

# Detection of Ultrawideband Pulses in Atmospheric Turbulence and Gaussian Noise

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**Abstract**— Scintillation of electromagnetic energy traversing the atmosphere is caused by refractive index inhomogeneities in the transmission path that cause phase shifts, giving rise to selective reinforcement or degradation of the energy across the beam. It is the object of this paper to determine the detection probability and false alarm rate for pulses of electromagnetic energy of varying width in the presence of atmospheric turbulence and Gaussian noise.

## I. INTRODUCTION

This Scintillation of electromagnetic energy traversing the atmosphere is caused by refractive index inhomogeneities in the transmission path that cause phase shifts, giving rise to selective reinforcement or degradation of the energy across the beam. The resulting energy distribution is log normal, characterized by a variance  $\sigma_E^2$  that is a function of the severity of atmospheric turbulence. The detection of ultrawideband (UWB) pulses is affected by atmospheric turbulence as well as by receiver and background noise, both of which have a Gaussian distribution. It is the object of this paper to determine the detection probability and false alarm rate for pulses of electromagnetic energy of varying width in the presence of atmospheric turbulence and Gaussian noise. Detection of ultrawideband/ultrashort pulses is relevant to the Army mission because communications, radar, and remote sensing systems increasingly make use of such signals. In making this calculation we assume that the parameter  $\sigma_E^2$  does not change during the pulse width and that the receiver has sufficient bandwidth to reproduce the UWB signal faithfully.

The distribution of energy in an atmospherically perturbed beam is [1]

$$P(E)dE = \frac{1}{\sqrt{2\pi}\sigma_E} \exp\left[-\frac{(\ln E - \ln E^*)^2}{2\sigma_E^2}\right] d(\ln E), \quad (1)$$

where  $P(E)dE$  is the probability of observing an energy between  $E$  and  $E + dE$  and  $E^*$  is defined by  $\ln E^*$  being the mean of  $\ln E$ . Now assume that this perturbed beam is incident on a receiver characterized by a threshold  $T$  and that the signal

exists in the presence of Gaussian noise of rms value  $\sigma_N$ . Under these conditions, the probability that the received signal-plus-noise in the presence of atmospheric turbulence will exceed the receiver threshold is [2]

$$P_D = \frac{1}{\sqrt{2\pi}\sigma_E} \int_0^\infty \exp\left[-\frac{(\ln E - \ln E^*)^2}{2\sigma_E^2}\right] d(\ln E) \cdot \frac{1}{\sqrt{2\pi}\sigma_N} \int_T^\infty \exp\left[-\frac{(\tau - E)^2}{2\sigma_N^2}\right] d\tau. \quad (2)$$

In this equation,  $\sigma_E$  is the log amplitude standard deviation corrected for saturation and aperture averaging effects and  $\tau$  is a variable of integration.

In the second integral, let  $X = (\tau - E)/\sqrt{2}\sigma_N$ , then the second integral becomes

$$\frac{1}{2} \operatorname{erfc}\left(\frac{T - E}{\sqrt{2}\sigma_N}\right). \quad (3)$$

Substituting  $\frac{\ln E - \ln E^*}{\sigma_E} = W$  in the second integral gives

$$\frac{1}{\sqrt{2}} \int_{-\infty}^\infty \exp(-W^2/2) dW. \quad (4)$$

The probability of exceeding threshold is then

$$P_D = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty \exp\left(-\frac{W^2}{2}\right) \cdot \operatorname{erfc}\left[\frac{T - E}{\sqrt{2}\sigma_N}\right] dW. \quad (5)$$

Now  $E = E^* \exp(\sigma_E W)$  from the above substitution, and  $E^* = \bar{E} \exp(-\frac{\sigma_E^2}{2})$ , where  $\bar{E}$  is the average value of  $E$  [3]. Now let  $\bar{E} = K E_0$ , where  $E_0$  is the nominal energy required to achieve a given detection probability and false alarm rate in the absence of turbulence.  $K$  is a constant equal

to the fraction of the nominal energy required for this performance. Making these substitutions we get

$$\frac{T-E}{\sqrt{2}\sigma_N} = \frac{T - KE_0 \exp\left(-\frac{\sigma_E^2}{2} + \sigma_E W\right)}{\sqrt{2}\sigma_N}. \quad (6)$$

The quantities  $T/\sigma_N$  and  $E_0/\sigma_N$  are the receiver threshold-to-noise (TNR) and signal-to-noise ratios (SNR), respectively, chosen at receiver design to give a desired detection probability and false alarm rate (FAR), and TNR is given by [4]

$$\frac{T}{\sigma_N} = \left[ 2 \ln \left( \frac{1}{2\sqrt{3}\tau \text{FAR}} \right) \right]^{1/2}. \quad (7)$$

Making these substitutions in Equation (5) we can perform the integration numerically for different values of  $\tau$ , SNR,  $K$ , and  $\sigma_E$ , thus resulting in a calculation of the probability of detection as a function of the false alarm rate for various parameters of interest including the pulse width  $\tau$ , not the integration variable above. Such a plot is shown in the figure below.

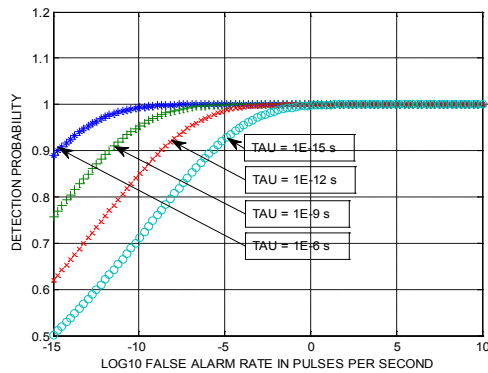


Figure 1. Detection probability vs false alarm rate for  $\sigma_E = 0.3$ , SNR = 7, and  $K = 2$ . This plot shows that detection probability is degraded for short pulses in atmospheric turbulence.

#### REFERENCES

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- [2] R. W. McMillan and N. P. Barnes, *Appl. Opt.* Vol. 15, p. 2501
- [3] A. Hald, *Statistical Theory with Engineering Applications*, Wiley, New York, 1952
- [4] *Electro-Optics Handbook* (RCA Defense Electronic Products, Burlington, MA, 1968)