

Application of Measurement Simulation and Least Squares Parameter Estimation in an Undergraduate Servomotor Lab

ROBERT DEMOYER, JR., MEMBER, IEEE, AND RICHARD V. HOUSKA

Abstract—Laboratory confirmation of theoretical results is important for student confidence in both analytical and experimental techniques. A frequent source of difficulty in obtaining such a confirmation is the improper handling of error-corrupted measurements. This paper discusses measurement simulation and least squares parameter estimation as applied in a dc servomotor laboratory.

Simulation illustrates how ideal measurements are obscured by noise, and how usable estimates are recovered from noisy data. Many control system parameters are estimated by the use of linear least squares. Motor time constant is estimated by the method of quasi-linearization combined with least squares. Applications of estimation statistics are described.

All measurement and estimation simulations are demonstrated graphically by plots showing both point of measured values and the least squares curve fits.

I. INTRODUCTION

A major component of systems engineering at the U.S. Naval Academy is the study of automatic control systems. There is a required sequence of three theoretical courses, the first two of which are paralleled by laboratory courses. The first of the laboratory courses is designed to reinforce theoretical concepts by both analog and digital simulation. The second of the laboratory courses provides the students with their first experience with actual servo-mechanism hardware as well as associated instrumentation.

Fig. 1 is a block diagram of an angular position control system, consisting of hardware manufactured by Feedback [1]. The dc armature-controlled servomotor is approximately modeled by a first-order transfer function [2] with the typical assumption that the electrical time constant is much shorter than the mechanical time constant. The overall objective of the study of this control system is to first determine component transfer functions, and then to compare analytically predicted to experimentally observed closed-loop response.

In order to determine transfer function parameters, error-corrupted measurement data must be processed. Error sources consist of precision error as well as unmodeled higher order effects and nonlinearities. At this point in the curriculum, the students have had a course in probability

and statistics. Because the course is theoretical, they typically are not comfortable with practical applications of statistics. This leads to a second objective of the study of this control system. Least squares parameter estimation is studied as a means to extract useful information from noisy data. A third objective is to enhance the students' skills in Fortran programming. While they are made aware of the existence of statistical programming packages, each individual is required to write all of the simulation, estimation, and display programs used in the course.

The sequence of events in the overall component parameter estimation process is summarized in Fig. 2. Initially, before experience with the laboratory equipment, the entire sequence is carried out by digital simulation. Gaussian precision error is generated by summing uniform random numbers. A visual confirmation of this application of the Central Limit Theorem is made by a histogram display. Simulated measurements are then made by adding the precision error to ideal measurements. A least squares parameter estimator produces the parameter estimate, estimate standard deviation, a table of measurement residuals, and residual standard deviation.

The simulation permits comparisons not possible when processing laboratory measurements. It is possible to verify that the 95 percent confidence region of the parameter estimate almost always includes the simulated true parameter value which, in a laboratory, is known only to nature. Likewise, the histogram and statistics of the measurement residual are comparable to those of the simulated precision error. Having made these consistency checks, the student is ready to process actual physical data, having confidence that the estimation program is correct. The parameter estimate standard deviation is useful to compute the 95 percent estimate confidence region. This result is useful in order to redesign the experiment to reduce the uncertainty of the estimate to an acceptable level. It is also useful to limit the number of significant digits which may be legitimately claimed for the estimate. The latter reduces students' tendency to write as many digits as the computer or calculator will produce.

It takes several weeks to develop the statistical background and discuss the programs to be written. As each concept is developed, the appropriate program is illustrated in class. While the students do their programming on a time-sharing system, all of the classroom demonstra-

Manuscript received October 11, 1984; revised July 7, 1985.

The authors are with the Department of Weapons and Systems Engineering, U.S. Naval Academy, Annapolis, MD 21402.

IEEE Log Number 8608273.

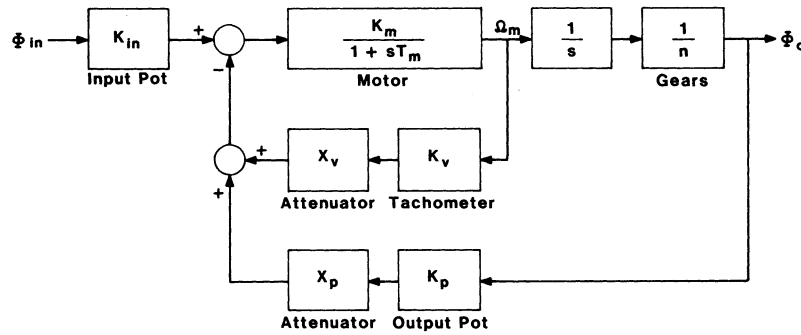


Fig. 1. Angular position control system.

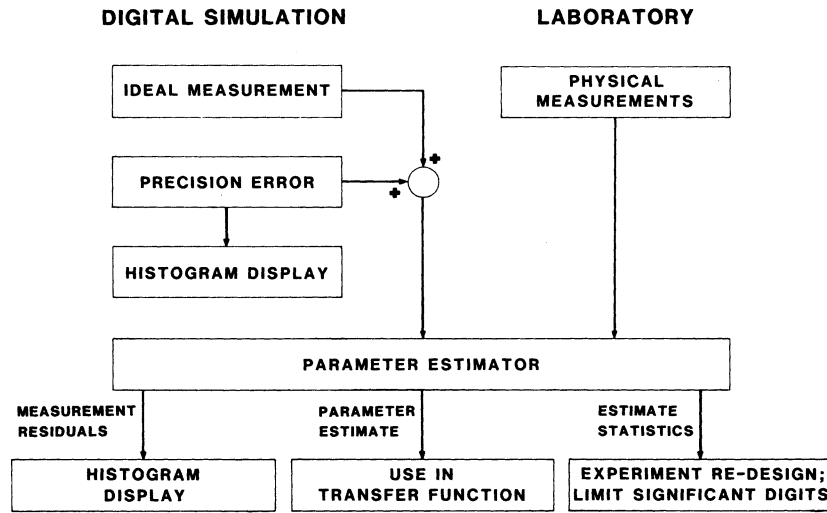


Fig. 2. Simulation and measurement sequence.

tions are carried out on a free-standing microcomputer system. An IMSAI S-100 bus computer system executes compiled Fortran-80 [3] code under the CP/M [4] operating system. All results are displayed in eight color graphics on an ICS 8001 color terminal [5]. The 19 in (48 cm) color screen can be seen at the back of the classroom. Displays are constructed rapidly at 9600 baud.

The following sections summarize the classroom statistical development [6] and illustrate typical simulated results. The computer-generated results are terminal print-outs and hard copies produced from a Tektronix 4051 graphics terminal, both connected to a time-sharing computer. The appearance of these results is typical of the student programs rather than the more vivid microcomputer-generated graphics.

II. LINEAR PARAMETERS ESTIMATION

Equation (1) expresses a vector of measurements y in terms of a linear combination of parameters β and independent variables V . The additive term ϵ represents measurement noise.

$$y = V\beta + \epsilon. \quad (1)$$

The matrix V of independent variables is defined as follows:

$$V = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1m} \\ 1 & x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix}. \quad (2)$$

There are then n measurements and additive noise terms and $(m + 1)$ parameters.

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}; \quad \epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}; \quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{m+1} \end{bmatrix}. \quad (3)$$

The i th measurement in the vector can be expanded as follows:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_m x_{im} + \epsilon_i; \quad 1 \leq i \leq n. \quad (4)$$

In order to mathematically model a linear device, the β terms which should be present must be chosen. This choice is made based upon the physical nature of the device being modeled, engineering judgment, or stepwise regression. All of the proportionality constants in Fig. 1

can be estimated by measurement equations of the form

$$y_i = \beta_i x_i + \epsilon_i; \quad 1 \leq i \leq n. \quad (5)$$

This form is also useful to estimate the motor time constant T_m when a derivative-type optimization method is used as described in Section IV.

At this point, rather than be restricted to this form, we will return to the more general form of (1).

The vector β contains the $(m + 1)$ parameters whose true values, obscured by the noise ϵ , are known only to nature in laboratory situations. The vector $\hat{\beta}$ is the estimate of β . When associated with V , it forms the expected measurement $E\{y\}$.

$$E\{y\} = V\hat{\beta}. \quad (6)$$

The measurement residual δy is then defined as the difference between the measurement and the expected measurement.

$$\delta y = y - E\{y\} = y - V\hat{\beta}. \quad (7)$$

The least squares estimate of β is the vector $\hat{\beta}$ which minimizes the sum square of the residuals.

$$SS = \delta y^T \delta y = \sum_{i=1}^n \delta y_i^2. \quad (8)$$

To minimize SS , its partial derivative is taken with respect to β , and the result, evaluated at $\beta = \hat{\beta}$, is set equal to zero

$$\left. \frac{\partial(\delta y^T \delta y)}{\partial \beta} \right|_{\beta = \hat{\beta}} = 0. \quad (9)$$

The least squares estimate $\hat{\beta}$ obtained from (9) can then be written as

$$\hat{\beta} = (V^T V)^{-1} V^T y. \quad (10)$$

The individual noise terms within ϵ are defined to be zero mean and white, with a variance of σ^2 :

$$\begin{aligned} E(\epsilon_i) &= 0; E(\epsilon_i \epsilon_j) = 0 & \text{for } i \neq j \\ &= \sigma^2 & \text{for } i = j. \end{aligned} \quad (11)$$

$\hat{\beta}$ is an unbiased estimator of β :

$$E(\hat{\beta}) = \beta. \quad (12)$$

In a simulation, β is specified, so β and the computed $\hat{\beta}$ must be comparable.

The sample variance of the residuals is an unbiased estimate of the error variance:

$$s_{\delta y}^2 = \frac{1}{(n - (m + 1))} \delta y^T \delta y \quad (13)$$

$$E[s_{\delta y}^2] = \sigma^2. \quad (14)$$

Again, in a simulation, σ^2 is specified, so σ^2 and the computed $s_{\delta y}^2$ must be comparable.

The covariance matrix of the estimate is

$$E[(\hat{\beta} - \beta)(\hat{\beta} - \beta)^T] = \sigma^2 (V^T V)^{-1}. \quad (15)$$

When a single parameter is being estimated, $(V^T V)$ reduces to a scalar, so the standard deviation of the estimate is given by

$$\sigma_{\hat{\beta}} = s_{\delta y} (V^T V)^{-1/2} \quad (16)$$

and can be numerically computed by its unbiased sample statistic, the parameter sample standard deviation

$$s_{\hat{\beta}} = s_{\delta y} (V^T V)^{-1/2}. \quad (17)$$

III. SIMULATION OF RANDOM NUMBERS

Measurements are simulated by adding noise to an ideal measurement whose parameter is specified. The objective here is, first, to generate physically realistic noise, and second, to develop a visual impression of the noise by means of histogram and scattergram. Visual examination is important. Later, when examining the measurement residuals of actual measurements, it is sometimes possible to pick out a systematic instrumentation problem by observing an unusually shaped histogram.

The derivation of the least squares estimate makes no assumption about the nature of the additive noise. However, typical nonsystematic precision noise is Gaussian because it is the cumulative error due to many sources. Furthermore, nonsystematic errors encountered from one measurement to the next are usually essentially independent. Both the Gaussian and the white properties of random numbers can be demonstrated by simulation.

According to the Central Limit Theorem, the sum of many random variables is approximately Gaussian, regardless of the distributions of the individual random variables. Computers typically have uniform random number generators, but not Gaussian. If u is a random variable, uniform from -0.5 to $+0.5$, its variance is $1/12$. If γ is defined to be the sum of p uniform random variables, then

$$\gamma = \sum_{k=1}^p u_k \quad (18)$$

and

$$E(\gamma) = 0 \quad \sigma_{\gamma}^2 = p/12. \quad (19)$$

An obvious choice for the value of p to produce pseudo-Gaussian random numbers is 12. The Central Limit Theorem is well satisfied, and the variance of the resulting sum is unity. Fig. 3 is a histogram of 1000 uniform random variables, while Fig. 4 is a histogram of 1000 pseudo-Gaussian random numbers, generated using $p = 12$. The familiar Gaussian bell-shaped curve is evident. These histograms are centered at the sample mean and extend three sample standard deviations above and below.

For the purpose of simulating Gaussian random errors with a specified standard deviation σ , the following is used:

$$\epsilon = \sigma \sum_{k=1}^{12} u_k. \quad (20)$$

Returning to sequential independence, it is, first, a realistic assumption, and second, required for the validity

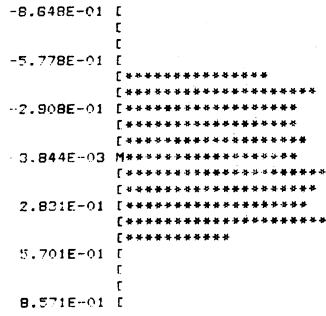


Fig. 3. Histogram of uniform random variables.

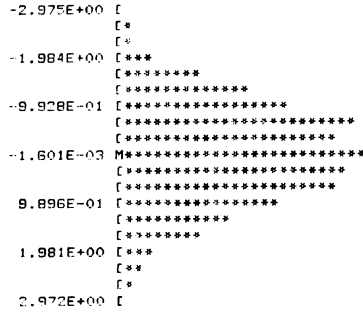


Fig. 4. Histogram of sum of 12 uniform random numbers.

of (17), the estimated standard deviation. The degree of independence of random variables can be visualized by examining the correlation. The correlation between two zero-mean random variables is given by

$$\text{Corr}(\alpha_i, \alpha_j) = \frac{E[\alpha_i \alpha_j]}{[E[\alpha_i^2] E[\alpha_j^2]]^{1/2}} \quad (21)$$

If the random variable α is defined as

$$\alpha_i = \rho \alpha_{i-1} + \sqrt{1 - \rho^2} \epsilon_i \quad (22)$$

where

$$\begin{aligned} E(\epsilon_i) &= 0 \\ E(\epsilon_i \epsilon_j) &= \sigma^2 \quad \text{for } i = j \\ &= 0 \quad \text{for } i \neq j \end{aligned} \quad (23)$$

then

$$\begin{aligned} E(\alpha_i) &= 0 \\ E(\alpha_i^2) &= \sigma^2 \end{aligned} \quad (24)$$

and

$$\text{Corr}(\alpha_{i-1}, \alpha_i) = \rho. \quad (25)$$

In other words, the correlation between successive random variables is ρ . If $\rho = 0$, the random variables are sequentially uncorrelated or independent.

A scattergram is a two-dimensional histogram of random numbers. Pairs of numbers form the coordinates in a plane. The numbers within the scattergram represent the number of points falling within square cells. Fig. 5 is a scattergram of 1000 simulated independent ($\rho = 0$) and random numbers. No pattern is apparent. However, when

Fig. 5. Scattergram: $\rho = 0.0$.Fig. 6. Scattergram: $\rho = 0.9$.

the simulated numbers are sequentially correlated, the scattergram reveals a defined pattern as seen in Fig. 6, where $\rho = 0.9$. The classroom demonstration produces a color scattergram of random variables of a specified correlation. It also numerically computes sample correlation which is comparable to the specified correlation.

IV. PARAMETER ESTIMATION APPLIED TO SIMULATED MEASUREMENTS

The intent here is to build student confidence in the use of least squares parameter estimation techniques, including the interpretation of estimation statistics. For each of the control system parameters shown in Fig. 1, a different experiment can be devised to obtain the required measurements. For this reason, the vector β reduces to a scalar β .

In the simulations, β and σ^2 are specified. If all goes well, $s_{\delta y}$ is comparable to σ , and most of the time, the approximate 95 percent confidence region of the estimate $\hat{\beta} + 2s_{\hat{\beta}}$ spans the true value β .

TABLE I
COMPARISON OF SIMULATED TO ESTIMATED QUANTITIES: MEASUREMENT OF A CONSTANT

Simulated Population		Measurement	Estimate			
Constant	Error	No. Sample	Parameter	Residual STD	Parameter STD	95 Percent Confidence Region
β_0	σ	n	β_0	$s_{\delta y}$	s_{β_0}	$\beta_0 \pm 2s_{\beta_0}$
10.0	1.0	10	10.19	1.27	0.402	10.19 ± 0.80
10.0	1.0	40	10.05	0.998	0.158	10.05 ± 0.32
10.0	1.0	160	9.98	0.954	0.075	9.98 ± 0.25

Estimation of a Constant

For the purpose of estimating parameters for the given servomotor configuration, the estimation of a constant is not necessary. However, it is useful to go through the exercise to see familiar results follow from the estimation equations, and to compare simulated population parameters to estimated parameters.

The form of the measurements is

$$y_i = \beta_0 + \epsilon_i \quad (26)$$

or

$$y = V\beta + \epsilon \quad (27)$$

where

$$V = [1 \ 1 \ \cdots \ 1]^T \quad (28)$$

$$\beta = \beta_0.$$

Due to the form of V , the general estimation expression

$$\hat{\beta} = (V^T V)^{-1} V^T y \quad (29)$$

reduces to the familiar, and intuitively pleasing, sample mean.

$$\hat{\beta} = \frac{1}{n} \sum_{i=1}^n y_i \quad (30)$$

Similarly, the single parameter estimate standard deviation, given by (16), reduces to

$$\sigma_{\beta_0} = \sigma/\sqrt{n}. \quad (31)$$

While this cannot be computed due to the unknown population standard deviation, the corresponding sample statistics can be:

$$s_{\beta_0} = s_{\delta y}/\sqrt{n} \quad (32)$$

where

$$s_{\delta y} = \left[\frac{1}{n-1} \delta_y^T \delta_y \right]^{1/2}. \quad (33)$$

These results are illustrated in Table I. Note the correspondence between the simulation population parameter β_0 and its estimate $\hat{\beta}_0$. Likewise, σ and $s_{\delta y}$ are comparable. Note also that as the sample size n is increased by a factor of 4 in successive experiments, the estimate uncertainty s_{β_0} is approximately halved.

Beyond determining the 95 percent confidence region or

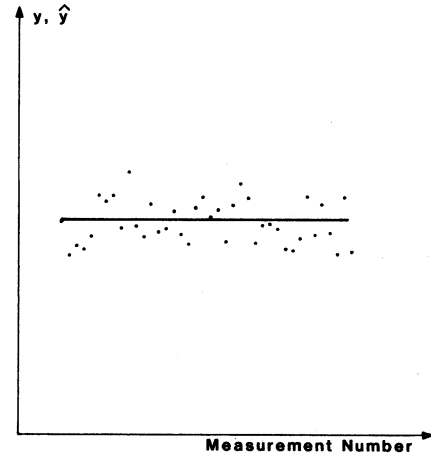


Fig. 7. Points are simulated $y_i = \beta_0 + \epsilon_i$; line is estimate, $y = \hat{\beta}_0$.

of the estimate, the estimate sample standard deviation has a further use. Students tend to record computed results with as much apparent precision as the computer has printed. For example, the first estimate in the table was printed as

$$\hat{\beta}_0 = 10.1902. \quad (34)$$

There is uncertainty associated with s_{β_0} , as its number of significant digits must be assumed. If two significant digits are assumed, then in this case, the 95 percent confidence region is the value of the estimate ± 0.08 . For this reason, it is legitimate to record $\hat{\beta}_0$ with only two places to the right of the decimal.

Fig. 7 illustrates the last simulation. The sample number is spaced horizontally, with measured values plotted vertically. The horizontal line represents the value of the least squares estimate of the constant. The residuals, the difference between the estimated constant and actual measurements, are the vertical displacements between the horizontal line and the individual points. The least square estimate has minimized the sum of the squares of these displacements.

Estimation of a Proportionality Constant

All of the parameters which are linearly related to their measurements in the given servomotor configuration are of the form

$$y_i = \beta_1 X_i + \epsilon_i \quad (35)$$

TABLE II
COMPARISON OF SIMULATED TO ESTIMATED QUANTITIES: MEASUREMENT OF A PROPORTIONALITY CONSTANT

Range		Estimate			
		Parameter	Residual STD	Parameter STD	95 Percent Confidence Region
x_{\min}	x_{\max}	β_1	$s_{\delta y}$	$s_{\beta 1}$	$\beta_1 \pm 2s_{\beta 1}$
-100	100	0.01008	0.1017	$1.74 \cdot 10^{-4}$	0.01008 ± 0.00034
0	100	0.01005	0.1017	$1.74 \cdot 10^{-4}$	0.01005 ± 0.00034
95	100	0.01001	0.1018	$1.04 \cdot 10^{-4}$	0.01001 ± 0.00021
195	200	0.01001	0.1018	$5.13 \cdot 10^{-5}$	0.01001 ± 0.00010
395	400	0.01000	0.1018	$2.55 \cdot 10^{-5}$	0.01000 ± 0.00005

$$y = V\beta + \varepsilon$$

where

$$V = [x_1 x_2 \cdots x_n]^T$$

$$\beta = \beta_1. \quad (36)$$

The general estimation equation

$$\hat{\beta} = (V^T V)^{-1} V^T y \quad (37)$$

reduces to

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}. \quad (38)$$

The estimate sample standard deviation reduces to

$$s_{\hat{\beta}} = s_{\delta y} \left/ \left[\sum_{i=1}^n x_i^2 \right]^{1/2} \right. \quad (39)$$

This last result indicates that while estimate uncertainty can be reduced by increasing sample size, the same can be more effectively done by choosing independent variables with large absolute values.

These results can be illustrated by simulation. In Table II, $\hat{\beta}_1$ compares to β_1 as does $s_{\delta y}$ to σ . Equation (38) shows that the parameter estimate uncertainty can be altered even if the number of measurements remains the same. The first two rows in the table show, as would be expected, that there is essentially no difference between using positive and negative or all positive independent variables. The third row shows a marked improvement resulting from the use of larger independent variables. If the independent random variables are doubled, the uncertainty of the parameter estimate is essentially halved, an effect demonstrated twice in the last rows of the table.

Fig. 8 shows simulated measurements also, with the measurement function evaluated at $\hat{\beta}_1$. The residuals are, again, the vertical displacement between the measurements and the line.

Nonlinear Estimation

While motor time constant can be inferred from the exponential rise in shaft rate due to a step input, the measured quantity is not linearly related to the parameter to be estimated.

$$y_i = A(1 - \exp(-x_i/\beta)) + \epsilon_i. \quad (40)$$

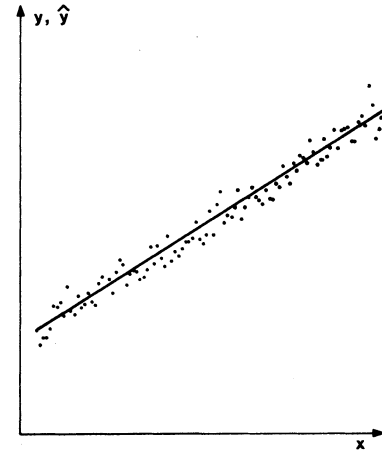


Fig. 8. Points are simulated $y_i = \beta_1 X_i + \epsilon_i$; line is expected function, $y = \beta_1 X$.

In this equation, y_i , x_i , and β represent, respectively, shaft rate, time, and time constant. There are many ways to deal with nonlinear estimation, but the one which fits well with the development to this point is the method of quasi-linearization combined with least squares [7].

Suppose the measurement equation is generalized:

$$y_i = f(x_i; \beta) + \epsilon_i. \quad (41)$$

The measurement can be linearized about the current estimate

$$y_i - f(x_i; \beta) = \left. \frac{\partial f(x_i; \beta)}{\partial \beta} \right|_{\beta = \hat{\beta}} (\beta - \hat{\beta}) + \epsilon_i. \quad (42)$$

This equation may be written in terms of measurement residual δy_i and the parameter estimate residual $\delta \beta$.

$$\delta y_i = \left. \frac{\partial f(x_i; \beta)}{\partial \beta} \right|_{\beta = \hat{\beta}} \delta \beta + \epsilon_i. \quad (43)$$

This is of the same form as (35), so the same estimation equations apply. However, rather than estimating the parameter directly, an estimate is made of the correction to the current parameter estimate. An initial guess of the parameter can be readily obtained from the raw data. The estimation algorithm in Fig. 9 usually converges within a few iterations.

Again, the estimation procedure can be demonstrated by simulation. The general function $f(x_i; \beta)$ is given by

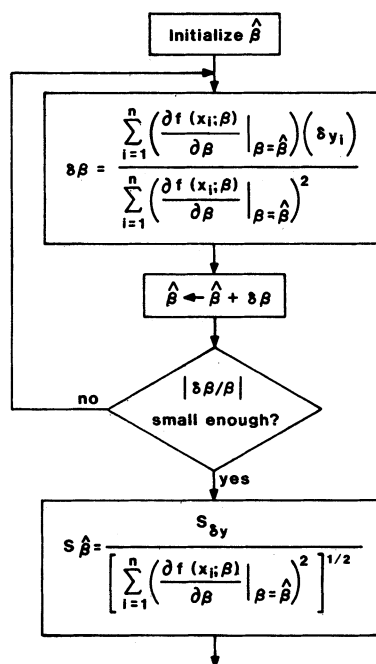


Fig. 9. Nonlinear estimation flowchart.

TABLE III
COMPARISON OF SIMULATED TO ESTIMATED QUANTITIES: MEASUREMENT TO DETERMINE TIME CONSTANT

Simulated Population		Measurements			Estimates			
Constant	Error	No. Samples	Range		Parameter	Residual STD	Parameter STD	95 Percent Confidence Region
β	σ	n	x_{\min}	x_{\max}	β	$s_{\delta y}$	$s\beta$	$\beta \pm 2s_{\beta}$
0.25	1	100	0	1	0.25026	1.0225	0.00414	0.2503 ± 0.0082
0.25	1	100	0.2	1	0.2499	1.0226	0.00417	0.2499 ± 0.0083
0.25	1	100	0.4	1	0.2498	1.0225	0.00522	0.250 ± 0.010
0.25	1	100	0	0.8	0.2505	1.0225	0.00377	0.2505 ± 0.0075
0.25	1	100	0	0.6	0.2503	1.0225	0.0034	0.2503 ± 0.0069
0.25	1	100	0	0.4	0.2500	1.0225	0.00329	0.2500 ± 0.0066
0.25	1	200	0	0.2	0.2493	1.0224	0.00391	0.2493 ± 0.0078

(40) where $A = 25$. In Table III, the simulated population parameters β and σ are to be compared to $\hat{\beta}$ and $s_{\delta y}$.

The derivative in the denominator of the expression for s_{β} obscures what is to be done to minimize the uncertainty of the estimate. The simulated results suggest that, for a given number of measurements, they are most effectively used over the span of one time constant. Fig. 10 illustrates simulated measurements and the corresponding least squares fit.

In order to estimate the motor time constant, the students are required to obtain a strip chart recording of a step response, hand digitize the data, transfer it to a data file, and then run their nonlinear estimation program. In the classroom demonstration, however, the microcomputer system carries out the experiment directly. The computer's reed relay starts the motor, samples are taken under control of the real-time clock, the motor is stopped, and the results are written to a file. The estimation program then reads the file, produces the necessary numerical results, and produces a color plot similar to Fig. 10. While the microcomputer implementation details are beyond the

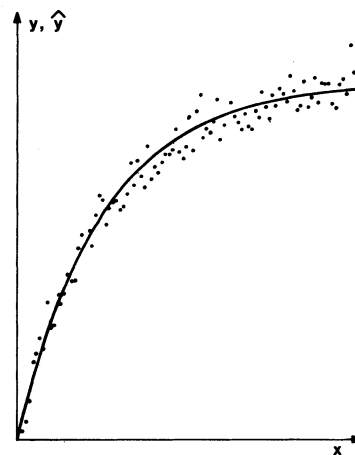


Fig. 10. Points are simulated $y_i = 1 - \exp(-X_i/\beta) + \epsilon_j$; line is expected function, $y = 1 - \exp(-X_i/\beta)$.

scope of the course, the advantages of automated experimentation are apparent.

An interesting alternative to obtaining the motor time constant by processing time response data is to process

frequency response data. In this case, both magnitude and phase are nonlinear in the parameter to be estimated. The students use the same fundamental nonlinear estimation technique, but with different functions and derivatives. The time constant estimates from both magnitude and phase data are then optimally combined to form a single estimate. The confidence band of the estimate formed in this way is comparable to that formed by the time response data. The time response technique is generally favored because it is less complex to carry out and is more easily automated by the microcomputer.

At this point, all of the tools are in place for the student to obtain good estimates of the control system parameters and to compute such closed-loop time response characteristics as percent overshoot, damped natural frequency, and settling time. Closed-loop laboratory experiments then compare favorably to predicted results.

V. CONCLUSIONS

The primary objective of the laboratory course is to confirm, by experimentation, analytically predicted closed-loop control system results. Parameters for this purpose are determined from experimental data taken from the individual components.

Prior to the introduction of statistical parameter estimation into the course, results were quite erratic. The students' background in probability and statistics was, and is, such that few could take the initiative to statistically minimize the effects of precision error, unmodeled higher order effects, and nonlinearities. The way the course is currently taught, numerical results are better, the students gain additional Fortran programming skills, and many gain a better appreciation for statistics. The measurement simulation is particularly useful for them to see how ideal measurements are obscured by noise, and then how good estimates are recovered from the noisy data.

The particular application in this paper is a dc servo-system. This set of programs is also used throughout the semester on ac components, hydraulic components, and various sensors. The application of the nonlinear estimation previously described is the estimation of the dc motor pole location by means of the estimate of the time constant. The same technique is applied later in the course to estimate the pole location directly by frequency response gain and phase information.

In the future, we plan to extend the statistical analysis to propagation of error. Having done this, it will be possible to show how component estimate errors affect closed-loop response errors.

Program listings will be provided by the authors upon request.

REFERENCES

- [1] *Modular Servo Type MS150 Manual*, Feedback Instruments Ltd., Sussex, England, 1976.
- [2] B. C. Kuo, *Automatic Control Systems*. Englewood Cliffs, NJ: Prentice-Hall, 1982.
- [3] *Microsoft FORTRAN-80 Version 3.2*, Microsoft, Bellevue, WA, 1978.
- [4] *An Introduction to CP/M Features and Facilities*, Digital Research, Pacific Grove, CA, 1978.
- [5] *The Intecolor User's Manual for 8000, 8300 and 8900 Series Intecolor Terminals*, Intecolor, Norcross, GA, 1980.
- [6] R. DeMoyer and R. V. Houska, "A statistical simulation approach for evaluating measurement and estimation techniques in an undergraduate servomotor lab," presented at the 1981 Summer Simulation Conf., Washington, DC, July 1981.
- [7] D. M. Himmelblau, *Process Analysis by Statistical Methods*. New York: Wiley, 1970.



Robert DeMoyer, Jr. (S'65-M'66) received the B.S. degree in electrical engineering from Lehigh University, Bethlehem, PA, in 1966, and the M.S. degree in systems science and the Ph.D. degree in systems engineering from the Polytechnic Institute of Brooklyn, Brooklyn, NY.

From 1966 to 1977 he worked for the General Electric Company, Philadelphia, PA. During this time, he was a Systems Engineer working on reentry vehicle projects. Since 1977 he has been on the Faculty of the United States Naval Academy,

Annapolis, MD, where he is currently Associate Professor of Systems Engineering. His current interests include microcomputer control and robotics.

Dr. DeMoyer is a member of Tau Beta Pi, Sigma Xi, Eta Kappa Nu, and Phi Eta Sigma. He is a Registered Professional Engineer in the Commonwealth of Pennsylvania.



Richard V. Houska was born in Chicago, IL, on January 26, 1946. He received the Bachelors and Masters degrees from Rutgers University, New Brunswick, NJ, in 1968 and 1972, respectively, and the Ph.D. degree from the University of Minnesota, Minneapolis, in 1978.

From 1968 to 1973 he was an Electrical Engineer for Lockheed Electronics Company; from 1973 to 1976, a Research Fellow at the University of Minnesota; and from 1976 to 1978, a Consulting Engineer with Systems Consultants, Washington, DC.

After a one-year visiting appointment in the Department of Electrical Engineering, University of Minnesota, in 1978, he joined the Faculty of the U.S. Naval Academy in the Weapons and Systems Engineering Department, Annapolis, MD, where he is now an Associate Professor. His professional interests are concerned with the practical application of control system concepts to computer and communication systems.