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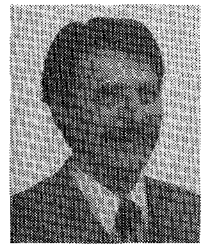

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An Overview on the Time Delay Estimate in Active and Passive Systems for Target Localization

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***Abstract*—Sonar and radar systems not only detect targets but also localize them. The process of localization involves bearing and range estimation. These objectives of bearing and range estimation can be accomplished actively or passively, depending on the situation. In *active sonar* or radar *systems*, a pulsed signal is transmitted to the target and the echo is received at the receiver. The range of the target is determined from the time delay obtained from the echo. In *passive sonar systems*, the target is detected from acoustic signals emitted by the target, and it is localized using time delays obtained from received signals at spacially separated points. Several authors have calculated the variance of the *time delay estimate* in the neighborhood of true time delays and have presented their results in terms of coherence function and *signal and noise autospectra*. Here we analyze these derivations and show that they are the same for the case of low *signal-to-noise ratio* (SNR). We also address a practical problem with a target-generated wide-band signal and present the Cramér-Rao lower bound on the variance of the time delay estimate as a function of commonly understood terms such as SNR, *bandwidth*, *observation time*, and *center frequency* of the band. The analysis shows that in the case of low SNR and when signal and noise autospectra are constants over the band or signal and noise autospectra fall off at the same rate, the minimum *standard deviation* of the time delay estimate varies inversely to the SNR, to the square root of the product of observation time and bandwidth, and to the center frequency (provided $W^2/12f_0^2 \ll 1$, where W = bandwidth and f_0 = center frequency of the band). The only difference in the case of a high SNR is that the standard deviation varies inversely to the square root of the SNR, and all other parameter relationships are the same. We also address the effects of different signal and noise autospectral slopes on the variance of the time delay estimate in passive localization.**

Manuscript received April 4, 1980; revised October 9, 1980. This work was supported by the Naval Underwater Systems Center's Wide Aperture Array Electronics Project.

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I. Introduction

SONAR and radar systems not only detect the targets but also find the location and velocity of the target. To locate a target using an *active system*, a pulse is transmitted to the target and the echo is received. The range of a target is determined using the *time delay* between the transmission of a pulse and the reception of its echo. To estimate this time delay, the system must determine the instant when the echo arrives. Generally, this is accomplished by matched filter or correlation where the "clean" reference signal, i.e., transmitted signal, is available. The time delay is estimated by measuring the peak of the output processor (matched filter or correlator), but exactly when this peak occurs is uncertain owing to the noise added to the echo signal.

To locate a target using a *passive system*, the sonar system receives a signal generated by the target and noise at spatially separated points. This system provides bearing and range information on a target by comparing its received signal at a multiplicity of widely spaced points along the length of its own ship or along a towed array. The target's bearing is determined by measuring the time differential for received signals at two locations. This time differential is obtained by cross correlating the received signals from these two points and measuring the time displacements of the correlogram peak. The range of a target is determined by measuring the difference of the time differential at two pairs of points.

The target's location may be determined either in the active or in the passive system by measuring the time delays that are obtained from correlation peaks. Therefore, the accuracy of the *time delay estimate* is critical to the accuracy of a target's bearing and range estimation.

Considerable research has been conducted to estimate the variance of the time delay estimate [1]-[7]. Helstrom [1], Woodward [2], and Wahlen [3] have presented the variance of time delay errors about true time delay, especially for an active system, where a clean reference deterministic signal is available in terms of signal energy to noise autospectral density and root-mean-square (rms) bandwidth. Knapp and Carter [4] have shown the variance of a time delay estimate in the neighborhood of true time delay in terms of coherence function. Hahn [5], Schultheiss [6], and Tomlinson and Sorokowsky [7] have presented the variance of the time delay estimate about unbiased mean time delay in terms of *signal and noise autospectra*.

Sometimes it is difficult for a user to determine whether the variance of time delay errors about a true time delay in the passive system obtained by several authors in terms of coherence function and signal and noise autospectra is the same or different [4]-[7]. An attempt to answer this question is made here. The results obtained by Knapp and Carter [4], Schultheiss [6], Tomlinson and Sorokowsky [7], and Hahn [5] are unified and the analysis shows that these results are the same for the case of low SNR (SNR $\ll$ 1), where the wide-band signal has a flat spectrum and the noise is white. Furthermore, the variance of time delay errors as a function of commonly understood terms such as SNR, *bandwidth*, *observation time*, *center frequency*, and ratio of the bandwidth to center frequency are calculated and presented. Also, a practical problem with the wide-band signal and noise in the passive system is addressed, and the effect of change of SNR, observation time, bandwidth, center frequency, the ratio of bandwidth to center frequency, and signal and noise autospectral falloff with frequency on the variance or *standard deviation* of time delay errors are investigated. In addition, the standard deviation of time delay errors in the passive and active systems are compared.

II. Variance of Time Delay Estimate

A. Active System

To measure the range of a target in an active system, it is necessary to estimate the time delay D at which the echo arrives at the receiver. If the time from the transmission of the pulse is measured, the range of the target is $PD/2$, where P is the speed of sound or electromagnetic wave. The received signal will consist of a deterministic signal that is corrupted with white noise of spectral density $N_0/2$.

It has been shown that the Cramér-Rao lower bound of variance of time delay errors about true time delay is [1]-[3]

$$\sigma_D^2 \geqslant \frac{1}{d^2\beta^2} \tag{1}$$

where

$$d^2 = \frac{2E}{N_0}, \tag{2}$$

$$\beta^2 = \frac{\int_{-\infty}^{\infty} \omega^2 |F(\omega)|^2 \, d\omega}{\int_{-\infty}^{\infty} |F(\omega)|^2 \, d\omega}, \tag{3}$$

E = energy of the signal $S(t)$, $F(\omega) = \int_{-\infty}^{\infty} S(t) e^{-j\omega t}\, dt$, and β = a measure of bandwidth.

Woodward [2] has shown that if the value of d^2 is about 8 or more, then the variance of the time delay estimate about true time delay can be estimated without ambiguity.

Here we assume that the signal autospectrum is two sided and extends from f_1 to f_2 Hz (and also $-f_1$ to $-f_2$ Hz) with spectral density of $S_0/2$ W/Hz. Then

$$\beta^2 = \frac{2\int_{f_1}^{f_2} (2\pi f)^2 S_0/2 \; 2\pi \, df}{2\int_{f_1}^{f_2} S_0/2 \; 2\pi \, df} = (2\pi)^2 (f_2^2 + f_1 f_2 + f_1^2)/3 \tag{4}$$

$$\sigma_D^2 \geqslant \frac{1}{\frac{2E}{N_0}\frac{4\pi^2}{3}(f_2^2 + f_1 f_2 + f_1^2)} = \frac{1}{\frac{2ST}{N_0(f_2 - f_1)}\frac{4\pi^2}{3}(f_2 - f_1)(f_2^2 + f_1 f_2 + f_1^2)} = \frac{3}{8\pi^2 T}\frac{1}{\text{SNR}}\frac{1}{(f_2^3 - f_1^3)} \tag{5}$$

where signal power $S = S_0(f_2 - f_1)$, noise power $N = N_0(f_2 - f_1)$, observation time = T, and signal-to-noise ratio = S/N = SNR. Therefore, the standard deviation of the time delay estimate about true time delay is

$$\sigma_D \geqslant \left(\frac{3}{8\pi^2 T}\right)^{1/2} \frac{1}{\sqrt{\text{SNR}}} \frac{1}{\sqrt{f_2^3 - f_1^3}}. \tag{6}$$

Equation (6) is valid for any SNR and may be written in terms of bandwidth W and center frequency f_0 as

$$\sigma_D = \left(\frac{1}{8\pi^2}\right)^{1/2} \frac{1}{\sqrt{\text{SNR}}} \frac{1}{\sqrt{TW}} \frac{1}{f_0} \frac{1}{\sqrt{1 + W^2/12 f_0^2}} \tag{7}$$

where

$$f_1 = f_0 - \frac{W}{2}, \quad \text{and} \quad f_2 = f_0 + \frac{W}{2}.$$

Equation (7) shows that σ_D is inversely proportional to the

1) square root of the SNR;
2) square root of the product of bandwidth and time;
3) center frequency and, also, is a function of the ratio of bandwidth and center frequency.

B. Passive System

No signal is transmitted in the passive system. The received signals are composed of signals generated by target and noise. It is assumed that the target signal and noise are not correlated and are a stationary random process. We calculate the variance of the time delay estimate in the neighborhood of true time delay using derivations provided by several authors as a function of SNR, center frequency, bandwidth, and observation time and then compare these results [4]-[7].

1) Time Delay Estimate at Low SNR, Approach 1: Knapp and Carter [4] have shown that the Cramér-Rao lower bound variance of the time delay estimate about the true value using the coherence function is

$$\sigma_D^2 \geqslant \left\{2T \int_0^{\infty} (2\pi f)^2 \frac{|\gamma(f)|^2}{[1-|\gamma(f)|^2]} df\right\}^{-1} \tag{8}$$

where $\gamma(f)$ is coherence function and T is observation time, and

$$|\gamma(f)|^2 = \frac{G_{ss}^2(f)}{[G_{ss}(f)+G_{nn}(f)]^2} \tag{9}$$

where $G_{ss}(f)$ is the signal autospectrum and $G_{nn}(f)$ is the noise autospectrum. Let

$$\frac{G_{ss}^2(f)}{[G_{ss}(f)+G_{nn}(f)]^2} \equiv \frac{S^2}{(S+N)^2} \tag{10}$$

and, then,

$$\frac{|\gamma(f)|^2}{[1-|\gamma(f)|^2]} = \frac{S^2}{(S+N)^2} \Bigg/ \left[1-\frac{S^2}{(S+N)^2}\right] \simeq (\text{SNR})^2 \tag{11}$$

when SNR $\ll 1$.

Substituting the value of

$$|\gamma(f)|^2/[1-|\gamma(f)|^2]$$

from (11) into (8), we get

$$\sigma_D^2 \geqslant \left\{2T \int_0^{\infty} (2\pi f)^2 \frac{S^2(f)}{N^2(f)} df\right\}^{-1}. \tag{12}$$

Assuming the signal and noise autospectra are constant over the band extending from f_1 to f_2 Hz with S_0 and N_0 W/Hz, respectively, we see that

$$\sigma_D^2 \geqslant \left(\frac{3}{8\pi^2 T}\right) \frac{1}{(S_0/N_0)^2} \frac{1}{(f_2^3-f_1^3)}. \tag{13}$$

Therefore, for low SNR, the standard deviation of the time delay estimate is

$$\sigma_D \geqslant \left(\frac{3}{8\pi^2 T}\right)^{1/2} \frac{1}{\text{SNR}} \frac{1}{\sqrt{f_2^3-f_1^3}} \tag{14}$$

where

$$S = S_0(f_2-f_1)$$

$$N = N_0(f_2-f_1).$$

2) Time Delay Estimate at Low SNR, Approach 2: Schultheiss [6] has shown that the minimum variance of time delay elements about the true time delay is given by

$$\sigma_D^2 \geqslant \frac{2\pi}{T}\left\{\int_0^{\infty} \frac{M\omega^2 S^2(\omega)/N^2(\omega)}{1+MS(\omega)/N(\omega)} d\omega\right\}^{-1} \tag{15}$$

where M is equal to the number of hydrophones or arrays that are utilized to measure the time delays. In case of a low SNR (SNR $\ll 1$), i.e., $MS(\omega)/N(\omega) \ll 1$ and $M = 2$, (15) reduces to

$$\sigma_D^2 \geqslant \frac{2\pi}{T}\left\{\int_0^{\infty} 2\omega^2 S^2(\omega)/N^2(\omega)\, d\omega\right\}^{-1}$$

$$= \frac{3}{8\pi^2 T} \frac{1}{(S_0/N_0)^2} \frac{1}{f_2^3-f_1^3}. \tag{16}$$

In (16), we have assumed that the signal and noise autospectra are flat, extend from f_1 to f_2 Hz, and yield

$$\sigma_D \geqslant \left(\frac{3}{8\pi^2 T}\right)^{1/2} \frac{1}{\text{SNR}} \frac{1}{\sqrt{f_2^3-f_1^3}} \tag{17}$$

where

$$S = S_0(f_2-f_1)$$

$$N = N_0(f_2-f_1).$$

3) Time Delay Estimate at Low SNR, Approach 3: Hahn [5] and Tomlinson and Sorokowsky [7] have shown that the variance of the time delay estimate about the true time delay is given by

$$\sigma_D^2 = \frac{\frac{1}{2\pi T}\int_{-\infty}^{\infty} \omega^2[N_1(\omega)N_2(\omega)+S(\omega)\{N_1(\omega)+N_2(\omega)\}]\, d\omega}{\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} \omega^2 S(\omega)\, d\omega\right]^2} \tag{18}$$

where $S(\omega)$, $N_1(\omega)$, and $N_2(\omega)$ are autospectra of the signal $S(t)$, noise $N_1(t)$, and noise $N_2(t)$, respectively.

At low SNR (SNR $\ll 1$), (18) may be approximated by

$$\sigma_D^2 \cong \frac{\frac{1}{2\pi T}\int_{-\infty}^{\infty} \omega^2 N_1(\omega) N_2(\omega)\, d\omega}{\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} \omega^2 S(\omega)\, d\omega\right]^2}. \tag{19}$$

Assuming that $N_1(\omega) = N_2(\omega) = N(\omega)$ and that the signal autospectrum $S(\omega)$ and noise autospectrum $N(\omega)$ are constant over the band extending from f_1 to f_2 Hz with densities $S_0/2$ and $N_0/2$, respectively, we can rewrite (19) as follows:

$$\sigma_D^2 \cong \frac{\frac{1}{2\pi T} 2 \int_{f_1}^{f_2} (2\pi f)^2 \frac{N_0^2}{4} 2\pi \, df}{\left[\frac{1}{2\pi} 2 \int_{f_1}^{f_2} (2\pi f)^2 \frac{S_0}{2} 2\pi \, df\right]^2}$$

$$= \left(\frac{3}{8\pi^2 T}\right) \frac{1}{(S_0/N_0)^2} \frac{1}{(f_2^3 - f_1^3)} \tag{20}$$

$$\sigma_D \cong \left(\frac{3}{8\pi^2 T}\right)^{1/2} \frac{1}{\text{SNR}} \frac{1}{\sqrt{f_2^3 - f_1^3}} \tag{21}$$

where

$$S = S_0(f_2 - f_1)$$
$$N = N_0(f_2 - f_1).$$

Notice that (14), (17), and (21) are the same. A comment may be in order. In terms of coherence function and signal and noise autospectra, the basic derivations [see (8) and (15)] for the Cramér-Rao lower bound of the variance of the time delay estimate about true time delay are the same as those shown in (13) and (16), and σ_D^2 is presented in terms of SNR, W, T, f_1, and f_2. Therefore, in the case of low SNR with constant signal and noise autospectra over the band, we can generalize that the standard deviation of the time delay estimate in the neighborhood of true time delay is the same as shown in (14), (17), and (21). The equations may be further simplified as a function of SNR, product of bandwidth and observation time, and center frequency:

$$\sigma_D = \frac{1}{\sqrt{8\pi^2}} \frac{1}{\text{SNR}} \frac{1}{\sqrt{TW}} \frac{1}{f_0} \frac{1}{\sqrt{1 + W^2/12f_0^2}} \tag{22}$$

or

$$\sigma_D \sim \frac{1}{\text{SNR}} \qquad T, W, f_0 \text{ constant}$$

$$\sim \frac{1}{\sqrt{T}} \qquad \text{SNR}, f_0, W \text{ constant}$$

$$\sim \frac{1}{f_0} \qquad \text{SNR}, T, W \text{ constant, and } W \ll f_0$$

$$\sim \frac{1}{\sqrt{W}} \qquad \text{SNR}, T, f_0 \text{ constant}$$

$$\sim \frac{1}{\sqrt{1 + W^2/12f_0^2}} \frac{1}{\sqrt{W}} \frac{1}{f_0} \qquad \text{SNR}, T \text{ constant.}$$

Notice that the only difference between an active and a passive system when estimating the variance of the time delay is the term $\sqrt{\text{SNR}}$ with the SNR, as shown in (6) and (21); all other terms are the same.

4) Time Delay Estimate at High SNR: So far the analytical results of standard deviation σ_D of the time delay estimate for low SNR have been presented. For completeness, the results for the standard deviation of the time delay estimate when the SNR is high (SNR $\gg$ 1) have been presented. It is now possible to show algebraically and by approximations that (8), (15), and (18) yield

$$\sigma_D \cong \left(\frac{3}{4\pi^2 T}\right)^{1/2} \frac{1}{\sqrt{\text{SNR}}} \frac{1}{\sqrt{f_2^3 - f_1^3}}. \tag{23}$$

Observe that the standard deviation of the time delay estimate in the case of high SNR, as shown in (23), varies inversely to the square root of the SNR, whereas it varies inversely to the SNR in the case of low SNR, as shown in (14), (17), and (21). Notice that (23) is similar to (6), which is valid for the active system. The only difference is that (23) is $\sqrt{2}$ times higher than (6). This is intuitively appealing because, in the case of the passive system (23), both the received signals are corrupted by noise; but, in the case of the active system (6), a clean reference or transmitted signal is available for correlation.

5) Effects of Signal and Noise Autospectral Falloff on Time Delay Estimate: Up to now, in the analysis of variance of the time delay estimate, we have assumed that signal and noise autospectra are constants over the band W, which extends from f_1 to f_2 Hz. However, in practice, neither the signal autospectrum nor the noise autospectrum in underwater acoustics is constant over the bandwidth. Therefore, we address a practical problem where signal and noise autospectra fall off and investigate the effects of spectral falloff on the variance of the time delay estimate.

Assuming that the signal and noise autospectra extend from f_1 to f_2 Hz and signal and noise autospectra fall off at the rate $10p$ dB/decade $\{S_0(f_1/f)^p\}$ and $10n$ dB/decade $\{N_0(f_1/f)^n\}$, we can rewrite (12) or (16) as follows:

$$\sigma_D^2 = \left\{2T \int_{f_1}^{f_2} (2\pi f)^2 f_1^{2p-2n} f^{2n-2p} \left(\frac{S_0}{N_0}\right)^2 df\right\}^{-1}$$

$$= \frac{1}{8\pi^2 T} \frac{1}{(S_0/N_0)^2} \frac{1}{f_1^{2(p-n)}} \frac{3 + 2(n-p)}{f_2^{3+2(n-p)} - f_1^{3+2(n-p)}},$$

$$(p - n) \neq 1.5, \quad \text{SNR} \ll 1 \tag{24}$$

$$\therefore \sigma_D = \left(\frac{1}{8\pi^2 T}\right)^{1/2} \frac{1}{S_0/N_0} \frac{1}{f_1^{p-n}} \frac{[3 + 2(n-p)]^{1/2}}{[f_2^{3+2(n-p)} - f_1^{3+2(n-p)}]^{1/2}},$$

$$(p - n) \neq 1.5, \quad \text{SNR} \ll 1 \tag{25}$$

where

$$S_0/N_0 = \text{SNR} \frac{1-p}{1-n} (f_1^{n-p}) \frac{f_2^{1-n} - f_1^{1-n}}{f_2^{1-p} - f_1^{1-p}}, \qquad p \neq 1, \quad n \neq 1$$

and

$$\sigma_D = \left(\frac{1}{8\pi^2 T}\right)^{1/2} \frac{1}{S_0/N_0} \frac{1}{f_1^3 \ln f_2/f_1},$$

$$\text{only for} \quad (p - n) = 1.5. \tag{26}$$

Notice that when n and p equal zero, (24) yields to (13), as expected. In other words, when n and p are equal to zero, which implies that the signal and noise autospectra are constants over the band, the variance of the time delay estimated is expected to be equal to (13), and it is.

Also observe that when n and p are equal, signal and noise autospectra are falling off at the same rate, and the variance σ_D^2 is the same as that in (13). So, the variance does not change when signal and noise autospectra fall off at the same rate. On the other hand, if the noise autospectrum falls off faster than the signal autospectrum, then the variance σ_D^2 decreases and the reverse is true if the signal autospectrum falls off faster than the noise autospectrum.

A comment may be in order. In derivations of (8) and (15) a shaping filter was utilized before correlation to obtain the Cramér-Rao lower bound of the variance of the time delay estimate; but in the case of (18), the shaping filter was not utilized to obtain the variance of the time delay estimate. However, if the shaping filter is utilized to obtain the minimun variance, then (18) will yield the same result shown in (13) or (20) [8].

III. Analysis Results

An investigation was made of the effects of SNR, bandwidth, observation time, and center frequency on the standard deviation of the time delay error about mean time delay, which is assumed to be unbiased. In Fig. 1, the standard deviation σ_D is plotted against the SNR in the range of -10 to -20 dB. The signal and noise autospectra are constants over the band $W = 4000$ Hz. The center frequency $f_0 = 4000$ Hz ($f_1 = 2000$ Hz and $f_2 = 6000$ Hz), and the observation time 60 and 120 s. It is seen from Fig. 1 that σ_D varies inversely with the SNR in the passive system, whereas σ_D varies inversely to the square root of the SNR in the active system. By doubling the integration time from 60 to 120 s, σ_D decreases, which is an improvement of 1.5 dB (10 log 2) in both the active and passive systems. It is found in Fig. 1 that σ_D is higher by a factor of $1/(\text{SNR})^{1/2}$ in the passive system compared with the result in the active system, as expected.

In order to see the effect of change in center frequency f_0 on standard deviation σ_D, the latter is plotted in Fig. 2 as a function of SNR with the center frequency as a parameter. Fig. 2 shows that σ_D decreases with increasing center frequency if W and T are held constant. In other words, if the constant bandwidth W is moved along the frequency line with increasing frequency, keeping other parameters such as SNR, T, and W constant, then σ_D will decrease. Most probably this is a result of an increase in oscillations due to increasing frequency. Therefore, it indicates that one can measure the position of the correlation peak more accurately if the center frequency is increased; i.e., the uncertainty in peak position is decreased with increasing frequency. Fig. 3 shows σ_D versus f_0 for SNR's of -10, -15, and -20 dB, with $W = 400$ Hz and $T = 120$ s. By doubling the center frequency, an improvement of about 3 dB (10 log 2) in σ_D is possible.

Fig. 4 shows the effect of change in bandwidth W on the standard deviation of the time delay estimate σ_D, which decreases with increasing bandwidth for constant T, f_0, and SNR.

So far, a result of the analysis of the variance of the time delay estimate when the signal and noise autospectra are constants over the band has been presented. Now, a result of the analysis for when the signal and noise autospectra fall off and how it affects the variance or standard deviation of the time delay estimate is shown.

Fig. 5 illustrates the standard deviation of the time delay estimate as a function of SNR in the range of -10 to -20 dB.

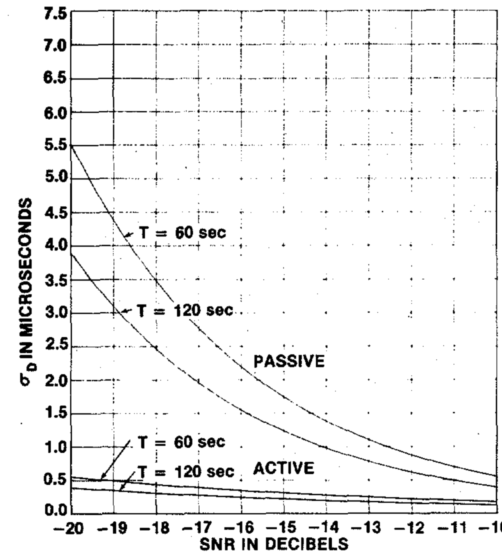


Fig. 1. Standard deviation of time delay estimate as a function of SNR for active and passive systems with different integration times. $W = 4000$ Hz and $f_0 = 4000$ Hz.

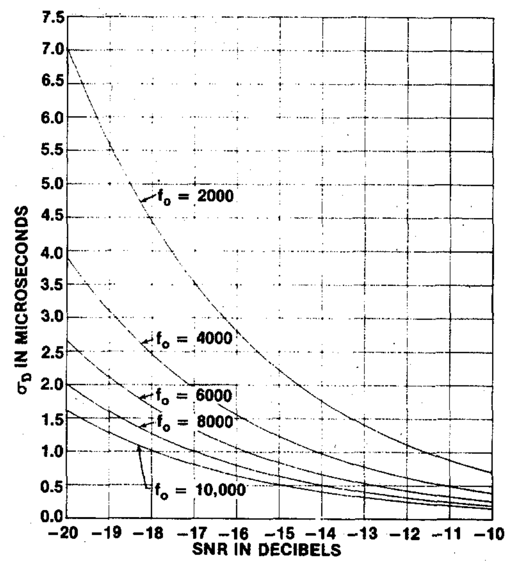


Fig. 2. Standard deviation of time delay estimate as a function of SNR with center frequency f_0 as a parameter. $W = 4000$ Hz and $T = 120$ s.

The integration time is 120 s and the bandwidth is 4000 Hz ($f_1 = 2000$ Hz and $f_2 = 6000$ Hz). Also, the figure shows that the spectral falloff does not affect σ_D when n and p are equal. In other words, as long as the autospectral falloff of signal and noise are equal, σ_D does not change compared with the standard deviation of the time delay estimate when signal and noise autospectra are constants over the band.

Fig. 6 shows the effect of signal autospectral falloff with frequency when the noise autospectrum is constant over the band. It is evident from the figure that σ_D increases with increasing signal autospectral falloff, as expected.

Fig. 7 shows the effect of noise autospectrum falloff, with the frequency keeping the signal autospectrum constant over the band on the standard deviation of the time delay estimate. It is seen from the figure that σ_D decreases with increasing falloff of the noise autospectrum, as expected.

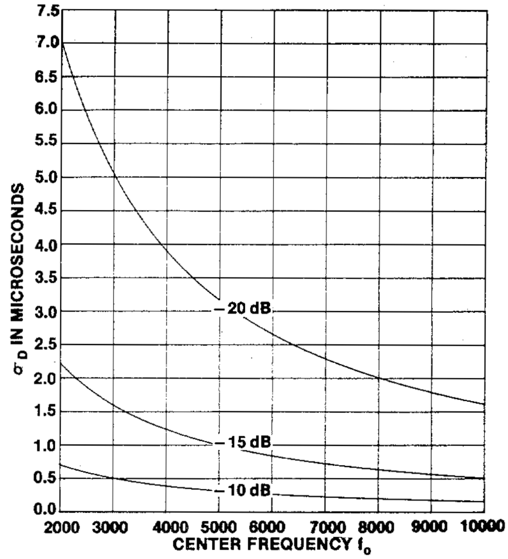


Fig. 3. Standard deviation of time delay estimate as a function of center frequency f_0 at different SNR's. T = 120 s and W = 4000 Hz.

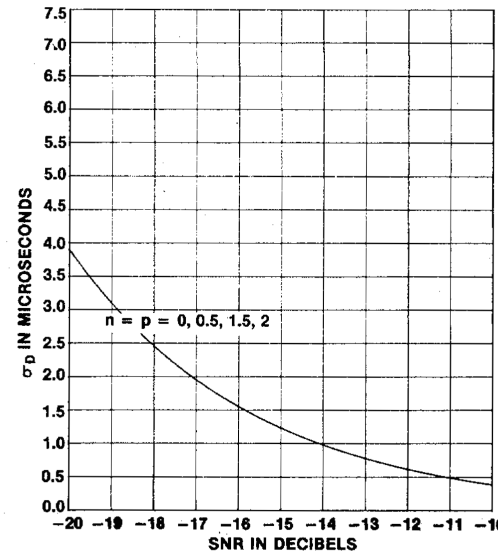


Fig. 5. Standard deviation of time delay estimate as a function of SNR. Signal and noise autospectral fall off at same rate. $n = p$, T = 120 s, W = 4000 Hz, and f_0 = 4000 Hz.

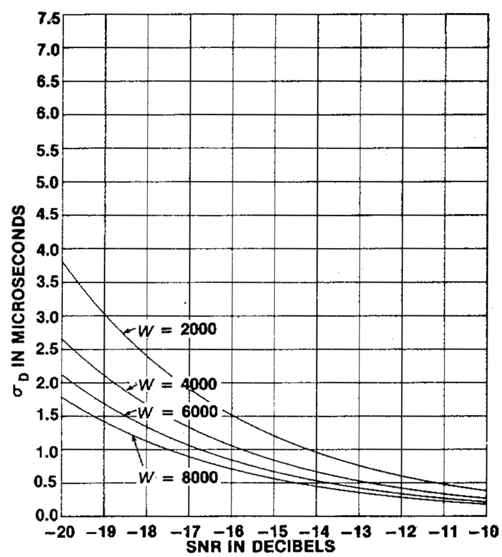


Fig. 4. Standard deviation of time delay estimate as a function of SNR with bandwidth W as a parameter. T = 120 s and f_0 = 6000 Hz.

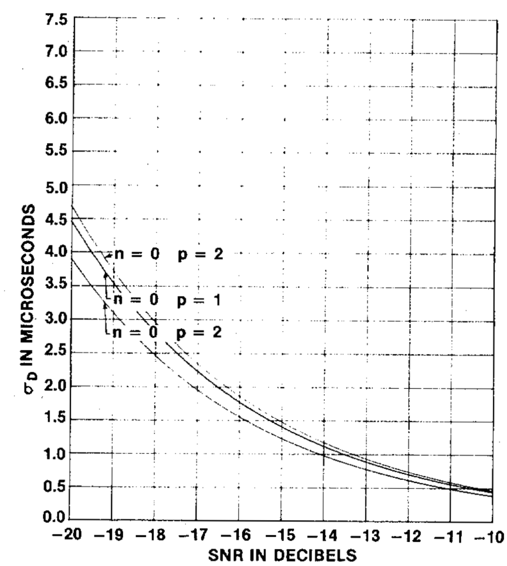


Fig. 6. Standard deviation of time delay estimate as a function of SNR. Noise autospectrum is constant over the band. Signal autospectrum falls off at a rate of 0, −10, and −20 dB/decade over the band. T = 120 s, f_0 = 4000 Hz, and W = 4000 Hz.

IV. Conclusion and Summary

An attempt has been made here to calculate the Cramér-Rao lower bound of variance for the time delay estimate about the time delay in active and passive systems as used in target localization. The derivations, obtained in various forms, have been analyzed and are presented here: standard deviation of time delay estimate in terms of commonly understood terms such as SNR, bandwidth, observation time, and center frequency. The noteworthy results for the standard deviation of the time delay estimate σ_D in the neighborhood of true time delay in the case of low SNR are:

- The standard deviation σ_D varies inversely to the square root of the SNR in the case of an active system, whereas σ_D varies inversely to the SNR in the case of the passive system provided observation time, bandwidth, and center frequency remain constant.
- σ_D varies inversely to the center frequency for constant W, T, SNR, and $W^2/12f_0^2 \ll 1$.
- σ_D varies inversely to the square root of bandwidth for constants SNR, f_0, T, and $W^2/12f_0^2 \ll 1$.
- σ_D remains constant so long as the signal and noise autospectra fall off at the same rate or are constant over the band.
- σ_D increases if the signal autospectrum falls off faster than the noise autospectrum.
- σ_D decreases if the noise autospectrum falls off faster than the signal autospectrum.

In the case of high SNR (SNR $\gg$ 1), the standard deviation of the time delay estimate varies inversely to the square root of the SNR, whereas σ_D varies inversely to the SNR in case of a low SNR. The effects of other parameters such as

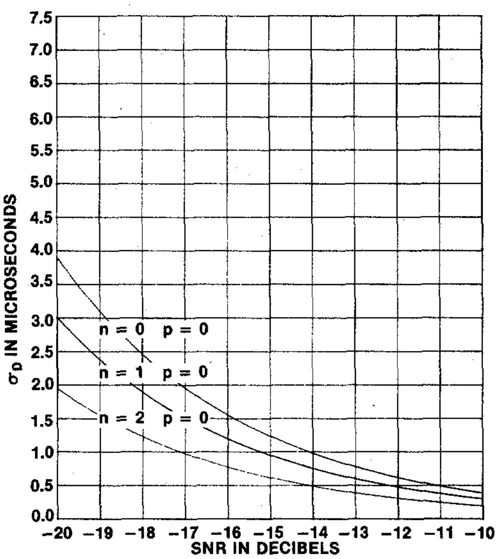


Fig. 7. Standard deviation of time delay estimate as a function of SNR. Signal autospectrum is constant over the band. Noise autospectrum falls off at a rate of 0, −10, and −20 dB/decade. $T = 120$ s, $f_0 = 4000$ Hz, and $W = 4000$ Hz.

T and W remain the same for σ_D in both low and high SNR's.

Future research will include validating the minimum variance of the time delay estimate using simulation or experimental results. Also an investigation of how useful is the Cramér-Rao bound of variance in predicting the performance of bearing and range estimation at low, high, and "in-between" values of SNR is deemed advisable.

Acknowledgment

The author wishes to thank Dr. G. C. Carter and Dr. A. H. Nuttall for their helpful discussions and criticism.

References

[1] C. W. Helstrom, *Statistical Theory of Signal Detection*. New York: Pergamon, 1968, pp. 274-319.

[2] P. M. Woodward, *Probability and Information Theory, With Applications to Radar*. New York: Pergamon, 1953, pp. 81-99.

[3] D. A. Wahlen, *Detection of Signals in Noise*. New York: Academic, 1971, pp. 337-339.

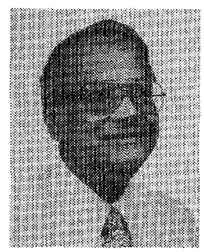
[4] C. H. Knapp and G. C. Carter, "The generalized correlation method for estimation of time delay," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-24, pp. 320-327, Aug. 1976.

[5] W. R. Hahn, "Optimum signal processing for passive sonar range and bearing estimation," *J. Acoust. Soc. Amer.*, vol. 58, pp. 201-207, July 1975.

[6] P. M. Schultheiss, "Locating a passive source with array measurements–A summary result," in *Proc. ICASSP '79*, Washington, DC, Apr. 24, 1979, pp. 967-970.

[7] G. Tomlinson and J. Sorokowsky, "Accuracy prediction report, Raploc error model," IBM, Manassas, VA, June 30, 1977, pp. B1-B7.

[8] V. H. MacDonald and P. M. Schultheiss, "Optimum passive bearing estimation in a spatially incoherent noise environment," *J. Acoust. Soc. Amer.*, vol. 46, pp. 37-43, July 1969.



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