Monitoring the U.S. Legal Unit of Resistance via the Quantum Hall Effect

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Abstract—**The quantum Hall effect is being used to monitor the resistances of the five 1-Ω Thomas-type resistors which define the U.S. legal unit of resistance, the ohm maintained at the National Bureau of Standards (Ω_{NBS}). Typically, the total one-standard-deviation (1σ) accuracy for the transfer between three different GaAs quantum Hall devices and the five 1-Ω resistors is ±0.05 ppm. Measurements to date provide the first direct evidence that the value of Ω_{NBS} is decreasing by about (0.05 ± 0.02) ppm per year.**

I. INTRODUCTION

THE quantum Hall resistance R_H of a two-dimensional electron gas is, under certain special conditions, quantized in units of h/e^2 [1]:

$$R_H(i) = \frac{V_H(i)}{I} = \frac{h}{e^2 i} \approx \frac{25\,812.80}{i}\,\Omega \qquad (1)$$

where V_H is the Hall voltage across the sample, h the Planck constant, e the elementary charge, and i is an integer quantum number. Equation (1) is written in absolute or International System (SI) units. It can be expressed in as-maintained laboratory units by replacing R_H and Ω by the quantized Hall resistance and ohm at the National Bureau of Standards ($(R_H)_{\mathrm{NBS}}$ and Ω_{NBS}, respectively), where Ω_{NBS} is the United States legal unit of resistance, and is defined in terms of the mean resistance of five 1-Ω Thomas-type resistors maintained at NBS. One measures the value of R_H in laboratory units, and then expresses it in SI units once the ratio ($\Omega_{\mathrm{NBS}}/\Omega$) has been determined. This ratio can be obtained in two ways: either from the calculable capacitor experiment [2], [3], or by combining the low-field gyromagnetic ratio of the proton, γ_p', and $2e/h$ via the Josephson effect [4]–[6]. Both approaches are currently being pursued at the NBS. (The value of Ω_{NBS} can, however, be expressed in terms of the SI resistance unit as realized at the National Measurement Laboratory (NML), Australia because one quantum Hall device and three 1-Ω resistors have been used as transfer standards between the two laboratories [7], [8]. Those measurements imply that the Ω_{NBS} was (1.341 ± 0.062) ppm smaller than the realization of the International System ohm (SI Ω) at NML in October, 1985 [8], and that h/e^2 is (0.40 ± 0.08) ppm larger than the 25 812.80 Ω nominal value. These results will be described in more detail elsewhere.)

II. EXPERIMENTAL DETAILS

Three high-quality quantum Hall effect devices are being used to monitor Ω_{NBS}. Each is a GaAs–$\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ heterostructure ($x = 0.29$) grown using molecular beam epitaxy by A. C. Gossard at AT&T Bell Laboratories, and then prepared into Hall bar geometries and mounted by D. C. Tsui at Princeton University. The devices are ~4.6 mm long and ~0.4 mm wide, and have three sets of Hall potential probes, with two sets symmetrically displaced ±1.0 mm along the channel from the center set. Two different sets of Hall probe pairs are used for each of the three devices, the center set, and an off-center set. The devices designated GaAs(7) and GaAs(8) have ~100 000 $\mathrm{cm}^2/(\mathrm{V \cdot s})$ zero magnetic field mobilities at 4.2 K, while the GaAs(9) device has a mobility of ~75 000 $\mathrm{cm}^2/(\mathrm{V \cdot s})$. (GaAs(9) was the device used in the NML transfer.) The epitaxially grown film thicknesses and doping density profiles of these devices are optimized for the $i = 4$ quantum Hall step, where $R_H(4) \approx 6{,}453.20\ \Omega$. The centers of this step occur at ~5.6 – 6.0 T for the three devices; the corresponding electron densities are ~$5.4 - 5.8 \times 10^{11}\ \mathrm{cm}^{-2}$.

A set of wire-wound reference resistors have been constructed to have values R_R within a few parts-per-million of the value of $R_H(4)$. They are hermetically sealed in silicone fluid-filled containers and placed in specially constructed, temperature-regulated air bath enclosures. The air temperature is controlled to within ±0.002°C of a nominal temperature of ~28°C.

III. QUANTUM HALL EFFECT MEASUREMENTS

Two different measurement systems are used to compare the quantum Hall voltages V_H with the voltage drops V_R across the wire-wound reference resistors: a manually-operated potentiometric comparator [9] and an automated and guarded resistance bridge [10]. Figs. 1 and 2 show simplified schematic diagrams of these two systems. The potentiometric system has a ±0.011-ppm random, or type

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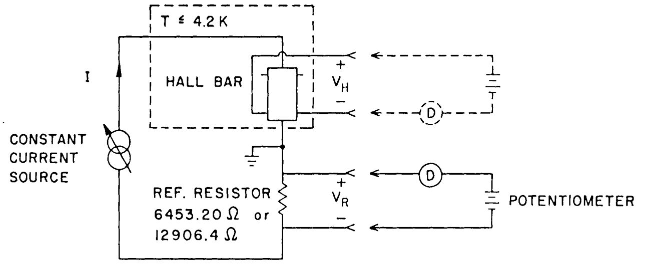

Fig. 1. A simplified schematic of the manually operated potentiometric comparator measurement system, where D is an electronic detector.

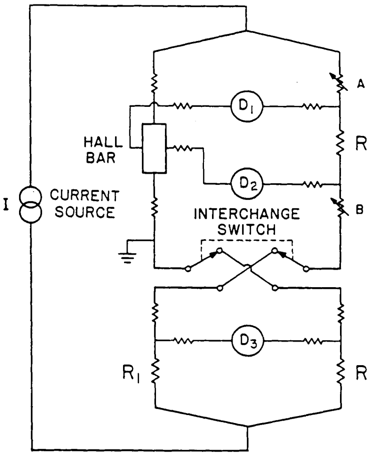

Fig. 2. A simplified schematic of the automated quantum Hall resistance bridge, which uses three electronic detectors.

A, uncertainty after a 1-h measurement period for a device current of 25.5 μA; the random uncertainty of the resistance bridge is typically ± 0.006 ppm for a comparable measurement period at 25.5 μA.

Both measurement systems have been used to compare the values of R_H with those of R_R for the two Hall probe sets on the three GaAs devices for both magnetic field directions. To be useful as a resistance standard, the Hall steps must be flat within the experimental resolution. All twelve quantum Hall steps are flat to within ± 0.01 ppm over a magnetic field range that is ~2 percent of the central field values when the devices are cooled to ~1.2 K. Fig. 3 of [9] shows a digital mapping of one of these $i = 4$ steps for GaAs(7).

IV. Quantum Hall Effect Systematic Measurement Uncertainties

In addition to the random measurement uncertainties, there are systematic corrections with associated systematic, or type B, uncertainties. One such correction is due to a measurement system offset error in which the value of the device under test depends on whether it is measured in the R_H position or the R_R position of the measurement circuit. The correction is determined by replacing the quantum Hall device with a 6,453.20-Ω reference resistor and then using either of the measurement systems to intercompare the resistor pairs. This offset error has been observed on both of the NBS measurement systems, as well as on the NML automated potentiometric comparator system which uses a different detector [7]. The source of the error is not understood, but it is probably associated with the electronic detectors. It does not seem to be due to dc leakage currents because the leakage resistances are $> 10^{12}$ Ω for both NBS measurement systems. It is also independent of the detector input current. The position-dependent measurement offset error is sometimes as large as (0.025 ± 0.016) ppm for the potentiometric system and (0.019 ± 0.011) ppm for the resistance bridge. The resistor interchange procedure to determine this offset correction is done each day that R_H is measured.

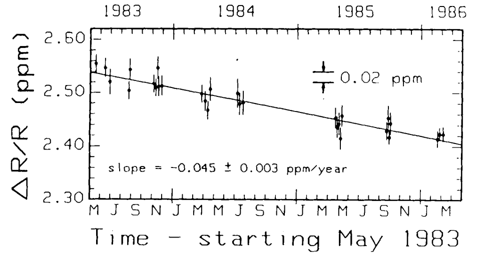

Fig. 3. Relative comparisons as a function of time of the resistance of the $i = 4$ steps of three different quantum Hall devices with that of a nominal 6,453.20-Ω wire-wound reference resistor ($\Delta R/R = (V_H - V_R)/V_R$. The value of this particular resistor is increasing by (0.045 ± 0.003) ppm per year.)

There is an uncertainty in calibrating the gains and linearities of the electronic detector–digital voltmeter pairs. Both the detector–digital voltmeter pairs used at NBS and at NML [7] *appear* to have gains which vary by a few tenths of a percent over the input voltage range. This nonlinearity is due to a 1-μV dead band of the digital voltmeters at zero volts. This problem can be avoided by either using digital voltmeters which have no dead band or by increasing the output voltages of the detectors. The voltmeters used in calibrating the detector gains must, of course, be the same ones that are used in the quantum Hall resistance measurements. There still remains, however, the problem of stability; the gains of the detector–digital voltmeter pairs vary by ~0.1 percent during a day if the room temperature is controlled to ~ ±1 °C. This instability typically contributes a ± 0.003-ppm uncertainty to the measurements for the potentiometric comparator system, and a ± 0.015-ppm uncertainty for the resistance bridge.

There is also a correction for the temperature dependence [11] of R_H for each Hall probe set of every quantum Hall device for both magnetic field directions. The corrections to the values of R_H for some devices are found to vary linearly with the minimum values of the voltage drop along the device, $V_x^{\min}$. These corrections can be quite significant. Reference [11, Figs. 3, 4] demonstrates that these linear relationships hold over at least four orders of magnitude change in $V_x^{\min}$ for GaAs(7) and GaAs(8).

Every quantum Hall device is unique; the effects reported in [11] are not always observed, nor are they necessarily the *only* temperature-dependent effects. For example, GaAs(9) has a nonlinear dependence on $V_x^{\min}$ similar to that for one Hall probe set of GaAs(7) [11]. This nonlinearity is probably due to the asymmetry of the

TABLE I
ESTIMATED 1σ (68-PERCENT CONFIDENCE LEVEL) UNCERTAINTIES FOR THE QUANTUM HALL RESISTANCE MEASUREMENTS, $R_H \rightarrow R_R$

Sources of Uncertainty	Uncertainties (ppm)	
	Potentiometric System	Bridge System
Random Measurement Uncertainty	0.011	0.006
Measurement Offset Error	0.016	0.011
Detector Gains and Linearity	0.003	0.015
Temperature Dependence Corrections	$\lesssim$ 0.002	$\lesssim$ 0.002
Current Dependence Corrections	< 0.001	< 0.001
ROOT-SUM-SQUARE TOTAL (ppm)	0.020	0.020

Hall step with respect to $V_x^{\min}$; thus the value of R_H in this case includes the effect of structure on the side of the step. In another example, the value of R_H is too small at higher temperatures for one Hall probe set of GaAs(9), but then becomes consistently *too large* by ~0.13 ppm over the temperature range 3.4 − 2.5 K before dropping to the "correct" value at 1.2 K. One could thus infer a temperature-independent (but incorrect) value of R_H over the 2.5–3.4-K temperature range. All three GaAs devices have temperature-dependent effects which are completely repeatable over many cool-downs from room temperature. To date the largest correction to R_H, necessary to extrapolate from the 1.2–0-K values, has been (0.026 ± 0.002) ppm.

No current dependence nor current breakdown phenomena [12] were observed for the three GaAs devices for $I \leq 25.5\ \mu$A, so no correction for finite current is required. Table I summarizes the assigned uncertainties; the total root sum square (rss) uncertainty for each measurement system is typically ±0.020 ppm.

V. QUANTUM HALL EFFECT RESULTS

Fig. 3 shows comparisons of $i = 4$ quantum Hall resistances of the three GaAs devices with that of a nominal 6,453.20-Ω reference resistor during a 34-month time period starting in May 1983. These data are independent of the Hall device, the Hall probe set, the magnetic field direction, and the measurement system once the appropriate offset, gain, and temperature-dependent corrections are made. A weighted linear least squares fit, which takes into account the total uncertainty of each measurement, shows that the resistance of this particular reference resistor is increasing at a rate of (0.045 ± 0.003) ppm per year. This unusually small and linear drift rate enables us to continuously monitor the reliability of the two measurement systems.

VI. STEP-DOWNS TO THE NBS OHM

To monitor the NBS ohm, the nominal 6,453.20-Ω reference resistors must be calibrated in terms of the set of five 1-Ω resistors which define Ω_{NBS}. This is done in two stages: the first uses a 6,453.20 to 100-Ω series/parallel Hamon network configuration [13] consisting of eight 800-Ω resistors plus a series-connected 53.2-Ω resistor; the second uses a 100 to 1-Ω Hamon network consisting of ten 10-Ω resistors. Transfers from 6,453.20 to 1 Ω_{NBS} are currently estimated to have an uncertainty of ±0.044 ppm [9].

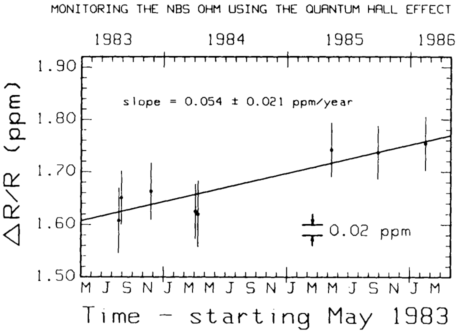

Fig. 4. Monitoring as a function of time the value of $R_H(4)$ expressed as a difference in ppm from a reference value of 6,453.20 Ω_{NBS}. (These data indicate that the U.S. legal ohm, Ω_{NBS}, is decreasing by ~ (0.05 ± 0.02) ppm per year.)

TABLE II
ESTIMATED 1σ (68-PERCENT CONFIDENCE LEVEL) UNCERTAINTIES FOR THE STEP-DOWNS TO THE U.S. LEGAL OHM, $R_H \rightarrow \Omega_{\mathrm{NBS}}$

Sources of Uncertainty	Uncertainties (ppm)
QHE Resistance Measurement Uncertainty	0.020
Resistance Scaling Uncertainty	0.044
Self-Heating of Reference Resistors	0.020
ROOT-SUM-SQUARE TOTAL (ppm)	0.052

The current used in the step-downs is 1.25 mA for the 6,453.20-Ω reference resistors, whereas it is 25.5 μA in the quantum Hall effect resistance comparisons. The reference resistors are maintained in constant-temperature air baths, which enhance the self-heating effect in the reference resistors. The self-heating increases the temperature of the silicone fluid at higher currents. This typically produces a (+0.02 ± 0.02)-ppm correction to the value of the 6,453.20-Ω reference resistors in the step-down procedure.

Measurements involving the entire sequence (quantum Hall resistance comparisons with nominal 6,453.20-Ω reference resistors and then step-downs to Ω_{NBS} have been made over a 31-month interval commencing in August 1983. Fig. 4 shows the results of these measurements to date. The total 1σ rss uncertainty is typically ±0.052 ppm for each datum, as indicated in Table II.

The data of Fig. 4 show the first direct evidence that Ω_{NBS} is decreasing with time. A weighted linear least squares fit yields a drift rate of (0.054 ± 0.021) ppm per year, but data must be accumulated over a longer time span in order to reduce the uncertainty and to verify that the drift is indeed linear.

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