

Data Fusion for Detecting Sonar Contacts from Multiple Sensors: The Two Sensor Case

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Abstract - The detection performance obtained by combining the output of two sensors is considered. Four combination methods are analyzed and compared: combine by summing the signals; make 0,1 decisions at the sensors and OR the decisions; use the 0,1 decisions in an AND detector; sum threshold signals to make detections. Summing the threshold signals provides a system designer with tradeoffs between message rate from the sensor to the detection processor, prob(false alarm), and prob(detection). The comparisons also show that the OR detector performance is much better than the AND performance if there is a large difference in signal levels at the two sensors*

I. INTRODUCTION

Fusing data from multiple sensors can improve the detection performance of a sonar system over that of the individual sensors. Data fusion techniques developed for other applications are enhanced and modified here for ocean engineering [1]. Applications for multi-sensor detection using fusion include: sequential detection of contacts on a single sensor; detecting contacts using sonar data processed in different ways (e.g., C.W. and F.M. sonar); and detecting contacts from dissimilar sensors (e.g., active sonar and passive acoustic processors).

Data association and tracking has been considered for multiple sensors using hidden Markov models [2]. The detection problem is considered in this paper; the association problem is not addressed. Detecting contacts using fused data from multiple sensors is a two hypothesis detection problem where the individual sensor outputs are the observations.

The term sensor is used in a very general way in this paper. It can refer to the returns from separate pings in an active sonar, signals from different beams in passive sensor, signals observed at significantly different times, signals from different processors, as well as the output of different acoustic sensors. The development is applicable to these or any other model in which two signals are processed from the same target.

II. APPROACH

First the distribution for the combined sensor outputs is found. Then a threshold is calculated such that a specified prob(false alarm) is achieved for the noise only case. The prob(detect) is found as a function of the signal levels at the two sensors. In this analysis, the noise is assumed to be Gaussian with a known mean and variance.

The two sensor case is considered so we can look at the data in two dimensions; thus, the relative performance can be seen. When more sensors are used, presenting the important differences between the processes is much more difficult.

The detection performance of a threshold processing system (referred to as THRESH) will be compared with the performance of fusion systems based on SUMming, on ORing, and on ANDing. These comparisons will be made assuming Gaussian distributions for the noise. $p(\text{mark})$ will be used in the comparison: this is just the probability that a sensor output exceeds its threshold in the noise case.

All Calculations were made using Mathematica, Version 2.0, on a Macintosh IIx.

A. Mathematical Preliminaries

Assume that the sensors have random variables with Gaussian distributions:

$$f_x(x_i) = \frac{1}{\sigma} \exp\left[-\frac{x_i - \bar{x}_i}{\sigma}\right], \quad (1)$$

where

$$\underline{Z}(t) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}t^2\right]. \quad (2)$$

Thus

$$F_x(x_i) = P\left[\frac{x_i - \bar{x}_i}{\sigma}\right], \quad (3)$$

where

$$\underline{P}(u) = \int_{-\infty}^u \underline{Z}(t) dt. \quad (4)$$

In the development which follows we will use

$$\underline{Q}(u) = 1 - \underline{P}(u) = \int_u^{\infty} \underline{Z}(t) dt \quad (5)$$

In order to simplify the notation in the following developments, we will normalize all sensor signals as follows:

$$y_i = \frac{x_i - \bar{x}_0}{\sigma} \quad (6)$$

\bar{x}_0 is the mean for the noise case. The sensor signals will be characterized by

$$\Delta_i = \frac{\bar{x}_i - \bar{x}_0}{\sigma} \quad (7)$$

assuming that the standard deviation σ is constant. Δ_i is the normalized deviation of the sensor signal from the noise case: $\Delta_i = 0$ for noise with no signal.

With this approach, data from dissimilar processes can be used to make the detection as long as the probability of

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observing the value u_i can be estimated under the noise hypothesis for processor i ; a condition which is usually met for sonar systems.

III. FUSION DETECTORS

All the detectors are analyzed in the same mathematical form so that the comparisons made below are consistent.

The SUM processor is used whenever all data are available for processing. The measurements are just added together and the distribution for the sum determined from the parameters of the individual outputs. Sum is included for comparison, and because this is the approach used when all data are sent to a central processor (the term "fusion center" is used in the fusion literature).

The AND and OR detectors use the sensor output which is either 0 or 1: if the sensor signal exceeds the threshold, 1 is sent; otherwise no value is sent. The AND and OR detectors combine the sensor outputs to make a system detection. For AND, both sensors must be 1 to call detection. For OR, if either sensor is 1 a detection is called.

The THRESH detector is based on the sum of the two sensor outputs, but in this case the outputs are thresholded. Thus either a 0 is received (the sensor signal is less than the sensor threshold), or the sensor amplitude is received.

In the mathematical analysis below, we use the following steps for each detector: (1) the $\text{prob}(\text{False Alarm})$ is found, that is, the $\text{prob}(\text{Detect})$ for $\Delta_1 = \Delta_2 = 0$; (2) $\text{prob}(\text{mark})$ is given, i.e. the probability that a message is sent from the sensor to the detector; (3) thresholds are determined for a specified PFA; and (4) $\text{prob}(\text{Det})$ is determined in terms of these parameters, Δ_1 , and Δ_2 .

A. Detection based on SUM

The detector inputs are the sensor signals and detection is based on the sum
 $z = y_1 + y_2$

The system reports a detection if $z \geq T_{sum}$. For Gaussian noise

$$\text{prob}(z \geq T_{sum}) = Q\left[\frac{T_{sum} - (\Delta_1 + \Delta_2)}{\sqrt{2}}\right]. \quad (8)$$

The design consists of determining the threshold so that the $\text{prob}(\text{False Alarm})$ meets a predetermined specification.

1) $\text{prob}(\text{False Alarm})$: A false alarm occurs when a detection is made in the noise case, $\Delta_1 = \Delta_2 = 0$; so

$$\text{prob}(\text{False Alarm}) = Q\left[\frac{T_{sum}}{\sqrt{2}}\right]. \quad (9)$$

2) $\text{prob}(\text{mark})$: since all data is processed, this probability is 1.

3) Threshold: The threshold is determined from the specified PFA and (9)

$$T_{sum} = \sqrt{2} Q^{-1}[PFA] \quad (10)$$

4) $\text{prob}(\text{Detection})$: The probability of detecting a target is the probability that z exceeds threshold; it is given by (8).

B Detection Based on AND

In this case the sensors either report 1 for a detection (threshold exceeded), or make no report (threshold not exceeded). The system reports a detection if both sensor 1 and sensor 2 report a detection.

The probability that sensor i reports a 1 for a detection is $\text{prob}(y_i \geq T_{and}) = Q[T_{and} - \Delta_i]$ and

$$\text{prob}(y_1 \geq T_{and} \wedge y_2 \geq T_{and}) = Q[T_{and} - \Delta_1]Q[T_{and} - \Delta_2] \quad (11)$$

1) $\text{prob}(\text{False Alarm})$: A false alarm occurs when both sensors report a detection for the noise case, $\Delta_1 = \Delta_2 = 0$. From (11)

$$\text{prob}(\text{False Alarm}) = Q[T_{and}]^2. \quad (12)$$

2) $\text{prob}(\text{mark})$: for an AND sensor this probability is \sqrt{PFA} .

3) Threshold: The Threshold is determined from the specified PFA by using (12)

$$T_{and} = Q^{-1}[\sqrt{PFA}] \quad (13)$$

4) $\text{prob}(\text{Detection})$: The probability of detecting a target, with the specified false alarm rate, is the probability that both sensors report 1. From (11)

$$\text{prob}(\text{Detect}) = Q[T_{and} - \Delta_1]Q[T_{and} - \Delta_2]. \quad (14)$$

C. Detection Based on OR

As in the case of the AND detector, each sensor either reports 1 for a detection (threshold exceeded), or makes no report (threshold not exceeded). However, for ORing, the system reports a detection if either sensor 1 or sensor 2 reports a detection.

The probability that sensor i reports a detection is

$$\begin{aligned} \text{prob}(y_i \geq T_{or}) &= Q[T_{or} - \Delta_i] \text{ and} \\ \text{prob}(y_1 \geq T_{or} \vee y_2 \geq T_{or}) &= Q[T_{or} - \Delta_1] + \\ &Q[T_{or} - \Delta_2] - Q[T_{or} - \Delta_1]Q[T_{or} - \Delta_2]. \end{aligned} \quad (15)$$

1) $\text{prob}(\text{False Alarm})$: A false alarm occurs when either sensor reports a detection for the noise case, $\Delta_1 = \Delta_2 = 0$; from (15)

$$\text{prob}(\text{False Alarm}) = 2Q[T_{or}] - Q[T_{or}]^2. \quad (16)$$

2) $\text{prob}(\text{mark})$: for an OR sensor, this probability is $\text{prob}(\text{mark}) = 1 - \sqrt{1 - PFA} = 5PFA$. (17)

3) Threshold: The threshold is determined from the specified PFA using (16)

$$T_{or} = Q^{-1}[1 - \sqrt{1 - PFA}]. \quad (18)$$

4) $\text{prob}(\text{Detect})$: The probability of detecting a target, with the specified false alarm rate, is the probability that either sensor signal exceeds the sensor threshold. From (15)

$$\begin{aligned} \text{prob}(\text{Detect}) &= Q[T_{or} - \Delta_1] + Q[T_{or} - \Delta_2] - \\ &Q[T_{or} - \Delta_1]Q[T_{or} - \Delta_2]. \end{aligned} \quad (19)$$

D. Detection based on Threshold Sum

The sensors considered here are similar to those for the AND and OR detector: if the sensor amplitude is less than the threshold, no value is sent. However, if the amplitude exceeds the threshold, the amplitude u_i is sent,

$$u_i = \begin{cases} y_i & y_i \geq T_{sens} \\ 0 & y_i < T_{sens} \end{cases} \quad (20)$$

The detection of a target is based on the sum of the two sensor outputs:

$$z = u_1 + u_2$$

The system reports a detection if $z \geq T_{sys}$.

The cumulative distribution function for z , $F(z)$, is the basis for analyzing the summed threshold detector. From appendix A

$$F_z(z) = q_1 q_2 \mu_0[z] + (q_2 P[z - \Delta_1] + q_1 P[z - \Delta_2] - 2q_1 q_2) \mu_0[z - T_{sens}] + \int_{2T_{sens}}^z C[t, \Delta_1, \Delta_2] dt \mu_0[z - 2T_{sens}] \quad (21)$$

where

$$q_i = P[T_{sens} - \Delta_i]. \quad (22)$$

The design consists of determining the two thresholds, T_{sens} for the sensor and T_{sys} for the system so that the $\text{prob}(\text{False Alarm}) = \text{PFA}$, and that (24) is satisfied.

1) *prob(False Alarm)*: A false alarm occurs when a detection is made in the noise case, $\Delta_1 = \Delta_2 = 0$, so from 21

$$\begin{aligned} \text{prob}(\text{False Alarm}) &= 1 - F[T_{sys}, \Delta_1 = 0, \Delta_2 = 0] \\ &= 1 - q_1 q_2 \mu_0[T_{sys}] - \\ &\quad (q_2 P[T_{sys}] + q_1 P[T_{sys}] - 2q_1 q_2) \mu_0[T_{sys} - T_{sens}] - \\ &\quad \int_{2T_{sens}}^{T_{sys}} C[t] dt \mu_0[T_{sys} - 2T_{sens}] \end{aligned} \quad (23)$$

2) *Sensor Threshold*: The sensor threshold is one of the two thresholds to consider. We assume that the sensor threshold is constrained by the message rate allowed. Appendix B shows how the allowed data rate from a sensor can be interpreted as the $\text{prob}(\text{mark})$.

The threshold is set so that the noise case gives the required $\text{prob}(\text{mark})$

$$T_{sens} = Q^{-1}[\text{prob}(\text{mark})] \quad (24)$$

In order to obtain a solution to (26) below, the values allowed for $\text{prob}(\text{mark})$ are constrained to be in the range $1 \geq \text{prob}(\text{mark}) \geq 1 - \sqrt{1 - \text{PFA}}$.

The greatest value corresponds to the SUM detector and the lowest value corresponds to the OR detector (from (17)).

3) *System Threshold*: This threshold is determined from the sensor threshold and the specified PFA. The system threshold is found from by solving (23) for T_{sys} where $\text{prob}(\text{False Alarm}) = \text{PFA}$, (25) is satisfied, and $T_{sens} =$ value from (24).

4) *prob(Detection)*: The probability of detecting a target is found from (21)

$$\begin{aligned} \text{prob}(\text{Detect}) &= 1 - F[T_{sens}, T_{sys}, \Delta_1, \Delta_2] \\ &= 1 - q_1 q_2 \mu_0[T_{sys}] - \\ &\quad (q_2 P[T_{sys} - \Delta_1] + q_1 P[T_{sys} - \Delta_2] - 2q_1 q_2) \mu_0[T_{sys} - T_{sens}] - \\ &\quad \int_{2T_{sens}}^{T_{sys}} C[t, \Delta_1, \Delta_2] dt \mu_0[T_{sys} - 2T_{sens}] \end{aligned} \quad (26)$$

IV. DISCUSSION

A. SIGNALS WITH EQUAL MEAN VALUES

This is a special case which is often the only one considered in a multisensor analysis. We will see that this view does not give the information required to choose between the different detectors (Fig. 1).

When the two sensors have equal values for Δ , the relative performance of each detector appears clear: the SUM detector is best; the AND detector is next; and the OR detector has the poorest performance. The performance of the THRESH detector can be anywhere between that for the SUM and that for the OR: If we set $p[\text{mark}] = 1$, then the THRESH detector is the same as the SUM detector. If we set $p[\text{mark}]$ equal to the value for the OR detector, then the THRESH detector is the OR detector. When $p[\text{mark}]$ is between these values, the curve will fall between the SUM and the OR curves.

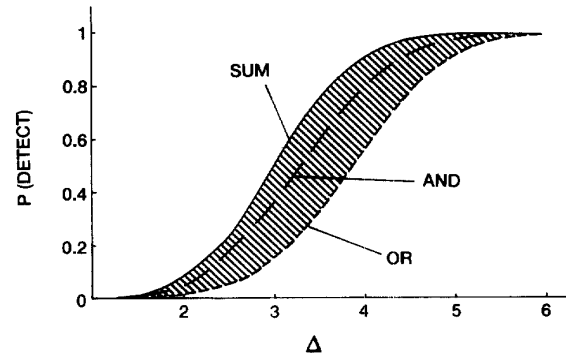


Fig. 1. $p(\text{detect})$ vs Δ for SUM, AND, and OR detectors with $\text{PFA} = 10^{-5}$, $\Delta_1 = \Delta_2$.

B. SIGNALS WITH DIFFERENT MEAN VALUES

Most sonar sensors or signals have variable mean values. If we consider active sonar, the signal amplitudes may have significant differences between different pings. In systems with widely separated sensors, again the mean values will be significantly different.

The OR and THRESH detectors perform better than the SUM when the difference between the sensor signal levels is very great. The SUM is essentially adding the noise from one sensor to the signal at the other sensor, when the levels are very different. Since the sensor outputs are 0 for the noise case, this does not happen in either the OR or THRESH detectors.

1) Comparison of AND and OR Detectors

Fig. 2 shows the relative performance of the AND and OR detectors when the two sensors have different signal levels. If the difference between the signal levels of the two sensors is not too great, AND will provide detections at smaller signal levels than the OR detector. However, if the signal level is small on one sensor, and large on the other, the OR detector will have a higher probability of detection than the AND detector. Finally if a large signal appears on one sensor, and noise on the other, OR will have a high probability of detection: the AND will make no detections at all.

However, if the sensor outputs are to be cross correlated for range estimates, then AND is the appropriate detector since the signal levels should be of approximately the same level for best results.

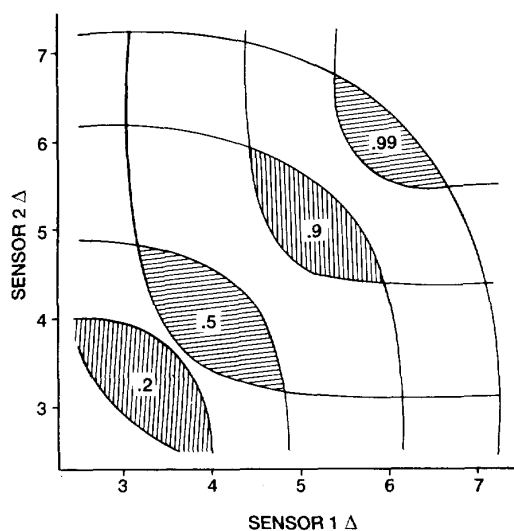


Fig. 2. $p(\text{detect})$ contour plots for AND, and OR detectors with $PFA = 10^{-6}$ and $\text{prob}(\text{detect}) = 0.2, 0.5, 0.9, \text{ and } 0.99$.

2) Comparison of SUM and OR Detectors

The $\text{prob}(\text{mark})$ for the OR detector in Fig. 3 is $\cong 0.5 \times 10^{-6}$. The OR performance is surprisingly good considering the relatively small amount of data used for the detection. When the amplitude difference between the two sensors is about 3 to 1, the OR is as good or better than the sum.

3) The THRESH Detector

Fig. 4 shows the THRESH detector $\text{prob}(\text{detection})$ contours for $\text{prob}(\text{mark}) = 0.001$. This value for $\text{prob}(\text{mark})$ is the same as that for the AND detector with $\text{prob}(\text{false alarm}) = 10^{-6}$. Comparison with Fig 2 shows that the THRESH detector has better performance than the AND for any sensor signal level.

Comparison of SUM and THRESH detectors show that the performance for SUM is not significantly better than that of the THRESH with $p(\text{mark}) = 0.1$; for a large difference in signal levels, THRESH is better. The THRESH works as well or better than the SUM detector with an order of magnitude less data.

4) Comparison of THRESH, and OR Detectors

When significantly large differences are present in the two sensor signal levels, either the OR or the THRESH detector should be used. Fig 5 compares the performance of these two detectors.

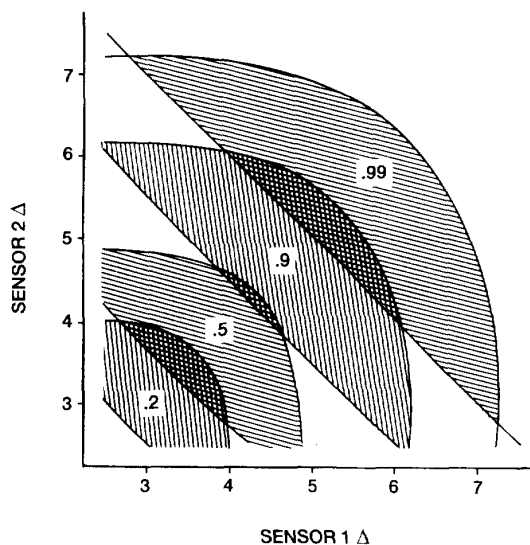


Fig. 3. $p(\text{detect})$ contour plots for SUM, and OR detectors with $PFA = 10^{-6}$ and $\text{prob}(\text{detect}) = 0.2, 0.5, 0.9, \text{ and } 0.99$.

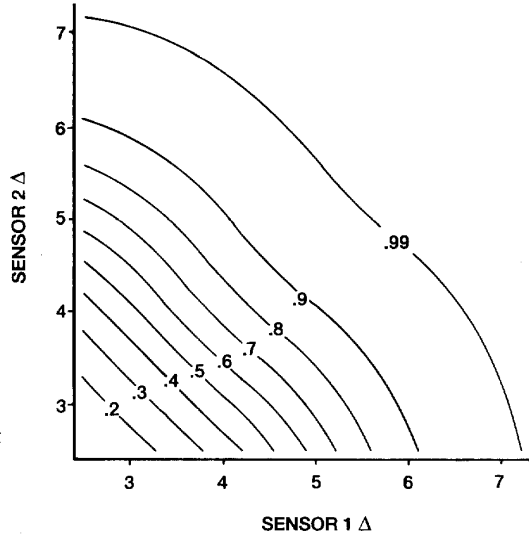


Fig. 4. $p(\text{detect})$ contour plots for THRESH detector with $PFA = 10^{-6}$ and $p(\text{mark}) = 0.001$.

The OR detector imposes the least load on the transmission system since only a 1 is sent, and the smallest data rate is required; for $\text{prob}(\text{false alarm}) = 10^{-6}$, the $\text{prob}(\text{message}) \approx 0.5 \times 10^{-6}$. The decision to use the OR detector means the PFA drives all parameters.

The THRESH detector allows the sensor and detector performance to be matched to the system requirements. If a maximum transmission rate is determined, the parameters of the THRESH detector can be tailored for this value to give the highest probability of success; the designer has more flexibility.

V. CONCLUSIONS

When a significant difference in signal amplitudes exists at two sensors, OR gives better detection performance than AND; if the amplitudes are approximately the same, AND is better.

For a given PFA, the performance of the THRESH detector depends on the sensor threshold, and the performance is bounded between the SUM detector and the OR detector.

Trade off between the data rates, $\text{prob}(\text{detect})$, and $\text{prob}(\text{False Alarm})$ can be made to meet system constraints in the THRESH detector.

The data transmission rate from the two sensors can be reduced by one to two orders of magnitude from the rate for the SUM detector with little degradation in performance if the sensor threshold detector is used.

The argument is often made that "all data should be sent to the fusion center because that is optimum." This is marginally true; sending 1% of the data to the fusion center can give performance which is not significantly degraded from that of sending all the data.

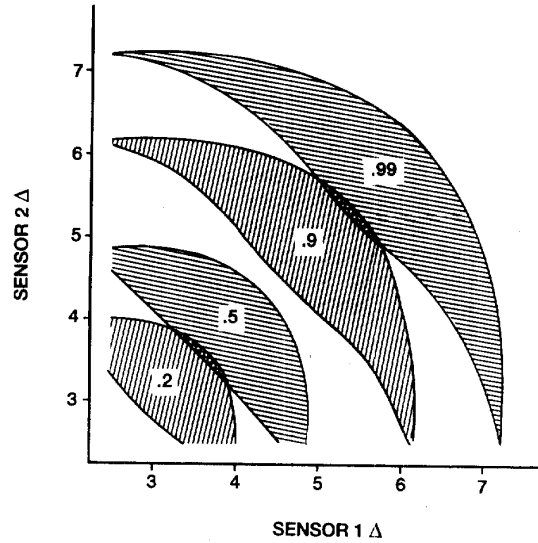


Fig. 5. $p(\text{detect})$ contour plots for THRESH and OR detectors with $PFA = 10^{-6}$ and $\text{prob}(\text{detect}) = 0.2, 0.5, 0.9, \text{ and } 0.99$.

APPENDIX A . DISTRIBUTION FOR THE SUM OF TWO THRESHOLD SENSOR OUTPUTS

1 . Distribution for the Sensor Output

The sensor output distributions will be found assuming the input to the threshold has a Gaussian distribution. Since

$$u_i = \begin{cases} y_i & y_i \geq T_{\text{sens}} \\ 0 & y_i < T_{\text{sens}} \end{cases}$$

the cumulative distribution for u_i is

$$F_u(u_i) = \underline{P}(T_{\text{sens}} - \Delta_i) \mu_0[u_i] + (\underline{P}(u_i - \Delta_i) - \underline{P}(T_{\text{sens}} - \Delta_i)) \mu_0[u_i - T_{\text{sens}}] \quad (\text{A1})$$

where

$$\delta_0[t - T] = \begin{cases} 1 & t = T \\ 0 & t \neq T \end{cases}, \quad \mu_0[t - T] = \begin{cases} 1 & t \geq T \\ 0 & t < T \end{cases}$$

$$q_i = \underline{P}[T_{\text{sens}} - \Delta_i]. \quad (\text{A2})$$

Using the above, we can write $F_u(u_i)$ as

$$F_u(u_i) = q_i \mu_0[u_i] + (\underline{P}(u_i - \Delta_i) - q_i) \mu_0[u_i - T_{\text{sens}}] \quad (\text{A3})$$

and the distribution function as

$$f_u(u_i) = q_i \delta_0[u_i] + \underline{Z}[u_i - \Delta_i] \mu_0[u_i - T_{\text{sens}}] \quad (\text{A4})$$

2. Distribution for the Sum of Two Threshold Sensor Outputs

The conditional distributions can now be determined from the sensor distributions:

$$\text{prob}[Z = z | u_1 = 0, u_2 = 0] = \begin{cases} 1 & z = 0 \\ 0 & z \neq 0 \end{cases},$$

$$\text{prob}[u_1 = 0, u_2 = 0] = q_1 q_2$$

$$\text{prob}[Z = z | u_1 = 0, u_2 \geq T_{\text{sens}}] = \begin{cases} Z[z - \Delta_2] & z \geq T_{\text{sens}} \\ 0 & z < T_{\text{sens}} \end{cases}$$

$$\text{prob}[u_1 = 0, u_2 \geq T_{\text{sens}}] = q_1(1 - q_2)$$

$$\text{prob}[Z = z | u_1 \geq T_{\text{sens}}, u_2 = 0] = \begin{cases} Z[z - \Delta_1] & z \geq T_{\text{sens}} \\ 0 & z < T_{\text{sens}} \end{cases}$$

$$\text{prob}[u_1 \geq T_{\text{sens}}, u_2 = 0] = (1 - q_1)q_2$$

$$\text{prob}[Z = z | u_1 \geq T_{\text{sens}}, u_2 \geq T_{\text{sens}}] = \begin{cases} C[z, \Delta_1, \Delta_2] & z \geq 2T_{\text{sens}} \\ 0 & z < 2T_{\text{sens}} \end{cases}$$

$$\text{prob}[u_1 \geq T_{\text{sens}}, u_2 \geq T_{\text{sens}}] = (1 - q_1)(1 - q_2).$$

where

$$C[z, \Delta_1, \Delta_2] = \frac{1}{2\sqrt{2}} Z\left[\frac{z - (\Delta_1 + \Delta_2)}{\sqrt{2}}\right] \times \left(\text{erf}\left[\frac{z - 2T_{\text{sens}} - (\Delta_1 - \Delta_2)}{2}\right] + \text{erf}\left[\frac{z - 2T_{\text{sens}} + (\Delta_1 - \Delta_2)}{2}\right] \right)$$

We will assume that

$$\int_{2T_{\text{sens}}}^{\infty} C[t, \Delta_1, \Delta_2] dt = (1 - q_1)(1 - q_2).$$

This has not been shown, but in the examples worked so far, the difference has been negligible.

The distribution function for z can be found from the probabilities given above, and the fact that the events are mutually exclusive:

$$f_z(z) = q_1 q_2 \delta_0[z] + (Z[z - \Delta_1]q_2 + Z[z - \Delta_2]q_2)\mu_0[z - T_{\text{sens}}] + C[z]\mu_0[z - 2T_{\text{sens}}] \quad (\text{A5})$$

The Cumulative Distribution Function (CDF), which is used to analyze the performance of the threshold detector, is the integral of the distribution function, i.e.,

$$F_z(z) = q_1 q_2 \mu_0[z] + (q_2 P[z - \Delta_1] + q_1 P[z - \Delta_2] - 2q_1 q_2)\mu_0[z - T_{\text{sens}}] + \int_{2T_{\text{sens}}}^z C[t, \Delta_1, \Delta_2] dt \mu_0[z - 2T_{\text{sens}}]. \quad (\text{A6})$$

APPENDIX B. DATA RATES FROM A SENSOR

Assume that a sensor requires 1 second to calculate the signal value. In this case the "continuous" process for the SUM detector would have a rate of 3600 messages per hour. The following table shows the message rates to the detectors using $PFA = 10^{-6}$ and assuming 1 second to calculate the sensor values.

Detector	P(mark)	rate
SUM	1	3600/hour
AND	10^{-3}	3.6/hour
OR	$.5 \times 10^{-6}$	10^{-3} /hour

The rate for the input to the THRESH detector can take any of the rates shown above. If the rate is set for 360 per hour, or $p(\text{mark}) = 0.1$, the detection probability is not significantly different from that for the SUM

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