

On-Line Accuracy Assessment for the Dual Six-Port ANA: Background and Theory

GLENN F. ENGEN, FELLOW, IEEE

Abstract—One of the major challenges confronting the microwave metrologist today is that of providing an accuracy assessment for the automatic network analyzer (ANA). This paper provides the background and theory for the recently developed on-line solution now in use with the six-port systems at the National Bureau of Standards (NBS).

I. INTRODUCTION

THE role of the automatic-network-analyzer (ANA) in the art of microwave metrology, as practiced today, is well known. The so-called "six-port" version of this instrument has been under development at the National Bureau of Standards (NBS) for the past 15 years and more recently has captured the attention of the international community of microwave metrologists. This interest is reflected by a bibliography which includes more than 100 entries. For all this interest, however, the problem of developing an accuracy statement for its performance has been largely unresolved. It is the purpose of this paper to provide the background and theory for the on-line accuracy assessment which has recently become operational for the dual six-port systems currently in use at NBS. Additional details of this development will be presented in a series of companion papers [1]–[4].

The "six-port" [5] is a measuring instrument that provides for the determination of a complex parameter (reflection coefficient) at its "measurement port" in terms of four scalar observations (power meter readings) at four of its remaining "ports." (The remaining "port" provides for connection to an external signal source.) The functional relationship between these power meter readings and the desired measurement result (reflection coefficient) involves 11 frequency-dependent parameters of the six-port network that is the heart of the measuring instrument and from which its name is derived.

In general terms, the six-port accuracy will be limited by uncertainties in the four power measurements. In addition, the accuracy will also be a function of the parameter value being measured, the 11 parameters that characterize the six-port, and the errors in their determination. Moreover, these 11 parameters are not necessarily invariant with time, temperature, mechanical stress, etc. Finally, the six-port model equations assume excitation at a

single frequency and in practice there are sometimes substantial deviations from this idealization. A further problem, which is common to all of microwave metrology, is that of nonideal connector performance. Although there is a substantial precedent for minimizing the effect of this component in the accuracy statement, this at best is difficult in the present context and in any case is probably unrealistic, especially if the nonideal connector performance is the predominate source of error. For the reasons just mentioned, it is obviously impossible to characterize the six-port accuracy by a single number. Instead, the on-line accuracy assessment provides an estimate of the combined effect of these different error sources on each individual measurement result. In general, all of the necessary information on which the error estimate is based is provided by the dual six-port response during the course of its calibration and subsequent use as a measuring instrument. It is necessary, however, to provide an independent evaluation of any sources of systematic error in the standards by which the six-port calibration is effected. Given this information, the contribution of this error source is factored into the error estimate.

II. SYNOPSIS OF PRIOR RESULTS

In common with other ANA's, the operation of the "six-port" requires a calibration prior to its use as a measuring instrument. As a minimum, this calls for a determination of the 11 parameters that characterize the six-port network (an additional or twelfth parameter is required for a power measurement application, but this is not explicitly considered here). As already noted, these 11 parameters permit one to determine the complex reflection coefficient Γ which obtains at the measurement port as a function of the four sidearm readings. When the measurement objective is attenuation or some other two-port parameter, the dual six-port [6] is convenient. This is comprised of a pair of six-ports and permits a simultaneous determination of the reflection coefficient values which obtain at both ports of the item under test, and under a number of different excitation conditions. The two-port parameters may then be determined from these measurement(s) results. As before, the primary interest is in the $P(\text{Power}) - \Gamma$ transform.

It has proven convenient to effect the $P - \Gamma$ transform in two steps. This is achieved by introducing a complex parameter w such that the $P - w$ transform involves five

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The author was with the National Bureau of Standards, Electromagnetic Fields Division, Boulder, CO 80303. He is now with Boulder Research Associates, Inc., Boulder, CO 80303.

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of the 11 parameters, while the $w - \Gamma$ transform involves the remaining six, which are in the form of three complex numbers, A , B , and C . The $w - \Gamma$ transform is given by

$$w = \frac{A\Gamma + B}{C\Gamma + 1}. \quad (1)$$

This is, of course, recognized as a linear fractional transform, which plays a major role in microwave metrology, and which, in particular, describes the operation of a four-port reflectometer. The $P - w$ transform, which provides the complex w , and as noted reduces the behavior to that of a four-port, is termed the "six-to-four-port reduction." This transform may be expressed by the following equations:

$$\frac{P_1}{P_0} = |w|^2 \quad (2)$$

$$\frac{P_2}{P_0} = \rho_2 |w - \lambda_2|^2 \quad (3)$$

$$\frac{P_3}{P_0} = \rho_3 |w - \lambda_3|^2. \quad (4)$$

Here, the P_i represent the sidearm power readings while the real ρ_2 , ρ_3 , λ_2 , and the complex λ_3 are the five parameters which characterize the $P - w$ transform. It is frequently convenient to represent this system of equations as shown in Fig. 1 where the solution for w is obtained from the intersection of three circles, and where the radii are proportional to $\sqrt{P_i/P_0}$, $i = 1, 2, 3$. It will also be recognized that the radii of any two circles determines the third to the extent of a choice between two possible values. This implies a functional relationship between P_1/P_0 , P_2/P_0 , and P_3/P_0 which may be represented in three-dimensional space as an elliptic paraboloid, and which is tangent to the $P_i/P_0 = 0$ planes [7]. The five "paraboloid" parameters, which characterize the $P - w$ transform, also determine the points of tangency and the direction of the paraboloid axis. Conversely, given a collection of measurement results (P_i/P_0 values) for a variety of different excitation conditions, it is possible to determine the paraboloid parameters by using a "surface" fitting procedure in the three-dimensional P_i/P_0 space.

In order to complete the calibration, one requires the A , B , and C that characterize the $w - \Gamma$ transform. In the dual six-port environment, the "thru-reflect-line" (TRL) [8] technique continues to be an attractive choice. This calls for observing the system response with the two measurement ports connected together (thru); next, with the same unknown termination connected, in turn, to each of the six-ports (reflect); and finally, with an unknown and arbitrary length of line inserted between them (line). Having thus completed the calibration, the "measurement mode" merely calls for observing the sidearm power values and applying the appropriate transforms to obtain the Γ value. As will be explained in what follows, an essen-

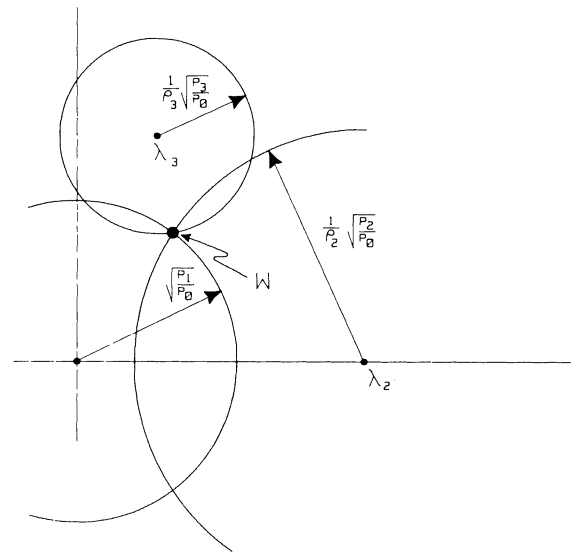


Fig. 1. The value for w is obtained from the intersection of three circles in the complex plane.

tial element in the accuracy assessment has been the availability of redundant information. Although a certain amount of this is already present in the solution outlined above, it is confined primarily to the $P - w$ transform. In the course of developing the accuracy assessment it has been found useful to add redundancy to the TRL technique from which the parameters of the $w - \Gamma$ transform are determined.

The assessment begins with an estimate of the power-meter error and the accuracy to which the 11 six-port parameters have been determined. These results are then combined to yield an estimate of the instrument performance in a measurement application.

III. $P - w$ TRANSFORM

Given a suitable collection of power meter *observations*, the primary objective in phase 1 of the calibration procedure is to obtain *estimates* of the five "paraboloid" parameters that characterize the $P - w$ transform. (Note that in accordance with the statistician's vocabulary, we *do not measure* anything! Rather, we *observe* the response of a measuring instrument (power meter) and subsequent to certain mathematical operations, obtain *estimates* of the parameter values of interest.) By the use of the appropriate statistical methods, however, it is also possible to simultaneously obtain an estimate of the power-meter error, and of the accuracy to which the paraboloid parameters have been determined. The foregoing is an exercise in that branch of statistics that carries the name "parameter estimation theory." Although this theory owes much of its original development to Gauss (1777-1855), its application in the field of microwave metrology has been slow. The details of its application to the present problem will be found in a companion paper [1]. This paper will limit itself to formulating the problem in such a manner that the theory can be conveniently applied.

Returning to (2)–(4), these may be written

$$\frac{P_i}{P_0} = \rho_i |w - \lambda_i|^2, \quad i = 1, \dots, 3 \quad (5)$$

where

$$\rho_1 = 1$$

$$\lambda_1 = \text{Im}(\lambda_2) = 0.$$

In a measurement application, the values of ρ_i and λ_i are assumed to be known parameters, having been previously determined in the calibration mode. Thus (5) represents a system of three scalar equations for the determination of a single complex parameter w . Because the system is over determined, and because of measurement errors in the P_i , it will not be possible, in general, to find a value for w that satisfies all three equations. Instead, one defines the function

$$\phi = \sum_{i=1}^3 \epsilon_i^2 \quad (6)$$

where

$$\epsilon_i = \frac{P_i}{P_0} - \rho_i |w - \lambda_i|^2, \quad i = 1, \dots, 3 \quad (7)$$

and chooses for w that value which makes ϕ a minimum.

In the calibration mode, the preceding technique is used to obtain an estimate of the ρ_i and λ_i , as well as the w values. In this context one has a collection of P values, which in a typical case correspond to 16 different excitation conditions or values of w . Let

$$\epsilon_{ij} = \frac{P_{ij}}{P_{0j}} - \rho_i |w_j - \lambda_i|^2, \quad i = 1, \dots, 3, \quad j = 1, \dots, 16 \quad (8)$$

and let

$$\phi = \sum_{i=1}^3 \sum_{j=1}^{16} \frac{\epsilon_{ij}^2}{\sigma_{ij}^2} \quad (9)$$

where σ_{ij} is a “weighting factor” whose role is described in the companion paper [1]. At this point, (8) represents a system of 48 equations, while the complex w_j represents 32 unknowns, and the ρ_i and λ_i an additional five. As before, one chooses these so that ϕ is a minimum. Although the details are reserved for the companion paper, it may be intuitively recognized that ϕ_{\min} may be used to obtain an estimate of the power meter error. This, in turn, leads to an estimate of the errors in the w_j , ρ_i , and λ_i , which are thus obtained.

Although the foregoing closely parallels an earlier treatment of the subject [9], the following distinctions should be recognized. As already noted, the ρ_i and λ_i are obtained in addition to the w_j . Moreover, instead of the individual values, the power ratios (P_{ij}/P_{0j}) are taken as the “observed” quantity. This reflects a change in the measurement technique where the P_{ij} and P_{0j} are observed

simultaneously, and although ideally constant, a repeat observation of the P_{0j} is made for each of the P_{ij} . For this reason the (random) errors in the P_{ij}/P_{0j} ratios are uncorrelated, which would not be the case if, as in the earlier environment, a single observation of P_{0j} was used to form the ratios.

An important characteristic of the formulation mentioned is that it explicitly includes the observed quantity (P_{ij}/P_{0j}) as a function of parameters to be estimated (in this case the excitation state w_j and the six-port parameters ρ_i and λ_i). In addition, the parameter ϵ_{ij} tells how much the observed P_{ij}/P_{0j} must be “adjusted” in order for (8) to be satisfied, or alternatively represents an assignment of “error” to it. These features are preserved in the treatment of other aspects of the problem which follows.

At this point in the procedure one has estimates of the paraboloid parameters and a collection of estimated w values which result from a set of excitation conditions, as defined below. In addition, however (and this is the new feature), one also has an indication (or estimate) of the accuracy to which these parameters have been estimated, and of the power-meter error.

IV. ACCURACY ASSESSMENT: $w - \Gamma$ TRANSFORM

Given the paraboloid parameters for the two six-ports which comprise the dual six-port, it is convenient to represent the remainder of the calibration problem as shown in Fig. 2. As noted, the foregoing procedure yields a collection of w values in addition to the paraboloid parameters. Each of these w values is related to the value of Γ which obtains at the measurement port by (1). Thus the remainder of the calibration problem is to determine the A , B , and C that characterize this transform for each of the six-ports. In the model provided by Fig. 2, these parameters are identified with a so-called “Error Box” or two-port. Using the subscripts 1 and 2 to represent the parameters for each six-port, the boundary conditions imposed by the thru, reflect, and line are $\Gamma_1\Gamma_1 = 1$, $\Gamma_1 = \Gamma_2$, $\Gamma_1\Gamma_2 = e^{-2\gamma l}$, respectively, where γ is the propagation constant for the line, and l is its length. It is convenient to define the cascade combination of the Error Boxes which results from the thru connection as the thru two-port. In a similar way, the line two-port is comprised of Error Box 1, the line, and Error Box 2 in cascade. As originally described [8], the TRL calibration consisted first of determining the parameters of the thru and line two-ports, each of which may be characterized by three complex parameters (A_t , B_t , \dots , C_t) where the subscripts t and l represent thru and line, respectively. The w_1 and w_2 associated with the reflect represent two additional observations which, in combination with the two-port parameters, represents eight measurement results. The unknowns are also eight in number and include three parameters for each of the Error Boxes, the Γ for the unknown reflect, and the line parameter γl . The information contained in the measurement results is, thus, just suffi-

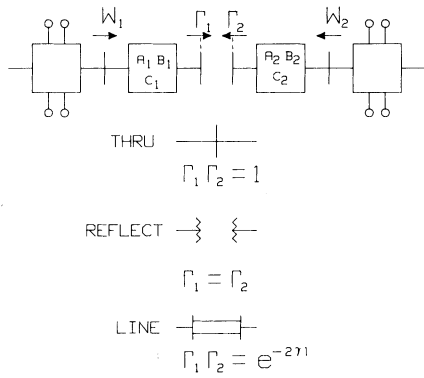


Fig. 2. The TRL calibration procedure calls for a number of different boundary conditions, as shown.

cient to determine the unknowns. Since redundancy is the key to the accuracy assessment, it has proven useful to augment this procedure by the use of additional reflects. Although somewhat of an abstraction, it is convenient to supplement the above description by the addition of a reflect two-port which consists of the two Error Boxes and a fictitious two-port inserted between them which enforces the $\Gamma_1 = \Gamma_2$ boundary condition. The TRL solution now consists of two steps. These include the determination of the parameters of the thru, reflect, and line two-ports; this is followed by a determination of the individual Error-Box parameters from these results.

The scattering equations for an arbitrary two-port are

$$b_1 = s_{11}a_1 + s_{12}a_2 \quad (10)$$

$$b_2 = s_{21}a_1 + s_{22}a_2 \quad (11)$$

where b_i and a_i are the emergent- and incident-wave amplitudes at the i th port, respectively. An application of these to the thru two-port gives

$$w_1 = \frac{b_1}{a_1} = s_{11} + s_{12} \frac{a_2}{a_1} \quad (12)$$

and

$$w_2 = \frac{b_2}{a_2} = s_{22} + s_{21} \frac{a_1}{a_2} \quad (13)$$

The treatment to follow requires only the product $s_{12}s_{21}$; thus, even apart from reciprocity considerations, one can assume $s_{12} = s_{21}$. Although in phase 1 the w_1 and w_2 were *estimated* from the power meter observations, they now assume the role of *observations*. Thus (12) and (13) yield the "observed" w_1 and w_2 as functions of the thru two-port scattering parameters s_{ij} and the excitation state a_2/a_1 .

In general a_2/a_1 is unknown so that (12) and (13) represent two equations in four unknowns. As it stands, the system is indeterminate; however, the dual six-port includes provision for providing additional excitation states, usually a total of four. Given the system response to these as well, one has a system of eight equations in seven

(complex) unknowns. These include s_{11} , s_{12} , s_{22} , and the four values of a_2/a_1 associated with the four excitation states. As before, it will not be possible to find values for these unknowns so that all eight equations are satisfied and, as explained in greater detail in [1], a least squares solution is again used which not only yields estimates of their value, but also estimates the accuracy of the estimate.

In the original solution the a_2/a_1 were eliminated from the problem yielding a system of four equations in three unknowns. Here, although they are of no further interest, they have been retained in order to satisfy the requirements outlined above in formulating the least squares solution.

Although the foregoing is immediately applicable to determining the parameters of the line two-port as well, its application to the reflect two-port calls for some caution. Let R_{11} , R_{12} , and R_{22} represent the scattering parameters of the reflect two-port. These, in turn, are functions of the A_1 , B_1 , \dots , C_2 which characterize the Error Boxes, but not of Γ . In this specific context and for reasons to follow, it is convenient to take $1/w_1$ rather than w_1 as the measurement observation provided by six-port number 1. Then by analogy with (12) and (13), one would like to be able to write

$$\frac{1}{w_1} = R_{11} + R_{12}G \quad (14)$$

$$w_2 = R_{22} + R_{12}/G \quad (15)$$

where G is some function of Γ . The question is now one of whether or not this is even possible, and if so to find the functional relationship between the R_{ij} and the A_1 , B_1 , \dots , C_2 .

Although the procedures used to find the solution are beyond the scope of this paper, it may be easily confirmed, by direct substitution, that the following is indeed a solution:

$$G = \frac{1 + C_2\Gamma}{A_1\Gamma + B_1} \sqrt{\frac{A_1 - B_1C_1}{A_2 - B_2C_2}} \quad (16)$$

$$R_{11} = \frac{C_1 - C_2}{A_1 - B_1C_2} \quad (17)$$

$$R_{22} = \frac{A_1B_2 - A_2B_1}{A_1 - B_1C_2} \quad (18)$$

$$R_{12} = \frac{\sqrt{(A_1 - B_1C_1)(A_2 - B_2C_2)}}{(A_1 - B_1C_2)} \quad (19)$$

As already noted, the parameter G plays the same role as a_2/a_1 in (12) and (13). Although in both cases these parameters are functions of the excitation conditions, there is an important difference. In the case of the thru and line, the different values of a_2/a_1 are the result of changes in the parameters of the dividing network that provides the excitation for the two six-ports [6]. For the reflect con-

nections, the different values of G are determined (in part) by the reflection coefficient Γ of the terminations used. Thus while the existing practice calls for four different terminations, which are the counterpart of the four different states of the dividing network, it is obviously a simple matter to increase the number of reflects as circumstances may warrant.

The rationale for choosing $1/w_1$ rather than w_1 as the “observed” parameter in (14) can now be explained. Although use of w_1 would have led to a result similar to (16)–(19), the term $C_1 - C_2$ that appears in the numerator of (17) would have become the denominator in all four expressions. Ideally, the C_1 and C_2 are zero, and there is no assurance that $|C_1 - C_2|$ may not become arbitrarily small. This could lead to an ill-conditioned solution. As formulated this potential source of trouble is avoided.

The theory associated with the first step in the TRL solution is now complete. In particular, as explained in [1], (12)–(13) or (14)–(15) become the basis for a system of equations which may be solved by least squares methods. As noted above, the w_1 and w_2 obtained under the different excitation conditions become the observations while the solution provides estimates of the scattering parameters for the thru, reflect, and line two-ports and, as before, an estimate of the accuracy of the “estimate.” At this point, these scattering parameter estimates become the measurement results or “observations” for the final step of the procedure which is to obtain an estimate of A_1 , B_1 , \dots , C_2 .

The functional relationship between the R_{ij} and the A_1, B_1, \dots, C_2 is already provided by (16)–(18). Let the scattering parameters for the thru and line two-ports be denoted by T_{ij} and L_{ij} , respectively. Then a simple application of network cascading theory yields:

$$T_{11} = \frac{B_1 - A_1 C_2}{1 - C_1 C_2} \quad (20)$$

$$T_{22} = \frac{B_2 - A_2 C_1}{1 - C_1 C_2} \quad (21)$$

$$T_{12} = \frac{\sqrt{(A_1 - B_1 C_1)(A_2 - B_2 C_2)}}{1 - C_1 C_2} \quad (22)$$

$$L_{11} = \frac{B_1 - A_1 C_2 e^{-2\gamma l}}{1 - C_1 C_2 e^{-2\gamma l}} \quad (23)$$

$$L_{22} = \frac{B_2 - A_2 C_1 e^{-2\gamma l}}{1 - C_1 C_2 e^{-2\gamma l}} \quad (24)$$

$$L_{12} = \frac{\sqrt{(A_1 - B_1 C_1)(A_2 - B_2 C_2)} e^{-2\gamma l}}{1 - C_1 C_2 e^{-2\gamma l}}. \quad (25)$$

Equations (17)–(25) now provide the “observed” R_{11} , \dots , L_{12} as functions of the Error-Box parameters A_1 , B_1 , \dots , C_2 and $e^{-2\gamma l}$. There are nine observations from which to obtain seven estimates and a least squares solution is again used [1]. This completes the calibration of the dual six-port and also provides an assessment of the

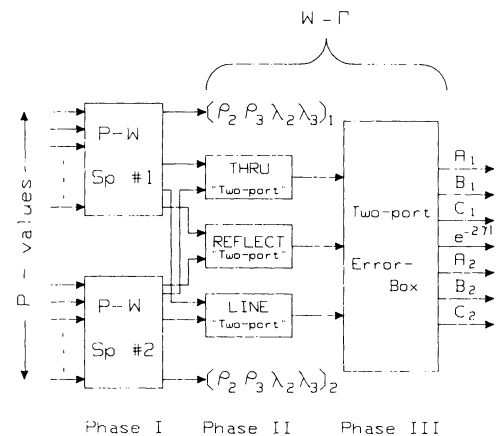


Fig. 3. The calibration procedure may be summarized as shown here.

accuracy to which the six-port parameters are known. See Fig. 3 for a summary.

V. ACCURACY ASSESSMENT: MEASUREMENT MODE

The question of ultimate interest is, of course, "What accuracy does the six-port provide as a measuring instrument?" This is obviously a function of the accuracy to which the parameters that characterize the six-port are known and the power-meter error. The foregoing theory, which is further developed in [1], provides this information.

To continue, the six-port theory required for assessing the accuracy in the measurement mode is already included in the foregoing treatment. The measurement of an unknown Γ , for example, is effected by the successive applications of the $P - w$ and $w - \Gamma$ transforms, (1)–(4). Since the calibration results include estimates of the accuracy for the power-meter readings and the six-port parameters, the remaining problem is an exercise in error propagation [1]. If the device to be measured is a two-port, (12) and (13) may be used where the Γ_1 and Γ_2 values obtained for the different excitation states are substituted for w_1 and w_2 .

VI. AN EXTENSION TO LOWER FREQUENCIES

Although, in principle, the foregoing is independent of the frequency of operation, it is desirable to maintain a minimum line length of 20–30 electrical degrees in order to assure a reasonably well-conditioned solution. As the frequency of operation becomes progressively smaller, the required line length is no longer convenient and an alternative solution is desirable.

Returning to (14) and (15), the values of G corresponding to the different excitation conditions are ordinarily discarded. However, if the Γ values for one of the terminations is known, then (16) can be used in place of (23)–(25) to form a over-determined system of seven equations for finding the six (complex) A_1, B_1, \dots, C_2 . It may be noted, in this context, that while (17)–(22) represent a system of six equations in six unknowns, they cannot be inverted because of a functional relationship be-

tween the R_{ij} and T_{ij} :

$$T_{22} - R_{22} + (T_{12}^2 - T_{11}T_{22}) \cdot R_{11} - (R_{12}^2 - R_{11}R_{22})T_{11} = 0. \quad (26)$$

The practical (and not surprising) implications of this are that it is impossible to complete the calibration with the use of the thru and reflect observations alone. One does require a standard, and this may be either in the form of a uniform section of line or alternatively a termination of known reflection (but with a value of other than ± 1).

VII. SUMMARY

As noted in the introduction, the purpose of this paper has been to formulate the six-port theory in such a way as to expedite the application of parameter estimation techniques. This, in turn, provides the foundation for the accuracy assessment which is described in further detail in the companion papers [1]–[4].

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