Inductance Calculations for Air Core Foil Wound Transformers

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ABSTRACT

Equations and methods for self and mutual inductance calculations of air core foil wound inductors or transformers are presented. In particular, current distribution and the related effects on inductance and coupling are analyzed and verified by experiments. The method of analysis is based on decomposing cylindrical foils into numerous loops which are characterized by self and mutual inductances. Matrix circuit analysis methods are used to solve the n-loop equations for the current distributions and to approximate a simplified equivalent circuit consisting of two coupled inductors. The effects were studied using two concentric cylinders. It was found that the effects are minimal when the lengths of the cylinders are approximately the same. When the cylinders are of significantly different lengths, large eddy currents develop in the longer cylinder which significantly affect the self and mutual inductances. In all cases the method of analysis agrees with experimental data.

I. INTRODUCTION

The concept of "inductance" is that of relating a flux to a current by a simple constant called "inductance". It is necessary to know or specify the geometry of both the current and the flux. When the geometry of the current and the flux is the same, the term self-inductance is used; when they are different, the term is mutual-inductance. It is necessary to know the inductances of a transformer in order to predict how it will perform in a circuit. Well known methods and procedures exist for calculating inductance when the conductors have small dimensions with respect to the over-all dimensions. For example, the use of a small round wire confines the current and thereby defines the geometry of the current which is necessary to accomplish accurate calculations. The most common method[1] for calculating inductance or flux is to calculate the line integral of the magnetic vector potential due to the current around the perimeter of the area through which the flux is to be evaluated. This was the method used by Lorenz in 1879 to calculate the inductance of a cylindrical current sheet with a uniform current distribution and is the basis of later work by Nagaoka which one finds in tabular form in Grover's[2] book on inductance calculations. The uniform current condition is consistent with a solenoid wound with small constant pitch wire. As a consequence of the uniform current distribution, the flux along the cylinder is non-uniform. The problem being dealt with here is the dual of that problem in that the current distribution along the cylinder is not uniform and in such a way as to result in a uniform flux. This situation is consistent with a cylindrical foil conductor being driven as a single turn with a voltage which is constant along the axis. It is assumed that the time scale of interest is such that no flux penetrates the cylinder thereby imposing the uniform flux and the non-uniform current conditions. Rohwein[3,4] has extensively used double tuned air core transformers wound with foil to charge pulse capacitors or pulse forming networks and it is this particular application which motivated this paper.

II. CALCULATION METHOD

The geometry used for this paper consists of two coaxial cylinders centered on axis as shown in Fig. 1a. Determination of the current distribution by means of integral equations is not feasible, at least by these authors, therefore the problem is approached by a finite element type of solution. The cylinders are sliced into loops of square cross-section and then converted to round cross-sections of equal area as in Fig. 1b. The self and mutual inductances of these equivalent loops are then calculated by means of equations[5] [1] and [2]. In [1] a is the radius of the wire and b is the radius of the loop. In [2] b and c are the respective radii of the two loops and z_{bc} is the axial spacing. The integrals are elliptic and are conveniently evaluated by means of mathematical computer software such as MATHCAD[6] or MATHEMATICA[7]. The results are very conveniently and compactly displayed in matrix notation as L11, L22, M12 and M21.

[1]

$$L_{Loop} := \mu \cdot b \cdot (b-a) \cdot \int_0^{\pi} \frac{\cos(\phi)}{\sqrt{b^2 + (b-a)^2 \cdot (1 - 2\cdot\cos(\phi))}} \, d\phi$$

[2]

$$M_{bc} := \mu \cdot b \cdot c \cdot \int_0^{\pi} \frac{\cos(\phi)}{\sqrt{(b-c)^2 + z_{bc} + 2\cdot b \cdot c \cdot (1 - \cos(\phi))}} \, d\phi$$

These same matrices can be used to solve both uniform and non-uniform current distribution problems depending upon the circuit connections defined by the problem. That is, the individual loops representing each cylinder can be interpreted by the circuit analysis formulation as being either series (uniform current) or parallel (non-uniform current) connected. The matrix equation pair corresponding to the two coupled arrays of loops are:

[3a] $\quad V1 = I1\, L11 - I2\, M12$

and

[3b] $\quad V2 = I1\, M21 - I2\, L22.$

The matrices L11 and L22 are the self inductance matrices of the cylinders, the main diagonal elements are the self-inductances of the loops and the off diagonal elements are the mutual-inductances between the loops respectively. Both matrices are square and symmetric. The matrices M12 and M21 are related by the fact they are transposes of each other. The elements are the mutual inductances between the loops of the two arrays of loops. These matrices are in general not square. Vectors I1 and I2 are the time derivatives of the currents. In keeping with standard transformer analysis, we use the open circuit and short circuit calculations to determine the inductances of an equivalent two inductance coupled circuit and arrive at L1e, L2e and M12e (see Fig. 2). To adapt equations [3a,b] to be consistent with cylindrical conductors, we recognize that both V1 and V2 are row vectors with equal elements respectively. For the short circuit calculations we simply set either V1 or V2 equal to zero. The open circuit case is represented by setting $\Sigma I1=0$ or $\Sigma I2=0$. Note that this does not mean that $I1\equiv 0$ or $I2\equiv 0$. That is, the individual elements of I1 and I2 represent eddy currents which affect the values of the equivalent circuit inductances but since their sum is zero, they do not flow external to the transformer terminals. The short circuit and open circuit current derivatives in matrix notation are given as:

[4a] $\quad I1sc2 = V1[L11 - M21 L22^{-1} M12]^{-1}$

[4b] $\quad I2sc1 = V2[G]$

[4c] $\quad I2oc2 = [V1 L11^{-1} M21 - \Gamma oc2\, V1' N2] G$

[4d] $\quad I1oc2 = [V1 + I2oc2\, M12] L11^{-1}$

where: $G = [M12 L11^{-1} M21 + L22]^{-1}$
V1' = magnitude of V1
N2 = unit row matrix of V2 dimension
I1sc2 = I1 with a short circuit on 2, etc.
$\Gamma oc2 = V2/V1$ (with oc on 2).

There are two more equations for I1oc1 and I2oc1 but they are redundant and not required to find the scalar values L1e, L2e, and M12e, but are required to determine the current distributions. Although the current derivatives are actually what is calculated in equations [4a,b,c,d], the distribution is the same as the current over the time duration of interest if no flux penetrates the conductors. Equations [4a,b,c,d] are related to the equivalent inductances in Fig. 2 as:

[5a] $\quad L1e = V1' / \Sigma\, I1oc2$

[5b] $\quad L2e = L1e\, [1 + V2/\Sigma\, I2sc1] / [1 + \Sigma\, I1sc2]$

[5c] $\quad M12e = [L2e - L1e\, V2 / \Sigma\, I2sc1]^{1/2}$

III CALCULATED AND MEASURED PERFORMACE

To illustrate the calculation method, a coaxial cylindrical transformer is analyzed. The #1 winding is a single turn which has a 0.05834 m mean radius and is 0.06827 m long with a conductor thickness of 800 μm. The #2 winding is axially centered on the #1 winding and is 0.2715 m long, with a 0.0516 m mean radius and conductor thickness of 1626 μm. The calculated current distributions are shown in Fig. 3 for both open and short circuit conditions. In Figs. 3a and 3c, the driven (primary) winding is #1 and in Figs. 3b and 3d winding #2 is driven. The calculated equivalent winding inductances are: Le1 = 31.15 nH, L2e = 63.13 nH, and M12e = 31.16 nH. Verification of these calculated values was not possible by direct measurement because of the inductance magnitudes, bridge range, and stray inductance problems. However, a modified

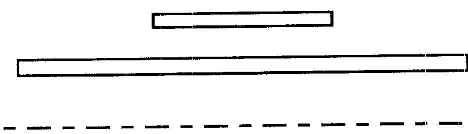

Fig. 1a, Concentric Foils

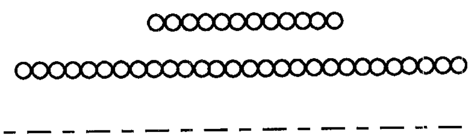

Fig. 1b, Equivalent Loop Configuration

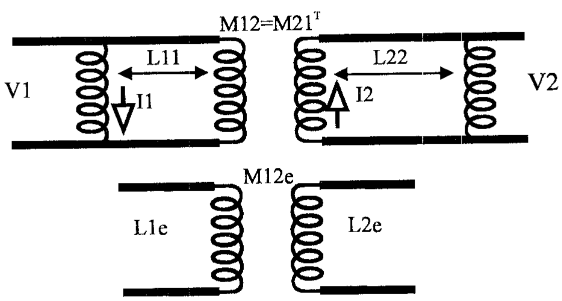

Fig. 2 Multi-Loop Model and Simplified Coupled Inductor Pair Equivalent

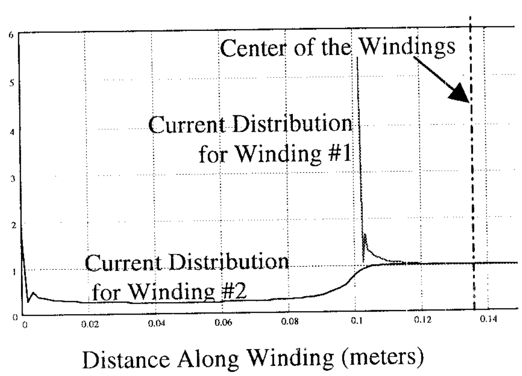

Fig. 3a Current Distribution with #1 Driven and #2 Shorted.

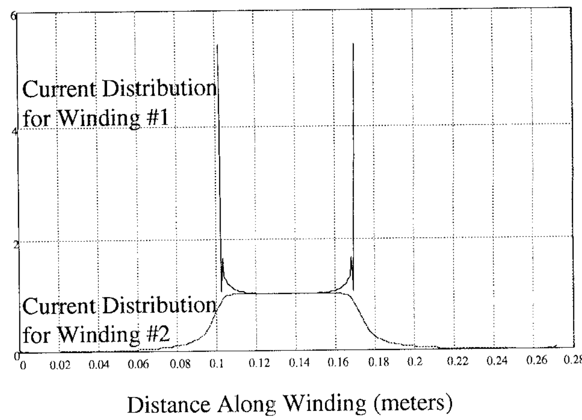

Fig. 3b. Current Distribution with #2 Driven and #1 Shorted.

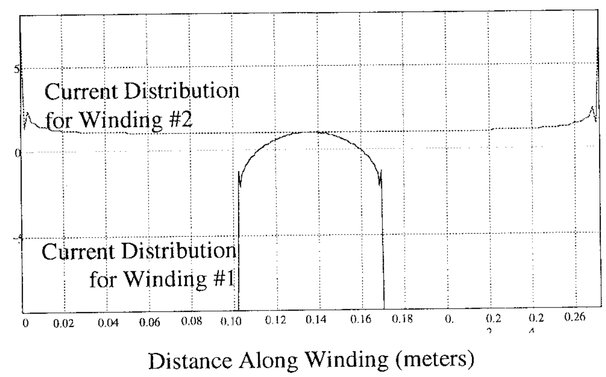

Fig. 3c. Current Distributions with #2 Driven and #1 Open.

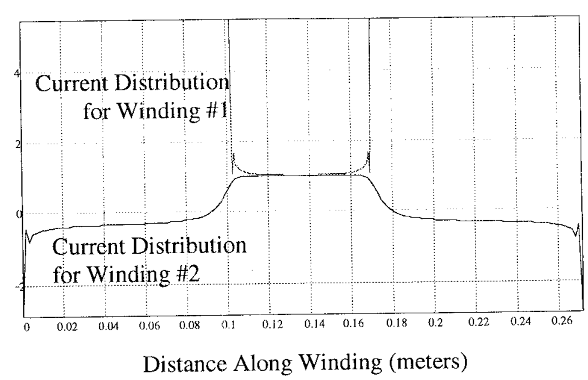

Fig. 3d. Current Distribution with #1 Driven and #2 Open.

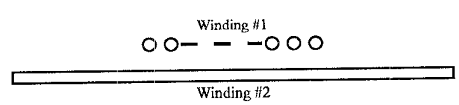

Fig.4 Modified Configuration
Winding #1 Wire Wound, 28 Turns
Winding #2 Foil, 1 Turn

configuration (as in Fig. 4) was fabricated and measured. By replacing the foil winding #1 with a wire wound winding consisting of 28 turns of 800 μm diameter wire with a pitch of 2483 μm, a coil length 0.0683 m, and mean radius 0.05834 m; the calculation method can be experimentally verified. By changing winding #1 to 28 turns of wire we accomplish a step-up of the inductances measured to the 10's of microhenrys range which can be accurately measured. This modified circuit is a combination of a wire winding (uniform current distribution) and a foil winding (non-uniform current distribution). The calculation method is valid for foil-foil, winding-foil, or winding-winding configurations. The objective is to experimentally verify that the method can predict the behavior of a foil winding including the non-uniform current effects. Thus, if one of the windings is foil and the calculations track the measurements, it establishes the viability of the method. To this end four different #2 foils were measured and calculated using the 28 turn wire wound #1 winding (see Fig. 4). The #2 foils were 0.0516 m mean radius with a 1626 μm thick wall. Two lengths were used (0.0683 m and 0.2715 m) and a short circuit and open circuit was used with each length. The open circuit was simply a slit along the axial length of the cylinder. This permitted accurate measurements to be taken on the #1 winding. The free space inductance of winding #1 (i.e. with no #2 winding in place) measured 87.12 μH and calculated 86.58 μH at 1 MHz. Measurements were then made with each of the four #2 windings in place. To carry out the calculations, the long #2 foil was decomposed into 166 sections and the short #2 foil into 41 sections. The calculated and measured values are:

0.2715 m Long #2 Cylinder

	calculated	measured
L#1 with sc#2	27.12 μH	30.7 μH
L#1 with slit#2	51.56 μH	52.6 μH
coupling k	0.688	0.680

0.0683 m Long #2 Cylinder

	calculated	measured
L#1 with sc#2	27.70 μH	31.45 μH
L#1 with slit#2	83.33 μH	81.87 μH
coupling k	0.817	0.785

IV. CONCLUSIONS

The measured self and mutual inductances agree with the calculated values very well when the widths of the primary and secondary windings are about the same. The agreement is still reasonable when the winding widths are vastly different. The circulating or eddy currents are considerable higher under this condition as well as the degree of the non-uniform current distribution and no doubt contribute to the error. It is concluded that the calculation method described in this paper does provide a more accurate method of analysis of foil windings than previous methods in that current distribution and eddy currents in the foils are accounted for. Although the information presented is restricted to single turn windings for simplicity of the presentation, the extension to multi-turn windings is obvious and straight forward and it is to be expected that the results thus obtained will be as accurate as those presented.

V. REFERENCES

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[3] Gerry J. Rohwein, "Design of Pulse Transformers for PFL Charging", Proceedings of the 2nd IEEE International Pulsed Power Conference, Lubbock, TX, June, 1979.

[4] Gerry J. Rohwein, "High Voltage Air Core Pulse Transformers", SAND 80-0451, August, 1984.

[5] William R. Smythe, op cit, p.335, p.339.

[6] MATHCAD is the registered trademark of MathSoft Inc, Cambridge MA.

[7] MATHEMATICA is the registered trademark of Wolfram Research, Champaign, IL.