

On Modified EMD: Selective Extrema Analysis

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Abstract—The Empirical Mode Decomposition (EMD) algorithm was introduced as the first step of the Hilbert-Huang Transform, proposed by Huang et al. (1998). EMD decomposes a signal into so-called Intrinsic Mode Functions (IMFs) in a systematic way. Since then, various versions of EMD have been developed, addressing weaknesses of the original EMD procedure and aiming to optimize the original algorithm in a number of ways. This paper proposes to use selective extrema analysis while generating IMFs with two goals. One is to reduce/control the number of IMFs a signal is decomposed into with a small decomposition error, and second is to make EMD insensitive to small variations in the analyzed signal. The proposed algorithm is applied to a gait signal and shown to consistently yield two IMFs, even in the presence of small disturbances.

Key words: HilbertHuang Transform (HHT), Empirical Mode Decomposition (EMD), Gait Analysis, Intrinsic Mode Functions (IMFs), Modified EMD

I. INTRODUCTION

The Fourier and wavelet transforms have been used for a long time as theoretical and mathematical tools to analyze the data in various domains. These techniques are well established and effective but are limited in following sense:

- Fourier analysis represents any signal as a sum of sinusoidal functions, thus the transform basis does not depend on the nature of the data. Because the frequency of each sinusoidal function must be time-independent, Fourier analysis is able to construct stationary data only. (Amplitude and frequency values of the resulting harmonic components are constant. That is, their values are constant over the entire initial signal. This means that if the nature of a given signal changes over an interval under consideration, such changes will not be reflected in the transform results. The results obtained in this case will only reflect a certain averaged state of the process as this transform is based on the assumption of stationarity of the initial data).
- A wavelet transform solves the problem associated with a possible non-stationarity of an analyzed process since every component resulting from a wavelet transform has parameters that determine its scale and level over time. But again, like the Fourier transform, wavelet transforms perform decomposition in a fixed basis of functions. However, the basis can be set *a priori*.

Besides Fourier and wavelets, several other transforms have been proposed, each with its own benefits and limitations. See [1], [2] for details of the transforms.

The Hilbert-Huang transform (HHT) offers a solution to both of these limitations. It uses an adaptive basis determined by the initial data and allows us to deal with non-stationary processes. The HHT is thus a data driven empirical approach. It is a two-step time-frequency analysis method for non-stationary signals. The first step is to use the Empirical Mode Decomposition (EMD) algorithm [2], which decomposes a signal into intrinsic mode functions (IMFs). The second step is to apply the Hilbert transform to the IMFs obtained in the first step, thus yielding an instantaneous frequency spectrum, which is not discussed here. See [2] for details.

For a given signal $s(t)$, let mx and mn be the sets containing the locations of all the maxima and minima. For $s(t)$ to be a candidate IMF, it should satisfy two requirements:

- (a) *The size of mx and mn combined and number of zero-crossings (ZC) must either be equal or differ at most by one (allowed to have exactly one zero between successive extrema). If $|\cdot|$ represents the cardinality of a set, then this condition can be written mathematically as*
$$\text{abs}(|mx| + |mn| - |ZC|) \leq 1 \quad (1)$$
- (b) *At any point, the mean value of the upper envelope (defined by the local maxima) and the lower envelope (defined by the local minima) is zero (zero local mean).*

An IMF is a simple oscillatory mode, as a counterpart to a simple harmonic function, but it can have variable amplitude and frequency along the time axis. Each successive IMF contains lower frequency oscillations than the preceding one. The definition of an IMF guarantees a well-behaved Hilbert transform of the signal. The EMD method operating in the time domain is thus adaptive and highly efficient. Since the decomposition is based on the local characteristic time scale of the data, it is especially useful in applications dealing with non-stationary data and nonlinear processes.

The original EMD was proposed as an algorithm in [2]. It uses sifting to systematically decompose a time series into IMFs. Since then, many variations have been developed. One of the important steps of EMD is to identify the sets mx and mn . EMD is highly sensitive to these extrema. Minor changes in the number and location of extrema and amplitude of the signal give rise to big differences in the number and shape of the IMFs. In applications where these minor changes do not matter, it would be desirable to modify the algorithm to obtain an approximate decomposition with an arbitrarily small decomposition error. We propose an alternative algorithm to EMD by adding what we call *selective extrema analysis* to the original EMD algorithm.

Selective extrema analysis uses a subset of the actual extrema obtained from the analyzed signal at every step of sifting. The final decomposition has number of IMFs no larger than original decomposition and is different from what we would obtain if the whole set of extrema was used for sifting. The extrema adjustment makes the modified decomposition insensitive to small changes in the signal, in the presence of sampling errors or slowly varying weak noise, and thus helps reduce the number of IMFs a signal is decomposed into. Without extrema adjustment, EMD might decompose two very similar signals into a different number of components and thus different looking IMFs, making it difficult to compare corresponding IMFs (as they would have different scales) and draw conclusions. Extrema can be adjusted to let EMD yield the same number of modified IMFs for somewhat different signals. This makes comparison of IMFs possible and convenient. In cases where a higher frequency noise is superimposed on a lower frequency signal, one or more low-order IMFs can be thrown away to remove noise from the signal and obtain a partial signal reconstruction; but in cases where the noise is insignificant and has corrupted only a few signal values, it might be useful to not generate extra IMF(s) accounting for small changes in the signal, or obtain a partial reconstruction. Instead, our modified decomposition can be used for analysis.

The paper is organized as follows. Section II describes previously proposed modifications to EMD found in literature. The original EMD algorithm is described in Section III. Our proposed modification is then described in Section IV. In Section V the technique is applied to a set of gait cycles collected using a body-worn sensor. Conclusions are then drawn in Section VI.

II. LITERATURE REVIEW

Several research works have modified the original EMD procedure for various purposes. The authors of [3] propose end mirror extensions and employing least square polynomials to lower the errors due to end effects in EMD. The algorithm proposed in [1] uses the above mirror extension technique to lower the errors due to end effects by restricting the splines from varying abruptly at the ends of gaps encountered in real data. In [4], [5], the authors propose the concept of a weak IMF by relaxing the requirement that an IMF should be zero mean. The authors of [5] prove that any function with simple zeros and extrema can be decomposed into a sum of two or fewer weak IMFs. The decomposition is constructive and it is not obtained by applying EMD.

Filters are also used with EMD. Filters with long impulse responses might mix features far apart in the original signal so may not be desirable to use for signals with transients, while lowpass compact support filters are not smooth enough and thus create artificial oscillations in higher-order IMFs. The authors in [6] propose compactly supported, infinitely differentiable, adaptive local iterative filters to find adaptive and stable decompositions of nonlinear and non-stationary signals. The authors in [7] propose a moving average based approach for rigorous mathematical analysis of alternative EMD and a more stable analysis.

III. ORIGINAL EMD

Throughout the paper we use the following notation:

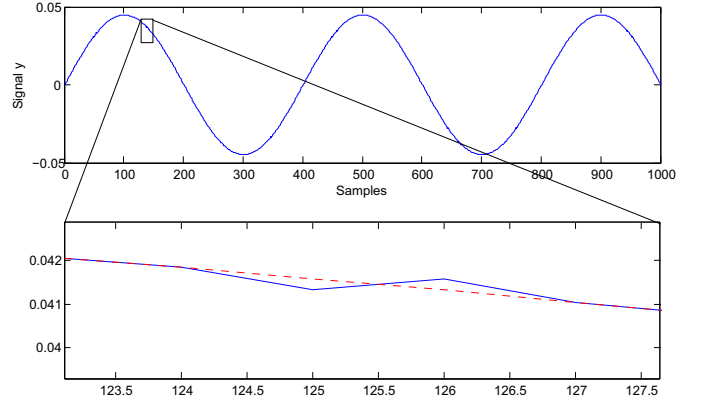


Fig. 1. Sinusoids with two sample values interchanged

- $s(t)$ is the analyzed signal sampled into N samples thus $\{t_1, t_N\}$ are the endpoints
- For simplicity of notation, we use $\{1, N\}$ instead of $\{t_1, t_N\}$ to denote endpoints
- mx and mn are the sets containing the locations of maxima and minima of $s(t)$, respectively
- the extrema are the locations of maxima and minima combined.
- IMF_i refers to the i th IMF.
- If the remainder of the sifting process is sufficiently small or a monotone, it is called residue (instead of IMF).

The procedure of extracting an IMF is called *sifting*, which starts with identifying the sets mx and mn and connecting all the local maxima as an upper envelope and local minima as a lower envelope using cubic splines. It is an iterative process, and the result of successive iterations depend on the results of an interpolation in the previous iteration. See [2] for details.

Identification of extrema is central to sifting. EMD is extremely sensitive to the extrema of the signal and even small changes in the location and number of extrema might make decomposition look different from one obtained in the absence of these changes/noise. The following example illustrates this point.

Consider a simple sinusoid

$$y(t) = \sin(\omega t). \quad (2)$$

The signal is essentially an IMF as it satisfies the definition of an IMF. Let us interchange two random samples of $y(t)$ to yield an extra hump in the signal, as shown in Fig. 1. Even though the difference between the signal amplitude at this pair is negligible, the original EMD generates a completely different decomposition (from the single IMF decomposition $s(t) = y(t)$), as shown in Fig. 2. With our proposed selective extrema analysis, modified EMD will ignore these two extrema and will not include them in generation of splines. Thus, unless the application demands to keep all the extrema to highlight this change of values, we can discard this extrema pair and still take $y(t)$ as the one (modified) IMF.

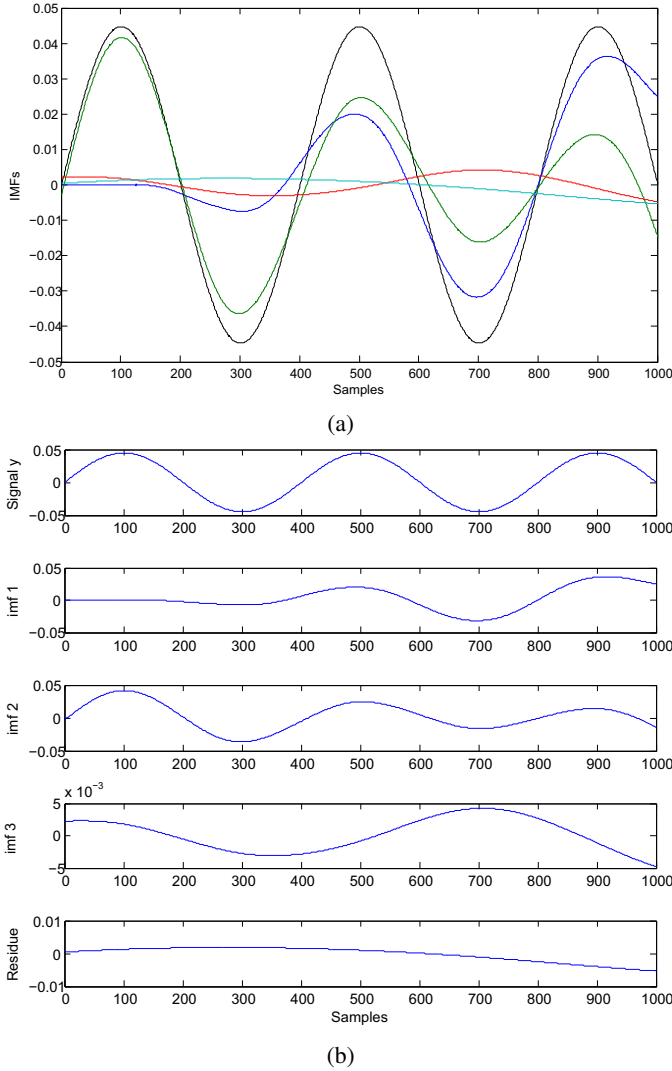


Fig. 2. IMFs of Fig. 1 without any extrema adjustment; (a) original signal and all IMFs and (b) individual IMFs identified

IV. PROPOSED SELECTIVE EXTREMA ANALYSIS

The algorithm proposed in [4] does not require IMFs to satisfy the local zero mean requirement, condition (b) above. We also relax the definition of IMFs in the sense that they might not satisfy (1) in condition (a). We propose to compare the heights of neighboring extrema using an exhaustive search. If the heights are close enough, i.e., if the difference between the heights of neighboring extrema is within a certain (preset or adaptive) threshold, we remove these extrema from the set of candidate extrema used for sifting. Thus the decomposition is different from the original EMD as IMFs might have more extrema than ZCs. Although the decomposition is changed, we gain two benefits. First, the modified algorithm lets EMD decompose signals into a fewer number of approximate basis functions. Second, in some applications, we need to compare the data before and after processing of some kind. If we get a different number of IMFs from pre and post processed datasets, it is difficult to compare the IMFs. However, if we are able to decompose both datasets into the same number of (approximate) IMFs, it is then possible to compare corresponding IMFs

and their spectra for changes and draw useful conclusions.

Initially, endpoints are neither a part of mx nor mn . Let us define a new set ϵ that contains mx, mn and the endpoints $\{1, N\}$. Since extrema adjustments might need to be done near endpoints, endpoints must be included in the set ϵ before extrema adjustment. As mx and mn are disjoint, ϵ can be written as

$$\epsilon = \{1, \text{sort}(mx \cup mn), N\} \quad (3)$$

Here sorting is done in ascending order of time indexes, and thus ϵ contains alternating locations of maxima and minima and is used for extrema adjustment.

We carry out adjustment in two steps, first working with groups of neighboring extrema triplets and then with groups of neighboring extrema pairs. We always throw away extrema in pairs to make sure elements of ϵ are alternating minima-maxima locations of the signal $s(t)$. Thus triplets look like $\{max, min, max\}$ or $\{min, max, min\}$ and, of course, pairs look like $\{max, min\}$ or $\{min, max\}$.

Step I. Pick the triplet which has closest vertical heights (i.e., three neighboring extrema at which the amplitude of the signal deviates the least). Double differentiation in MATLAB can help identify such a triplet, for example. Check if this deviation is within the threshold. If it is not, there is no need to analyze any further triplet, as, obviously in this case, the deviation in signal values at all the other triplets will be larger than the threshold used for extrema adjustment; move to Step II. But if the amplitude of the signal at this triplet deviates within the threshold, we can throw away one of the $\{min, max\}$ or $\{max, min\}$ pairs in the triplet, whichever has a smaller vertical distance. Note this is a relative comparison and hence, no threshold is needed here. Thus we replace ϵ with remaining extrema. We repeat Step I using this pruning process until all sets of triplets have been considered, exhaustively.

Step II. Using the same sequential exhaustive search of step I, but now with extrema pairs instead of triplets, we eventually remove all unnecessary extrema pairs.

It is possible that extrema adjustment occurs near the edges of the signal, so we include boundaries in ϵ . The algorithm works as follows near the boundaries. In Step I, if a candidate triplet is found for adjustment that includes any of the edges, it keeps the endpoint(s) and throws away the other pair instead. (This won't be a problem as the deviation at this triplet is already within the threshold). In Step II, if it finds an extrema near the boundary such that deviations of signal amplitude at this $\{extrema, endpoint\}$ pair is within the threshold, it discards that extrema (not the endpoint). If we throw away the endpoint(s) and instead keep the other maxima or minima near the edges, we generate an extraneous extrema pair when we replicate the signal for boundary adjustment. In the end, we can obtain modified maxima and minima mx_1 and mn_1 from the remaining elements in ϵ for sifting.

A. Setting the Threshold

Any reasonably small threshold works and often helps reduce the number of IMFs. The IMFs defined in (1) guarantee a well-behaved Hilbert transform. So, setting a higher threshold and over adjusting the extrema will not be useful, as it

might lead to a large decomposition error, yielding incorrect decomposition and meaningless spectra.

B. Mirror Extension for Boundary Condition

The mirror extension technique of [5], [8] is used with the EMD algorithm in order to properly extend interpolation to the edges of the signal. Mirror extension repeats the image of the signal to the left and right and transforms the sets mx and mn into extended sets. Doing mirror extension after extrema analysis is efficient because it avoids the need to perform extrema analysis on extended versions of sets mx and mn .

C. Stopping Criterion

Ideally, the sifting should be stopped when the result of sifting satisfies the definition of a candidate IMF. In practice, some stopping criterion should be used to avoid infinite loops. Any suitable stopping criterion can be used. For simulation purposes, we used a Cauchy type of convergence test, as proposed in [2]. Sifting should be stopped when the standard deviation of the result of consecutive sifting, σ , becomes smaller than a preset tolerance level, T (from (5.5) in [8]). If $\sigma < T$ is satisfied, the result of sifting should be declared an IMF. The sifting process is then repeated on the residue.

The stopping criterion is extremely important in the EMD procedure as it affects the number and the shape of the IMFs. A Cauchy type of convergence test is the most commonly used stopping criterion. We can also limit the maximum number of iterations allowed for sifting to obtain each IMF, or place a tolerance on the energy of the signal obtained after each iteration, or their combination [9].

D. Proposed Modified Algorithm

The new sifting process is as shown in Algorithm 1.

Algorithm 1

- Step 1.** Identify the sets mx and mn
 - Step 2.** Generate the set ϵ using (3)
 - Step 3.** Perform extrema analysis to throw some of the elements in ϵ and generate the modified sets mx_1 and mn_1
 - Step 4.** Carry out boundary adjustment on mx_1 and mn_1
 - Step 5.** Connect all the local maxima using as the upper envelope $U(t)$
 - Step 6.** Repeat Step 5 for the local minima to produce the lower envelope $L(t)$
 - Step 7.** Find their mean: $m(t) = 0.5(L(t) + U(t))$
 - Step 8.** Find the residual signal: $h(t) = s(t) - m(t)$
 - Step 9.** Repeat sifting if stopping criteria is not met
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V. APPLICATION TO GAIT CYCLES ANALYSIS

Gait patterns are both non-linear and non-stationary signals and they can be analyzed using empirical mode decomposition [10], [11]. A 24-cycle walk segment was collected using a body sensor node, TEMPO3.1, a wearable motion capture system developed at the University of Virginia [12]. This sensor captures both linear and angular acceleration. The signal used for analysis is the Z plane signal from the gyroscope, which measures rotation about the central axis of the ankle. For

showing the effectiveness of the selective extrema adjustment technique and its insensitivity to small changes in the signal, data was collected from a randomly chosen young healthy subject having a uniform walk. Characterization of the data using other tools, such as dynamic time warping (DTW) [13], Euclidean distance metric, and correlation between the cycles, is given in Fig. 3.

Since most of the cycles in the walk segment look similar to each other and are highly correlated, ideally we expect to get a similar EMD decomposition for all the cycles. We show that even with an arbitrary small threshold, the modified algorithm is able to decompose most of the cycles into an IMF and a residue, as compared to the original algorithm, which decomposes most of the cycles into two IMFs and a residue. The same threshold is used to decompose all the cycles. If, even with the extrema adjustment, a cycle is decomposed into more than one IMF (and possibly a residue), or if the IMFs look different from the IMFs of other cycles, we can look closely at the cycle and find abnormalities, if any. Thus extrema adjustment can be used to indicate the presence of some kind of abnormal behavior.

Fig. 4 illustrates the extrema removal process for an example gait cycle. Extrema that are close in value (in amplitude and time) are successively removed, as shown. Endpoints (1st and 33rd samples) were also included in the pruning process.

Fig. 5 shows the result of modified EMD on the number and shape of IMFs. This was tried for a number of walk segments; results for two randomly chosen cycles (cycle 4 and 5) of one segment are shown. Extrema adjustment helps reduce the number of IMFs. Using the original EMD method, we can throw away the first IMF from cycle 5 and get a smoother cycle; but we cannot always do this, as some cycles contain just 2 IMFs. Instead, by employing extrema adjustments, we get a uniform decomposition on all the cycles without the need to throw away information from signals.

Fig 6 shows the number of IMFs generated for the entire gait segment. Some cycles still have 3 IMFs (2 IMFs and a residue), even after our modification. Upon further investigation, we noticed that using a higher value of the threshold for extrema adjustment makes all the gait cycles decompose into one IMF and a residue. This is fine for the walk segment considered, as cycles look similar to each other. But in general, setting a bigger threshold might not be desired, as it will let the algorithm produce fewer abrupt and meaningless IMFs with larger decomposition error.

The examples shown illustrate the potential of extrema adjustment to generate modified EMD. The above modification can be employed on data with small sampling errors or with noise in the form of small changes in the signal values. If the signal is itself just one or two IMFs plus possibly a residue, throwing away higher IMFs to remove noise is not a good idea as one might lose important information embedded in those higher IMFs. Instead, we can use our modified version of EMD to get an approximate decomposition and preserve all the signal.

VI. CONCLUSION

In this paper, we present a simple but effective modification of the original EMD algorithm. Since EMD is sensitive to the

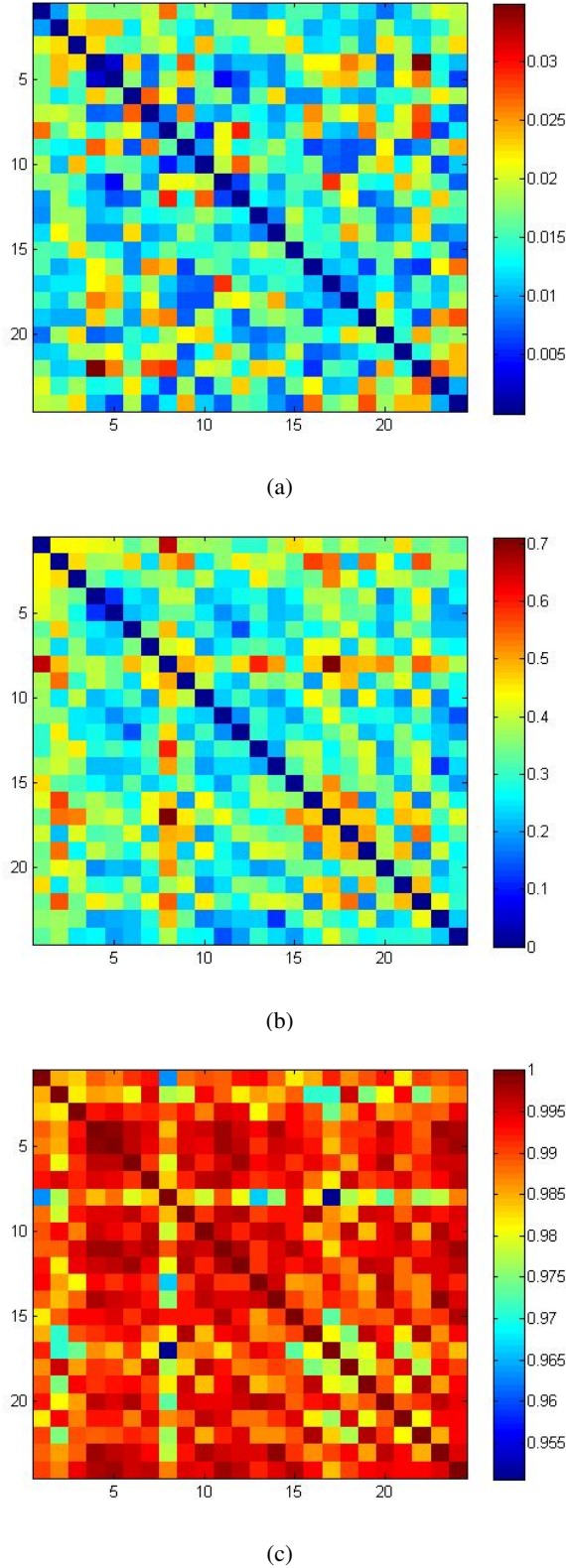


Fig. 3. Characterization of gait signal using standard signal processing tools; (a) DTW and (b) Euclidean distance between Cycles, (c) Correlation between Cycles

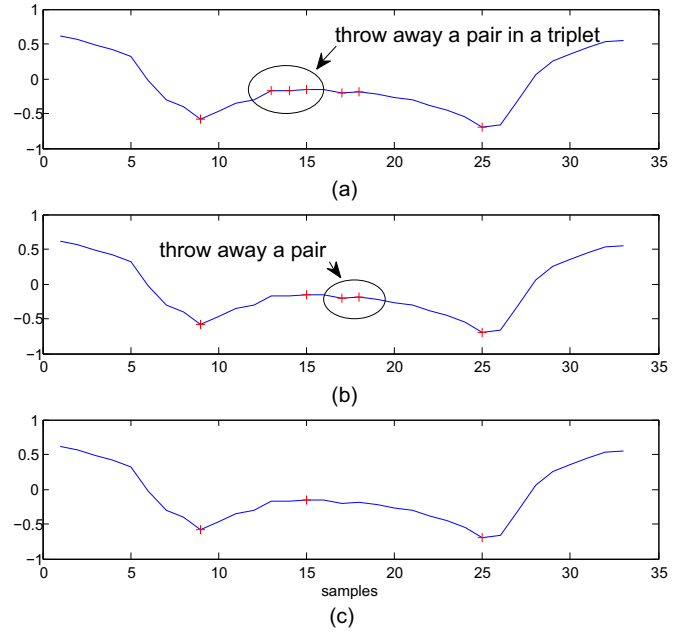


Fig. 4. Extrema removal process for one example gait cycle; (a) original signal with all extrema marked as red pluses, (b) after the first cycle of extrema removal, (c) the final set of extrema marked with red pluses.

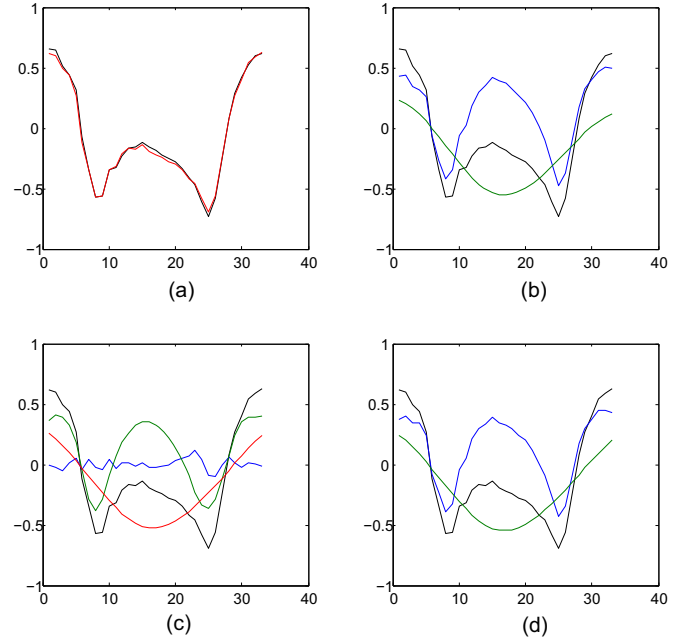


Fig. 5. EMD for a gait cycles; (a) original cycle 4 in black and cycle 5 in red, (b) actual IMFs of cycle 4, (c) actual IMFs of cycle 5, (d) IMFs of cycle 5 after extrema adjustment

location and number of extrema and small changes in the data, by including extrema adjustment into the sifting procedure, a modified but more controlled decomposition can be obtained. The extent to which extrema adjustment can be carried out can vary depending on the application and accuracy of the decomposition needed. The threshold for extrema adjustment can either be preset by the user or be made to vary as sifting progresses. A small threshold always works and often helps

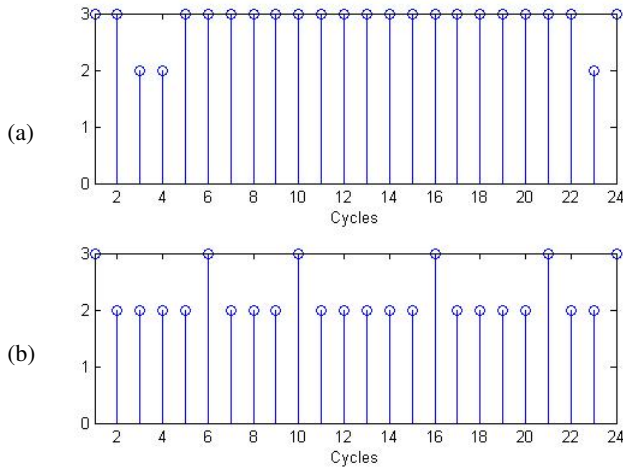


Fig. 6. Number of IMFs generated by EMD, last IMF is residue (a) without any adjustment, (b) with extrema adjustment

reduce the number of IMFs generated. In future work we plan to develop a more adaptive, data driven threshold selection technique for extrema adjustment for more robust *selective extrema analysis* by eliminating the need to preset a threshold.

ACKNOWLEDGMENT

This work is supported by the National Science Foundation under grant IIS-1231712.

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