

ERROR MODELS FOR STILLING WELL - FLOAT TYPE TIDE GAGES

H. H. Shih

Engineering Development Office
Ocean Technology & Engineering Services/NOAA
Rockville, Maryland 20852

D. L. Porter

National Ocean Survey/NOAA
Rockville, Maryland 20852

ABSTRACT

Tidal measurements are subject to various errors. Because of the increasing emphasis on the accuracy of the data obtained by the Tide and National Water Level Network, the knowledge of these errors is of great importance. This paper summarizes the development of error models pertinent to tide measurement systems utilizing stilling wells and floats. The major error sources addressed include wind waves, water currents, float response, water density variation and marine fouling.

Laboratory experiments including a wave tank, tow tank, and oscillatory water tunnel tests were also conducted to validate the wave, current and float response models. The error models developed can be useful tools for evaluating tide measurement systems utilizing stilling wells and floats and can also provide guidance to future design modifications to reduce tidal measurement errors.

1. INTRODUCTION

Little work has been done to evaluate the overall accuracy of the Analog-to-Digital Recorder (ADR) tide measurement system utilizing a stilling well and float. Cross¹ analyzed the water level response inside the stilling well to small amplitude sinusoidal wind waves. He also presented an equilibrium model to calculate the steady state water level 'set-down' induced by wind waves. Noye² presented refined computational procedures for Cross' models and verified experimentally the small amplitude wave response model. These models, though giving a first order approximation to each phenomenon, lack the ability to describe the detailed dynamic features of the water level inside the stilling well. The effect of water current was demonstrated in experiments by Halliwell and Perry³, Friday⁴ and recently by Whitsell⁵. However, the range of these data is not adequate for practical application and no data exists to describe the effect of oscillatory wave orbital velocities. Lennon⁶ pointed out the effect of water density variations. The effect of float lag caused by the friction of recording mechanism was reported by the instrument manufacturer, but no analysis has been done to evaluate the dynamic effect of float response on tidal data. Marine fouling on the stilling well and orifice degrades tide data to a greater extent

and has received attention recently from the Tidal Data analysis group⁸.

Other factors, which are not directly related to the ADR but are involved in the tide measurement system, such as; staff gage observation and comparison, leveling and bench mark comparison, and sampling frequency and interval, have received little attention^{9,10}.

The purpose of this study is to investigate the nature of errors and to develop analytical techniques to quantify these errors.

2. ERROR SOURCES

As shown in Figure 1, an ADR tide measurement system typically consists of an ADR for measuring and recording the time series of water levels, a staff gage for water level comparison, calibration and correlation of the recorded data to bench marks, and bench marks for datum recovery.

The ADR is a float driven water level measurement device. The stilling well provides a protected environment for the float and the orifice acts as a hydraulic filter to reduce the high frequency water level oscillations caused by ambient wind waves. The ADR records the punched binary-decimal code on foil-backed paper tape. Typically the water level is sampled at six-minute intervals. The orifice is usually located about 5 to 6 feet below the lowest probable tide and high enough above the sea bottom to prevent silting. In areas where water is subject to freezing, a 2 to 5 foot kerosene column is placed inside the stilling well.

The error sources related to the ADR tide measurement system are shown schematically in Figure 2. This study addresses only the major hydro-mechanical error sources related directly and uniquely to the stilling well/orifice and float system. These error sources are wind wave, water current, float response, density variation, and marine fouling.

3. ERROR MODELS

A. Errors Introduced By Wind Waves

Wind waves provide an oscillatory exciting force to the water column inside the stilling well. There are three types of phenomenon arising from the interaction between wave and stilling well/orifice that affect the quality of the tide data.

- The lowering of the mean water level inside the stilling well due to the oscillatory wave pressure excitation,
- The instantaneous water level fluctuation (filtered by the orifice) around this mean due to the wave pressure excitation, and
- Additional lowering of the mean water level inside the stilling well due to the interaction between wave orbital velocity and the stilling well-orifice.

Figure 3 is a definition sketch of the analytical model, in which d_w is the inside diameter of the stilling well, d_o is the orifice diameter, d is the water depth, and Y_o is the orifice submergence. H and T are the wave height and period, respectively. Using the mean sea level (MSL) as the horizontal reference axis and y as the vertical axis with positive downward, the equation of motion for the fluid inside the stilling well can be written as:

$$\frac{D\vec{V}}{Dt} = -\frac{1}{\rho} \nabla p - \nabla U - \vec{R} \quad (1)$$

Where \vec{V} is the velocity vector of the fluid particle, ρ is the fluid density, p is the local pressure, U is the gravitational potential ($U = -gy$), R is the resistance to fluid motion, ∇ denotes the vector operator and D/Dt represents the total time derivative. Utilizing the orifice equation and the continuity equation, equation (1) can be integrated¹¹. The final governing equation for the water level inside stilling well can be written as:

$$Y_o \frac{d^2 y}{dt^2} + \left[\frac{1}{2C_d} \left(\frac{dw}{d_o} \right)^4 + \frac{f Y_o}{2 dw} + K \right] \left| \frac{dy}{dt} \right| \frac{dy}{dt} + gy = -\frac{P(t)}{\rho} \quad (2)$$

where C_d is the Orifice discharge coefficient, and \vec{V} is approximated by dy/dt . f is the friction coefficient for flow along stilling well and the term K represents the effect of an orifice shroud or intake structures. The term $P(t)$ represents the wave pressure at the orifice. A second order Stoke's Wave¹² is used in this simulation. Typically this type of wave has a sharp narrow crest and a shallow wide trough which bears some resemblance to the actual shallow water waves.

The governing equation (2) is a second order non-linear differential equation and can be solved most readily by numerical method once a proper initial value is prescribed. A Runge-Kutta numerical integration technique is used to integrate this equation. The detailed computer programming procedures are described in¹¹.

Sample numerical results are presented in Figures 4 and 5. In Figure 4, the water level error, 'set-down' ($\Delta Y = Y$), is plotted against water depth (d) for various orifice submergences (Y_o), orifice discharge coefficients (C_d), wave heights (H) and wave periods (T). Figure 5 is a dimensionless plot of water level response inside the stilling well to ambient waves. The filtering effect of the orifice is clearly shown in this plot.

Similar to the effect of steady (or slow varying) water current (Section 3.B) the interaction between oscillatory wave orbital velocities and stilling well/orifice will affect the water level inside the stilling well. This contribution is not included in the analytical model given above. The wave tank experiment conducted for the error model validation will be used to derive this information.

B. Error Due To Water Current

Water current passing around the stilling well/orifice often creates a low pressure at the orifice due to the Bernoulli effect. As shown in Figure 3 this corresponds to a drop of the water level inside the stilling well (ΔY , the 'draw-down'). The magnitude of ΔY depends largely on the local flow pattern around the orifice which in turn depends upon the ambient water velocity (V) at the orifice level, the orifice submergence (Y_o), the distance between orifice and sea bottom (b), the stilling well diameter (d_w), the orifice geometry and surface conditions, the water properties (density ρ , and dynamic viscosity μ) and the gravitational force (g). Expressed in non-dimensional parameters, the functional relationship is

$$\frac{\Delta Y}{V^2/2g} = F \left(Re, Fr, \frac{Y_o}{dw}, \frac{b}{dw}, \text{Orifice Geometry} \right) \quad (3)$$

where $Re = \rho V d_w / \mu$, the Reynolds number, is a measure of the viscous effect, $Fr = V / g d_w$, the Froude Number, is a measure of the surface effect, Y_o / d_w is a measure of the two dimensionality of the flow along the stilling well, and b / d_w is a measure of the sea bottom effect.

In most practical cases, the effect of Fr and b / d_w are small. Figure 6 show the relationship for a conical griffice. The plots are derived from Reference^{3,5} and the tow tank experiment.

C. Error Due To Float Response

The forces acting on the float are the float weight, the water pressure on the float, the viscous damping, the float inertia, and the float line tension.

The governing equation for the float response can be derived as¹¹:

$$\frac{d^2 x}{dt^2} + 2n \frac{dx}{dt} + p^2 x = \frac{F_o}{M_t} \cos(\omega t + \phi) - \frac{P_s \pm R_f}{M_t} \quad (4)$$

where x = Float displacement from MWL
 y = Wave surface from MWL = $y_0 \cos \omega t$
 ω = Wave circular frequency
 m = Float Mass
 m' = Float added mass
 P_s = Negator spring force
 R_f = Friction from recording mechanism
 (+ fall, - rise)
 M_t = Total Mass = $m + m' + I_0/a$
 b_t = damping coefficient = $2nM_t$
 p = natural frequency of free oscillation
 $= [c/M_t]^{1/2}$
 c = restoring coefficient = $\rho g A$
 F_0 = exciting force = $y_0 [(c - m'\omega^2)^2 + (b\omega)^2]^{1/2}$
 σ = phase angle between exciting force and Wave
 $= \tan^{-1} [-b\omega/(c - m'\omega^2)]$

Equation (4) is a linear ordinary differential equation and the solution consists of a superposition of three parts:

$$x_1 = \frac{F_0}{M} \frac{1}{\sqrt{(p^2 - \omega^2)^2 + 4n^2\omega^2}} \cos(\omega t + \phi)$$

represents the steady state response. ϕ is the phase lag between wave and float response.

$$x_2 = e^{-nt} [B_1 \sin \sqrt{p^2 - n^2} t + B_2 \cos \sqrt{p^2 - n^2} t]$$

represents the transient response to a prescribed initial condition, and

$$x^3 = -\frac{P_s \pm R_f}{\rho g A}$$

is the bias due to friction of the recording mechanism.

The damping coefficient and the added mass coefficient for the disc type or spar type float can be obtained from references^{13,14}.

Figure 7 shows typical float response to wave amplitude ratio. Depending upon the nature of the wave, the float motion affects the tide measurement. The constant bias due to friction of the recording mechanism is inversely proportional to the float diameter and agrees with the data reported in Reference⁷.

D. Error Due To Density Variation

Density difference between fluid inside and outside the stilling well creates a water level bias. Kerosene is often added to the stilling well to prevent freezing and is an obvious source of error. The density of the water column inside the stilling well could also differ from the ambient. This is most likely to occur in estuaries or river inlets where flow of fresh water exists. The fresh water inflow and the seawater intrusion at high tide create a density stratified water column inside the stilling well which yields a mean density value lower than the ambient.

The total difference in water level (ΔY) can be calculated as:

$$\Delta Y = Y_k \left(1 - \frac{\rho_k}{\rho_{amb}}\right) + Y_w \left(1 - \frac{\rho_w}{\rho_{amb}}\right) \quad (5)$$

where
 ρ_{amb} = Mean density of ambient water
 Y_k = Height of kerosene column
 Y_w = Height of water column inside stilling well
 ρ_k = Density of kerosene
 ρ_w = Mean density of water inside stilling well

As an example, Figure 8 shows the density profile and associated water level difference derived from the salinity data at the mouth of the Columbia River, Washington¹⁵.

E. Error Due To Marine Fouling

Marine fouling inside stilling well and orifice increases flow resistance, decreases orifice effective area and alters the orifice normal discharge coefficients for inflow and outflow. These could seriously degrade the tide data.

Gradual orifice fouling can be simulated easily and the results can be used to quantify the progressive nature of degradation. Irregular fouling is difficult to analyze. However, the mathematical model can still be used to assist in the characterization of the fouling phenomenon.

The governing equation for tide under fouling conditions can be derived in a way similar to that for the wave model. Neglecting the inertial and frictional resistance and using an exponential orifice clogging rate the equation becomes:

$$\frac{dh}{dt} = \pm C_d \left(\frac{dy}{dw}\right)^2 \exp(-bt) \sqrt{2g|y-h|} \quad (6)$$

where h = water level inside stilling well
 y = ambient tide level at stilling well
 b = orifice fouling rate

Note that C_d for inflow and outflow can be significantly different for irregular fouling. The tide level y is approximated as:

$$Y = a_0 + \sum_{i=1}^7 f_i a_i \cos[\omega_i t + (V_0 + U)_i - K_i] \quad (7)$$

where i is the number of tide constituents and seven major ones are used here which include three diurnal (K_1 , O_1 , P_1) and four semi-diurnal (M_2 , S_2 , N_2 , K_2) tide constituents.

The constants f_2 , a_i , K_i , W_i and $(V_0 + U)_i$ are obtained from Reference^{16,17}.

Figure 9 shows comparisons between computer simulations and that observed at the Indian River, Delaware, Tide Station Test Facility.

4. ERROR MODEL VALIDATION

Limited physical model experiments were conducted to validate the error models. These included a wave tank experiment to validate the predicted response of water level inside a stilling well to ambient waves; a tow tank experiment to validate the draw-down of water level inside a stilling well due to water current; and an oscillatory water tunnel experiment to validate the dynamic

response of floats to water level movement. The detailed description of these experiments and results will be documented in a separate engineering report. Figure 10 and 11 show the experimental set-up and typical strip chart data record of the wave tank experiment.

5. CONCLUSION

Error sources for the ADR tide measurement system utilizing stilling well/orifice and float have been identified. The major error sources were investigated and analytical models were formulated. Results are presented by typical parametric curves which show the effects of the controlling physical parameters. These error models, when validated, can be used to evaluate the performance of either existing or new tide station designs. In addition, they have provided inputs for conducting sensitivity studies of measurement errors on tidal datum determination and tide prediction.

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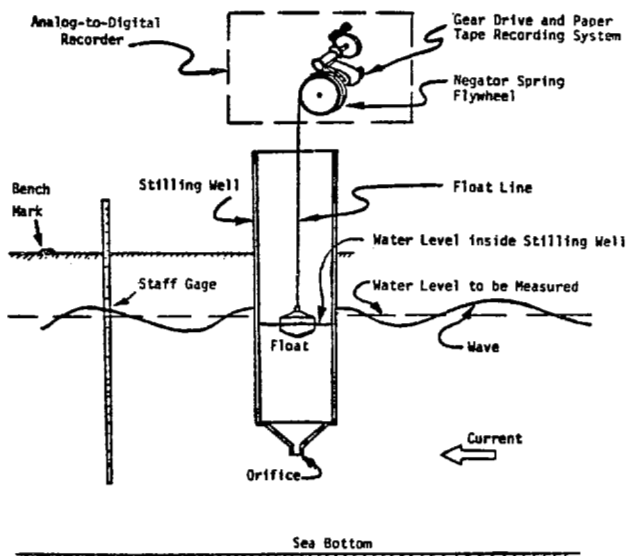


Figure 1 Schematic of ADR Tide Measurement System Elements Utilizing Stilling Well and Float

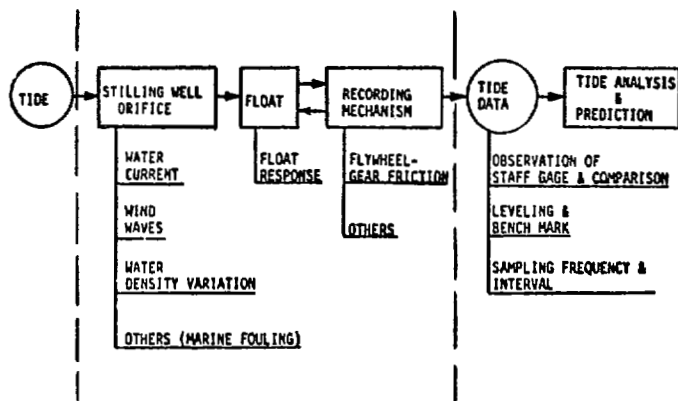


Figure 2 Schematic Diagram of Error Sources ADR Tide Measurement System Utilizing Stilling Well and Float

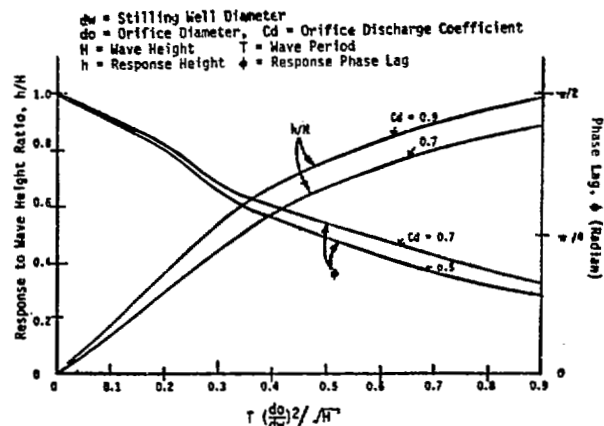


Figure 5 Response of Water Level Inside Stilling Well/Orifice to Waves

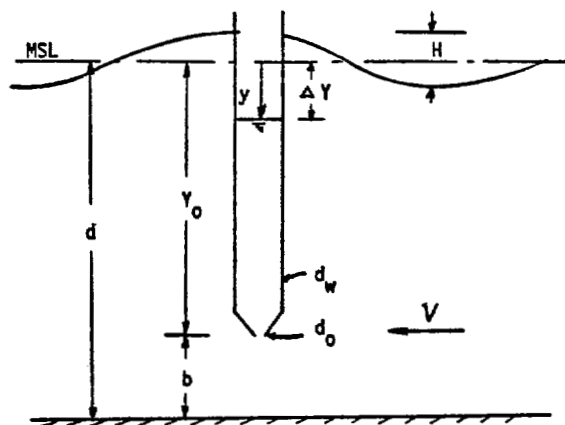


Figure 3 Definition Sketch for Wave and Current Error Models

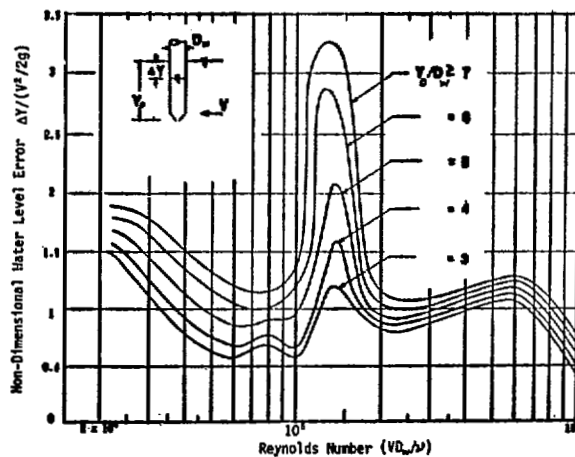


Figure 6 Water Level Error Due to Current for Stilling Well with Conical Orifice

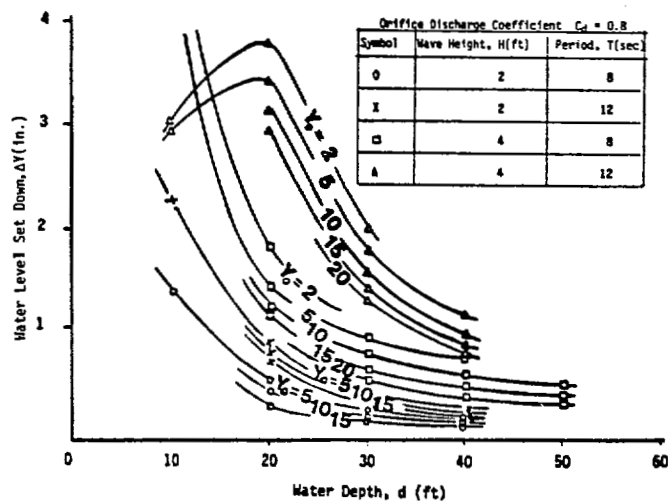


Figure 4 Water Level Error Due to Waves vs Water Depth

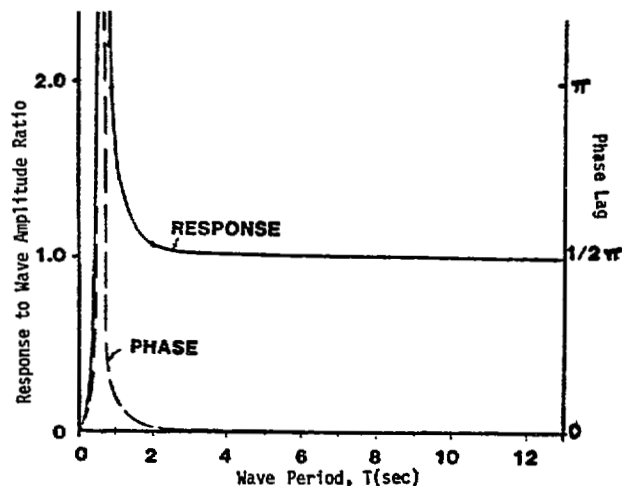


Figure 7 Float Response vs Wave Period (8 1/2 inch diameter, 4 1/2 lbs float)

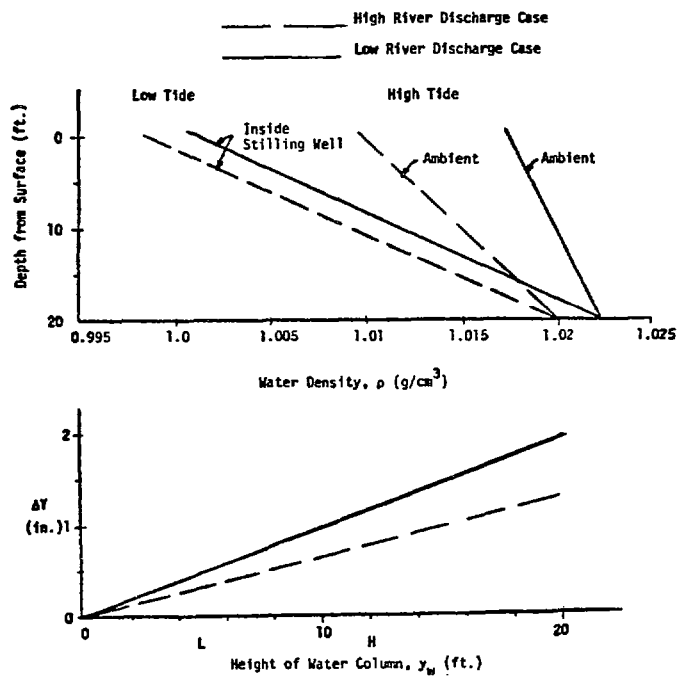


Figure 8 Density Profile And Associated Water Level Error

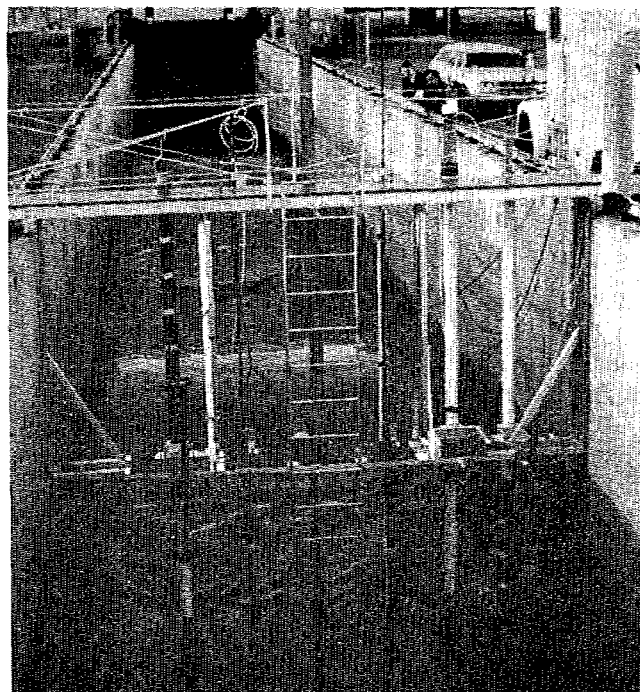


Figure 10 Experimental Set-up of Wave Error Model Verification Experiment

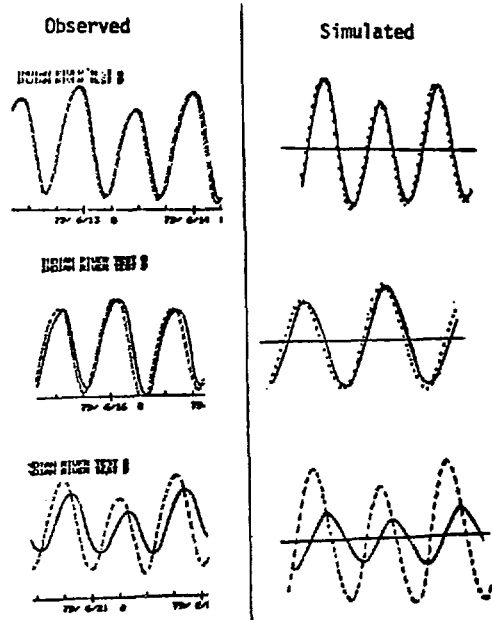


Figure 9 Computer Simulation of Marine Fouling Effect on Tide (— Fouled Orifice, --- Clean Orifice)

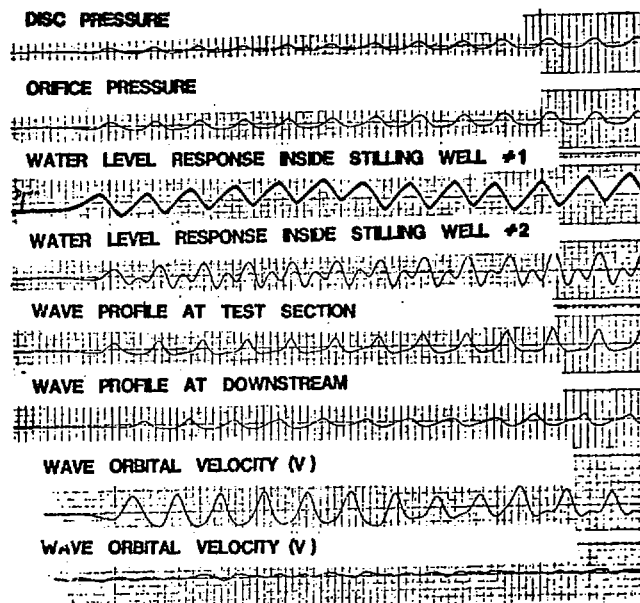


Figure 11 Sample Data Out-put from Wave Error Model Verification Experiment