

QUASI-STATIC ANALYSIS: A METHOD FOR PREDICTING GRASP STABILITY

J. W. JAMESON* - L. J. LEIFER*

Dept. of Mech. Engr., Design Div., Stanford, Ca. 94305

ABSTRACT

This paper introduces a general method for predicting the stability of grasps obtained by articulated mechanical hands. Two contact models are considered: point contact with friction, and a distributed form called the "soft finger contact". The method is applicable to a large class of statically indeterminate, as well as determinate, cases. Results from several examples are included.

1.0 INTRODUCTION

In a broader effort to enhance the capability of computer controlled manipulation, many researchers have investigated the mechanics of grasping with articulated mechanical hands. Most of these efforts were concerned with the application of net forces and the control of fine motions of the object. A prerequisite for the large majority of these manipulation tasks, however, is that the grasp be stable. Grasp stability criteria have been incorporated into objective functions for maximization of stability with respect to joint torques^{6,10,11} and for the determination of stable grasp positions^{6,10}.

While some researchers have investigated grasp stability for statically determinate systems, none of their methods are applicable to statically indeterminate systems. The latter case includes a large class of practical grasps, and hence is a primary motivation for the present technique. One example is the "parallel-jaw" gripper, which relies on distributed contact for a secure grasp.

When an object is fixed with respect to the hand, the grasp is presently defined as "stable", otherwise the grasp is "unstable". However, the phrase "grasp stability" has taken on at least two meanings in the literature which are more specific. Some authors considered only whether the object will slip for a particular grasp^{3,9,10}. Other authors^{1,4,5,6,14} also considered whether the object will return to an equilibrium position after a perturbation of its position with respect to the hand, this perturbation resulting from either a sliding motion^{1,5,6,14}, or from compliance in the elements of the hand⁴. Presently the former criteria is referred to as "first-order" stability and the latter as "second-order" stability. Although quasi-static analysis addresses only first-order stability, the method is applicable to second-order methods since the latter incorporate first-order stability criteria (note that second-order stability presupposes first-order stability).

Another characteristic of quasi-static analysis is the requirement that the object be immobile with respect to the hand when sliding is prohibited at all contacts between the object and hand. However, the method might be extended for other cases as well.

Suppose a hand H is stably grasping an object B such that B is fixed with respect to H , and the magnitude of the external load on B is zero. Now suppose the magnitude of the external load is slowly increased (keeping its direction constant) until

the object slips. For quasi-static analysis it is assumed that the magnitude of the external load, *as the object is sliding, is the maximum that the grasp can withstand in that direction*. It is also assumed that the inertial forces are negligible in the analysis of the sliding motion. This approach has been considered with regard to grasp stability but only for the case of frictionless contacts³. Also related to this approach is the analysis of object motion on a planar surface under the influence of frictional forces and "pushing" loads (e.g. Mason¹³). Related to a lesser extent is previous work on "holding devices", which considers the relative mobility of two rigid objects in multi-point contact^{12,16,18}.

By considering all possible motions of the object with respect to the hand, it is possible, for quasi-statically determinate systems, to determine the corresponding set of external loads for which the grasp is stable. In general, the external load can be represented by a 6×1 vector called the *external wrench*, w_e , defined as

$$w_e \equiv (f_e^T, m_e^T)^T, \quad (1)$$

where:

f_e external force on the object
 m_e external moment on the object

Refer to Hunt⁷ for a more detailed discussion on wrenches. Hence, for quasi-static analysis it is possible to determine a set \mathcal{W} defined as follows:

Definition 1 The *external wrench stability region* $\mathcal{W} \subset \mathbb{R}^6$ is defined as a set of points in w_e -space for which a given grasp is stable, i.e., if the applied external wrench is denoted by w_e^* , the grasp is stable for $w_e^* \in \mathcal{W}$.

In general, it is impractical to develop the entire stability region for a particular grasp. Instead, the goal is to find a set of external *wrench intensities* for which the grasp is stable. That is, if the external wrench is represented as

$$w_e = s \hat{w}_e, \quad (2)$$

where s is the wrench intensity and \hat{w}_e is a unit wrench, the goal is to find the set of values of s such that the grasp is stable. Note that \hat{w}_e represents a line in w_e -space (\mathbb{R}^6) and s designates a point on this line. Only five parameters are needed to completely specify \hat{w}_e , therefore its six elements must be dependent. For the example considered at the this paper it is assumed that the external load is a pure force f_e acting at a point x_e on the object. For these cases

$$\hat{w}_e = \begin{pmatrix} \hat{f}_e \\ x_e \otimes \hat{f}_e \end{pmatrix} \quad (3)$$

If $\mathcal{P}_{\hat{w}_e}$ is defined as the set of points on the line corresponding to \hat{w}_e , i.e., if $\mathcal{P}_{\hat{w}_e} = \{w_e : w_e \in \mathbb{R}^6, \hat{w}_e \in \mathbb{R}^6, \hat{w}_e^T w_e = \text{constant}, s \in \mathbb{R}, w_e = s \hat{w}_e\}$, and $\mathcal{S}_{\hat{w}_e}$ is a subset of $\mathcal{P}_{\hat{w}_e}$ for which the grasp is stable, then $\mathcal{S}_{\hat{w}_e} = \mathcal{P}_{\hat{w}_e} \cap \mathcal{W}$. Furthermore, the surfaces corresponding to the boundaries of \mathcal{W} are gener-

ated by the boundaries of $S_{\hat{w}_i}$ for all \hat{w}_i . Note that $S_{\hat{w}_i}$ is denoted as the "stability range".

For statically determinate systems, the set \mathcal{W} as predicted by quasi-static analysis is never conservative, i.e., assuming the contact models are accurate, there are never points outside of \mathcal{W} which also correspond to stable grasps. For statically indeterminate systems, a given grasp could resist a greater range of external loading than is predicted by the quasi-static analysis and still not violate any of the assumptions regarding the contact models—even though the coefficients of static and dynamic friction are assumed to be equal. This is because it is possible for a given grasp to resist all external loads as developed by considering all possible contact forces (and/or moments) that satisfy the friction constraints without regarding the kinematics. The latter set of external loads is generally larger than, and contains, the set of external loads predicted by the quasi-static analysis (for stable grasping). In other words, while a grasp may be stable where the quasi-static analysis predicts it to be unstable, it may also be unstable. The quasi-static method yields the most conservative result.

2.0 CONTACT MODELS

In this section we briefly review force and kinematic models for point contacts with friction, and a distributed form of contact called the "soft finger". See the references^{10,17} for more detailed discussions of these models. In general, the object, as well as the hand links, are assumed to be rigid. The exception is that for a soft finger contact, the bodies are assumed to be deformable in the vicinity of the contact in order to obtain a finite contact area.

The surfaces in the vicinity of the i^{th} contact are characterized by position and orientation, the latter being represented by the *contact normal* (\hat{n}_i), which is defined as a unit vector which points in a direction normal to the plane tangent to the surfaces at the i^{th} contact. The direction of the normal is defined as inward with respect to the object (or outward with respect to the finger).

CONTACT FREEDOMS

The number of parameters needed to completely describe the relative changes of position and orientation of two rigid bodies in contact at *only* the i^{th} location is denoted as the *contact freedom*, f_i . For point and soft finger contacts, the value of f_i depends on whether sliding occurs at the contact.

For point contacts, when no sliding occurs, $f_i = 3$, the contact essentially behaves as a "ball-and-socket" joint. For this case, three angular velocities, corresponding to three mutually perpendicular axes which pass through the contact point, completely describe the relative motion. When sliding occurs, $f_i = 5$, since translation is allowed in the plane perpendicular to the contact normal.

The soft finger contact is similar to the point contact model except when no sliding occurs, $f_i = 2$ because rotation is prohibited about the axis parallel to the contact normal (due to the finite area of contact). Kinematically, this case corresponds to a "u-joint" with rotation axes perpendicular to the contact normal.

For quasi-static analysis, it is necessary to recognize two "modes of sliding" for the soft finger contact corresponding to $f_i = 3$ and $f_i = 5$. The possible motions are the same as for the related point contact cases, except that rotation about the axis parallel to the contact normal requires sliding to occur. Note that the motion and force descriptions for the soft finger are approximations based on the fact that the moment exerted by a soft finger (described shortly) is only significant when the axis of rotation of the object with respect to the link (which it is in contact with) passes near the center of the contact area. See the reference article¹⁰ for a more detailed discussion of these

approximations.

For quasi-static analysis it is convenient to provide a descriptor for the type of motion at each contact corresponding to a gross motion of the object with respect to the hand:

Definition 2 The *contact state vector* is defined as $\xi = \xi_1, \dots, \xi_{n_c}$, where $\xi_i \in B, P, U$, $\xi_i = B$ corresponds to $f_i = 3$ (ball-and-socket motion), $\xi_i = P$ corresponds to $f_i = 5$ (ball-and-socket plus translational motion), $\xi_i = U$ corresponds to $f_i = 2$ (u-joint motion), and n_c is the total number of contacts.

CONTACT FORCES AND MOMENTS

For all contacts ($i = 1, \dots, n_c$) it is assumed that contact is maintained, i.e.,

$$f_i \cdot \hat{n}_i \geq 0 \quad \forall i, \quad (4)$$

where f_i is the force exerted on the object through the i^{th} contact. Note that eq. (4) applies regardless of whether sliding occurs or not at the contacts.

Assuming point contacts with Coulomb friction, the condition for no sliding at the i^{th} contact ($\xi_i = B$) can be written as¹⁰:

$$f_i \cdot f_i - (1 + \mu^2)(f_i \cdot \hat{n}_i)^2 < 0 \quad \forall i : \xi_i = B \quad (5)$$

where μ is the coefficient of friction. The set of points in f_i -space which satisfy eqs. (4) and (5) comprise the so-called friction cone, the central axis of which is parallel to \hat{n}_i and passes through the contact point. If sliding occurs at the i^{th} contact, then f_i must lie on the friction cone and eq. (5) must be replaced with:

$$f_i \cdot f_i - (1 + \mu^2)(f_i \cdot \hat{n}_i)^2 = 0 \quad \forall i : \xi_i = P \quad (6)$$

For a soft finger contact a couple (m_i) about an axis parallel to \hat{n}_i can be exerted on the object through the (i^{th}) contact, as well as a force f_i through the center of the contact area. Thus, the condition for no sliding ($\xi_i = U$) at the i^{th} contact must be of the form:

$$S(f_i, m_i) < 0 \quad \forall i : \xi_i = U. \quad (7)$$

For example, the following approximate expression was derived for S , where the contacting bodies are assumed to be elastic—and the contacting surfaces spherical—in the vicinity of the contact¹⁰:

$$S = (m_i)^2 + (0.35)(\alpha f_i \cdot \hat{n}_i)^{2/3} [f_i \cdot f_i - (1 + \mu^2)(f_i \cdot \hat{n}_i)^2], \quad (8)$$

where

$$\alpha = \frac{3}{16} \left(\frac{\rho_{o,i} \rho_{f,i}}{\rho_{o,i} + \rho_{f,i}} \right) \left(\frac{1 - \nu_{o,i}^2}{E_{o,i}} + \frac{1 - \nu_{f,i}^2}{E_{f,i}} \right), \quad (9)$$

and,

$\rho_{o,i}, \rho_{f,i}$ radii of curvature
 $\nu_{o,i}, \nu_{f,i}$ poisson ratios
 $E_{o,i}, E_{f,i}$ elastic moduli

where the "o" and "f" subscripts refer to the object and the finger surfaces, respectively. Eq. (8) is used for the example given in Sec. 7.

If sliding occurs for a soft finger contact and $\xi_i = P$ ($f_i = 5$), then the condition on f_i is the same as for a point contact (eq. 6). The condition for m_i is simply:

$$m_i = 0 \quad \forall i : \xi_i = P. \quad (10)$$

If sliding occurs for a soft finger contact and $\xi_i = B$ ($f_i = 3$), then the f_i and m_i must satisfy the relation:

$$S(f_i, m_i) = 0 \quad \forall i : \xi_i = B. \quad (11)$$

3.0 STATIC ANALYSIS

The equilibrium equations for the object can be written as:

$$Wc = -w_c. \quad (12)$$

where

$$c \equiv (f_1^T, \dots, f_{n_c}^T)^T \quad \text{for point contacts,} \quad (13)$$

$$c \equiv (f_1^T, m_1, \dots, f_{n_c}^T, m_{n_c})^T \quad \text{for soft finger contacts,} \quad (14)$$

W $6 \times 3n_c$ ($6 \times 4n_c$) matrix for point (soft finger) contacts.

The static equilibrium equations for the hand can be written as

$$J^T c = t, \quad (15)$$

where:

$$\begin{aligned} t & n_\ell \times 1 \text{ vector of joint torques} \\ J^T & n_\ell \times 3n_c \text{ } (n_\ell \times 4n_c) \text{ matrix for point (soft finger) contacts.} \end{aligned}$$

Note that J^T denotes the transpose of J ; the latter is sometimes referred to as the "hand Jacobian matrix". Equations (12) and (15) are essentially the same formulations introduced by Salisbury¹⁷. If w_c and t are specified, then c is uniquely determined, and the system is statically determinate, when the following matrix is full rank:

$$Y = \begin{pmatrix} W \\ J^T \end{pmatrix} \quad (16)$$

Related to the determinacy of the forces is the *mobility*, M , which is the number of independent parameters needed to specify the position of every body in the hand/object system:

$$M \geq 6(1 - n_c) + n_\ell + \sum_{i=1}^{n_c} f_i, \quad (17)$$

where f_i is the contact freedom for the i^{th} contact. For static analysis, it is assumed no sliding occurs at any of the contacts and thus $\xi_i = B$ ($f_i = 3$) for point contacts and $\xi_i = U$ ($f_i = 2$) for soft finger contacts (for $i = 1, \dots, n_c$). The inequality for eq. (17) applies for cases where constraints on the motion of the object are not independent, which implies a statically indeterminate system. As we shall see, this also implies that the system is quasi-statically indeterminate.

For the analysis in this paper it is assumed that the equality in eq. (17) applies. When this is the case, and no sliding is allowed, the system is statically determinate for $M = 0$, and indeterminate for $M < 0$. When $M > 0$, the system is over-constrained and the joint torques must be dependent.

Note that a grasp is (first-order) stable when eqs. (4) and (5) are satisfied for $i = 1, \dots, n_c$. For statically determinate systems, these conditions can be checked directly since all contact forces and moments can be determined.

4.0 THE MOTION CONSTRAINTS

Most of the constraints for quasi-static analysis have already been developed in previous sections. To complete the set, we need to find the relation between gross motions of the

object (with respect to the hand) and the direction of the forces at the contacts.

For contacts where $\xi_i = B$ the component of the contact force in the plane perpendicular to the contact normal must be anti-parallel to the relative velocity ($v_{t,i}$) of the adjacent points in contact:

$$f_i \cdot (v_{t,i} \otimes \hat{n}_i) = 0 \quad \forall i : \xi_i = P \quad (18)$$

$$f_i \cdot v_{t,i} \leq 0 \quad \forall i : \xi_i = P. \quad (19)$$

If v_i is the velocity of the object at the i^{th} contact point and $v_{\ell,i}$ is the velocity of the link at this point, then

$$v_{t,i} = v_{\ell,i} - v_i. \quad (20)$$

Recall that the mobility, M , is the number of independent parameters needed to specify the position of every body in the hand/object system. Hence, the contact locations can be expressed as a function of M generalized coordinates q_1, \dots, q_M . If these functions are differentiated with respect to time one finds that the velocities (v_i and $v_{\ell,i}$) at the contacts where sliding occurs can each be expressed as a linear combination of $\dot{q}_1, \dots, \dot{q}_M$, where the dots indicate differentiation with respect to time. Thus, we can write

$$v_{t,i} = A_i \dot{q}, \quad (21)$$

where

$$\dot{q} \equiv (\dot{q}_1, \dots, \dot{q}_M)^T, \quad (22)$$

and A_i is a constant $M \times 3$ matrix. We can also express \dot{q} as

$$\dot{q} = |\dot{q}| \hat{q}, \quad (23)$$

where $|\dot{q}|$ is the vector norm of \dot{q} , i.e.,

$$|\dot{q}| \equiv \sqrt{\dot{q}^T \dot{q}},$$

and

$$\hat{q} = (\hat{q}_1, \dots, \hat{q}_M)^T, \quad (24)$$

where

$$\hat{q}_j = \dot{q}_j / |\dot{q}|, \quad \text{for } j = 1, \dots, M \quad (25)$$

(note that $\hat{q}^T \hat{q} = 1$). Combining eqs. (18), (21) and (23), we see that $|\dot{q}|$ cancels and thus the following two equations replace eq. (18):

$$f_i \cdot (A_i \hat{q} \otimes \hat{n}_i) = 0, \quad \forall i : \xi_i = P \quad (26)$$

$$\hat{q}^T \hat{q} = 1. \quad (27)$$

If H_i is defined as follows:

$$H_i \equiv N_i A_i, \quad (28)$$

where

$$N_i \equiv \begin{pmatrix} 0 & \hat{n}_{x,i} & -\hat{n}_{y,i} \\ -\hat{n}_{x,i} & 0 & \hat{n}_{z,i} \\ \hat{n}_{y,i} & -\hat{n}_{z,i} & 0 \end{pmatrix} \quad \text{and} \quad \hat{n}_i \equiv \begin{pmatrix} \hat{n}_{x,i} \\ \hat{n}_{y,i} \\ \hat{n}_{z,i} \end{pmatrix}, \quad (29)$$

then eq. (26) can be written as

$$f_i^T H_i \hat{q} = 0 \quad \forall i : \xi_i = P. \quad (30)$$

5.0 THE GENERAL QUASI-STATIC CONSTRAINTS

For convenience, the complete set of quasi-static equality and inequality constraints are presented in eqs. (31–38) (for point contacts) and eqs. (39–49) (for soft finger contacts). Note that eqs. (38) and (49) are obtained by combining eq. (19) with eqs. (21) and (23). Also note that $\xi_i^{m,p}$ is the contact state vector for the i^{th} contact, where the type of motion of the object is indicated by the (m,p) superscript (this shall be clarified further in Def. 6).

General Quasi-Static Constraints for Point Contacts

$$Wc = -s\hat{w}_e \quad (31)$$

$$Jc = t \quad (32)$$

$$f_i \cdot f_i - (1 + \mu^2)(f_i \cdot \hat{n}_i)^2 = 0 \quad \forall i : \xi_i^{m,p} = P \quad (33)$$

$$f_i^T H_i q^* = 0 \quad \forall i : \xi_i^{m,p} = P \quad (34)$$

$$q^{*T} q^* = 1 \quad (35)$$

$$f_i^T \hat{n}_i \geq 0 \quad \forall i \quad (36)$$

$$f_i \cdot f_i - (1 + \mu^2)(f_i \cdot \hat{n}_i)^2 < 0 \quad \forall i : \xi_i^{m,p} = B \quad (37)$$

$$f_i^T A_i q^* \leq 0 \quad \forall i : \xi_i^{m,p} = P \quad (38)$$

General Quasi-Static Constraints for Soft Finger Contacts

$$Wc = -s\hat{w}_e \quad (39)$$

$$Jc = t \quad (40)$$

$$f_i \cdot f_i - (1 + \mu^2)(f_i \cdot \hat{n}_i)^2 = 0 \quad \forall i : \xi_i^{m,p} = P \quad (41)$$

$$m_i = 0 \quad \forall i : \xi_i^{m,p} = P \quad (42)$$

$$S(f_i, m_i) = 0 \quad \forall i : \xi_i^{m,p} = B \quad (43)$$

$$f_i^T H_i q^* = 0 \quad \forall i : \xi_i^{m,p} = P \quad (44)$$

$$q^{*T} q^* = 1 \quad (45)$$

$$f_i^T \hat{n}_i \geq 0 \quad \forall i \quad (46)$$

$$f_i \cdot f_i - (1 + \mu^2)(f_i \cdot \hat{n}_i)^2 < 0 \quad \forall i : \xi_i^{m,p} = B \quad (47)$$

$$S(f_i, m_i) < 0 \quad \forall i : \xi_i^{m,p} = U \quad (48)$$

$$f_i^T A_i q^* \leq 0 \quad \forall i : \xi_i^{m,p} = P \quad (49)$$

Note that if the total number of equations is equal to the total number of unknowns for eqs. (31–35), or eqs. (39–45), there will be several distinct roots, in particular, distinct values of s , such that these equations are satisfied. Hence, the following definition:

Definition 3 Let λ be a vector containing all the unknowns for the general quasi-static equality constraints (eqs. 31–35 for point contacts or 39–45 for soft finger contacts), where $\lambda \equiv (s^T, c^T, q^{*T})^T$. Then a grasp is *quasi-statically determinate* when there exists a finite number of λ 's such that the general quasi-static equality constraints are satisfied.

Thus, we seek to show that:

$$n_E = n_U. \quad (50)$$

where:

n_E total number of equations

n_U total number of unknowns

In the following only soft finger contact are considered since the point contact case is essentially a restricted case of the former. For the number of unknowns we have the following:

$$n_U = M + 4n_c + 1, \quad (51)$$

where:

M number of components of q^*

$4n_c$ number of components of c

1 for the external wrench, s

To find the expression for the mobility (M) let

$$\sum_{i=1}^{n_c} f_i = 3n_b + 5n_p + 2n_u, \quad (52)$$

where:

n_b number of contacts where $\xi_i = B$

n_p number of contacts where $\xi_i = P$

n_u number of contacts where $\xi_i = U$

and note that

$$n_p = n_c - n_b - n_u. \quad (53)$$

Combining eqs. (52) and (53) with eq. (17), we obtain

$$M \geq 6 - n_c - 2n_b + n_\ell - 3n_u. \quad (54)$$

Requiring that the equality hold for eq. (54) and combining with eq. (51) yields

$$n_U = 7 + 3n_c - 2n_b + n_\ell - 3n_u. \quad (55)$$

The total number of equations is

$$\begin{aligned} n_E &= 6 + n_\ell + 2n_p + n_u + (n_p + 1) \\ &= 7 + 3n_c - 2n_b + n_\ell - 3n_u, \end{aligned} \quad (56)$$

where the enumeration is as follows:

6 equilibrium equations for the object (eq. 39)

n_ℓ equilibrium equations for the gripper (eq. 40)

$2n_p$ constraints for $\xi_i = P$ (eqs. 41 and 42)

n_u constraints for $\xi_i = U$ (eq. 43)

$n_p + 1$ motion constraints (eqs. 44 and 45)

Thus, when all of the mobility constraints are independent, i.e., when the equality holds for eq. (54), then $n_E = n_U$ identically; the same condition is easily shown for point contacts as well.

To narrow the class of grasps which are quasi-statically determinate, it is most convenient to make the following definitions:

<i>fixed joint</i>	position-controlled joint
<i>movable joint</i>	single degree-of-freedom torque-controlled joint
<i>finger</i>	open-loop kinematic chain of rigid links connected by movable joints
n_ℓ	number of movable joints
n_f	number of fixed joints

Definition 4 A *qs-grasp* is defined as a system of N_f fingers ($N_f \in [1, \infty]$) and a rigid palm, such that (1) the mobility satisfies the condition $M \leq 0$ when sliding is prohibited at all contacts, (2) all the mobility constraints are independent, (3) each finger is connected to the palm by either a fixed or movable

joint, and $n_f \in [0, 1]$, and (4) each link is in contact with the object at, at most, one location.

The above definition also applies to hands with higher degree-of-freedom joints since these can be considered as concatenations of single degree-of-freedom joints connected by links of zero length.

Theorem 1 A grasp is quasi-statically determinate if and only if it is in the class of qs-grasps.

Proof The "if" was shown in the previous discussion, i.e., $n_E = n_U$ when all the mobility constraints are independent. The "only if" is addressed in the following.

Addressing condition (1) in Def. 4, note that if $M > 0$ when sliding is prohibited at all contacts, the object can move for any value of the external wrench intensity (s), violating the present definition of a stable grasp for quasi-static systems. Condition (2) reflects the fact that $n_E > n_U$ when the mobility constraints are dependent (see eq. 51). The following addresses conditions (3) and (4).

Suppose two links are connected by a fixed joint and each link contacts the object at one location (on each link). For this case the object can rotate with respect to the link pair about an axis which passes through both contact locations ($\xi_i = B$ at each contact). Since the mobility associated with this motion is the same when the link pair is connected by a movable joint, the fixed joint introduces a dependent mobility constraint. This is the same as saying that a dependent mobility constraint exists when two (or more) contacts are on one link (or rigid body).

Suppose more than one finger is attached to the palm by a fixed joint. Then the corresponding links (connected to the palm with fixed joints), along with the palm, become effectively one rigid body. Hence, either more than one of the links associated with the fixed joints are in contact with the object and a dependent mobility constraint exists, or some of the said links do not contact the object. For the latter case, the links effectively become part of the palm, effectively making the link and joint "invisible", effectively yielding a qs-grasp. Note that a similar argument applies to replacing movable joints in the finger with fixed joints. Hence, if a grasp is not in the class of (effective) qs-grasps, it is quasi-statically indeterminate. ■

6.0 SLIP MODES

Further conditions on the possible motions of the object arise from mobility considerations. One such condition stems from the fact that in some cases there is more mobility than required to develop all possible contact forces for a given slip mode. This condition is recognized by distinguishing those elements of n_E and n_U that arise solely from the motion constraints. Again, considering the case for soft finger contacts (eqs. 39–45), these constraints account for M elements of n_U and $(n_p + 1)$ elements of n_E . Now let eq. (50) take the form

$$n'_E + n_k = n'_U, \quad (57)$$

where n'_E and n'_U are the total number of constraint equations and unknowns, respectively, without consideration of the motion constraints. Thus, we have

$$n_k = n_p + 1 - M, \quad (58)$$

or, combining eq. (58) with eqs. (53) and (54), assuming the equality holds for the latter, we have

$$n_k = 2n_c + n_b + n_\ell - 5. \quad (59)$$

Note that n_k is the number of scalar functions

$$F_j(f_i, m_i) = 0, \quad \text{for } j = 1, \dots, n_k, i \in [1, n_c] \quad (60)$$

obtained by eliminating the (M) components of q^* from eqs. (44) and (45). When $n_k = 0$, each f_i (for $i = 1, \dots, n_c$) can be found from all the remaining constraint equations, i.e., all constraint equations except eqs. (44) and (45). If $n_k < 0$, however, the same equations would be needed to solve for $|n_k|$ elements of q^* , which is impossible since these equations do not contain any elements of q^* . Thus, we have the condition

$$n_k \geq 0, \quad (61)$$

or, combining eqs. (61) and (59) and solving for n_b :

$$n_b \geq 5 + n_\ell - 2n_c - 2n_u. \quad (62)$$

For the object to slip we must also have $M \geq 1$, and from eq. (54) we see that when the equality holds, $M \geq 1$ when $n_b \leq (5 - n_c + n_\ell - 3n_u)/2$. Hence, for soft finger contacts, we have the allowable range for n_b :

$$5 + n_\ell - 2n_c - 2n_u \leq n_b \leq \frac{5 - n_c + n_\ell - 3n_u}{2}. \quad (63)$$

A similar enumeration for point contacts yields:

$$5 + n_\ell - 2n_c \leq n_b \leq \frac{5 - n_c + n_\ell}{2}. \quad (64)$$

Definition 5 A slip mode corresponds to a particular value of n_b for point contacts, or to a particular combination of n_b and n_u for soft finger contacts.

Note that eq. (63) is the same as eq. (64) for $n_u = 0$. Hence, the slip modes for point contacts case also apply for soft finger contacts.

To solve the general quasi-static constraint equations, it is necessary to identify the contact states corresponding to an m,p-permutation:

Definition 6 A slip mode permutation, or m,p-permutation, is defined as the contact state vector $\xi^{m,p}$, where the m,p superscript refers to the p^{th} permutation of the m^{th} slip mode.

The number of unique $\xi^{m,p}$'s for the m^{th} slip mode is the number of ways to arrange n_b "B-objects", n_u "U-objects", and $(n_c - n_b - n_u)$ "P-objects", taken together, where the order is not regarded among each of the two subclasses of objects, but is regarded otherwise. Hence, the number of permutations for the m^{th} slip mode (P_m) is, for soft finger contacts,

$$P_m = \frac{n_c!}{n_{b,m}!n_{u,m}!(n_c - n_{b,m} - n_{u,m})!} \quad (65)$$

where $n_{b,m}$ and $n_{u,m}$ are the values of n_b and n_u corresponding to the m^{th} slip mode. Similarly, for point contacts:

$$P_m = \frac{n_c!}{n_{b,m}!(n_c - n_{b,m})!} \quad (66)$$

If P_t is defined as the total number of slip mode permutations, then

$$P_t = \sum_{m=1}^M P_m, \quad (67)$$

where M is the total number of possible slip modes. As an example refer to Table 1, where all the possible slip mode permutations are given for a qs-grasp with two movable joints and one fixed joint. Note that there are two distinct slip modes for this case (i.e., $M = 2$).

A subset of qs-grasps which includes most practical cases is defined as follows:

Definition 7 A *fully contacting grasp*, or *fc-grasp*, is defined as a *qs-grasp* for which every link of the hand contacts the object.

Hence, for a fc-grasp (recall that n_f is the number of fixed links),

$$n_c = n_\ell + n_f, \quad (68)$$

and combining with eqs. (63) and (64) yields, for the possible slip modes,

$$5 - 2n_u - n_\ell - 2n_f \leq n_r \leq \frac{5 - n_f - 3n_u}{2} \quad (69)$$

$$5 - n_\ell - 2n_f \leq n_r \leq \frac{5 - n_f}{2}, \quad (70)$$

where eqs. (69) and (70) apply to soft finger contacts and point contacts, respectively. The slip modes and the corresponding number of kinematic constraints (n_k) for a fc-grasp are summarized in Tables 2 and 3. The footnotes (*) in Table 3 refer to a "parallel-jaw gripper", which is composed of two links connected to the palm by prismatic joints (actually, one of the joints may be fixed). Furthermore, the (flat) gripping surfaces are parallel, and travel in directions perpendicular to the gripping surfaces. For a detailed explanation of these footnotes, see the reference article¹⁰

7.0 THE STABILITY RANGE

Recall that the stability ($S_{\hat{w}_e}$) is defined as the set of values of the external wrench intensity (s) for which the grasp is stable. Points in $S_{\hat{w}_e}$, on the other hand, are those which satisfy all the general quasi-static constraints *except* for the equality constraints eq. (33) (point contacts) or eqs. (41) and (43) (soft finger contacts), which must be replaced by

$$f_i \cdot f_i - (1 + \mu^2)(f_i \cdot \hat{n}_i)^2 \leq 0 \quad \forall i: \xi_i^{m,p} = P \quad (71)$$

$$S(f_i, m_i) \leq 0 \quad \forall i: \xi_i^{m,p} = B, \quad (72)$$

Table 1 Slip Mode Permutations ($\xi^{m,p}$) for $n_c = 3$ and $n_\ell = 2$

contact (i)	$\xi_i^{1,1}$	$\xi_i^{1,2}$	$\xi_i^{1,3}$	$\xi_i^{2,1}$	$\xi_i^{2,2}$	$\xi_i^{2,3}$
1	B	P	P	B	B	P
2	P	B	P	B	P	B
3	P	P	B	P	B	B

Table 2 n_r (n_k) for FC-Grasp with Point Contacts (or for Soft Finger Contacts and $n_u = 0$)

$n_f \backslash n_\ell$	1	2	3	4	≥ 5
0	-	-	2 (0)	1 (0) 2 (1)	0 (0) 1 (1) 2 ($n_\ell - 3$)
1	2* (0)	1 (0) 2 (1)	0 (0) 1 (1) 2 (2)	0 (1) 1 (2) 2 (3)	0 (2) 1 (3) 2 ($n_\ell - 1$)

*(for soft finger contacts only)

where eq. (71) applies for point contacts and eqs. (71) and (72) apply for soft finger contacts.

A *proper root* for the quasi-static equality constraints is one which satisfies the remaining inequality constraints. If $S_{\hat{w}_e}$ is an interval, i.e., a contiguous set of points on \mathbb{R} , then the maximum number of proper roots is two. Note that if the stability region (\mathcal{W}) is convex, then $S_{\hat{w}_e}$ will be an interval (for any \hat{w}_e). It is presently hypothesized that this is always the case for point contacts. However, if there is doubt as to which side the points of $S_{\hat{w}_e}$ lie on with respect to a proper root, neighboring points can always be checked using eqs. (71) and (72).

In the following example, $S_{\hat{w}_e}^{m,p}$ denotes the stability range for the m,p-permutation, and s_ℓ and s_h refer to the lower and upper bounds of $S_{\hat{w}_e}$.

EXAMPLE—3R3F-HAND

Figure 1 shows the "3R3F-hand" grasping a polyhedral object, where "3R3F" denotes three revolute joints and three fingers. Note that the palm, as well as the fingers, are represented by ellipsoids. Due to limited space, only a few results for this example are given for qualitative evaluation. See the reference article¹⁰ for a more detailed analysis.

The external load on the object is the force $s\hat{f}_e$ acting at point \mathbf{x}_e with respect to the frame described by the three mutually perpendicular unit vectors $\hat{\mathbf{b}}_x$, $\hat{\mathbf{b}}_y$, and $\hat{\mathbf{b}}_z$. Some of the parameters for this hand (and grasp) are:

coef. of frict. (μ)	1.5
finger lengths	two inches
joint torques	$t_1 = 2\text{lb-in}$, $t_2 = 2\text{lb-in}$, and $t_3 = 4\text{lb-in}$

Note that the joint axes are indicated in Figure 1 by the lines marked J_1, J_2 , and J_3 (none of the joint axes are parallel since this would yield a dependent mobility constraint).

Now we consider the case of the 3R3F-hand with point contacts. Table 2 reveals that there is only one slip mode possible for this case, which corresponds to $n_b = 2$. The system is statically determinate and thus there are no kinematic constraints for this case ($n_k = 0$), and furthermore, $n_p = 1$. Thus, for each permutation, the equality constraints (eqs. 31–35) reduce to only a quadratic in s for each of the slip mode permutations.

Figure 4-2 shows the bounds of $S_{\hat{w}_e}$ for the case of a horizontal force $s\hat{f}_e$ acting on the object at location \mathbf{x}_e , where

$$f_e = s \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{x}_e = \begin{pmatrix} 0.4 \\ y_e \\ -1.3 \end{pmatrix}, \quad (73)$$

the range of the abscissa is ($1 \leq y_e \leq 3$), and the units of s are pounds.

Table 3 n_r (n_k) for Soft Finger Contacts and $n_u = 1$

$n_f \backslash n_\ell$	1	2	≥ 3
0	--	1* (0)	0 (0) 1 ($n_\ell - 2$)
1	0 (0) 1** (1)	0 (1) --	0 (2) --

* (singular for parallel-jaw gripper)

** (can be ignored for parallel-jaw gripper)

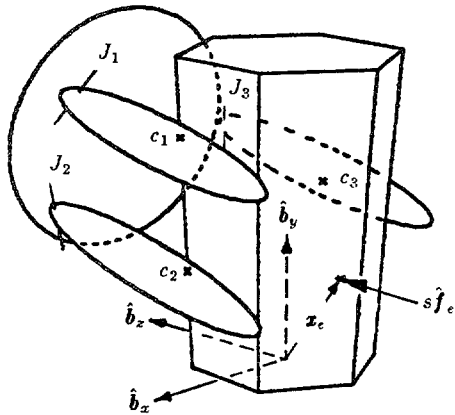


Figure 4-1 3R3F-hand and the external force f_e acting at x_e

Now consider the case of the 3R3F-hand with soft finger contacts. This case reveals how soft finger contacts can enhance a particular grasp as compared to a similar grasp with point contacts. Figure 4-3 shows the stability bounds for this case along with the bounds for the corresponding point contact case. See the reference article¹⁰ for a more detailed discussion of the analysis.

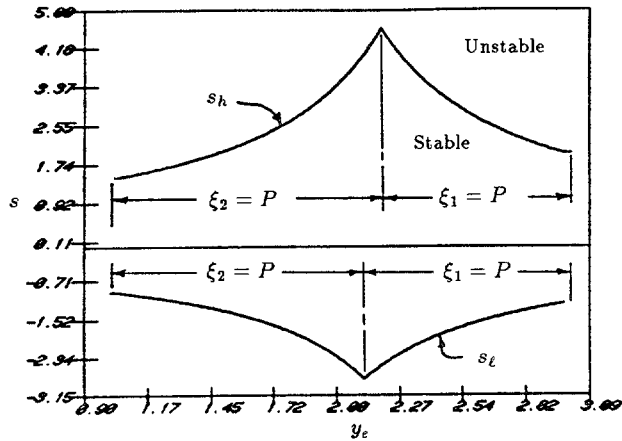


Figure 4-2 Bounds of S_w for the case of Fig. 4-1 (point contacts)

8.0 CONCLUSION: THE STANFORD/JPL HAND

To conclude we consider how the quasi-static method might be applied to articulated hands, which are capable of manipulating an object while maintaining a secure grasp. As an example we use the Stanford/JPL hand (Salisbury¹⁷), which has three fingers, each composed of three one degree-of-freedom links (thus making a total of nine degrees-of-freedom). If this hand is in point contact with the object at the outer link of each finger, then this hand has the capability of controlling either the

- (1) net force (f_e) and moment (m_e) on the object, or
- (2) three rotational and three translational instantaneous displacements of the object with respect to the palm

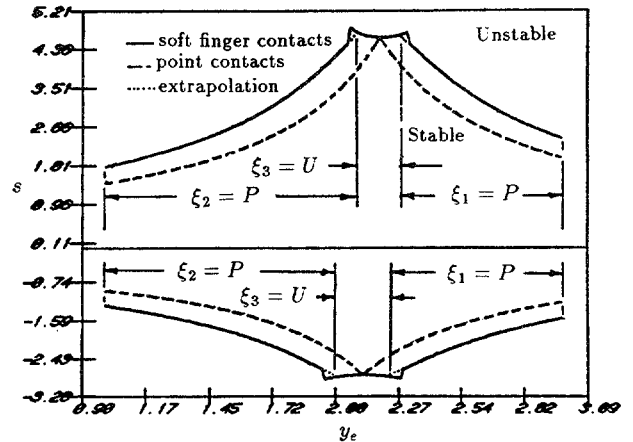


Figure 4-3 Bounds of S_w for the case of Fig. 4-1 (soft finger contacts)

of the hand, or

- (3) three rotational and three translational stiffnesses of the object with respect to the palm of the hand, or
- (4) any combination of the above, with the total number of quantities being controlled equal to six (each quantity must be associated with an independent freedom of the object).

Salisbury investigated (1), (2), and (3) in connection with the control of articulated hands. In another paper¹⁵, the "hybrid force/position control" was introduced for manipulator applications, which essentially represents a combination of (1) and (2). For both of these investigations the controlled quantities were with respect to arbitrary cartesian reference frames defined with respect to the hand (palm). Because of the requirement that $M \leq 0$, only case (2) above can be addressed by the present quasi-static method.

Note that controlling the position of a (movable) link effectively fixes the associated joint in terms of the static equilibrium equations (although the joints may move). An assumption, however, is that for joints under position control, sufficiently large torques can be provided by the actuators such that the desired positions can be maintained. The position control should probably have integral feedback, otherwise there will be a stiffness relation between the joint torque and the joint angle (which effectively yields a constant torque when considering only instantaneous displacements).

Thus, by having six of the nine joints of the hand under position control and the remaining under constant torque control, the case is essentially the same as the example above (with point contacts), for which only three quadratics are needed to find the stability bounds. Another possibility is to have seven joints under position control and two under torque control.

Finally, the quasi-static method could be used to assess grasp stability with the Stanford/JPL hand even if every link, as well as the palm, contacts the object (yielding, at most, ten contacts). Predicting stability for such "power grasps" are beyond the capability of purely static analysis. However, it is clear that the complexity of the computations for quasi-static analysis becomes appreciable due to the large number of slip mode permutations for such cases. A useful consideration for future work is the simplification of the quasi-static method when dealing with a large number of contacts.

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