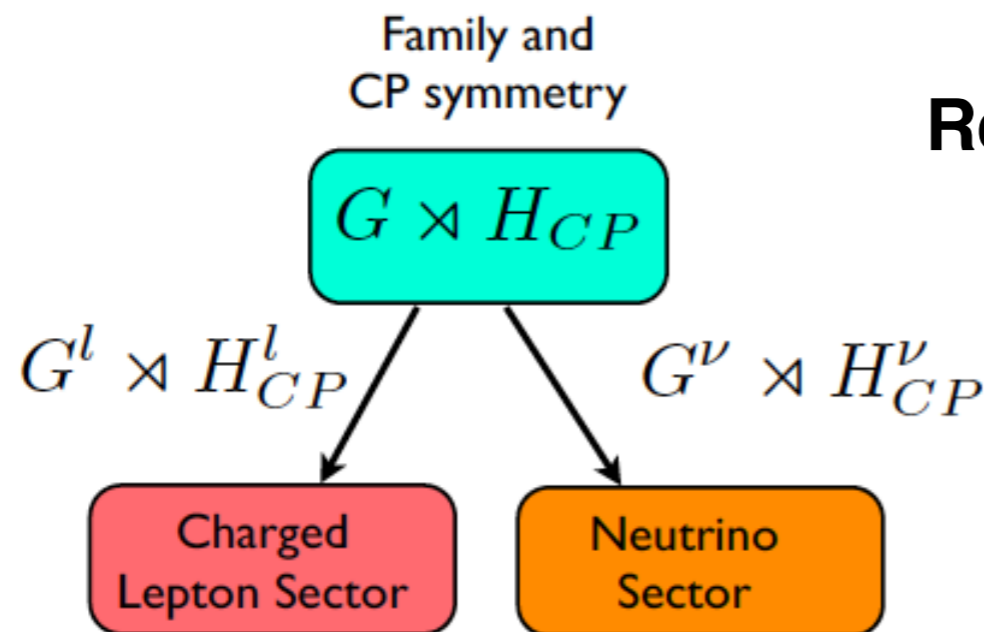
Grimus, Rebelo '95

automorphisms of discrete family symmetry:

Holthausen et al. '12; Feruglio et al. '12; Chen et al. '14; Ding et al. '14; Branco et al. '15; ...

$$X \rho(g)^* X^{-1} = \rho(g') \quad (\text{consistency condition})$$

family symmetry



Residual/generalized CP symmetries

existence of “CP basis”

Holthausen, Lindner, Schmidt '12

group classification

Chen et al. '14

Lots of interesting recent work along these lines!

many recent papers! see King '17 for review

Residual/CP symmetries (model-independent approach)

LE, Garon, Stuart '15; LE and Stuart '16

Assumptions: Majorana neutrinos, full Klein symmetry preserved

$$U_\nu^T \mathcal{M}_\nu U_\nu = \mathcal{M}_\nu^{\text{diag}} \quad \text{invariant if } U_\nu \rightarrow U_\nu Q_\nu \quad Q_\nu = \text{Diag}(\pm 1, \pm 1, \pm 1) \\ \text{Det } Q_\nu = 1$$

From these, obtain diagonal Klein generators $G_{i=0,1,2,3}^{\text{diag}}$

$$(G_i^{\text{diag}})^T \mathcal{M}_\nu^{\text{diag}} G_i^{\text{diag}} = \mathcal{M}_\nu^{\text{diag}}$$

Then obtain **Klein generators**: $G_i = U_\nu G_i^{\text{diag}} U_\nu^\dagger \quad G_i^T \mathcal{M}_\nu G_i = \mathcal{M}_\nu$
(reconstruct from MNSP for diagonal charged leptons)

For generalized CP operators in neutrino sector:

from above and $X_\nu G_i^* - G_i X_\nu = 0$ Feruglio et al. '12, Holthausen et al. '12

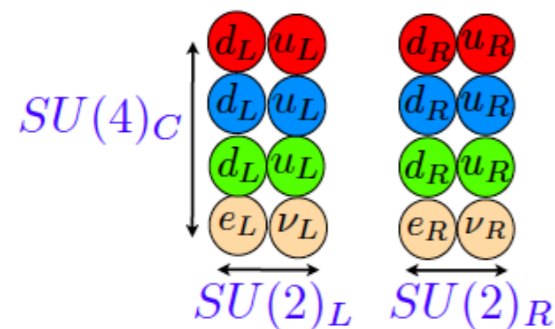
$$\longrightarrow X_i X_i^* = G_0 \quad X_0 X_i^* = G_i \quad X_i X_j^* = G_k$$

Similar approach for **charged lepton** generalized CP:

but need to be careful of phase redefinition degrees of freedom

SUSY GUTs and String Models: Top-Down

SUSY GUTs: explicit realizations of these scenarios (+ quark sector)



recent example: **SUSY Pati-Salam**

Poh, Raby, Wang '17

$$SU(4)_C \times SU(2)_L \times SU(2)_R \quad \mathcal{D}_3 \times U(1) \times \mathbb{Z}_2 \times \mathbb{Z}_3$$

can achieve consistency with LHC, neutrino data

(26-parameter fit)

String Models:

variety of possibilities, not necessarily just minimal Type I seesaw

→ ν_R candidates often not pure gauge singlets

explorations of Type I seesaw in heterotic orbifolds [Giedt et al.](#); [Buchmuller et al.](#);...

braneworlds: exponentially suppressed Yukawas

see e.g. [Langacker for reviews](#)

“Mixed” scenarios with seesaw and R-parity violation

e.g. G2 models

[Acharya et al. '16](#);...

Conclusions

Neutrino data has led to a renaissance for SM flavor puzzle

Model-building starting point question: Dirac or Majorana?

Many ways known to suppress neutrino mass scale

often a tradeoff between minimality/naturalness and testability

For 3 active neutrinos only:

mixings: anarchy or symmetry

symmetry approach: paradigm shift to discrete non-Abelian groups

many examples (top-down and bottom-up)

but still seeking **compelling, complete, testable** theories

More data (atmospheric angle, Dirac CP phase,...) will help enormously!

If sterile neutrinos confirmed: paradigm shifts again!

Stay tuned!