

# Critical Distance Method for Predicting the Tail Part of the Threshold Voltage Distribution

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**Abstract**—As a source of the tail of the statistical distribution of MOSFET threshold voltage distributions, we identified how the potential chain extending from one width edge of the channel to the other width edge creates an abnormally large  $V_T$  shift, using 3-D TCAD simulations. The critical distance between two adjacent fixed charges in a potential chain that is required to block the channel current has been identified, and its dependence on parameters has been verified. Using the critical distance, the probability of the occurrence of the worst case scenario, in which source to drain current is blocked by the potential chain, for 1 billion transistors has been predicted through percolation method.

**Keywords**—  $V_T$  distribution, statistical tail, abnormal  $V_T$ , potential chain, critical distance, percolation

## I. INTRODUCTION

Because of shrinking of device size down to 10 nm, the statistical variations due to the traps and dopants becomes more severe. Moreover, it is also problematic that there are an abnormally large threshold voltage ( $V_T$ ) shift that may belong to the 6-sigma tail of the statistical distribution [2]-[5]. It has been reported that chained traps and dopants forming a cluster of potential peaks on the channel surface results in the tail distribution. Potential peaks are formed by various sources of statistical variability such as random traps in the oxide or on the Si-SiO<sub>2</sub> interface, random dopants in the near-interface substrate, metal gate granularity, and line-edge roughness [6]. When the sources are positioned to have a “critical distance” from each other potential peaks due to each source, they form a chained potential barrier. If this barrier extends from one width edge of the channel to another, it blocks the channel current from source to drain; therefore, the threshold voltage increases significantly.

The atomistic simulation can predict the worst case scenario, however, it is intractable to calculate over 1 billion (1Gbits) transistors in order to predict the 6-sigma tail. In this paper, we propose the “critical distance method” for predicting the tail part of the  $V_T$  distribution efficiently, which does not require 1 billion atomistic simulations. The method consist of three steps; 1) determine the critical distance to form the potential chain. 2) calculate the probability of the worst case in which the potential chain is present, considering the critical distance. 3) categorize the devices that has a potential chain as

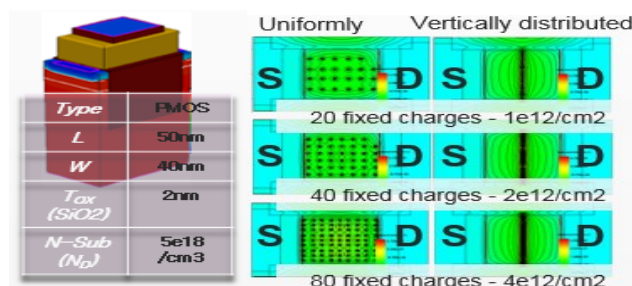


Fig. 1. Fixed charges are strategically distributed in the Si/SiO<sub>2</sub> interface of the pMOSFETs. For the same number of fixed charges, vertically distributed cases are compared to the uniformly distributed case to identify large  $V_T$  shifts of the statistical tail of cells.

the 6-sigma tail and conduct atomistic simulations only for those selected devices.

At first, we verified how the  $V_T$  varies significantly when the current-blocking-potential chain is formed, using a simulation in which various potential fluctuation sources are substituted for fixed charges. From the trend of the  $V_T$  variation, we could identify that there is the critical distance that gives the worst case. For the step 1), we describe how the critical method can be determined considering device parameters such as the gate bias, doping, and oxide thickness. It is investigated via the channel surface potential and the carrier density fluctuation. For the step 2), the distance between sources of potential fluctuation becomes a criterion for classifying the worst case (WC), in which a potential chain extends from one width edge to another. Using the critical distance, the probability of the occurrence of the WC was calculated using percolation theory. This is the “critical distance method” we suggest for predicting the statistical tail. Using this method, we identified the probability of a percolation for 1 billion transistors on various size of devices.

## II. ABNORMAL $V_T$ SHIFT IN MOSFET: WORST CASE

The abnormality of the  $V_T$  shift due to the distribution of traps and dopants in the WC has been identified using an atomistic 3-D device simulation with a pMOSFET of  $L/W/T_{ox} = 50 \text{ nm}/40 \text{ nm}/2 \text{ nm}$ , under conditions of continuous channel doping of  $5 \times 10^{18} \text{ cm}^{-3}$  and varying the density of the fixed charges ranging from  $1 \times 10^{11}$  to  $1 \times 10^{13} \text{ cm}^{-2}$ , equal to a number of fixed charges from 0 to 80. The density of fixed charges has

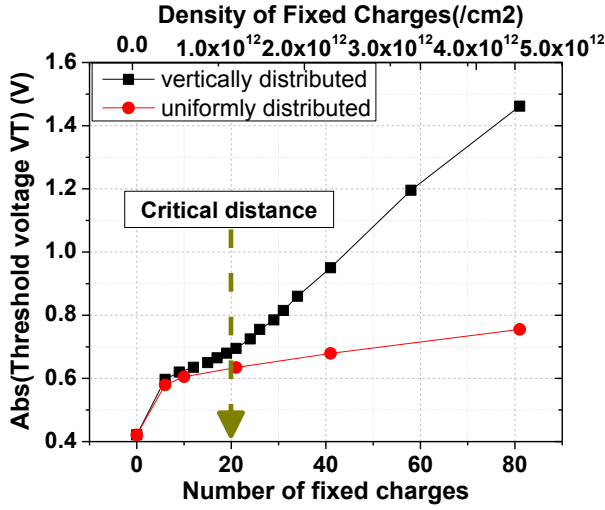


Fig. 2.  $V_T$  value versus the number of fixed charges. This figure shows the difference between the two cases. The two curves begin to deviate from a point, at which the fixed charges are at a critical distance in a vertical line.

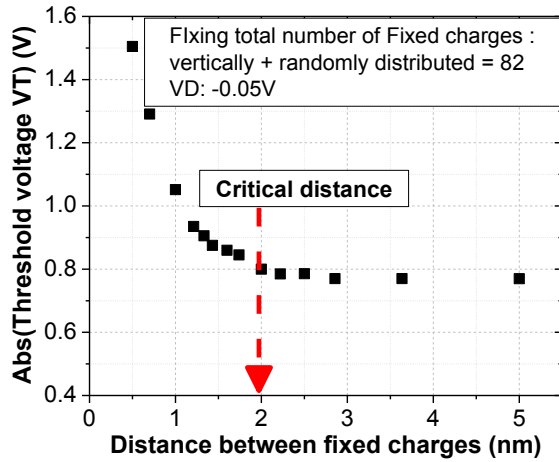


Fig. 3. The total number of fixed charges is fixed at 82; some of them are positioned in a vertical line and the others are distributed randomly. Increasing the number in a vertical line indicates the beginning of the  $V_T$  shift; thereafter, fixed charges come closer to each other than the critical distance.

been determined as an alternative to various potential fluctuation sources, not only for oxide interface traps but also dopants in the near-interface substrate.  $V_T$  values of devices with fixed charges that are uniformly distributed in the Si/SiO<sub>2</sub> interface have been compared to that with fixed charges distributed in a straight line that extends from one width edge to another (Fig. 1).

The difference between the fixed charge concentration dependent  $V_T$  variations of uniformly distributed cases and vertically distributed (worst) cases has a critical point, at which the two curves start to deviate (Fig. 2). In the uniformly distributed case,  $V_T$  increases linearly, depending on the increase in the number of fixed charges. In the vertically distributed case, however, the slope of the  $V_T$  variation versus the number of fixed charges becomes steeper after the number

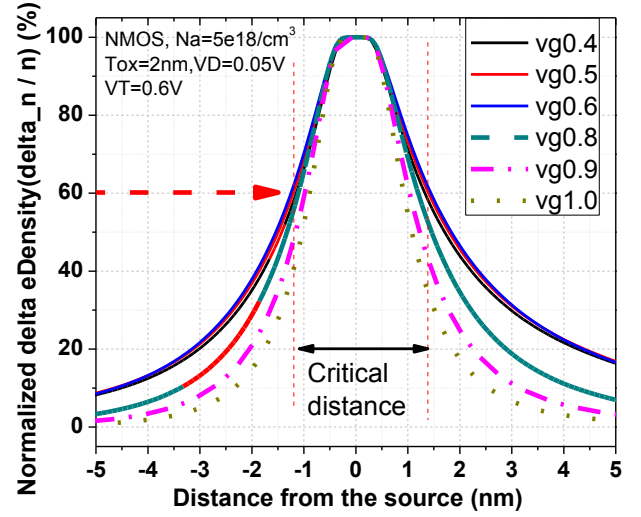


Fig. 4. Gate bias dependence of the critical distance. The diameter of the 60% area from the charge fixed at the center position shows the bias independence in the sub- $V_T$  regime; however, it decreases as the bias increases in the above- $V_T$  regime because of charge screening.

of fixed charges exceeds a critical value. A current-blocking-potential chain is assumed to be formed when the two curves begin to deviate; for this reason, in this case, the distance between fixed charges in the straight line is expected to be the critical distance.

In Fig. 3, the effect of the distance between the fixed charges in a potential chain is investigated for a total number of fixed charges of 82. Some of the 82 fixed charges are distributed in a straight line, extending vertically in the direction of the source-drain current from one width edge to another; the remaining fixed charges are distributed randomly over the entire channel area of the Si-SiO<sub>2</sub> interface. By increasing the ratio of the number of fixed charges in a vertical line, the  $V_T$  variation due to the distance between fixed charges in a vertical line exhibits an exponential increase after the distance between every fixed charge becomes smaller than the critical distance, 2nm, as shown in Fig. 2.

### III. CHARACTERISTICS OF THE CRITICAL DISTANCE

Depending on the distances between potential peaks due to fixed charges, the channel carrier density at the midpoint decreases when the two fixed charges get closer to each other; therefore, the critical distance can be defined as the distance from a fixed charge at which the normalized carrier density,  $\Delta n/n$  (which is the ratio of changed carrier density to carrier density without any fixed charge at the same position), reaches a certain ratio. This critical distance can change depending on various conditions such as the operating voltage, the structure of the device, and the manufacturing process. The parameter dependence of the critical distance has been examined using observations of variation of the  $\Delta n/n$  value in nMOSFETs. Similar to pMOSFETs, the device size  $L/W/T_{ox} = 50 \text{ nm}/40 \text{ nm}/2 \text{ nm}$ , and the continuous channel doping concentration is  $5 \times 10^{18} \text{ cm}^{-3}$ . Variation of  $\Delta n/n$  around a negative fixed charge that is positioned at the center of the channel area has been investigated.

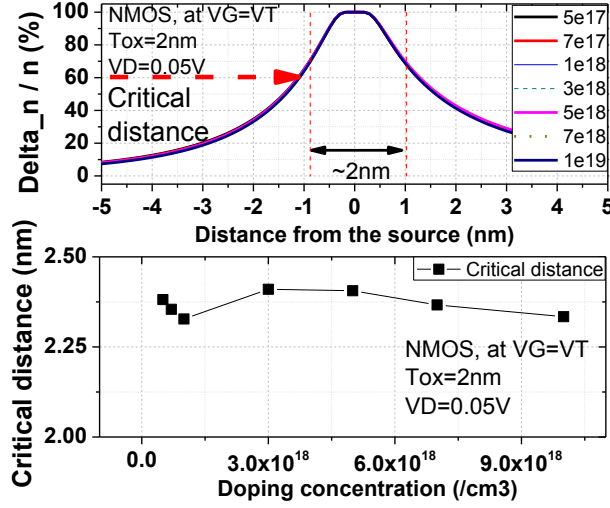


Fig. 5. Doping concentration dependence of the critical distance. The critical distance is almost independent of the doping concentration.

#### A. Gate Bias Dependence

In Fig. 4, beginning from the maximum of 100%,  $\Delta n/n$  decreases with increasing distance from the fixed charge at the center position. When the distance reaches 1 nm, it means the distance between two fixed charges becomes 2 nm, which is same as the critical distance we found in the simulation described above, and  $\Delta n/n$  reaches roughly 60%. Therefore, the variation of the diameter of the circle of  $\Delta n/n=60\%$ , based on the change in gate voltage from sub- $V_T$  to above- $V_T$ , has been investigated. In the sub- $V_T$  regime, the critical distance is unchanged by bias variation. In the above- $V_T$  regime, however, the critical distance decreases due to the increase in the gate voltage. Screening of the fixed charges by channel electrons, which are formed in the above- $V_T$  condition, causes the bias dependence of the critical distance in the above- $V_T$  regime (in contrast to the bias independence in the sub- $V_T$  regime).

#### B. Doping Concentration Dependence

Fig. 5 shows that the critical distance is not influenced by the doping concentration. Changing the doping concentration from  $5 \times 10^{17}$  cm<sup>-3</sup> to  $1 \times 10^{19}$  cm<sup>-3</sup> results in the depletion width varying from 10 nm to 20 nm. In spite of the doping variation, the depletion width is maintained at over 10 nm. Therefore, using image charge theory, we see that the effect of the fixed charge on the MOSFET channel surface is negligible, as a result of the long distance between the fixed charge and the equipotential surface, which is the bottom end of the depletion region.

#### C. Oxide Thickness Dependence

On the other hand, the critical distance decreases as the oxide thickness decreases, as shown in Fig. 6. For a fixed charge at the Si-SiO<sub>2</sub> interface, the distance between a fixed charge and the equipotential surface is almost the same as the oxide thickness, because of the very thin polysilicon gate depletion region. As the oxide is much thinner than the

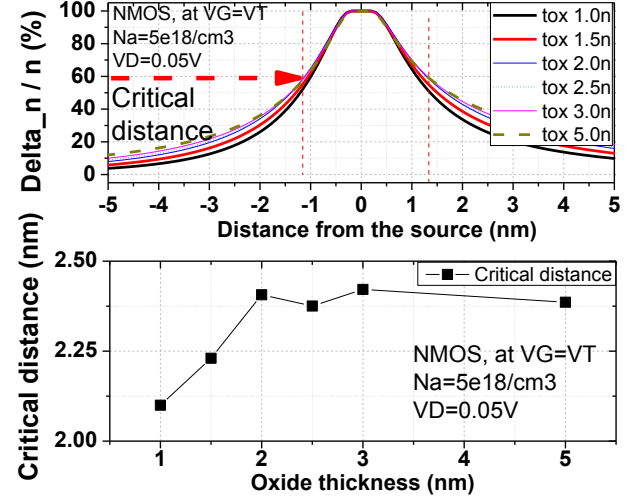


Fig. 6. From an image charge point of view, the equipotential surface close to the single charge position in the Si-SiO<sub>2</sub> interface causes the oxide thickness to have a stronger effect on the critical distance than the doping concentration.

substrate depletion region, the charge effect of one fixed charge on the channel surface is more sensitive to variation in the oxide thickness than variation in doping concentration.

### IV. PROBABILITY OF A PERCOLATION

Using the above parameter-dependent characteristics of the critical distance, we can fix the maximum approximate distance between fixed charges to form a current-blocking-potential chain in each device. The distance will depend on conditions such as bias, oxide thickness, and doping concentration. Then, we can simply calculate the probability of the occurrence of the worst case using percolation theory, i.e., by calculating the probability of a percolation in the channel surface divided by the size of the mesh cells.

Percolation theory, which is usually used for calculating the generation probability of oxide interface traps in a linked percolation path [7,8] extending from the Si-SiO<sub>2</sub> interface to the gate/oxide interface, has been applied to calculate the generation probability of a potential chain in the channel surface. As shown in Fig. 7, the percolation probability has been analyzed by considering not only the straight forward direction from one width edge to another, but also the left and right diagonals from the straight forward direction. Additionally, the jumped linkage between percolation meshes in the straight forward direction has been considered with fine meshing of the channel surface, where the mesh size is half the critical distance, 1 nm.

The probability that a fixed charge exist in a mesh cell is obtained by multiplying the area of a mesh by the density of the fixed charge in the channel's surface. The probability of a percolation formed by  $n$  fixed charges is, therefore, the  $n^{\text{th}}$  power of the probability of one mesh cell having a fixed charge. For percolation to form, at least, a minimum number of fixed charges must exist in the channel surface. If fixed charges in the channel are insufficient, the probability of percolation generation becomes zero for that concentration of fixed charge

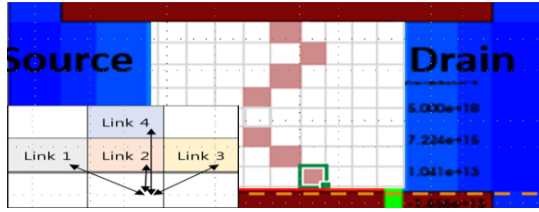


Fig. 7. Current blocking potential-chained percolation considering 4 linkages between nearby meshes. Mesh size is half the critical distance.

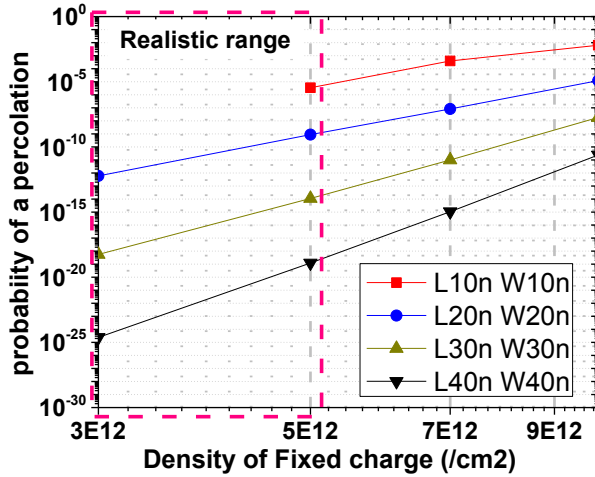


Fig. 8. Density of fixed charge dependent probability of a percolation varies according to the scaling down of the device size.  $3\text{e}12$  to  $5\text{e}12$   $\text{cm}^{-2}$  of fixed charge is realistic range, considering dopants and oxide traps. For  $3\text{e}12$   $\text{cm}^{-2}$ , L10n\*W10n, fixed charges are less than the minimum number to form a percolation, the probability of a percolation become '0'.

and channel size. By considering four linkages between nearby fixed charges, all possible percolations are counted through permutations and combinations. For example, calculating the probability of a percolation in  $40\text{ nm (W)} \times 50\text{ nm (L)}$  device with  $1\text{-nm}$  percolation mesh is the same as the problem finding all distributions of 4 colors of 40 marbles placed in a straight line (ignoring the order of same-colored marbles).

The variation on the probability of the percolation from one width edge to another due to the density of the fixed charge is calculated for various sizes of devices (Fig. 8). For a  $40\text{ nm} \times 40\text{ nm}$  and  $30 \times 30\text{ nm}$  device size, the probability of worst case is negligible (less than  $10^{-10}$ ) even under the highly stressed condition. As the device size is smaller than  $20\text{ nm} \times 20\text{ nm}$ , there are chance for the worst case among 1 trillion devices (probability becomes more than  $10^{-9}$  as shown in Fig. 8). The probability for a  $10\text{ nm} \times 10\text{ nm}$  device size with a fixed-charge density  $5 \times 10^{12}$   $\text{cm}^{-2}$  go as far as  $3.44 \times 10^{-6}$ , which means 3.44 transistors out of one million transistors have a percolation.

In this way, the number of devices which belong to the 6-sigma tail can be predicted.

## V. CONCLUSION

The critical distance method has been proposed to predict the probability in the tail of the statistical distribution of the threshold voltage. For the statistical tail, a potential chain, which blocks the source-drain current, creates a large, significant  $V_T$  shift, when the distance between fixed charges becomes the critical distance. The device parameter dependence of the critical distance has been investigated. Using the critical distance, the probability of the generation of a potential chain has been calculated using percolation theory. Using our method, the tail of the distribution for a 1-billion-transistor chip can be predicted with reasonable computational resources.

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