Multiband Jamming Strategies with Minimum Rate Constraints

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***Abstract*— We consider a channel with N parallel sub-bands. There is a single user that can access exactly k channels, while maintaining some minimum rate at each accessed channel. The transmission takes place in the presence of a jammer which can access at most m channels. We cast the problem as an extensive-form game and derive the optimal power allocation strategies for both the user and the jammer. We present extensive simulation results regarding convergence of rates, effect of changing the number of accessed bands for the user and the jammer, and the minimum rate constraint.**

I. Introduction

Reliable communication in the presence of a malicious jammer has attracted considerable amount of research. [1] finds the worst additive noise for a communication channel satisfying a covariance constraint and derives the saddle points corresponding to equilibrium distribution of noise and transmitted signals. For the special case of memoryless channels, [2] shows that Gaussian codebooks for both jammer and user satisfy the equilibrium conditions based on min-max problem. [3] considers the case where the jammer can eavesdrop on the channel and use the information obtained to perform correlated jamming. Consequently, [3] examines the existence of a simultaneously optimal set of strategies for the users and the jammer. A multiuser, multi-tone version of [1] is considered in [4], where a generalized water-filling algorithm is proposed for the user and the jammer power allocation.

In [5], the authors consider a jammed single-hop wireless network with N independent channels, n non-cooperating users and m non-colluding jammers. The transmission is assumed to be noiseless and non-faded. The model assumes that whenever a jammer hits an occupied channel, the rate of this channel drops directly to zero. Depending on whether each occupied channel is jammed or not, the jammers and the users change their frequency bands according to a fixed transmission strategy. [5] calculates the steady state normalized rate by formulating the system model as a Markov chain, where the throughput can be obtained from the stationary distribution.

In this paper, we consider an extension of the work in [5]. We consider a noisy, fading channel model of N parallel channels. Our system has one user that can access exactly k channels subject to the constraint that the minimum rate at each accessed channel should at least be θ. The motivation of the minimum rate constraint is that in some communication systems like broadcasting systems, the system may be forced to convey information for specific number of channels and the rate of transmission may be lower bounded by service requirements. Consequently, the communication system may not be able to simply switch off some channels in order to maximize its rate, but rather it may be forced to use exactly k channels with individual rates of at least θ in each channel. The transmission is disrupted by a jammer who is able to access at most m channels. Although our model deals with a single user and a single jammer, it can be thought of as a generalization of the work in [5], since it permits cooperation in the transmitter and jammer sides. Instead of fixing the strategies of the jammer and the user and analyzing the corresponding rate as in [5], we derive the optimal power allocation policies for the jammer and the user under transmission and jamming power constraints and a minimum rate constraint for each used channel. We cast the problem as an extensive-form game, where the jammer and the user take turns to respond to each other's strategy. Our model admits a softer version of the jamming effect, where the jammer decreases the rate of a channel down to a level θ. Once the rate decreases to this level, this sub-channel contributes zero rate to the throughput, and therefore, there is no need for the jammer to expend any more jamming power to decrease the rate to zero.

In this paper, we first show that the problem under the minimum rate constraints is concave in the user power allocation policy and convex in the jammer power allocation policy. Next, we determine the optimal channel selection strategy for the transmitter, and derive the corresponding optimal user power allocation strategy over the selected set of channels. The optimal allocation strategy is a modified water-filling algorithm where weaker channels are provided with sufficient power to maintain the minimum rate constraint. We show that the optimal power allocation strategy for the jammer is a generalized water-filling algorithm. We observe that the jammer does not target channels that barely satisfy the minimum rate constraint. We provide the conditions under which the jammer chooses not to jam a specific channel. We verify our theoretic findings with extensive simulation results. We observe that an equilibrium may not be obtained in case of partial band utilization. We discuss also the effects of changing the number of accessed bands for the user and the jammer, and the minimum rate constraint, on the system performance.

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II. System Model

Consider a system with N parallel channels. Assume that there is a user who can access *exactly* k of these channels to send its message to the receiver in the presence of a malicious jammer who can access *at most* m channels to inflict the maximum hurt on this transmission. We consider the case where the user and the jammer encode their signals in response to each other, i.e., they are involved in a perfect information extensive-form game [6].

The user begins its transmission by choosing the best possible k channels to send its message with the highest possible rate. The user has a minimum rate constraint θ on each channel it uses. The user performs power allocation along the set of channels $\mathcal{S}_u$ that it chooses for transmission.

The jammer chooses a jamming power allocation strategy j_i such that the jammer pulls the rate of the k channels below θ, and hence these channels are no longer active and the user is forced to leave these channels for worse channels. Consequently, the jammer performs the following optimization problem subject to the total jamming power constraint J

$$\begin{aligned} \min_{j_i,\mathcal{S}_j} \quad & \sum_{i\in\mathcal{S}_u} \left[\frac{1}{2}\log\left(1+\frac{h_i^2 p_i}{1+g_i^2 j_i}\right)-\theta\right]^+ \\ \text{s.t.} \quad & \sum_{i\in\mathcal{S}_j} j_i \le J \\ & |\mathcal{S}_j| \le m \end{aligned} \tag{1}$$

where h_i, g_i are the channel gains from the user and the jammer, respectively, to the receiver over the ith channel, p_i is the power of the user in the ith channel, and $|\mathcal{S}_u|, |\mathcal{S}_j|$ are the sizes of the transmission set $\mathcal{S}_u$ and the jamming set $\mathcal{S}_j$, respectively.

Whenever the rate of any channel is below θ, the user chooses another channel to replace the failed channel. That means that the user considers channel i completely jammed whenever $R_i < \theta$, where R_i is the rate of the ith channel. Thus the set $\mathcal{S}_u$ should be updated by replacing channel $i \in \mathcal{S}_u$ by a new channel from $\mathcal{S}_u^c$. The user performs the following power allocation strategy over the updated $\mathcal{S}_u$ set of channels in response to the jamming strategy $\{j_i\}_{i\in\mathcal{S}_j}$

$$\begin{aligned} \max_{p_i,\mathcal{S}_u} \quad & \sum_{i\in\mathcal{S}_u} \frac{1}{2}\log\left(1+\frac{h_i^2 p_i}{1+g_i^2 j_i}\right) \\ \text{s.t.} \quad & \sum_{i\in\mathcal{S}_u} p_i \le P \\ & \frac{1}{2}\log\left(1+\frac{h_i^2 p_i}{1+g_i^2 j_i}\right) \ge \theta, \quad i\in\mathcal{S}_u \\ & |\mathcal{S}_u| = k \end{aligned} \tag{2}$$

where P is the total power of the user.

III. Optimality Conditions

In this section, we derive the optimality conditions for the transmitter and jammer optimization problems. Consequently, we provide some structural properties of the optimal solution.

A. Convexity-Concavity Property

We start our discussion by considering the objective (payoff) functions of both the transmitter and the jammer. Although the objective functions of the two problems are different, we can cast the transmitter's payoff function to have the same optimal solution as the jammer's payoff function under the minimum rate constraint. Define the following objective $R(\mathbf{p},\mathbf{j})$

$$R(\mathbf{p},\mathbf{j}) = \sum_{i\in\mathcal{S}_u} \left[\frac{1}{2}\log\left(1+\frac{h_i^2 p_i}{1+g_i^2 j_i}\right)-\theta\right]^+ \tag{3}$$

where $\mathbf{p} = (p_1,\ldots,p_N)$, and $\mathbf{j} = (j_1,\ldots,j_N)$ are the power allocation strategies for the transmitter and jammer, respectively.

Lemma 1 *Maximization of* $R(\mathbf{p},\mathbf{j})$ *is equivalent to maximization in (2) under the minimum rate constraints.*

Proof: Since subtracting a constant term $\sum_{i\in\mathcal{S}_u}\theta = |\mathcal{S}_u|\cdot\theta$ does not change the optimal solution of the problem, the optimal power allocation strategy of the problem in (2) is the same as the optimal power allocation of the objective function $\sum_{i\in\mathcal{S}_u}\left[\frac{1}{2}\log\left(1+\frac{h_i^2p_i}{1+g_i^2j_i}\right)-\theta\right]$. From the minimum rate constraints $\frac{1}{2}\log\left(1+\frac{h_i^2p_i}{1+g_i^2j_i}\right)-\theta \ge 0, \quad i\in\mathcal{S}_u$. Consequently, $\sum_{i\in\mathcal{S}_u}\left[\frac{1}{2}\log\left(1+\frac{h_i^2p_i}{1+g_i^2j_i}\right)-\theta\right]^+ = \sum_{i\in\mathcal{S}_u}\left[\frac{1}{2}\log\left(1+\frac{h_i^2p_i}{1+g_i^2j_i}\right)-\theta\right]$, and the two objective functions are equivalent. ■

The following lemma states the convexity-concavity property of the payoff function $R(\mathbf{p},\mathbf{j})$.

Lemma 2 $R(\mathbf{p},\mathbf{j})$ *is concave in* $\mathbf{p}$ *and convex in* $\mathbf{j}$ *under the minimum rate constraints.*

Proof: For the convexity in $\mathbf{j}$, we do not need the minimum rate constraint. Consider the following function $f(j)$

$$f(j) = \frac{1}{2}\log\left(1+\frac{h^2p}{1+g^2j}\right)-\theta \tag{4}$$

The function $f(j)$ is convex in j for $j \ge 0$ [2]. Define $g(j) = \max\{f(j),0\}$. Since, the maximum of two convex functions is convex [7], g is convex in j. Since the sum of convex functions is convex, $R(\mathbf{p},\mathbf{j})$ is convex in $\mathbf{j}$. In addition, since $\left[\frac{1}{2}\log\left(1+\frac{h^2p}{1+g^2j}\right)-\theta\right]^+ = \frac{1}{2}\log\left(1+\frac{h^2p}{1+g^2j}\right)-\theta$ under the minimum rate constraint as in Lemma 1, it is concave in p. Thus, $R(\mathbf{p},\mathbf{j})$ is concave in $\mathbf{p}$. ■

B. Transmitter Side Problem

In this section, we consider the solution of the transmitter's optimization problem in (2). We begin by identifying $\mathcal{S}_u$ in the next lemma.

Lemma 3 *The transmitter chooses* $\mathcal{S}_u$ *such that it includes the highest* k *channels in the normalized signal to jamming and noise ratio (SJNR) sense.*

Proof: We define the normalized SJNR at the ith channel as

$$q_i = \frac{h_i^2}{1 + g_i^2 j_i} \quad (5)$$

Now, without loss of generality assume that the channels are ordered in the sense of normalized SJNR. Assume for the sake of contradiction that $\mathcal{S}_u^* = \{1, 2, \ldots, k-1, k+1\}$, i.e., we choose the $(k+1)$th instead of the kth largest SJNR, with optimal power distribution $\mathbf{p}^*$. Since $\log(1+x)$ is monotone in x, it is clear that with the same power p_k^*, we have

$$\log\left(1 + \frac{h_k^2 p_k^*}{1 + g_k^2 j_k}\right) > \log\left(1 + \frac{h_{k+1}^2 p_k^*}{1 + g_{k+1}^2 j_{k+1}}\right) \quad (6)$$

If p_k^* is feasible when using the $(k+1)$th channel, it satisfies the minimum rate constraint when using the kth channel. Hence, the total rate can be increased with the same optimal power allocation and this contradicts the optimality of $\mathcal{S}_u^*$. ■

The next theorem characterizes the optimal strategy of the transmitter in response to the jammer's strategy.

Theorem 1 *The optimal power allocation strategy $\mathbf{p}^*$ of the transmitter in response to the jammer's strategy $\mathbf{j}$ under the minimum rate constraint θ is given by*

$$p_i^* = \begin{cases} \frac{1}{q_i}(e^{2\theta} - 1), & i \in \mathcal{S}_u, q_i \le 2\lambda e^{2\theta} \\ \frac{1}{2\lambda} - \frac{1}{q_i}, & i \in \mathcal{S}_u, q_i > 2\lambda e^{2\theta} \\ 0, & i \in \mathcal{S}_u^c \end{cases} \quad (7)$$

where $q_i = \frac{h_i^2}{1+g_i^2 j_i}$ is the normalized SJNR at the ith channel, $\mathcal{S}_u$ is the set of channels corresponding to the highest normalized SJNR, and λ is chosen such that $\sum_{i\in\mathcal{S}_u} p_i = P$.

Proof: The optimal $\mathcal{S}_u$ is obtained by Lemma 3. The Lagrangian of the optimization problem in (2) is given by

$$\begin{aligned}\mathcal{L} =& -\frac{1}{2}\sum_{i\in\mathcal{S}_u}\log\left(1 + \frac{h_i^2 p_i}{1 + g_i^2 j_i}\right) + \lambda\left(\sum_{i\in\mathcal{S}_u} p_i - P\right) \\ &+ \sum_{i\in\mathcal{S}_u}\mu_i\left(\theta - \frac{1}{2}\log\left(1 + \frac{h_i^2 p_i}{1 + g_i^2 j_i}\right)\right) \quad (8)\\ =& -\frac{1}{2}\sum_{i\in\mathcal{S}_u}\log(1 + q_i p_i) + \lambda\left(\sum_{i\in\mathcal{S}_u} p_i - P\right) \\ &+ \sum_{i\in\mathcal{S}_u}\mu_i\left(\theta - \frac{1}{2}\log(1 + q_i p_i)\right) \quad (9)\end{aligned}$$

The optimality conditions are given by

$$-\frac{1}{2}\frac{q_i}{1 + q_i p_i^*} + \lambda - \frac{\mu_i}{2}\frac{q_i}{1 + q_i p_i^*} = 0 \quad (10)$$

If on the ith channel the minimum rate constraint is satisfied with equality, i.e., $\frac{1}{2}\log(1 + q_i p_i) = \theta$ and $p_i^* = \frac{1}{q_i}(e^{2\theta} - 1)$. Since $\mu_i \ge 0$ from (10), we have

$$-\frac{1}{2}\frac{q_i}{1 + q_i p_i^*} + \lambda = \frac{\mu_i}{2}\frac{q_i}{1 + q_i p_i^*} \quad (11)$$

Hence, the condition of satisfying constraint with equality is

$$-\frac{1}{2}\frac{q_i}{1 + q_i p_i^*} + \lambda \ge 0 \quad (12)$$

which further implies $q_i \le 2\lambda e^{2\theta}$. On the other hand, if the minimum rate constraint is a strict inequality, then $\mu_i = 0$ in (10), we have

$$\frac{q_i}{1 + q_i p_i^*} = 2\lambda \quad (13)$$

which implies

$$p_i^* = \frac{1}{2\lambda} - \frac{1}{q_i} \quad (14)$$

and this occurs if $q_i > 2\lambda e^{2\theta}$. ■

We note that for the special case of $k = N$, $\theta = 0$, (7) reduces to the classical water-filling strategy in [2].

C. Jammer Side Problem

In this section, we consider the jammer side optimization problem in response to the transmitter power allocation strategy. The epigraph form of the jammer's problem (1) is

$$\begin{aligned}\min_{j_i,\mathcal{S}_j,t_i} \quad & \frac{1}{2}\sum_{i\in\mathcal{S}_u} t_i \\ \text{s.t.} \quad & t_i \ge 0 \\ & j_i \ge 0 \\ & \frac{1}{2}\log\left(1 + \frac{h_i^2 p_i}{1 + g_i^2 j_i}\right) - \theta \le t_i \\ & \sum_{i\in\mathcal{S}_j} j_i \le J \\ & |\mathcal{S}_j| \le m \end{aligned} \quad (15)$$

For a fixed $\mathcal{S}_j$, (15) becomes a convex optimization problem. The following theorem derives the optimal power allocation strategy for the jammer in response to the transmitter strategy.

Theorem 2 *The optimal power allocation strategy $\mathbf{j}^*$ of the jammer in response to the transmitter's strategy $\mathbf{p}$ under the minimum rate constraint θ is given by*

$$j_i^* = \begin{cases} -\frac{1}{g_i^2} + \frac{w_i}{2}\left(\sqrt{1 + \frac{2\mu_i}{\lambda w_i}} - 1\right), & i \in \mathcal{S}_j,\ \mu_i g_i^2 r_i \le 2\lambda \\ 0, & \textit{otherwise} \end{cases} \quad (16)$$

where $w_i = \frac{h_i^2 p_i}{g_i^2}$ is the to signal to jamming ratio (SJR) of the ith channel, $r_i = \frac{h_i^2 p_i}{1+h_i^2 p_i}$ is the useful signal ratio, μ_i is the Lagrange multiplier corresponding to the rate constraint and λ is chosen such that $\sum_{i\in\mathcal{S}_j} j_i = J$.

Proof: The Lagrangian of the optimization problem is

$$\begin{aligned}\mathcal{L} =& \sum_{i\in\mathcal{S}_u} t_i + \sum_{i\in\mathcal{S}_u}\mu_i\left(\frac{1}{2}\log\left(1 + \frac{h_i^2 p_i}{1 + g_i^2 j_i}\right) - \theta - t_i\right) \\ &+ \lambda\left(\sum_{i\in\mathcal{S}_u} j_i - J\right) - \sum_{i\in\mathcal{S}_u}\nu_i t_i - \sum_{i\in\mathcal{S}_u}\eta_i j_i \end{aligned} \quad (17)$$

The optimality conditions are

$$\mu_i + \nu_i = 1 \quad (18)$$

$$\frac{\mu_i}{2}\left[\frac{g_i^2}{1+h_i^2p_i+g_i^2j_i} - \frac{g_i^2}{1+g_i^2j_i}\right] + \lambda - \eta_i = 0 \quad (19)$$

When the jammer does not jam a channel, i.e., $j_i = 0$, then $\eta_i \geq 0$, and from (19) we have

$$\frac{\mu_i}{2}\left[\frac{g_i^2}{1+h_i^2p_i} - g_i^2\right] + \lambda = \eta_i \geq 0 \quad (20)$$

which implies

$$\mu_i g_i^2 \frac{h_i^2p_i}{1+h_i^2p_i} \leq 2\lambda \quad (21)$$

which further implies

$$\mu_i g_i^2 r_i \leq 2\lambda \quad (22)$$

On the other hand, if the jammer jams the ith channel, then $\eta_i = 0$ and hence (19) becomes

$$\frac{\mu_i}{2}\left[\frac{-g_i^2h_i^2p_i}{(1+h_i^2p_i+g_i^2j_i)(1+g_i^2j_i)}\right] + \lambda = 0 \quad (23)$$

which is equivalent to

$$2\lambda(1+h_i^2p_i+g_i^2j_i)(1+g_i^2j_i) = \mu_i g_i^2h_i^2p_i \quad (24)$$

which is quadratic in j_i. By expressing the roots of this quadratic equation in explicit form, we obtain

$$j_i = -\left(\frac{1}{g_i^2} + \frac{h_i^2p_i}{2g_i^2}\right) + \sqrt{\frac{h_i^4p_i^2}{2g_i^4} + \frac{\mu_i h_i^2 p_i}{2\lambda g_i^2}} \quad (25)$$

$$= -\frac{1}{g_i^2} + \frac{w_i}{2}\left(\sqrt{1+\frac{2\mu_i}{\lambda w_i}} - 1\right) \quad (26)$$

with $w_i = \frac{h_i^2p_i}{g_i^2}$. ■

In the following lemmas, we investigate some properties of the jammer's constraints. The first lemma deals with the constraint of $\frac{1}{2}\log\left(1+\frac{h_i^2p_i}{1+g_i^2j_i}\right) - \theta \leq t_i$.

Lemma 4 *The constraint $\frac{1}{2}\log\left(1+\frac{h_i^2p_i}{1+g_i^2j_i}\right) - \theta \leq t_i$ should be satisfied with equality and hence $\mu_i \neq 0$.*

Proof: There are two cases to be considered. The first case is $t_i^* > 0$. Assume for sake of contradiction that the constraint is strict for the optimal j^*, t_i^*, i.e., $\frac{1}{2}\log\left(1+\frac{h_i^2p_i}{1+g_i^2j_i^*}\right) - \theta < t_i^*$ and $t_i^* > 0$. In this case, we can decrease the value of t_i until the equality holds. This is feasible and decreases the objective function and hence we have contradiction that t_i^* is optimal.

On the other hand, if $t_i^* = 0$, then if the constraint is also strict, then one can decrease j_i^* such that $\frac{1}{2}\log\left(1+\frac{h_i^2p_i}{1+g_i^2j_i^*}\right) - \theta = 0$. This is feasible under the total jamming power constraint, while the objective function will not increase. Hence, the constraint is satisfied by equality in all cases. ■

The following lemma concerns about the total jamming power constraint.

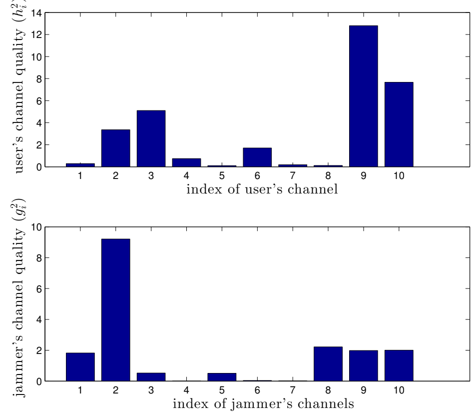

Fig. 1. Channel gains.

Lemma 5 *The total jamming power constraint should be satisfied with equality.*

Proof: First, if $J - \sum_{i\in\mathcal{S}_j} j_i^* = \Delta > 0$ and there exists $t_l^* \neq 0$ for some $l \in \mathcal{S}_j$, we let $j_l = j_l^* + \Delta$, which is a feasible power allocation strategy. Then, by the monotonicity of $\log$, we can have $t_l < t_l^*$. Moreover, if $t_i^* = 0$, $\forall i \in \mathcal{S}_j$, then any power allocation strategy is optimal and hence we restrict ourselves to satisfy the jamming power constraint with equality. ■

The following lemma states the conditions under which the jammer does not jam the ith channel.

Lemma 6 *The jammer chooses not to jam the ith channel if the SNR of the channel is low or the jammer's channel gain is low. More specifically, $j_i^* = 0$ if $\mu_i g_i^2 r_i \leq 2\lambda$.*

Proof: The proof follows from the optimality conditions derived in Theorem 2. $r_i = \frac{h_i^2p_i}{1+h_i^2p_i} = \frac{1}{1+\frac{1}{h_i^2p_i}}$, which is a monotone function in $h_i^2p_i$, i.e., the SNR of channel i. The SNR also controls μ_i, since from (18) we have $\mu_i + \nu_i = 1$. If $t_i^* = 0$ (which corresponds to the case where the channel barely satisfies the θ constraint), then $\nu_i \geq 0$, which means that $\mu_i \leq 1$ (in contrary to $\mu_i = 1$ for the channels that exceed rate θ). Hence, if channel SNR or jammer's channel gain decrease, the product $\mu_i g_i^2 r_i$ also decreases and the jammer chooses not to jam this channel, since it carries little rate or does not hurt the main link as much. ■

IV. Simulation Results

In this section, we present some simulation results for the presented system model. In all simulations, we fix $N = 10$. The user and the jammer repeat their encoding over 10 transmission blocks each, i.e., 10 encoding frames. We use fixed channel gains, which are shown in Fig. 1.

A. Power Allocation Results

We choose $m = k = N = 10$, and $\theta = 0.1$. In this case, we have an equilibrium in the sense that neither the user nor the jammer changes its power allocation, since the

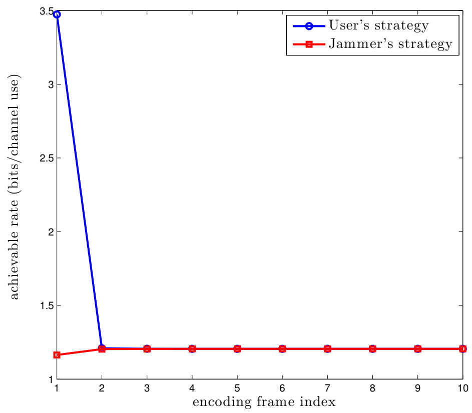

Fig. 2. Equilibrium of achievable rates under $P = J = 10$, $\theta = 0.1$, and $m = k = N = 10$ at each encoding frame for the user and the jammer.

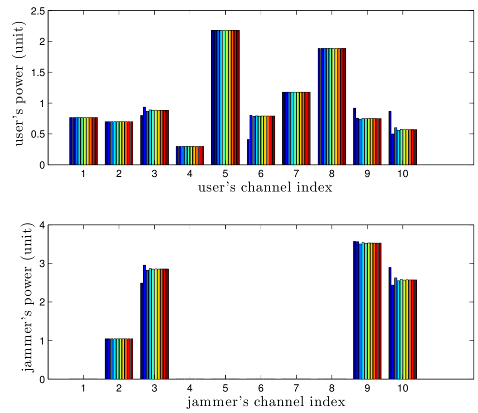

Fig. 3. Equilibrium power allocation for the user and the jammer under $P = J = 10$, $\theta = 0.1$, and $m = k = N = 10$.

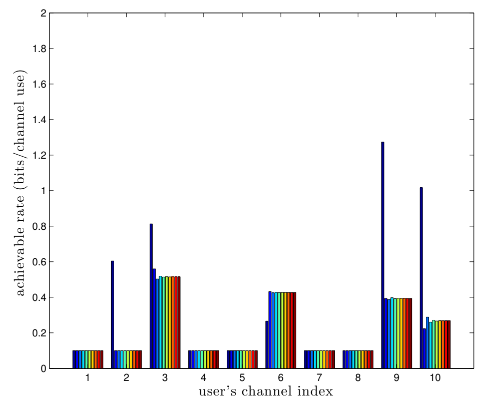

Fig. 4. Equilibrium achievable rates for $P = J = 10$, $\theta = 0.1$, and $m = k = N = 10$.

strategies achieve their optimal payoff functions as shown in Fig. 2. In Fig. 3, we show the power strategies of the user and the jammer at each channel. The colored bars represent the encoding frame power. We note that for the channels $\{3, 6, 9, 10\}$, the user applies ordinary water-filling in the sense that the higher the channel gain, the higher the transmitted power. We see slight variations in power distribution along these channels over time, because the noise levels change due to the jamming power. For the rest of the channels, i.e., channels $\{1, 2, 4, 5, 7, 8\}$, we see an inverse behavior, where the worse the channel gain, the higher the power injected by the user to maintain the required minimum rate θ. Hence, we have fixed power distribution along these channels. We note that the jammer does not waste its power on the channels that barely achieve θ, since any power $\epsilon > 0$ drives the rate on these channels to zero. Consequently, the jammer concentrates on good channels, i.e., the set $\{3, 6, 9, 10\}$. However, for channel 6, we note that the channel quality from jammer to receiver is bad. Hence, jammer uses channel 2 instead which has the maximum channel gain and in the meanwhile carries rate larger than θ for the first two encoding frames. The corresponding rates of every channel is given in Fig. 4.

We note that not all model settings lead to equilibrium. More specifically, when $m, k < N$, the user can possibly move from the jammed band to other channel which was initially worse in order to increase its rate. This potentially leads to an oscillations between multiple sets of channels with different payoffs and hence no equilibrium can be achieved. Figs. 5, and 6 show an example of this non-equilibrium case with $N = 10, m = k = 2$ with same channel gains. The figures show that the user and jammer jump between two set of channels $S_1 = \{2, 3\}$ and $S_2 = \{9, 10\}$. Hence, we have missing bars in the power allocation and the achievable rates oscillate.

B. Effect of the Minimum Rate Constraint

In Fig. 7, we investigate changing the minimum rate constraint θ. We consider the achievable rate after jammer determines its encoding strategy. We choose that $m = k = N = 10$ for $\theta = \{0.05, 0.1, 0.13\}$. We note at first that if we increase $\theta > 0.13$, the problem becomes infeasible for the channel gains under discussion. We also note that as θ increases, the achievable rate decreases. This is because the user must provide excessive power in the bad channels to maintain rate θ at each bad channel. This decreases the available power for other channels.

C. Effect of the Number of Channels

In Fig. 8, we investigate the effects of changing the number of accessed channels by the user and the jammer. We consider the case where $m = k$. In another words, we consider the special case where the jammer has the ability to jam all the channels of the user. Since equilibrium may not be obtained, we use the average rate over all encoding intervals, i.e., $\bar{R} = \frac{1}{T}\sum_{t=1}^{T} R(t)$ where T is the total encoding intervals for the user and the jammer, we take $T = 20$, and $R(t)$ is the achievable rate in the tth encoding interval. From Fig. 8, we note that the average rate increases until $k = 5$, because the dominant effect until that point is that we are adding channels to $\mathcal{S}_u$ that can achieve rates larger than θ. However, after we reach $k = 5$, the problem is more confined since we are forcing the user to inject power in bad channels to barely achieve rate θ and this sacrifices from the maximum rate that

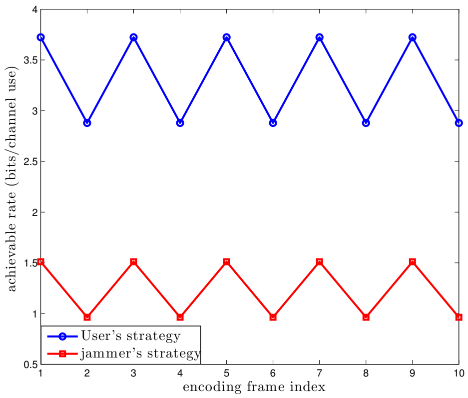

Fig. 5. Achievable rates at every encoding frame for the non-equilibrium instance $m = k = 2, N = 10$ and $\theta = 0.1$.

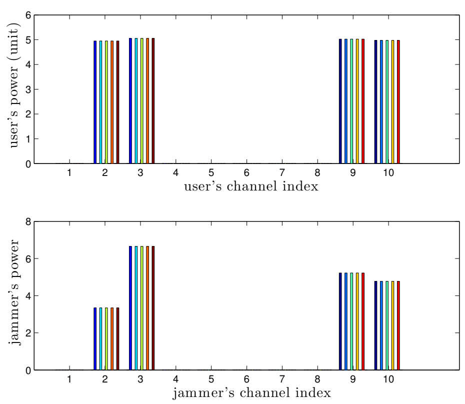

Fig. 6. Non-equilibrium power allocation for the case $N = 10, m = k = 2$ and $\theta = 0.1$.

the user can achieve if it follows the ordinary water-filling and switches off these bad channels. In Fig 9, we fix $k = 6$ and we investigate the effect of changing the number of accessed channels for the jammer. We note that the as m increases, we have monotone non-increasing graph which shows the fact that as m increase, the degrees of freedom of the jammer to hurt the user increases.

REFERENCES

[1] S. Diggavi and T. Cover, "The worst additive noise under a covariance constraint," *IEEE Trans. on Info. Theory,*, vol. 47, no. 7, pp. 3072–3081, Nov 2001.

[2] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. John Wiley & Sons, 2012.

[3] S. Shafiee and S. Ulukus, "Mutual information games in multiuser channels with correlated jamming," *IEEE Trans. on Info. Theory,*, vol. 55, no. 10, pp. 4598–4607, Oct 2009.

[4] R. Gohary, Y. Huang, Z.-Q. Luo, and J.-S. Pang, "A generalized iterative water-filling algorithm for distributed power control in the presence of a jammer," *IEEE Trans. on Signal Processing,*, vol. 57, no. 7, pp. 2660–2674, July 2009.

[5] P. Wang and B. Henz, "Performance analysis of jammed single-hop wireless networks," in *IEEE MILCOM*, November 2013.

[6] S. Tadelis, *Game Theory: An Introduction*. Princeton Univ. Press, 2013.

[7] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge Univ. press, 2004.

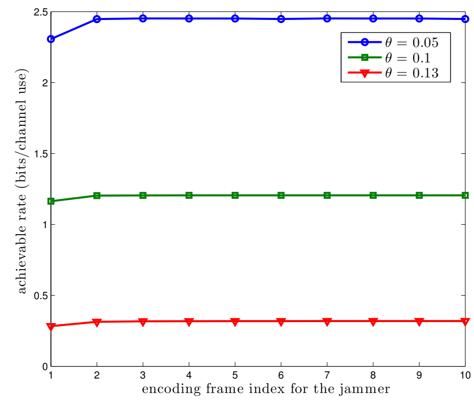

Fig. 7. Effect of changing θ for $m = k = N = 10$.

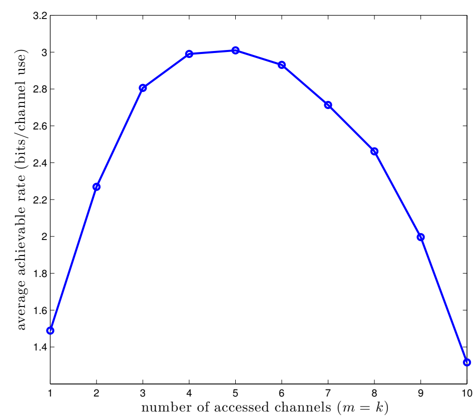

Fig. 8. Effect of changing m, k such that $m = k, N = 10$ and $\theta = 0.1$.

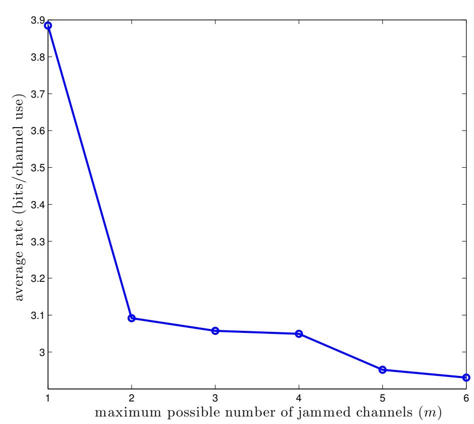

Fig. 9. Effect of changing m with fixed $k = 6$.