Comment on: Sequential Tests of Hypotheses for System-Reliability Modeled by a 2-Parameter Weibull Distribution

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Key Words **— MIL-STD-781, Repairable system, Weibull distribution, Weibull renewal process**

Reader Aids **—**
Purpose: Commentary
Special math needed for explanations: None
Special math needed to use results: None
Results useful to: Reliability theorists and practitioners

Abstract **— A different point of view is presented on the content of the original paper.**

I. INTRODUCTION

Ref. [1] pointed out some limitations of the research literature on MIL-STD-781C [2]. The paper being commented upon here [3] was one of nine cited in [1]. These comments are directed to [3] since a) it was published recently in these Transactions, b) it has the word *system* in its title and, as explained in [1], virtually any *system* will be repairable under MIL-STD-781C test conditions, and c) the model *implicitly* assumed in [3] is a particularly unrealistic *a priori* model for a repairable system.

Harter, Moore, Wiegand [3], henceforth HMW, present very valuable results for sequential testing when the data consist of times to failure which are *s*-independent and identically distributed (i.i.d.) samples from a 2-parameter Weibull distribution. However, it never states the assumption that the data must be i.i.d. for the results to hold. If HMW had been posed in terms where the i.i.d. assumption was a reasonable consequence of the physical situation, this would not be a major oversight. For example, if nominally identical nonrepairable items are tested under nominally identical conditions, the i.i.d. assumption follows directly. HMW, however, usually refers to *system reliability* — eg, this term is in the title. Moreover, the only testing document cited in HMW, MIL-STD-781C, is intended for the testing of repairable systems [1]. Since most real systems are designed to be repairable, HMW applies to such systems only under the unstated i.i.d. assumption. In addition, even when the times between successive failures of a repairable system *are* i.i.d., it will be shown that the Weibull distribution is a particularly implausible distribution to assume as an underlying model.

II. AN IMPLAUSIBLE MODEL

In some cases it is plausible to assume that each repair results in renewal. Even then, however, the properties of the Weibull distribution imply that it is not a very suitable candidate for modeling the times between successive failures of a repairable system. The force of mortality, $h_X(x)$, of the distribution of a random variable, X, is:

$$h_X(x) \equiv \mathrm{pdf}(x)/\mathrm{Sf}(x). \tag{1}$$

In the special case where the times between failures are Weibull distributed, such that $\mathrm{Sf}\{x\} = \exp - [(x/\theta)^K]$, it is simple to show that

$$h_X(x) = \frac{K}{\theta^K} x^{K-1}. \tag{2}$$

Therefore, for $K < 1, = 1, > 1$ the force of mortality is monotonically decreasing from ∞ towards 0, constant, and monotonically increasing from 0 towards ∞, respectively. Figure 1 shows how the force of mortality jumps from a value approaching 0, back up to ∞, when a repair (which is assumed to be performed instantaneously in the figure) occurs, for $K < 1$. In this case, the system is *bad*-as-new. Since the phrase, good-as-new is usually considered to be synonymous with renewal, this is an indication that even basic concepts have not been thought through properly, when dealing with a sequence of times between failures.

The model (with drastic jumps whenever an action follows failure) whenever $K \neq 1$, would be much more plausible when the action is replacement rather than repair, since the latter usually involves the replacement of only a small proportion of a system's constituent parts. For example, a renewal process with Weibull distributed times between successive failures would be a reasonable model for the successive light bulbs placed in a socket. In the case of a repairable system it is likely that, given a renewal process model, the Weibull distribution could not be rejected on statistical grounds. That is, the Weibull distribution is flexible enough, given the small sample sizes usually available in reliability studies, to be unlikely to be rejected by a goodness-of-fit test. Nevertheless, physical reasoning makes it an implausible candidate for modeling repairable system reliability. An argument could be made that the $K < 1$ case is not unreasonable since repair often seems to trigger additional immediate failures. Morever, this is consistent with the fact that a single bad part in *series* — which itself might have been damaged by the

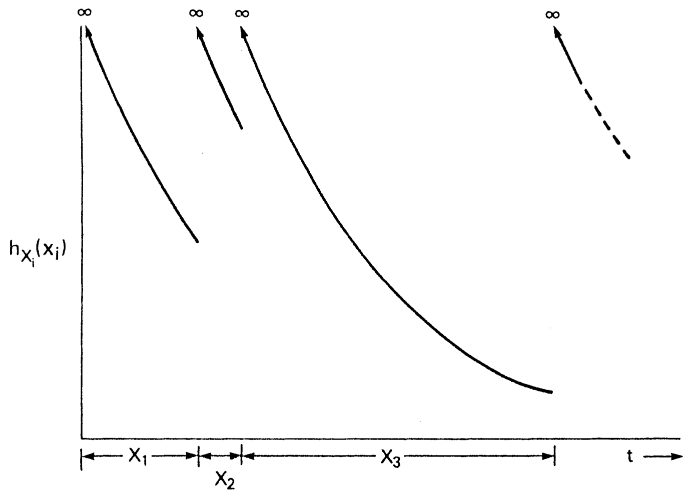

Fig. 1. Variation of force of mortality for Weibull renewal process with $K < 1$. x_i is the *local time* measured from repair $i - 1$ and t is *global time*, viz, system total operating time, regardless of its failure history.

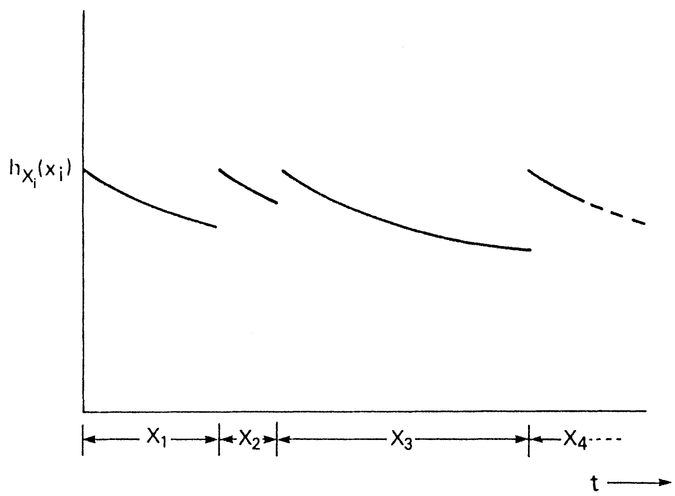

Fig. 2. Variation of force of mortality for a physically more plausible renewal process model for a repairable system. The x_i and t have the same interpretation as in figure 1.

previous failure and/or repair action — will cause another system failure, as soon as it fails. In contrast, the implication of the $K > 1$ case is that another failure is very unlikely in a short time interval after repair; this implies that *all* wearing, *series* parts are renewed during each repair. Even for the $K > 1$ case, however, it is important to state that renewal is being assumed, rather than discussing MIL-STD-781C as if it applied to non-repairable items.

When a renewal process, with nonexponential times between successive failures, is the appropriate model for a repairable system, a distribution with the properties:

$$0 < h_X(0) < \infty$$

$$0 < h_X(\infty) < \infty,$$

would be a more plausible generalization of

$$h_X(x) = constant, \quad 0 \leqslant x \leqslant \infty.$$

Figure 2 is a pictorial representation of a renewal process with such properties.

III. DISCUSSION & SUMMARY

If HMW had specifically referred to the testing of nonrepairable items, there would be no reason to object to it. For example, a better approach would have been to discuss MIL-STD-690B [4], rather than MIL-STD-781C. That is, modifications to MIL-STD-690B, which does deal with the testing of nonrepairable items with exponentially distributed time to failure, could have been considered. Alternatively, HMW could have stated the assumption that every repair restores the system to its original condition. Under this alternative, there could have been (at least a brief) discussion of the implausibility of Weibull renewal when a failed item is repaired, rather than replaced.

HMW is about extending the results of MIL-STD-781C to a different model than the one assumed in the Standard. Since the Standard applies solely to repairable systems, its model is the homogeneous Poisson process, in spite of what is stated in the Standard's title and scope [1]. The HMW alternative, therefore, is a renewal process with Weibull distributed times between successive failures. The HMW results could be applied directly to the testing of nonrepairable items. Under unstated — and usually unrealistic — conditions the HMW results could also be applied to repairable systems. As written, however, HMW perpetuates the widespread misconception that the times between successive failures of a repairable system are necessarily i.i.d. This *a priori* i.i.d. assumption is particularly ironic since it is often hoped that it does not hold, ie, it is hoped instead that reliability is growing.

REFERENCES

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Harold Ascher: For biography see pp 402 in this issue.

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