

Communications

An Exact Method of Integration for Vector Potentials of Thin Dipole Antennas

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Abstract—A general method for exact integration of vector potentials of thin dipole antennas characterized by various current distributions is developed. Such solutions are shown to be independent of the usual far-field restrictions involving dipole length, observation point distance, and wavelength. Their convergence is rapid in the induction and near-field regions.

I. INTRODUCTION

Radiation or far-field approximate expressions for the vector potentials of thin linear dipole antennas with various current distributions are well known [1]. Less familiar are expressions that are valid in the induction and near fields of such antennas. One such expression can be found exactly assuming a sinusoidal current distribution [2], but a general method for deriving near-field vector potentials for arbitrary current distributions does not appear available. Since many applications require knowledge of the near fields, it is necessary to address this problem in a general way.

We show that infinite series solutions can be derived by performing the integrals for the vector potentials exactly. The method is developed in detail in the next section using a uniform current dipole for simplicity. Later it is indicated that the method can be used for a number of "simple" function current distributions [3]. By performing several variable transformations on the original integral, the integrand of the vector potential can be represented as an infinite series of Bessel functions, whose arguments do not depend on the variable of integration. At this point, the integration can be performed and the new variables transformed back into the original spherical coordinates. This Bessel function form of the potential is noteworthy in that it satisfies our intuition concerning the fields of a linear conductor possessing azimuthal symmetry. Such solutions are completely general and independent of the usual restrictions involving the wavelength, observation point distance, and dipole length. Their convergence is very rapid in the induction- and near-field regions.

II. GENERAL METHOD

The vector potential of a uniform current dipole can be written in the form

$$A_z = \frac{1}{4\pi} \int_{-L}^L \frac{e^{-ikR}}{R} I(z') dz', \quad (1)$$

where $k = 2\pi/\lambda$, $I(z') = I_0$, and

$$R = (r^2 - 2rz' \cos \theta + z'^2)^{1/2}. \quad (2)$$

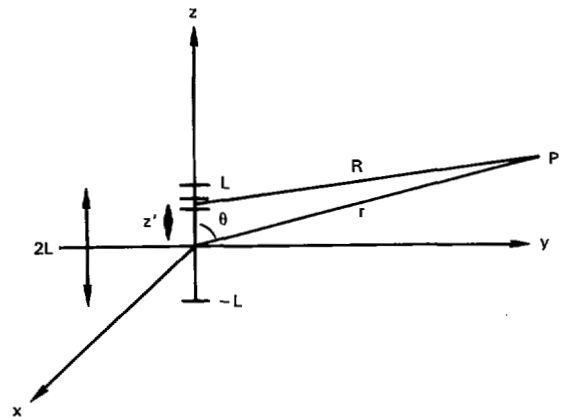


Fig. 1. Coordinate system of a small dipole.

Using the usual far-field approximation, $R \sim r$ (see Fig. 1), the vector potential becomes

$$A_z = \frac{I_0}{4\pi} \frac{e^{-ikr}}{r} (2L), \quad (3)$$

where $2L$ is the total dipole length and A_z obeys the restrictions $2L \ll r$, $2L \ll \lambda$. However, if (2) is not approximated, then

$$dz' = \frac{RdR}{\pm(R^2 - r^2 \sin^2 \theta)^{1/2}}. \quad (4)$$

Substitution of (4) into (1) gives

$$A_z = \frac{I_0}{4\pi} \int \frac{e^{-ikR}}{\pm(R^2 - a^2)^{1/2}} dR \quad (5)$$

with $a = r \sin \theta$ and $0 \leq \theta \leq \pi$. A further variable change,

$$R = a \cosh \alpha, \quad (6)$$

results in

$$A_z = \pm \frac{I_0}{4\pi} \int e^{-ika \cosh \alpha} d\alpha. \quad (7)$$

This integrand can be represented in terms of an infinite series of Bessel functions as

$$e^{-ika \cosh \alpha} = J_0(ka) + 2 \sum_{n=1}^{\infty} (-i)^n J_n(ka) \cosh n\alpha. \quad (8)$$

Upon integration with respect to α , (7) becomes

$$A_z = \pm \frac{I_0}{4\pi} \left[J_0(ka) \alpha + 2 \sum_{n=1}^{\infty} \frac{(-i)^n}{n} J_n(ka) \sinh n\alpha \right]. \quad (9)$$

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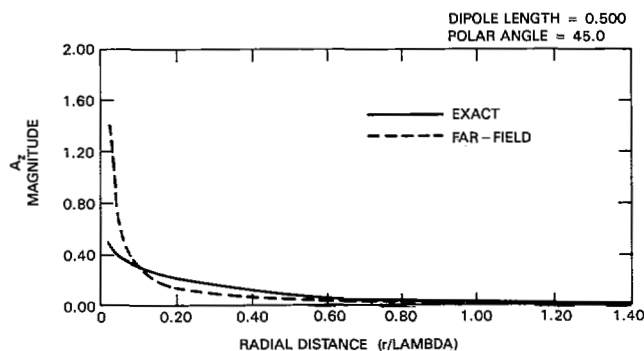


Fig. 4. Comparison of exact and far-field vector potentials; uniform current distribution.

complicated current distributions. Sinusoidal, exponential, and polynomial distributions have been investigated and can be integrated using the general method [6]. Vector potential solutions for several common current distributions are given in [6].

IV. CONCLUSION

A general method has been developed for exact integration of vector potentials for thin dipole antennas characterized by various current distributions. An infinite series solution for the uniform current distribution has been determined. Extrapolation from this case indicates that the method can be used for a number of current distributions to produce solutions that converge well in the induction- and near-field regions of thin dipole antennas.

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Statistical Processing Method of Sidelobe Peaks for Earth-Station Antennas

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Abstract—For coordination studies and for the assessment of mutual interference between radiocommunication-satellite systems and between earth stations and radio-relay stations sharing the same frequency band,

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the method which represents off-axis radiation characteristics of the earth-station antenna is desirable. For this purpose, International Radio Consultative Committee (CCIR) Report 391-4 describes a statistical processing method of sidelobe peaks which was adopted at the XVth Plenary Assembly in 1982. This statistical processing method is based on the slope of reference radiation pattern, while the old one, which has been used, is based on the absolute peak value. Therefore, the results of the statistical evaluation using the current (new) method may differ from that using the old method. According to the measured data on a Cassegrain antenna of 13 m in diameter at about 12 GHz, it is shown that the worst 10 percent value of sidelobe peaks of the new processing method, which is the level exceeded by 10 percent of the peaks, is statistically about 0.8 dB lower than that of the old method.

I. INTRODUCTION

For coordination distance calculations and the assessment of mutual interference between earth stations in the fixed satellite service and between earth stations and terrestrial radio-relay stations, the reference radiation pattern for the earth-station antenna is proposed in International Radio Consultative Committee (CCIR) Recommendation 465-1 [1] and Report 391-4 [2]. When the antenna diameter/wavelength ratio D/λ exceeds 100, the reference radiation pattern is represented as follows:

$$G = 32 - 25 \log \phi \quad (\text{dBi}), \quad \text{for } 1^\circ < \phi \leq 48^\circ \\ = -10 \quad (\text{dBi}), \quad \text{for } 48^\circ < \phi \leq 180^\circ, \quad (1)$$

where G is the gain relative to an isotropic antenna and ϕ is the angle between the axis of the main beam and the direction in question. This reference radiation pattern can be expected to represent the level exceeded by 10 percent of the sidelobe peaks of actual large earth-station antennas [2].

Annex II of CCIR Report 391-4 [2] describes a statistical processing method for evaluating sidelobe peaks in order to represent radiation characteristics of an individual earth-station antenna. In this method, the statistical processing of sidelobe peaks is carried out by taking into account the slope of $-25 \log \phi$ of the reference pattern, while the old one, which has been used, is based only on the absolute peak amplitude. Therefore, results of the statistical evaluation using the current (new) method may differ from that using the old method. Some of submitted statistical data for actual antennas have been evaluated by the new method, but some done by the old one. In this communication, according to measured data on a 13 m Cassegrain antenna which is designed to cover the 500 MHz bandwidth at 14 GHz and the 250 MHz bandwidth at 12 GHz and located at Kashima in Japan, the difference of the evaluation is estimated from the statistical point of view.

II. STATISTICAL PROCESSING METHOD OF ANTENNA SIDELobe PEAKS

A. Statistical Processing Methods

In Figs. 1(a) and 1(b), statistical expressions of the sidelobe peak distribution are shown by using the new and the old statistical processing methods. In the new method, the difference between each sidelobe peak and the level of the reference radiation pattern at the off-axis angle where the peak exists is calculated, and each sidelobe peak is evaluated from this difference. The statistical data such as maximum value, worst 10 percent (the level exceeded by 10 percent of the peaks), median value, best 10 percent and minimum value are shown in the middle of sample angular window on a logarithmic scale $\log \phi$. In other words, each sidelobe peak is evaluated from the peak