TABLE II
MASS AND FORCE DATA FOR THE HIGH-FIELD EXPERIMENT

flux density at r_0 = 250 mm :	31.25 mT
force at suspended coil :	9.8 N
mass of suspended coil :	2.2 kg
mass of suspending apparatus:	2 kg
ratio of force to dead load:	0.25

The suspended coil is wound on a glass ceramic former using an aluminum wire. The masses calculated for the coil and the apparatus for positioning and suspending the coil should be of an order as given by Table II. A ratio of force to dead load of 0.25 should be attainable, and it should, therefore, be possible to measure the force with almost the same accuracy as that with which a 1-kg comparison can be performed, even with 16 W dissipated in the suspended coil.

The low-field NMR experiment will be done using the same equipment as that described in detail in [7]. It does not seem overlay optimistic to hope that with these experiments the ampere can be realized with an uncertainty of 10^{-6} or less.

ACKNOWLEDGMENT

The authors would like to thank Prof. Dr. H. Capptuller for many helpful suggestions and discussions.

REFERENCES

[1] W. Schlesok, "Vorschlag zur Absolutbestimmung des Ampere mit Hilfe von Kernresonanzverfahren," *Metrologia*, vol. 6, no. 2, pp. 44-46, 1970.

[2] W. Chiao, R. Liu, and P. Shen, "The absolute measurement of the ampere by means of NMR," *IEEE Trans. Instrum. Meas.*, vol. IM-29, pp. 238-242, 1980.

[3] W. Schlesok, "Progress in the realization of electrical units at the Board for Standardization, Metrology and Goods Testing (ASMW)," *IEEE Trans. Instrum. Meas.*, vol. IM-29, pp. 248-250, 1980.

[4] P. Vigoureux and N. Dupuy, "The gyromagnetic ratio of the proton," NPL-Rep. Qu 26, 1973.

[5] P. T. Olsen and E. R. Williams, "A more accurate determination of γ_p' through improved dimensional measurement techniques," *IEEE Trans. Instrum. Meas.*, vol. IM-23, pp. 302-305, 1974.

[6] K. Weyand, "Proposal for realizing the ampere via combined two NMR experiments," Com. Consult. d'Elect., BIPM, Dec. 13, 1983.

[7] K. Weyand, "Ein neues Verfahren zur Bestimmung des gyromagnetischen Koeffizienten des Protons," PTB-Rep. E-26, 1984.

Determination of the SI Volt at the PTB

VOLKMAR SIENKNECHT AND TORSTEN FUNCK

***Abstract*—Work on an electrostatic measuring system for the determination of the SI volt and the conversion factor K_V at the PTB is reported. The system consists of a so-called voltage balance and a generator for the dc measuring voltage. There is a need for the development of such a device with an uncertainty below 1 part in 10^6 because measurements with existing voltage balances, and results obtained by other methods show discrepancies of several parts in 10^6. In order to reduce the uncertainty and to simplify the apparatus a special measuring method was developed. The voltage/force transducer of the PTB voltage balance is formed from two coaxial cylindrical electrodes and the generated force is substituted for the force of gravity on a weight of 2 g. The electrode voltage is composed of a constant part of 10186 V, derived in a 1000-fold stepup from 10 standard cells, and a much smaller variable part used for balancing the scale beam by means of a control loop. Taking the root sum of squares the total relative uncertainty of the SI volt and of K_V is expected to be less than 4 parts in 10^7.**

Manuscript received August 23, 1984.

The authors are with the Physikalisch-Technische Bundesanstalt, Bundesallee 100, D-3300 Braunschweig, Federal Republic of Germany.

I. INTRODUCTION

AT THE PTB a measuring device for the determination of the SI volt and of the conversion factor of the volt,

$$K_V^{\text{PTB}} = \frac{V_{\text{PTB}}}{V_{\text{SI}}} \tag{1a}$$

i.e., the ratio of the as-maintained volt-V_{PTB} to the SI volt-V_{SI} is under construction. A need for the development of such a device with an uncertainty below 1 part in 10^6 exists, since results reported up to now [1], [2] show discrepancies of several parts in 10^6.

The device consists of the so-called voltage balance for the voltage measurement in SI volts and a generator for the dc measuring voltage, the value of which is known in terms of V_{PTB}. The main part of the voltage balance is a capacitive voltage-to-force transducer whose capacitance, $C(s)$, varies with the position s of the movable electrode. The value of the

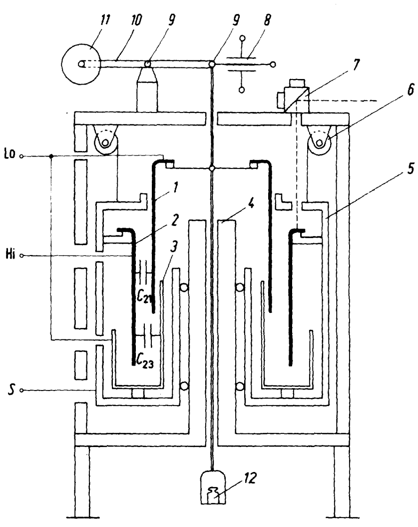

Fig. 1. Schematic of the PTB balance. 1) Suspended electrode (Lo). 2) High voltage electrode (Hi). 3) Guard electrode (S). 4) Column of displacement unit. 5) Carriage of displacement unit. 6) Driving device for displacement unit. 7) Interferometer for displacement measurement. 8) Position sensor (differential capacitor). 9) Strip spring joints. 10) Balance beam. 11) Counter weight. 12) Substitution weight.

electrostatic force F_e acting on this electrode in the direction of s is obtained by equating electrical energy dW_e and mechanical energy dW_m:

$$dW_e = \tfrac{1}{2}\, U^2\, dC$$

$$dW_m = F_e\, ds$$

$$F_e = \frac{1}{2}\, U^2\, \frac{dC}{ds} \tag{2}$$

where U is the voltage across the electrodes. If the force F_e and the slope of $C(s)$ are known in SI units, the voltage U can be calculated in SI volts:

$$U = \sqrt{2\, F_e\, \frac{ds}{dC}}. \tag{3}$$

The force measurement is performed by comparing F_e with the force due to gravity on a mass m by means of a balance. As a result, the conversion factor is obtained as the quotient of the numerical values of U in SI volts and in volts as maintained at the PTB:

$$K_V^{\mathrm{PTB}} = \frac{\{U\}_{\mathrm{SI}}}{\{U\}_{\mathrm{PTB}}}. \tag{1b}$$

II. THE VOLTAGE BALANCE

For the implementation of the voltage/force transducer coaxial, cylindrical electrodes were chosen (Fig. 1) because their manufacture and alignment presented fewer problems. The displacement unit for the axial motion of the outer high voltage electrode relative to the suspended electrode is also constructed coaxially. This has advantages for mechanical precision and rigidity. The (unwanted) reduction of the effective change of capacitance with displacement due to the central column is less than 20 percent and, therefore, tolerable.

The diameters of the transducer electrodes are $D_1 = 126$ mm and $D_2 = 142$ mm; the diameter of the guard electrode is $D_3 = 75$ mm (Fig. 1), which results in a capacitance change of nominally 0.54 pF/mm. For this value at the electrode voltage $U = U_{\mathrm{const}} = 10186$ V the generated force equals the force F_m due to gravity on a 2-g mass, minus the buoyancy of the nitrogen atmosphere. To achieve the nominal capacitance change of 20 pF the electrode must the moved by $\Delta s = 53$ mm. The measurement of the generated force is performed with a substitution balance with strip spring joints. The inner electrode is suspended from the load side joint with the distance between joint and electrode as small as possible. This reduces the deflection torque $M_d(\alpha)$ on the suspended electrode caused by the unwanted horizontal electrostatic force due to imperfect electrode alignment.

On the other hand, the distance between center of mass and joint is large enough to produce a restoring torque $M_r(\alpha)$, which fulfills the stability condition [3],

$$\frac{dM_r}{d\alpha} > \frac{dM_d}{d\alpha} \tag{4}$$

where α is the angular deviation from the vertical.

The balance beam length between the joints is $L = 200$ mm. The restoring torque of the plate springs at both joints results in a sensitivity of 10 mm/g. The capacitive sensor for the balance beam inclination has a resolution $\delta s = 2$ nm with a lever arm of 400 mm, which results in a resolution of weighing of 0.1 μg. To maintain stability in the electrode surface properties the balance is housed in a case containing dry nitrogen.

III. 10-kV dc VOLTAGE SUPPLY

The voltage U_{const} is derived by a two-stage 1000-fold stepup from the voltage U_{tr} of the transfer standard composed of 10 saturated standard cells connected in series (Fig. 2).

The setting of the U_{const} supply and the auxiliary supply is performed automatically by a feedback loop consisting of a null detector and a regulator, the output of which is connected is series with the supply. The commercially-available, self-calibrating resistive voltage divider t_1 (Fluke 752A) has a ratio of 1 : 100. The divider t_2 with a ratio 1 : 10 consists of one hundred 100-kΩ, wire-wound resistors with a temperature coefficient of $5 \times 10^{-7}\ \mathrm{K}^{-1}$ and is arranged in 10 segments. The U_{const} supply is also segmented to provide the guard potentials for the divider segments. With the symbols in Fig. 2, the constant voltage is given by

$$U_{\mathrm{const}} = \frac{1}{t_1\, t_2}\, U_{\mathrm{tr}}. \tag{5}$$

IV. MEASURING METHOD

Due to mechanical imperfections and finite electrode lengths, the change of capacitance C with displacement s is not exactly constant but varies slightly with s. Therefore, the uncertainty achievable with the differential measuring method according to (3) is limited, because small changes dC and ds have to be measured. For this reason an integral method, [4], is advantageous and, therefore, applied, because the changes can be enlarged to ΔC and Δs.

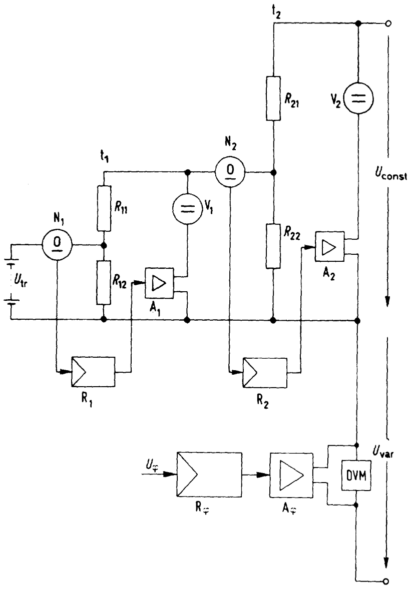

Fig. 2. Generation of the supply voltage.

A_1, A_2, A_φ : amplifiers N_1, N_2 : null detectors
R_1, R_2, R_φ : regulators V_1, V_2 : de voltage generators

$R_{11} = 3.6\ \mathrm{M\Omega}$ $R_{21} = 9\ \mathrm{M\Omega}$
$R_{12} = 0.4\ \mathrm{M\Omega}$ $R_{22} = 1\ \mathrm{M\Omega}$

$U_{\mathrm{tr}} = 10.186\ \mathrm{V}$
$U_{\mathrm{const}} = 10.186\ \mathrm{kV}$
$U_{\mathrm{var}} = -100\ \mathrm{V} \cdots + 100\ \mathrm{V}$.

To maintain the balance beam in a stable position when moving the high voltage electrode by Δs, the total force at the load-side joint must remain constant. This requires an additional force generator [4]. In order to avoid the need for such a mechanical device we achieve force equilibrium by continuously adjusting the voltage across the electrodes to such a value that the force F_e itself remains constant in spite of small variations of dC/ds with displacement s [5]. The adjustment is performed automatically by a regulator for the balance beam position (Fig. 3).

It is now disadvantageous, however, that linking the variable electrode voltage U to the transfer voltage U_{tr} requires a divider with a variable division ratio [6]. To overcome this problem, we realize the total voltage U in two parts: a constant voltage U_{const}, which can be linked to the transfer standard by means of a fixed ratio divider, and a much smaller variable voltage U_{var} (less than 1 percent of U_{const}), generated by the position regulator Fig. 2 and Fig. 3 so that

$$U = U_{\mathrm{const}} + U_{\mathrm{var}}(s). \tag{6}$$

$U_{\mathrm{var}}(s)$ is measured with a commercial DVM in terms of the as-maintained volt when the electrode is moved and the ratio,

$$R(s) = \frac{U_{\mathrm{var}}(s)}{U_{\mathrm{const}}} \tag{7}$$

required for $F_e = \mathrm{const}$ is calculated. The value of U_{const} is then evaluated according to the following equations:

$$\frac{1}{2} U_{\mathrm{const}}^2 \int_{\Delta s} dC = F_e \int_{\Delta s} \frac{ds}{(1+R)^2} \tag{8}$$

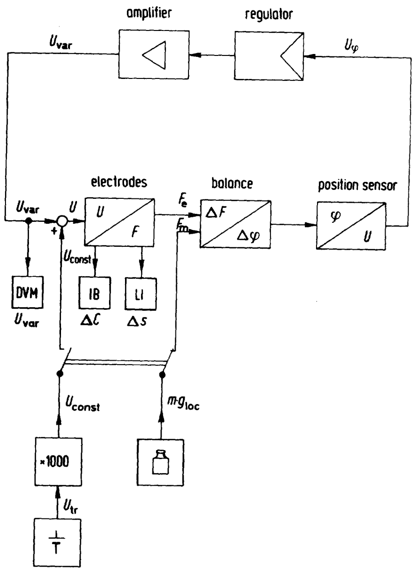

Fig. 3. Block diagram of the measuring device.

U : voltage across electrodes
U_{const} : constant part of U
U_{var} : variable part of U
U_{tr} : voltage of transfer standard
F_e : electrostatic force
m : substitution weight
g_{loc} : local value of acceleration due to gravity
F_m : substitution force
φ : inclination of balance beam
U_φ : output voltage of position detector
DVM : digital voltmeter
IB : inductive bridge
LI : laser interferometer.

$$\frac{1}{2} U_{\mathrm{const}}^2 \Delta C = F_e \Delta s (1 + D) \tag{9}$$

$$D = \frac{1}{\Delta s} \int_{\Delta s} (-2R + 3R^2 - 4R^3 + \cdots)\, ds \tag{10}$$

$$U_{\mathrm{const}} = \sqrt{2(1+D) F_e \frac{\Delta s}{\Delta C}}. \tag{11}$$

When the buoyancy of the environmental gas with density ρ_g is taken into accout, the force comparison leads to the equation

$$F_e = m\, g_{\mathrm{loc}} \left(1 - \frac{\rho_g}{\rho_m}\right) \tag{12}$$

where ρ_m is the density of the substitution mass material and g_{loc} the local acceleration due to gravity. We thus obtain the final equation for the evaluation of U_{const} in SI units:

$$U_{\mathrm{const}} = \sqrt{2(1+D)\, m\, g_{\mathrm{loc}} \left(1 - \frac{\rho_g}{\rho_m}\right) \frac{\Delta s}{\Delta C}}. \tag{13}$$

V. MEASUREMENTS AND UNCERTAINTIES

According to (5) and (13), the voltage U_{const} is known both in terms of the as-maintained volt and in terms of SI units, and the conversion factor can be calculated from (1b) with $U =$

TABLE I

Parameter	$u_p \times 10^6$	$u_p \times 10^6$ Related to U_{const}
I) m	0.5	0.25
g_{loc}	0.05	0.025
ρ_g	300	0.02
ρ_m	200	0.01
force comparison	0.25	0.125
Δs	0.12	0.06
ΔC (dc value)	0.2	0.1
D	10	0.05
II) U_{tr}		0.02
comparison 10.186 V		0.01
t_1		0.1
comparison 1018.6 V		0.01
t_2		0.1

$u_K = \sqrt{\Sigma u_p^2} = 0.34 \times 10^{-6}$

U_{const}. To determine U_{const} in SI volts the values of Δs and ΔC must be measured in SI units, and for the evaluation of the integral D, (10), the ratios $R(s)$ at force equilibrium have to be recorded for an appropriate number of s-values. Furthermore, the values of m and g_{loc} must be known in SI units. To determine the ratio ρ_g/ρ_m, the value of ρ_m is required, and a given fixed-point value of ρ_g must be corrected for the actual temperature and pressure. The value of ρ_g is also needed in order to determine the wavelength of the He-Ne laser for the interferometric measurement of the electrode displasement Δs.

To determine U_{const} in V_{PTB} according to (5), the divider ratios t_1 and t_2 have to be measured and the voltage U_{tr} of the transfer standard must be calibrated in terms of the as-maintained volt, which is realized by means of a Josephson voltage standard using the recommended CCE value $2e/h = 483\ 594.0$ GHz/V, [7]. Voltage intercomparisons have to be performed at the 10 V and the 1000 V level.

Table I shows the 1-σ values of the partial relative uncertainties u_p due to the various parameters for the determination of U_{const} in SI volts (part I) and in V_{PTB} (Part II). Taking the root sum of squares the total relative uncertainty of the determination of the SI volt and also of the conversion factor K_V will be less than 4 parts in 10^7.

REFERENCES

[1] N. Elnekavé and A. Fau, "Determination absolute du volt et du coefficient 2e/h au L.C.I.E.," *Com. Consultat. d'Electricité*, 16^{e} session, *Doc. CCE/83-2*, 1983.

[2] A. F. Dunn, "Rapport du comité consultatif d'electricité," 16^{e} session, 1983.

[3] V. Sienknecht and P. Klinzmann, "Die Gleichgewichtslagen der pendelnd aufgehängten Meßelektrode einer Spannungswaage," *PTB-Mitteilungen*, vol. 90, pp. 14-19, 1980.

[4] T. Yamazaki, K. Shida, and M. Kanno, "Absolute measurement of voltage by an electrostatic energy-changing method," *IEEE Trans. Instrum. Meas.*, vol. IM-21, pp. 372-375, 1972.

[5] H. Schrader, "Las unidades electricas, su interdependencia y realización," *Carta metrologia*, vol. 2, pp. 43-56, 1980.

[6] V. Bego, "Naponska vaga," *Zbornik JUKEM 1980*, vol. 3, pp. 87-106, 1980.

[7] J. J. Denton, "The numerical value of the constant 2e/h," *Com. Consultat. d'Electricité*, 13^{e} session, Doc. CCE/72-24, 1972.

A Multistate Reflectometer

L. C. OLDFIELD, J. P. IDE, AND E. J. GRIFFIN

***Abstract*—Precise measurement of complex reflection coefficient and power sensor effective efficiency is reported using a reflectometer comprising two detectors and two directional couplers, one coupler having connected to it a reflector able to switch to different stable states. Results were obtained in three waveguide bands spanning 8.2-26.5 GHz.**

I. INTRODUCTION

THE RELATIVE intensity and phase difference between two propagating waves can be measured at optical wavelengths using the Michelson interferometer. The microwave analogy of this, employing two detectors and not using frequency translation is known [1]. The calibration of a 4-port reflectometer to allow for all spurious reflections has been described, but the measurement involves connection of the device under test via a series of precision waveguide spacers [2]. In this paper we show that changing transmission and reflection within the reflectometer in a repeatable way obviates the need for spacers, but allows the same calibration method to be used [3]. By calibrating the reflectometer for at least three different stable states of transmission, it can be used for precise measurement of complex reflection coefficient Γ and of relative effective efficiency of power sensors.

The technique is applicable at any wavelength for which suitable standards are available.

II. DESCRIPTION

Fig. 1 illustrates the multistate reflectometer. The detectors are temperature compensated thermistor sensors, one indicat-

Manuscript received August 20, 1984; revised October 3, 1984.

The authors are with the Microwave Measurements Division, Royal Signals and Radar Establishment, Malvern, Worcs. WR14 3PS, England.