

Biological Restraint on the Izhikevich Neuron Model Essential for Seizure Modeling

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Abstract—Izhikevich model of a neuron allows for simulation of spiking pattern that mimics known biological subtypes. When a current within a range typical for biological experiments is injected into the cell the firing pattern produced in the simulation is close to that observed biologically. However, once these neurons are embedded into a network, the level of depolarization is controlled only by the synaptic depolarization received by the simulated connections. Under these conditions there is no limit on the maximum firing rate produced by any of the neurons. Here we introduce a modification of the Izhikevich model to restrict the firing rate. We demonstrate how this modification affects the overall network activity using a simple artificial neural network. The proposed restraint on the Izhikevich model is particularly important for larger scale simulations or when the frequency dependent short-term plasticity is used in the network. Although maximum firing rates are most likely exceeded in simulations of seizure-like activity we show that restriction of neuronal firing frequencies impacts even small networks with moderate levels of activity.

I. INTRODUCTION

Network approach to computational brain modeling requires a careful choice of all components of the network to make sure that they are not only biologically suitable and computationally stable by themselves but also when connected together in a network. Moreover, it is crucial to realize what range of activity is going to be modeled and foresee possible issues for computational stability of the network and the neuron model used. This is especially critical when modeling epileptiform or synchronous activity that can result in greater excitation than under normal conditions.

In this paper, we discuss the need to add an upper bound on the firing rate of the Izhikevich neuron model [1]. Since this model is capable of producing firing patterns exhibited by real biological neurons being computationally simple at the same time, it has been used in a wide range of applications, e.g. modeling of multisensory processing [2], [3], racing car controllers [4], character recognition [5], and in large scale simulations of the thalamocortical circuits [6]. In addition, there are multiple hardware circuit implementations of this model [7], [8].

Although it is a very successful and popular neuron model, it obviously is only a simplification of real neural

cell dynamics and therefore it is necessary to understand its restrictions to apply it properly. One of the limitations is that the model can produce spikes with arbitrarily high frequency, which is not a biologically feasible behavior. Note that this issue is not unique to this neuron model, e.g., the firing rate of the leaky integrate-and-fire neuron model is proportional to the value of input [9]. The paper is organized as follows. In Section II we briefly introduce the Izhikevich neuron model and discuss the proposed modification. Then, in Section III we give an example of the impact of the proposed modification on a simple network.

II. IZHIKEVICH NEURON MODEL AND ITS MODIFICATION

The Izhikevich neuron model [1] was derived using bifurcation methods [10] to reduce many biophysically accurate Hodgkin-Huxley-type equations to obtain a simplified system of two differential equations

$$\begin{aligned} v' &= 0.004v^2 + 5v + 140 - u + I \\ u' &= a(bv - u) \end{aligned} \quad (1)$$

with the condition

$$\text{if } v \geq 30, \text{ then } \begin{cases} v \leftarrow c \\ u \leftarrow u + d, \end{cases} \quad (2)$$

where v represents the membrane potential of the neuron and u is a membrane recovery variable (both are functions of time), I is the value of the input to the neuron (in this formulation, proportional to synaptic or injected current), and a , b , c and d are dimensionless parameters. The membrane potential v has mV scale and the time has ms scale in this model.

Similar quadratic models have been introduced ([11], [12]) but the Izhikevich model seems to be the most frequently used. One of the reasons is that it has been clearly demonstrated [13] that various spiking patterns can be easily obtained by different settings of parameters a , b , c , and d .

Since in this model the change in the voltage v depends linearly on input, the frequency of generated spikes is not bounded. A spike is always generated when the condition in equation (2) is satisfied. In cases of a powerful input (I in equation (1)), a neuron can spike arbitrarily fast, which does not occur biologically. Each subtype of neuron has a maximum rate at which firing is observed biologically.

The relationship between the simulated firing rate and the input amplitude is shown in Figure 1. We consider four different neuron subtypes: regular spiking (RS), intrinsically bursting (IB), fast-spiking (FS), and low-threshold spiking (LTS) neurons. These well-studied subtypes have distinct

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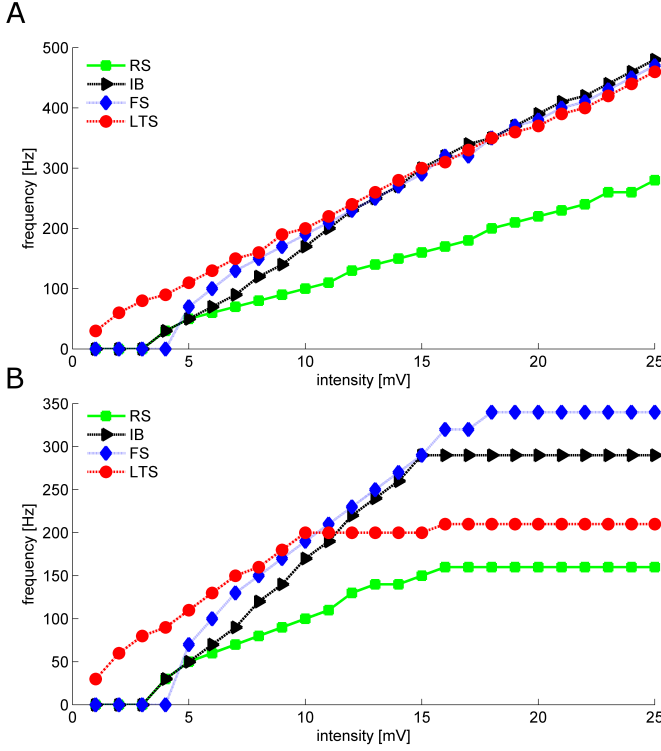


Fig. 1. Frequency of spikes versus intensity (amplitude) of the input of the original (A) and modified (B) Izhikevich neuron model. Maximal firing frequencies were assumed to be 160 Hz, 300 Hz, 350 Hz, and 212 Hz for RS, IB, FS, and LTS neurons respectively. The following parameters were used to generate different neuron types: RS: $a = 0.02$, $b = 0.2$, $c = -65$, $d = 8$, IB: $a = 0.02$, $b = 0.2$, $c = -55$, $d = 4$, FS: $a = 0.1$, $b = 0.2$, $c = -65$, $d = 2$, LTS: $a = 0.02$, $b = 0.25$, $c = -65$, $d = 2$. Frequency was averaged over 100 ms of stimulation.

firing patterns and constitute majority of the neurons in the cortex.

It is well known that FS neurons fire at frequencies much higher than RS or LTS neurons. Observed maximum rates range from 300-500 Hz [15], [16], [19], [20]. It is also well known that due to their ability to burst, IB cells fire at much higher frequencies than RS cells. Observed maximum rates of intraburst frequencies range from 300-500 Hz [19], [17], [18]. Maximum rates for RS and LTS cells have been reported in the ranges of 150-200 and 200-250 Hz, respectively [15], [16], [19], [20], [21]. For the example used here, we have set the maximum rates of IB, RS, FS, and LTS to 300, 160, 350, and 212 Hz, respectively.

Figure 1A shows that using the standard Izhikevich model, for most neuron subtypes their maximum firing frequency is exceeded with an input in the range of $I = 10-16$. Such a level of input is not unusual under normal conditions and is surely exceeded in scenarios with extremely powerful excitation, e.g., simulations of inhibitory blockade (biologically achieved by, e.g., application of bicuculline [22], [23]) where the strength of inhibitory neurons is decreased, or situations of synchronous activity of excitatory neurons which generate synchronized input to post-synaptic neurons.

To address this problem, we propose preventing the gen-

eration of an action potential if the time from the previous spike is shorter than given by the maximum firing frequency for that neuronal subtype. In this case, the membrane voltage is reset to 30 mV for computational stability. In other words, equation (2) is replaced by

$$\begin{aligned} &\text{if } v \geq 30 \text{ and } t - t_{prev} \geq \tau_{min}, \text{ then (spike generated)} \\ &\quad \begin{cases} v \leftarrow c \\ u \leftarrow u + d, \end{cases} \quad (3) \\ &\text{else if } v \geq 30, \text{ then (no spike)} \\ &\quad v \leftarrow 30 \end{aligned}$$

Here t_{prev} is time of the previous spike and τ_{min} is interspike interval in milliseconds given by the maximum firing frequency.

Since the modification imposes an absolute refractory period, it obviously assures an upper bound on the maximum firing frequency of a neuron. The impact of this modification on the firing rates of different neuron models is shown on Figure 1B. The frequency increases with increasing amplitude of input until it reaches the predefined firing frequency, which is more biologically feasible behavior than in the original neuron model (Figure 1A).

III. NETWORK DYNAMICS

To illustrate the importance of the maximum firing rate in a network dynamics, we use an artificial network introduced in [1]. This network consists of 800 regular spiking (RS) neurons and 200 fast-spiking (FS) neurons, thus the ratio of excitatory to inhibitory neurons is 4 : 1. All neurons are interconnected with the strengths of synaptic connections chosen randomly from the interval $(-1, 0)$ in case of inhibitory connections and from $(0, 0.5)$ for excitatory connections. In addition to the synaptic input, each neuron receives a noisy input with Gaussian distribution (mean value of zero, variance equal to five and two for excitatory and inhibitory neurons respectively). We will refer to this network as the original network.

We compared this network with one with the modified neuron model (see Figure 2). The maximum firing frequency is 160 Hz and 350 Hz for RS and FS neurons respectively, as described above. To achieve heterogeneity, we varied these frequencies in range of $\pm 10\%$. Two different scenarios were tested: increasing the input to neurons and blocking inhibitory synapses. Results of these simulations are presented in Figure 2. All simulations were calculated with a time step of 1 ms.

First, the input to the neurons was artificially increased by changing the mean value of the noise to two for all neurons. Note that this change was quantitatively bigger for FS than for RS neurons (change to 40% and 67% of the initial variance for these types respectively). Interestingly, although the network with modified neuron model (Figure 2D) is less synchronized than the original network (Figure 2C), there is still some synchrony. Indeed, analysis of artificially generated EEG (see Figure 3), which was computed as a sum

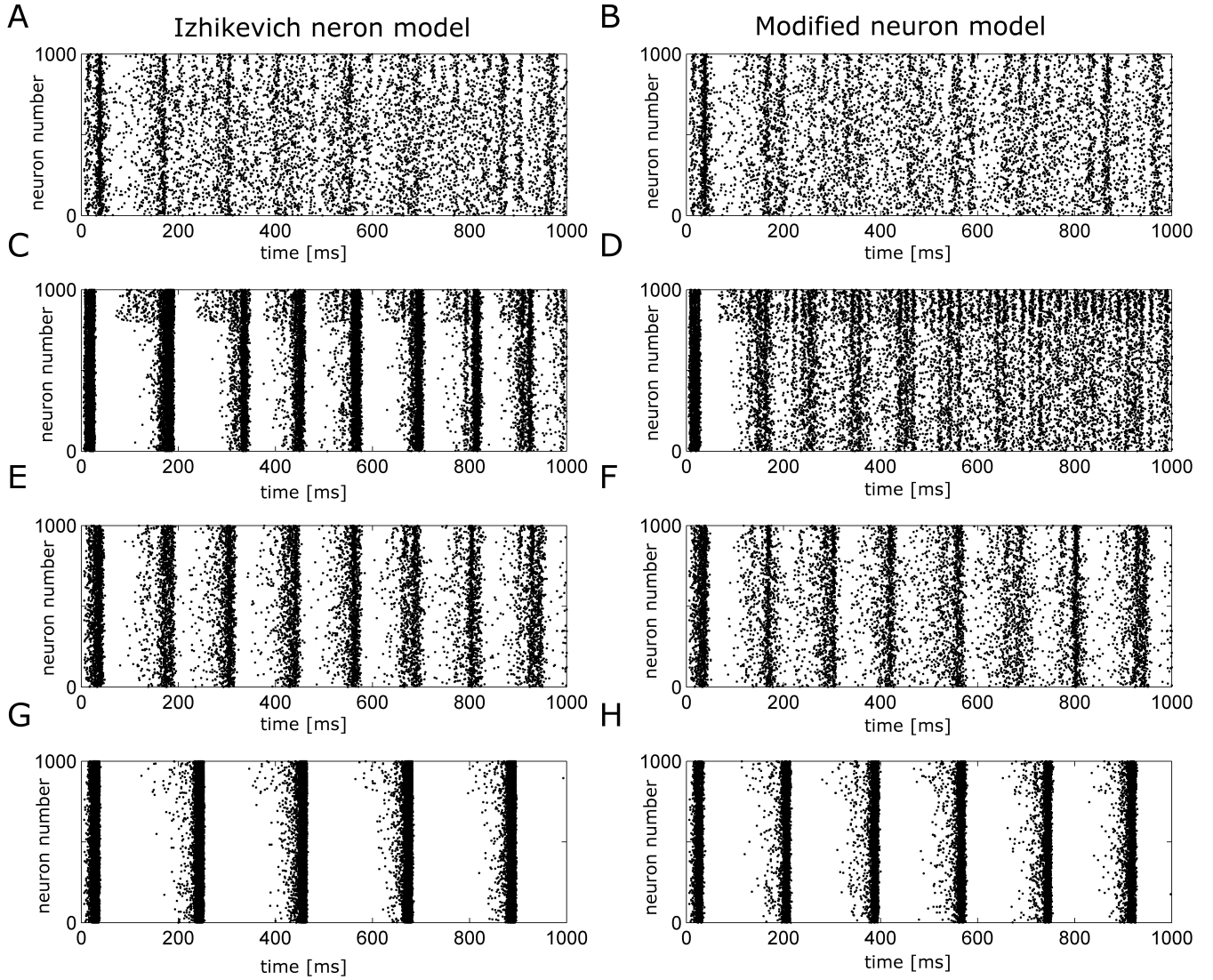


Fig. 2. Comparison of spike activity of two networks across different scenarios. The horizontal axis is time and the vertical axis is the neuron number, with each dot representing a single action potential and each row the activity from one neuron. The network is taken from [1] and consists of regular-spiking (neuron numbers 1-800) and fast-spiking (neuron numbers 801-1000) neurons. (A-B) The original (left) and modified (right) networks with a normal level of activity. (C-D) The variance of the noisy input increased to the value of two for all neurons. (E-F) 20% inhibitory blockade. (G-H) 50% inhibitory blockade.

of all inputs to the excitatory neurons [24], shows that the modified network exhibits oscillations with 43 Hz frequency that corresponds to the gamma band, contrary to the original network that oscillates with 10 Hz frequency. This result is consistent with findings that increased input to FS neurons results in generating gamma oscillations [25].

Secondly, we kept the level of input as in the original network but we blocked inhibitory neurons, that is reduced their strengths by 20% and 50%. We see that in the case of 20% inhibitory blockade the network with the modified neuron model (Figure 2F) is less synchronized than the original network (Figure 2E). With higher inhibitory blockade the network is synchronized (Figure 2H) but with slightly different frequency compared to that in the original network (Figure 2G). This demonstrates that this modification strongly impacts network behavior. In [26] and [27] we

presented a multi-column, multi-layer cortex model that uses the modified neuron model. It has been demonstrated that under inhibitory blockade conditions the network generates local field potentials (LFP) that are comparable with experimentally measured ones.

IV. CONCLUSIONS

Biologically accurate firing patterns of neurons are important for obtaining reliable frequencies of network oscillations. This paper both emphasizes the problem and proposes a pragmatic solution.

We introduced a simple modification of the Izhikevich neuron model that restricts firing rate of neurons and demonstrated how this modification affects the overall network behavior using a simple artificial neural network. The restraint imposed on the Izhikevich model is especially important

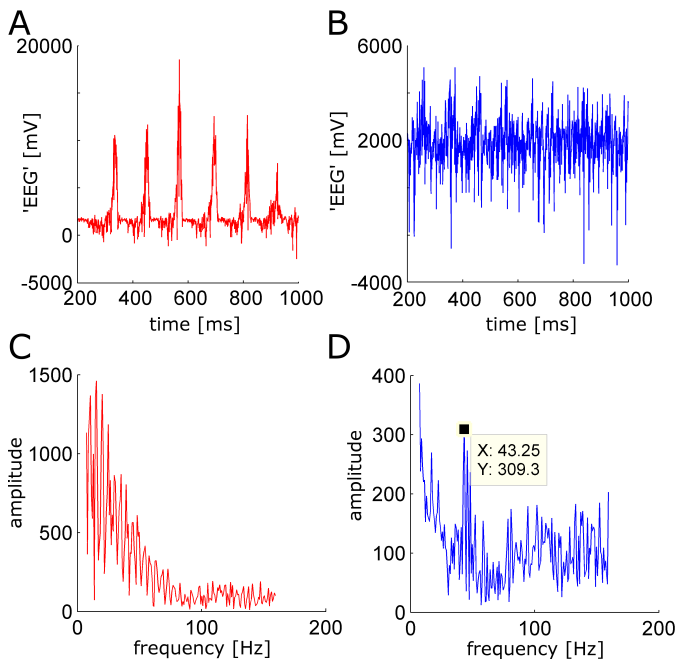


Fig. 3. Comparison of frequencies generated with use of the original (red) and modified (blue) neuron models in case of increased input to the network (compare Figure 2C-D). (A-B) artificial EEG generated for both networks. (C-D) Amplitude of Fourier transform of both signals.

in large scale simulations, or when a frequency-dependent mechanism, such as short-term plasticity, is used within the network. Although maximum firing rates are most likely exceeded in simulations of seizure-like activity, or other conditions that promote excessive excitation, we have shown that restriction of neuronal firing frequencies impacts even small networks with moderate amplitude of input.

REFERENCES

- [1] E. M. Izhikevich, "Simple Model of Spiking Neurons", IEEE Trans. on Neural Networks, vol. 14, pp. 1569-1572, Nov. 2003.
- [2] H. K. Lim, L.P. Keniston, K.J. Cios, "Modeling of Multisensory Convergence with a Network of Spiking Neurons: A Reverse Engineering Approach," IEEE Transactions on Biomedical Engineering, vol.58, no.7, pp.1940-1949, July 2011.
- [3] H. K. Lim, L.P. Keniston, J.H. Shin, B.L. Allman, M.A. Meredith, K.J. Cios, "Connectional parameters determine multisensory processing in a spiking network model of multisensory convergence," Exp Brain Res, vol.213, pp.329-339, 2011.
- [4] E. Yee, and J. Teo, "Evolutionary spiking neural networks as racing car controllers", 11th International Conference on Hybrid Intelligent Systems (HIS), pp. 411 - 416, 2011.
- [5] M. A. Bhuiyan, R. Jalsutram, and T. M. Taha, "Character recognition with two spiking neural network models on multicore architectures", Computational Intelligence for Multimedia Signal and Vision Processing, CIMSVP '09. IEEE Symposium on, pp: 29 - 34, 2009.
- [6] E. M. Izhikevich, and G. M. Edelman, "Large-scale model of mammalian thalamocortical systems", PNAS, vol. 105, no. 9, pp. 3593-3598, Mar. 2008.
- [7] A. Van Schaik, C. Jin, A. McEwan, and T. J. Hamilton, "A log-domain implementation of the Izhikevich neuron model," Proceedings of 2010 IEEE International Symposium on Circuits and Systems (ISCAS), pp.4253,4256, May 30 2010-June 2 2010.
- [8] A.S. Demirkol, and S. Ozoguz, "A low power VLSI implementation of the Izhikevich neuron model", IEEE 9th International New Circuits and Systems Conference (NEWCAS), pp.169-172, 26-29 June 2011.
- [9] P. Dayan, and L. Abbott, "Theoretical Neuroscience : Computational and Mathematical Modeling of Neural Systems", MIT Press, Cambridge, MA, USA, Dec 2001
- [10] E.M. Izhikevich, "Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting". The MIT Press, 2007.
- [11] P.E. Latham, B.J. Richmond, P.G. Nelson, and S. Nirenberg, "Intrinsic dynamics in neuronal networks. I. Theory". Journal of Neurophysiology, 83(2), 808-827, 2000.
- [12] D. Hansel, and G. Mato. "Existence and stability of persistent states in large neuronal networks." Physical Review Letters 86.18: 4175-4178, 2001.
- [13] E. M. Izhikevich, "Which Model to Use for Cortical Spiking Neurons?", IEEE Trans. on Neural Networks, vol. 15, pp. 1063-1070, Sep. 2004.
- [14] M. Shanahan, "Dynamical complexity in small-world networks of spiking neurons", Phys Rev E Stat Nonlin Soft Matter Phys, vol. 78, no. 4, pp.041924:1-7, Oct. 2011.
- [15] D. A. McCormick, B. W. Connors, J. W. Lighthall, and D. A. Prince, "Comparative electrophysiology of pyramidal and sparsely spiny stellate neurons of the neocortex.", J Neurophysiol., 54(4), pp. 782-806, Oct 1985.
- [16] A. Agmon, and B. W. Connors, "Correlation between intrinsic firing patterns and thalamocortical synaptic responses of neurons in mouse barrel cortex.", J Neurosci., vol. 12(1), pp. 319-29, Jan 1992.
- [17] Y. Chagnac-Amitai, and B. W. Connors. "Synchronized excitation and inhibition driven by intrinsically bursting neurons in neocortex." JJ Neurophysiol., vol. 62.5, pp. 1149-1162, 1989.
- [18] P. Schwindt, J. A. O'Brien, and W. Crill. "Quantitative analysis of firing properties of pyramidal neurons from layer 5 of rat sensorimotor cortex." J Neurophysiol., vol. 77.5, pp.2484-2498, 1997.
- [19] B. W. Connors, and M.J. Gutnick. "Intrinsic firing patterns of diverse neocortical neurons." Trends in neurosciences, 13.3, pp. 99-104, 1990.
- [20] Y. Kawaguchi, and S. Kondo, "Parvalbumin, somatostatin and cholecystokinin as chemical markers for specific GABAergic interneuron types in the rat frontal cortex.", Journal of Neurocytology, vol 31, pp. 277-287, 2002.
- [21] A. L. George, K. M. Jacobs, "Altered intrinsic properties of neuronal subtypes in malformed epileptogenic cortex.", Brain Res., vol. 16; pp. 1374:116-28, Feb 2011.
- [22] Y. Chagnac-Amitai, and B. W. Connors, "Horizontal spread of synchronized activity in neocortex and its control by GABA-mediated inhibition.", J Neurophysiol., 61(4):747-58, April 1989.
- [23] G. G. Hwa, and M. Awoli, "NMDA receptor antagonists CPP and MK-801 partially suppress the epileptiform discharges induced by the convulsant drug bicuculline in the rat neocortex.", Neurosci Lett., vol. 98(2), pp. 189-93, Mar 1989.
- [24] D. Cosandier-Rimele, I. Merlet, F. Bartolomei, J. M. Badier, F. Wendling, "Computational modeling of epileptic activity: from cortical sources to EEG signals", J Clin Neurophysiol, vol. 27, pp. 465-470, 2010.
- [25] R. D. Traub, D. Contreras, M. O. Cunningham, H. Murray, F. E. N. LeBeau, A. Roopun, A. Bibbig, W. B. Wilent, M. J. Higley, and M. A. Whittington, "Single-Column Thalamocortical Network Model Exhibiting Gamma Oscillations, Sleep Spindles, and Epileptogenic Bursts", J Neurophysiol, vol. 93, pp. 2194-2232, Nov. 2005.
- [26] B. Strack, K. M. Jacobs, and K. J. Cios, "Study of inhibition influence on epileptic seizures with a network of spiking neurons." Society for Neuroscience Abstracts, 37: 622.25, 2011.
- [27] B. Strack, K.M. Jacobs, and K.J. Cios, "Simulating lesions in multi-layer, multi-columnar model of neocortex ". Neural Engineering (NER), 2013 IEEE EMBS Conference on. IEEE, 2013.