

# Optimal Scheduling in Frequency-agile Wireless Networks

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**Abstract**—We consider the problem of spectrum-based minimum-length scheduling of point-to-point links in a spatial TDMA (STDMA) wireless network where each node is equipped with a frequency agile radio capable of configuring the bandwidth on a per-slot basis. The problem formulation integrates the activation of multiple sets of links in the network, in a hybrid fashion (both time and frequency domain), while taking into account the Signal-to-Interference and Noise Ratio (SINR) at each receiver. We assume uniform (fixed) transmission power at all nodes and propose an algorithm based on a column generation approach, which allocates a common spectrum band for links that satisfy the SINR constraints at the receivers, and different spectrum bands for interfering links, in order to generate a link schedule that minimizes the schedule length. For the formulated problem, we show that this column generation based approach can converge to a globally optimal solution.

## I. INTRODUCTION

Scheduling in wireless networks has been studied extensively on a graph-theoretic basis, and more recently, using cross-layer formulations by explicitly considering the signal-to-interference and noise ratio (SINR) at the receivers [1]–[3]. In wireless networks, scheduling as a medium access mechanism, avoids collisions and retransmissions due to collisions, which are typical in contention-based access control methods. While Spatial-TDMA [4] based scheduling schemes achieve higher throughput by scheduling multiple transmissions to be simultaneously active in the TDMA time-slots, their spectral efficiency can be further improved by allowing these sets of links to take advantage of spectrum sharing and dynamic spectrum access (DSA), especially when nodes are equipped with frequency-agile radios. In this paper, we develop such an STDMA scheduling scheme, for which the link activation details in each time slot can be described in the following way: links that satisfy their respective signal-to-interference and noise ratio (SINR) constraints are assigned a common spectrum band, while interfering links are allotted their own spectrum bands, subject to their traffic requirements and maximum bandwidth requirements.

Scheduling in wireless ad hoc networks has been considered in [1] by Hajek and Sasaki and in [2], [3] by Ephremides and Truong. Several innovative algorithms have been proposed in the literature for generating minimum-length wireless link schedules. Earlier works in the area of contention-graph based scheduling algorithms [5], [6] overcompensated for interference in the vicinity of transmitting and receiving nodes, and therefore, resulted in a lower throughput than actually possible. Recent works that incorporated the physical interference model

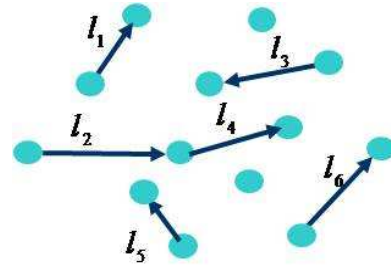


Fig. 1. Wireless network with links shown that need to be scheduled.

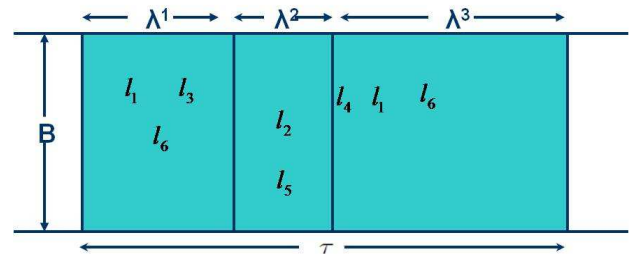


Fig. 2. Traditional Spatial-TDMA based Minimum Length Schedule. Simultaneous link activations based on satisfying SINR constraints, and utilize the common bandwidth  $B$ .

[7], [8] for wireless scheduling are therefore more accurate, although more complicated as well. In the context of STDMA-based scheduling algorithms, Bjorklund *et al.* [9] showed that the basic node and link assignment problems are NP-hard. In their formulation of the so-called node and link assignment optimization problems, the scheduler assigns at least one time slot to each node or link such that the number of time slots is minimized. Using set-covering formulations, the resulting mixed integer programming problems are relaxed into linear programming problems and solved using a column generation approach. However, except in [10], specific traffic demands on links were not taken into account.

Recent advances in software defined radio (SDR) and cognitive radio (CR) technologies have provided a greater flexibility in terms of dynamic allocation and access of the available spectrum in wireless networks, including the dynamic configuration of the channel bandwidth that is allotted to each transmitter. This flexibility has manifested itself in the design of the next generation of wireless standards such as the IEEE 802.16 standard [11], where the channel size can be chosen from a set of fixed profiles that are defined under the standard. Such fixed channelization schemes are a legacy of traditional radio design that focuses on a predefined spectrum band. By

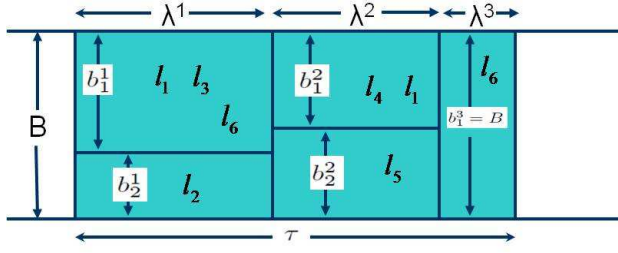


Fig. 3. Spectrum based Minimum Length Schedule.

using such fixed channelization schemes, wireless networks can be easily optimized by using an off-line calculation and allocation of the available spectrum. However, these schemes will not work in the case of ad hoc networks that employ DSA, as the available spectrum at nodes may not be known *a priori*. The availability of spectrum may also vary with users as well (for example, cognitive radio networks). Furthermore, while significant work has been done in the facilitation of dynamic spectrum access, relatively little has been done in the development of efficient link scheduling algorithms that take advantage of spectrum sharing. This paper focuses on addressing this technical void.

#### A. Our Contribution

Our earlier work in the area of wireless link scheduling [12], [13] focused on computing the minimum-length schedule such that certain end-to-end traffic demands are satisfied, all the while ensuring that the SINR threshold is either met or exceeded at the receivers of all active links. As an example, for the wireless network shown in Figure 1 with six links, the corresponding minimum-length schedule is depicted in Figure 2, where multiple links are allowed to transmit simultaneously in each slot, as long as the cumulative transmissions do not violate the SINR threshold criterion at the respective receivers. In this paper, we extend our previous work by exploiting the frequency agility of modern radios, and incorporating the idea of spectrum sharing into dynamic link scheduling. The solution approach of generating the spectrally efficient minimum-length schedule is based on column generation, and concurrently activates not-only links that satisfy the SINR constraints over a common spectrum band, but can also potentially activate some of the interfering links by allocating them different spectrum bands, as shown in Figure 3. The algorithm generates a feasible spectrum sharing policy that not only determines the duration of link activation but the amount of spectrum as well, that is to be utilized by the link whenever it is active. Figures 2 and 3 are explained further in Section II. Note that the schedule length for the spectrum based scheduling scheme is at least as good as that of the traditional scheduling schemes discussed earlier.

The rest of the paper is organized as follows. In Section II, we discuss the network and communication model that is used in the formulation of the spectrum-based minimum-length scheduling problem, which is presented in Section III. We

propose the column-generation based solution procedure in Section IV and discuss the computation of a lower bound in Section V. In Section VI, we provide final concluding remarks.

## II. COMMUNICATION MODEL

We model a multi-hop wireless network as a set of (directed) links  $\mathcal{E}$  and a set of stationary nodes  $\mathcal{N}$ . The set of links  $\mathcal{E}$  constitutes the network topology, i.e., link  $(i, j) \in \mathcal{E}$  exists if node  $i$  can communicate directly with node  $j$ , i.e., the corresponding signal-to-noise ratio (SNR) in the absence of any other interference source exceeds a specific threshold. Therefore, the graph representation of the wireless network is based on whether a node can reach another node when transmitting in isolation for a given power, noise level and channel gain.

We denote  $P_i$  as the transmission power for node  $i$ ,  $G_{ij}$  as the gain of the radio channel between nodes  $i$  and  $j$ , and  $\eta_j$  as the thermal noise at receiver  $j$ . The SINR at receiver  $j$  due to transmission from node  $i$  in the presence of other transmissions is given by:

$$SINR_{ij} = \frac{P_i G_{ij}}{\eta_j + \sum_{k \neq i, j} P_k G_{kj}}. \quad (1)$$

The channel gain  $G_{ij}$  is assumed to be calculated by the widely used free-space model  $G_{ij} = d_{ij}^{-\alpha}$ , where  $d_{ij}$  is the distance between nodes  $i$  and  $j$ , and  $\alpha$  is the path loss index. For each link  $(i, j)$ , we also assume that data is coded separately and the receivers consider unintended receptions as noise (i.e., single-user gaussian channel). Based on these assumptions, the Shannon capacity of link  $(i, j)$  over a frequency band  $b_{ij}$ , is given by:

$$c_{ij} = b_{ij} \log_2(1 + SINR_{ij}). \quad (2)$$

where  $SINR_{ij}$  is the signal-to-interference and noise ratio at the receiver  $j$  of link  $(i, j)$ . However, it is generally understood that most communication schemes will achieve lower rate, which depends on target bit error-rate, modulation and coding schemes. We are not concerned here with the capacity issue and use (2) only selectively for bounding purpose.

Given a set of links  $M$ , all links in  $M$  can be activated concurrently by satisfying the following transmission constraints: (i) such simultaneous activation does not violate the minimum SINR required for communication, i.e., the SINR threshold  $\gamma$  is satisfied at the receivers of all links in  $M$ , i.e.,  $SINR_{ij} \geq \gamma$ , (ii) links that cause interference to other links (i.e., violate the SINR constraints) are de-conflicted by allocated them different spectrum bands, and finally, (iii) links that cause a node to transmit and receive simultaneously are allotted different spectrum bands. We assume that a radio cannot transmit and receive simultaneously in the same band of spectrum. The link set  $M$  satisfying the transmission constraints described above is called a “feasible matching”, or simply, a *matching*.

The communication model that is used in this paper not only considers the SINR constraints at the receivers by accounting for all the secondary transmissions as interference, but specifically uses this information to decide whether a link

should be allowed to share a spectrum band with other links or be assigned a different band. We define a schedule as a finite indexed collection  $\mathcal{S} = (M^s, \lambda^s, \phi^s, s \in Z^+)$ , where the continuous quantity  $\lambda^s \geq 0$  is the duration associated with the matching  $M^s$ , and  $\phi^s = \{b_{ij}^s\}$  is the spectrum policy associated with each  $s$ .

It should be noted that this hybrid time-frequency domain based definitions of the schedule as well as the matching are in contrast to those defined in [12], where only the time domain was taken into consideration. This is clearly evident from comparing Figures 2 and 3. In Figure 2, which depicts a traditional minimum-length schedule, each slot can have multiple links activated simultaneously, with all the links utilizing the entire available bandwidth  $B$ , whereas Figure 3 shows how links can be scheduled in both time and frequency domain, based on the SINR at the receivers and link traffic demands.

The spectrum policy  $\phi$  defines the bandwidth  $b_{ij}^L \leq b_{ij} \leq b_{ij}^U$  that each link  $(i, j)$  is allowed to use, where  $b_{ij}^U$  is the upper bound or the maximum amount of spectrum that the link  $(i, j)$  can be allotted, while  $b_{ij}^L$  is the minimum amount of spectrum that can be allotted. In this paper, we choose  $b_{ij}^L = 0$ . However, any other value can be chosen for this design parameter, which has a direct impact on the number of different spectrum bands that can be allotted in each time slot. Moving on to  $b_{ij}^U$ , in frequency-agile wireless networks where the nodes are capable of DSA, these upper bounds correspond to the available spans of unused spectrum (the so-called *white spaces*) that the secondary users can take advantage of, and they could be different for different users. For the sake of simplicity, in this paper, we assume that the spectrum upper bound is common for all nodes and is given by  $B$  i.e.,  $b_{ij}^U = B, \forall (i, j) \in \mathcal{E}$ . However, our work can be easily extended to the general case as well.

The schedule length  $\tau$  of the schedule  $\mathcal{S}$  is defined as

$$\tau = \sum_s \lambda^s. \quad (3)$$

Each link  $(i, j) \in \mathcal{E}$  has associated with it certain non-negative traffic demand  $f_{ij}$ , that the schedule should satisfy. It should be noted that a link may be active in one or several time slots based on the number of matchings that the link is a part of. The goal is to determine the spectrum sharing policy along with the duration of activation, so that the overall schedule length  $\tau$  is minimized, given the location of the nodes, the link traffic demands and the SINR threshold (discussed in detail in Section IV).

### III. THE SPECTRUM-BASED MINIMUM-LENGTH SCHEDULING PROBLEM

The spectrum-based minimum-length scheduling problem, involves computing the schedule  $\mathcal{S} = (M^s, \lambda^s, \phi^s, s \in Z^+)$  that minimizes the schedule length  $\tau = \sum_s \lambda^s$ , such that the traffic demands  $f_{ij}$  of all the wireless links  $(i, j) \in \mathcal{E}$  are satisfied.

As described in Section II, each link  $(i, j) \in \mathcal{E}$  has a specific traffic demand of  $f_{ij}$  bits per frame that need to be transmitted across the link, where the frame length is not specified *a priori*. As part of the optimal schedule, each matching  $M^s$  indexed by  $s \in Z^+$ , is active for a duration of  $\lambda^s$ , and each link  $(i, j)$  that is part of the matching  $M^s$  transmits at a rate of  $c_{ij}^s$  bits/sec, which is computed based on the amount of spectrum  $b_{ij}$  allocated to it in the slot  $s$ , as described in Equation (2). Thus, a link  $(i, j)$  is active during all the time slots in a frame in which  $(i, j) \in M^s$ , and the overall data that is transmitted in the duration for which the link  $(i, j)$  is active, must be at least  $f_{ij}$ . With minor adjustments to the model, the traffic demand can also be expressed in terms of bit rate rather than bit volume.

For a given wireless network, once the set of all possible feasible matchings denoted by  $\mathcal{M}$  are computed, (i.e., any  $M^s \in \mathcal{M}$ ), the Spectrum-based Minimum-Length Scheduling Problem [SMLS] can be easily formulated as a linear programming (LP) problem having a very simple constraint structure as shown below.

[SMLS]:

$$\text{Minimize: } \tau = \sum_{1 \leq s \leq |\mathcal{M}|} \lambda^s \quad (4)$$

subject to:

$$\begin{aligned} \sum_{1 \leq s \leq |\mathcal{M}|} c_{ij}^s \lambda^s &\geq f_{ij}, \quad \forall (i, j) \in \mathcal{E} \\ \lambda^s &\geq 0, \quad \forall s = 1, \dots, |\mathcal{M}|. \end{aligned} \quad (5)$$

However, the complexity of the problem lies in the computation of the set of all feasible matchings  $\mathcal{M}$ . The total number of matchings that would have to be enumerated in order to compute an optimum may be as large as  $2^{|\mathcal{E}|}$ , with the added complexity that there are infinite ways of assigning the spectrum bands to the links in each matching. Therefore, a straightforward solution of the [SMLS] problem is not computationally efficient. However, this complexity can be reduced by eliminating those matchings from the problem formulation that are inefficient (and thus unlikely to be used in the optimal schedule). For example, matchings where links interfere with most other links, so that they all have to be assigned separate spectrum bands. Based on such observations, one could significantly reduce the number of feasible matchings, thus impacting the complexity of the problem. One could use other heuristic approaches in order to generate valid matchings that have a high probability of being part of the optimal solution.

An alternative approach to explicitly enumerate all the matchings and checking for feasibility would be to solve the [SMLS] problem in such a way that the matchings are computed in an iterative manner so that columns thus generated have a potential to improve the current solution at every iteration. Careful observation of the problem formulation shows that the matchings constitute columns in the linear programming problem. Utilizing an iterative column generation approach, we show in the next section, how the [SMLS]

problem can be solved by selectively enumerating only those matchings that contribute towards an optimum.

#### IV. COLUMN GENERATION BASED SOLUTION PROCEDURE

Column generation is an iterative approach for solving huge linear or integer programming problems, where the number of variables are too large to be considered explicitly. In the column generation approach, the original problem is decomposed into a master problem and a subproblem. The master problem and subproblem could be either linear or integer programs depending on the problem formulation. The strategy of this decomposition procedure is to operate iteratively on two separate, but easier-to-solve, problems. The key idea of the solution approach is to sequentially improve the current solution by solving the subproblem that identifies a single new variable (a column) during every iteration, and adding it to the master problem, until the algorithm terminates at or close to the optimal solution. During each iteration, the algorithm tries to determine whether any variables exist that have a *negative reduced cost* (in the case of a minimization problem) and adds the variable with the “most negative reduced cost” to the master problem.

##### A. Master Problem

The master problem [MP] is a restriction of the original [SMLS] problem, which uses only a subset of columns (matchings) indexed by  $s \in \{1, \dots, |\mathcal{M}|\}$ . Recall that the formulation of the [SMLS] problem includes the entire set of feasible matchings  $\mathcal{M}$ , even though, we know that most of the matchings will not be a part of the optimal solution, and will have duration  $\lambda^s = 0$ . Therefore, [MP] is first initialized with any feasible schedule  $\mathcal{S}$  that satisfies the link demands of all the links in  $\mathcal{E}$ . Section IV-C discusses different ways of initializing the schedule  $\mathcal{S}$ . The master problem [MP] can now be formulated as follows.

[MP]:

$$\text{Minimize: } \tau = \sum_{s \in \mathcal{S}} \lambda^s \quad (6)$$

subject to:

$$\sum_{s \in \mathcal{S}} c_{ij}^s \lambda^s \geq f_{ij}, \quad \forall (i, j) \in \mathcal{E} \quad (7)$$

$$\lambda^s \geq 0, \quad \forall s \in \mathcal{S}.$$

Since this formulation optimizes over a subset  $\mathcal{S}$  of all feasible matchings, it is a restriction of the original [SMLS] problem. Hence, an optimal solution to [MP] provides an upper bound  $UB$  for the [SMLS] problem.

##### B. Generating Feasible Matchings

As discussed earlier, we need to solve the master problem [MP] at the beginning of every iteration of the column generation-based solution procedure, at which point, we need to conclude whether that the current solution is optimal. If this is not the case, we need to identify a *new* matching that, when added to the master problem (i.e., enter into the basis in a revised simplex algorithm), can improve the current solution.

As discussed in Section III, each matching constitutes a single column in the master problem [MP]. Based on the theory of linear programming and the revised simplex algorithm [15], this can be achieved by examining whether any new column (that is not currently in [MP]), has a negative reduced cost.

Let us denoting the dual variables corresponding to (7) by  $(\bar{\omega}_{ij})$ . The reduced cost  $\bar{z}_k$  for any column  $k$  in the master problem [MP] can be expressed as follows:

$$\bar{z}_k = 1 - \sum_{(i,j) \in \mathcal{E}} \bar{\omega}_{ij} c_{ij}^k. \quad (8)$$

Therefore, in order to find a new column having the most negative reduced cost, we solve the subproblem defined as

$$\text{Minimize}_{k \in \mathcal{M} \setminus \mathcal{S}} \bar{z}_k,$$

or equivalently,

$$\text{Maximize}_{k \in \mathcal{M} \setminus \mathcal{S}} \sum_{(i,j) \in \mathcal{E}} \bar{\omega}_{ij} c_{ij}^k. \quad (9)$$

Here, the term  $\mathcal{M} \setminus \mathcal{S}$  refers to the set of all columns that are in  $\mathcal{M}$  and are not a part of  $\mathcal{S}$ . This subproblem can be referred to as the scheduling subproblem, because it aids in identifying a new matching that could be a part of the optimal schedule. Based on the optimal solution to the scheduling subproblem, a non-negative reduced cost implies that current solution to [MP] is indeed the optimal solution to the [SMLS] problem. Otherwise, the new matching that is obtained by solving the subproblem is added to the current schedule  $\mathcal{S}$ , and the master problem [MP] is re-optimized.

We now present the formulation of the sub-problem. Recall that the source nodes of all active links in the matching are set to transmit at their maximum power  $P_{max}$ , with the condition that the SINR of all the active links that share a spectrum band in the matching exceeds a *fixed* threshold  $\gamma$ . Note that those links that violate the SINR constraint are allotted different spectrum bands. Associated with  $\gamma$  is a transmission rate  $c_{ij} = b_{ij} \log_2(1 + \gamma)$  at which the active link  $(i, j)$  would be allowed to transmit.

First, we define  $x_{ij}$  as a binary variable associated with the link  $(i, j)$ , for each  $(i, j) \in \mathcal{E}$ , such that

$$x_{ij} = \begin{cases} 1, & \text{if link } (i, j) \text{ is allotted the common spectrum band} \\ 0, & \text{if link } (i, j) \text{ is allotted a different spectrum band,} \end{cases} \quad (10)$$

Given a set of dual variables  $(\omega_{ij})$  (obtained from the master problem), a new matching can be generated by solving the corresponding subproblem shown below.

[SP]:

$$\text{Maximize: } \sum_{(i,j) \in \mathcal{E}} \omega_{ij} c_{ij}$$

subject to:

$$(\eta_j + \sum_{k \neq i,j} G_{kj} P_{max} - \gamma^{-1} G_{ij} P_{max}) x_{ij} + \sum_{k, m \neq i,j} G_{kj} P_{max} x_{km} \leq \sum_{k \neq i,j} G_{kj} P_{max}, \forall (i,j) \in \mathcal{E} \quad (11)$$

$$\sum_{j:(i,j) \in \mathcal{E}} x_{ij} + \sum_{j:(j,i) \in \mathcal{E}} x_{ji} \leq 1, \forall i \in \mathcal{N} \quad (12)$$

$$c_{ij} = b_{ij} x_{ij} \log_2(1 + \gamma), \quad \forall (i,j) \in \mathcal{E} \quad (13)$$

$$b_{ij} x_{ij} + \sum_{(m,n) \neq (i,j)} b_{mn} (1 - x_{mn}) \leq B, \forall (i,j) \in \mathcal{E} \quad (14)$$

$$x_{ij} \in \{0, 1\}, 0 \leq b_{ij} \leq B \quad \forall (i,j) \in \mathcal{E}.$$

In [SP], the constraint set (11) guarantees that the SINR threshold is satisfied for all active links that share a common spectrum band in the matching. Due to space constraints, some of the details leading to the formulation of constraint set (11), which were already discussed in [12] are not presented here. The constraint set (12) ensures that links that cause a node to transmit and receive simultaneously are allotted different spectrum bands, while the constraint set (14) ensures that the bandwidth constraint is satisfied, that is, sum of the bandwidth allotted to the links whose SINR exceeds  $\gamma$  (i.e., links for which  $x_{ij} = 1$ ), and the bands allotted to other links for which  $x_{ij} = 0$  does not exceed the maximum bandwidth  $B$ .

As it stands, the sub-problem [SP], which is a mixed-integer non-linear programming (MINLP) problem is difficult to solve directly, because of the product terms  $b_{ij} x_{ij}$  in constraints (13) and (14). Therefore, in the rest of this section, we show how these can be relaxed to a linear constraint set.

For the product terms  $b_{ij} x_{ij}$ , we first generate nonlinear implied constraints by taking the products of bounding terms of the decision variables, and subsequently linearize these constraints by the means of variable substitutions, where ever the product term appears in the problem. This automatically creates outer linearizations that approximate the closure of the convex hull of the feasible region.

For instance, we let  $y_{ij} = b_{ij} x_{ij}$ , and since  $b_{ij}$  and  $x_{ij}$  are each bounded by  $0 \leq b_{ij} \leq B$  and  $0 \leq x_{ij} \leq 1$ , we can generate the following relational constraint:

$$[0 \leq b_{ij} \leq B] * [0 \leq x_{ij} \leq 1]$$

Substituting  $y_{ij}$  for the product term  $b_{ij} x_{ij}$ , the following relaxed linear constraints can be generated:

$$\begin{cases} y_{ij} \geq 0 \\ b_{ij} - y_{ij} \geq 0 \\ Bx_{ij} - y_{ij} \geq 0 \\ Bx_{ij} + b_{ij} - y_{ij} \leq B. \end{cases} \quad (15)$$

The resulting subproblem after including the constraint set (15), is a mixed-integer linear problem (MILP) the can be

easily solved using the widely known *branch-and-bound* algorithm which is commonly used to solve a whole plethora of integer programming problems. For the sake of completeness, the branch-and-bound is briefly described in Section IV-D.

#### C. Generating the Initial Feasible Solution

In order to pass down a set of cost coefficients from the master problem [MP] to the subproblem, the initial set of matchings in  $\mathcal{S}$  must provide a feasible solution to the original problem [MLSP]. For this purpose, one can initialize  $\mathcal{S}$  with a set of matchings, where each matching contains exactly one single link that occupies the entire spectrum  $B$ . This corresponds to the traditional pure TDMA scheduling. Other heuristic and greedy approaches can also be applied with the aim of generating a variety of possible matchings.

#### D. Branch-and-Bound Framework

Branch-and-bound is an iterative relaxation algorithm [14], which seeks to provide an  $\epsilon$ -optimal solution to an integer programming (IP) problem by partitioning the original solution space into smaller sub-hyperrectangles [14], and thereby solving the smaller sub-problems. Here  $\epsilon$  denotes an arbitrarily small, prescribed constant reflecting our tolerance for the optimality of the final solution. In branch-and-bound, the original integer problem is first relaxed using a suitable *relaxation technique* to obtain an easier-to-solve, lower-bounding problem. In our approach, we choose to relax the integer variables  $x_{ij} \in \{0, 1\}$  to continuous variables  $0 \leq x_{ij} \leq 1$ , in order to obtain a linear programming (LP) relaxation of the MILP problem (see Section IV-B). The optimal solution to this LP relaxation provides an upper bound  $UB$  for the original (maximization) problem. During each iteration, the original problem, is partitioned into sub-problems, each having a smaller feasible solution space, based on the solution provided by the LP relaxation. This branching process is carried out recursively to obtain two new sub-problems at each sub-problem of the *branch-and-bound tree*. The partitioning of the original solution space, i.e., the *branching rule* is carried out on the  $x_{ij}$  variables with fractional values in the optimal solution of the LP-relaxation. The best integer solution found so far in terms of the  $x_{ij}$  variables, provides the lower bound  $LB$  for the original (maximization) problem. The algorithm terminates when the lower bound  $LB$  is within  $\epsilon$  of the upper bound  $UB$ .

### V. LOWER BOUND

During the execution of the column generation algorithm, in order to get an idea of how close the algorithm is to finding the optimal solution, and to provide a means for terminating the algorithm, we need to evaluate the lower bound and the upper bound at each iteration. Since the master problem discussed in Section IV-A has the current best solution at every iteration, it provides the upper bound  $UB$  for the [SMLS] problem. In this section, we discuss how to compute the lower bound  $LB$ .

It should be noted that, for large problems, we can terminate the procedure once the optimality gap (gap between the lower

and upper bounds) approaches a value within  $\epsilon\%$ , instead of solving to absolute optimality (the tail-end convergence rate in obtaining the optimal solution can be very slow). It should also be noted that the lower bounds generated based on the discussion above need not be monotone, and one should therefore maintain the best (highest) lower bound in order to compute the optimality gap at every iteration.

*Proposition 1:* Let us consider any restricted master problem [MP], and let the optimal dual solutions obtained with respect to (6) be given by  $\bar{\omega} \equiv (\bar{\omega}_{ij}, (i, j) \in \mathcal{E})$ , with an optimal objective value  $\nu$ . Then a lower bound  $LB$  for [SMLS] is given by

$$LB = \frac{\bar{\omega}^T \mathbf{f}}{1 - \nu}.$$

*Proof:* We have, by duality, that any dual solution  $(\omega)$  to constraint set (7) in [SMLS] must satisfy the following:

$$\begin{aligned} \sum_{\{i,j\} \in \mathcal{E}} \omega_{ij} c_{ij}^s &\leq 1, \quad \forall s \\ \omega_{ij} &\geq 0, \quad \forall \{i, j\} \in \mathcal{E}. \end{aligned} \quad (16)$$

Using the dual vectors  $\bar{\omega}_{ij}$  obtained via the master problem [MP], and solving the subproblem we compute a new column having the most negative reduced cost, i.e.,

$$\begin{aligned} \nu &= \text{Min} \left\{ 1 - \sum_{\{i,j\} \in \mathcal{E}} \bar{\omega}_{ij} c_{ij}^s, \right. \\ &\Rightarrow \sum_{\{i,j\} \in \mathcal{E}} \bar{\omega}_{ij} c_{ij}^s \leq (1 - \nu), \quad \forall s, \end{aligned} \quad (17)$$

where  $\nu \leq 0$ . Moreover, by duality in [MP], we have that

$$\bar{\omega}_{ij} \geq 0, \quad \forall \{i, j\} \in \mathcal{E}.$$

From Equations (16) and (17), we conclude that  $\omega_{ij}$ , given by,

$$\omega_{ij} = \frac{\bar{\omega}_{ij}}{1 - \nu}, \quad \forall \{i, j\} \in \mathcal{E}$$

is a dual feasible solution to [SMLS], with the objective value given by

$$\omega^T \mathbf{f} = \frac{\bar{\omega}^T \mathbf{f}}{1 - \nu}, \quad (18)$$

which yields a lower bound  $LB$  for [SMLS]. ■

## VI. CONCLUSION

In this paper, we reviewed the problem of spectrum-based minimum-length scheduling in wireless networks that employ frequency-agile radios. We formulated the problem as a cross-layer optimization problem with consideration of simultaneous activations at the link layer and spectrum sharing at the physical layer. We proposed a solution procedure based on column generation, and showed that this method not only actually converges to an optimal solution, but also provides a control on the final optimality gap achieved, if required.

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