Engineering optical forces in waveguides and cavities based on optical response

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We present a new treatment of optical forces, revealing that the forces in virtually all optomechanically variable systems can be computed exactly and simply from only the optical phase and amplitude response of the system. This treatment, termed the response theory of optical forces (or RTOF), provides conceptual clarity to the essential physics of optomechanical systems, which computationally intensive Maxwell stress-tensor analyses leave obscured, enabling the construction simple models with which optical forces and trapping potentials can be synthesized based on the optical response of optomechanical systems.

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With recent advances in nanophotonics, the mass and the dimensions of optical devices have been miniaturized to the degree that device tuning through optical actuation is possible at micro- to milli-watt power levels through use of greatly enhanced optical forces at the nano-scale [1–7]. Given the technological relevance of optomechanical interactions, a general analytical formalism capable of handling such complex optical systems is essential to tailoring optical forces in nanoscales for use in technologies based on optomechanical actuation/transduction. In this paper, we present a new treatment of optical forces, revealing that the forces in virtually all optomechanically variable systems can be computed exactly and simply from only the optical phase and amplitude response of the system. This treatment, termed the response theory of optical forces (or RTOF), provides conceptual clarity to the essential physics of optomechanical systems, which computationally intensive Maxwell stress-tensor analyses leave obscured. We show that this formalism enables the construction of simple models with which optical forces and potentials can be synthesized based on the optical response of optomechanical systems.

Conservation of energy and photon number form the basis for RTOF, the theory of which is completely described in Ref [2]. The simplest statement which captures the essential physics of the response theory of optical forces is: “If work is performed against an optically induced force, the energy of the electromagnetic wave (responsible for the optical force) must change by an amount equivalent to the work done”. Therefore, from knowledge of the change in electromagnetic energy of the system (which we will show, can be related to the mechanically variable optical response of the system), and the change in a mechanical degree of freedom through which the work is performed, one can compute the optical forces generated in the system.

RTOF applies to lossless single port and multiport systems. We begin summarizing the results of RTOF in the special case of a lossless system (seen in Fig 1(a)) with one input and one output, whose effect on the transmitted light is purely dispersive. The optical response of this device is assumed to be variable through motion of the generalized spatial coordinate, q. We assume that monochromatic light carrying power P_i, of frequency, ω, excites the system. For this special (lossless) case, the incident wave simply experiences a coordinate dependent phase-shift, $\phi(q)$, in traversing the device. In the limit of adiabatic (or gradual) motion of the coordinate, the change in electromagnetic energy of the system (ΔU_{EM}), due to a change in coordinate of Δq, can be shown to be a [2]

$$\Delta U_{EM}(q) = -\frac{P_i}{\omega}\int_{q_o}^{q_o+\Delta q} \frac{d\phi}{dq'}\cdot dq' = -\frac{P_i}{\omega}\cdot \phi(q)+\alpha. \quad (1)$$

Here, q_o is an arbitrary point of origin. Since the change in energy of the system corresponds to mechanical work performed through motion the generalized coordinate, q, $\Delta U_{EM}(q)$ can be interpreted as the optomechanical potential $U_{eff}(q)$ of the system. Dropping the superfluous constant term, α, the potential is simply

$$U_{eff}(q) = -\frac{P_i}{\omega}\cdot \hbar\phi(q). \quad (2)$$

Thus, in the special case of a purely dispersive system, we see that the optomechanical potential is given by the phase-change imparted on the transmitted wave as the generalized coordinate varies. It is important to note that this expression is valid provided that the coordinate q is static, or quasi-static (i.e. under very gradual motion with respect to the photon-lifetime of the system). Dividing both sides of Eqn. 2 by Δq and taking the limit as $\Delta q \to 0$, the optical forces acting on the coordinate, q, are found to be

$$F_q = -dU_{eff}/dq. \quad (3)$$

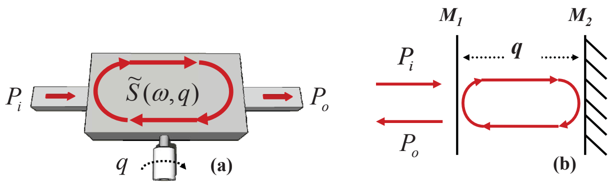

Fig. 1. (a) Generic reflectionless one-port optomechanically variable system. (b) A system of this form is Gires-Tournois interferometer. Here, q is taken to be mirror separation.

This result is valid for any reflectionless one-port system provided that it is lossless. Furthermore, through numerous examples, we have shown that this expression yields identical results to those found through exact Maxwell stress tensor and closed-system analysis [1,2,4].

Generalization of this theory can also be made for the treatment of systems with multiple input and output ports. For example see Fig. 2. In this case, the response of the system can be naturally treated through a coordinated dependent scattering matrix of the form $\tilde{S}(\omega, q)$. For simplicity, the multi-port system is schematically illustrated in Fig. 2(b), showing spatially separated input and output ports to the device (however, in reality they needn't be). We assume that N monochromatic input signals, of frequency ω, enter the multi-port system from the left with powers specified by $P_{i,k}$. Again assuming that the system is lossless, one can show that in the limit where q is static, the analogous effective optical potential of the coordinate q is,

$$U_{eff}(q) \quad = \quad -\frac{\hbar}{\omega}\cdot \int \left[\sum_k P_{o,k}(q) \cdot \frac{d\phi_{t,k}}{dq} \right] \cdot dq. \quad (4)$$

Here, $P_{o,k}(q)$ is the power $\phi_{t,k}(q)$ is the phase of wave transmitted by the k^{th} output port. Similarly, the optical force acting on the coordinate, q becomes becomes,

$$F_q \quad = \quad -\frac{dU_{eff}}{dq} = \frac{\hbar}{\omega} \cdot \sum_k P_{o,k}(q) \cdot \frac{d\phi_{t,k}}{dq}. \quad (5)$$

The above is a remarkably general form of the force and potential for a lossless optomechanically variable circuit with N inputs and N ouputs. No knowledge of the internal workings of this system is necessary in order to compute the optomechanical force and potential that it will create. For fixed input conditions, we need only know the amplitude and phase of the transmitted waves as the generalized optomechanical coordinate, q, is varied.

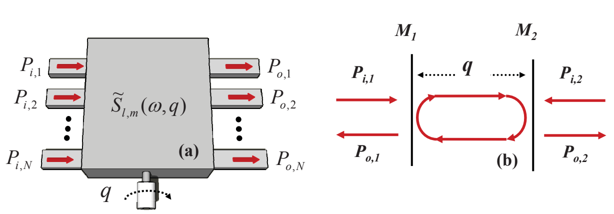

Fig. 2. Lossless mech. variable system with N inputs and outputs.

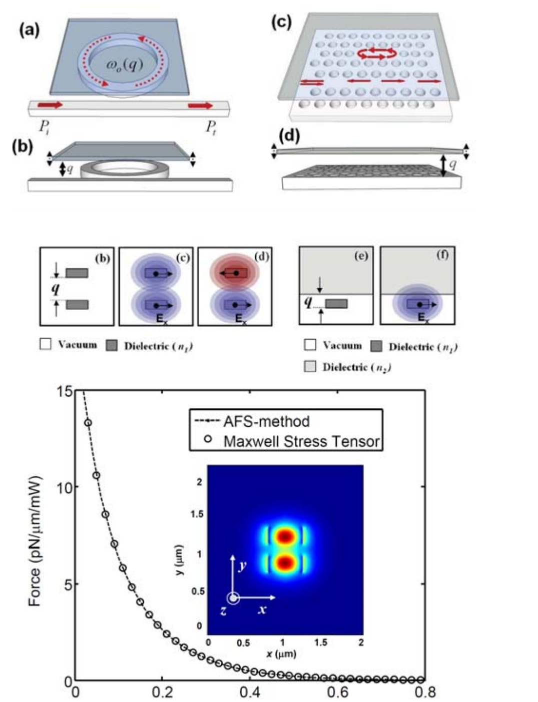

Fig. 3. Microring, photonic crystal and waveguide systems that will be treated using RTOF.

Through this presentation, we will treat the optical forces generated in nontrivial multi-port cavity and waveguide systems (seen in Fig. 3), demonstrating exact equivalence between RTOF and Maxwell stress tensor methods. In addition to the examples seen in Fig. 3, we will discuss the use of RTOF to map the stability diagram of nonlinearly tunable optomechanical cavities and waveguide systems using through use of the effective optical potential in Eqns 2 and 4.

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