

Degrees of Freedom of Completely-Connected Multi-Way Interference Networks

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Abstract—This paper considers a fully-connected interference network with a relay in which multiple users equipped with a single antenna want to exchange multiple unicast messages with other users in the network by sharing the relay equipped with multiple antennas. For such a network, the optimal degrees of freedom (DoF) are derived by providing both converse and achievability. Further, considering single-antenna relays in the three-user fully-connected interference network, it is shown that three distributed relays with a single antenna is sufficient to achieve the optimal DoF. A major implication of the derived DoF results is that a relay with multiple antennas or distributed relays employing a single antenna increases the capacity scaling law of the multi-user interference network when multiple directional information flows are considered, even if the networks are fully-connected and all nodes operate in half-duplex. These results verify the intuition that the relay is useful in increasing DoF for the multi-way the interference network.

I. INTRODUCTION

Multi-way communication using an intermediate relay is a promising wireless network architecture with applications including cellular networks, sensor networks, and device-to-device communications. The simplest multi-way relay network model is the two-way relay channel [1]-[3] in which a two-user pair wish to exchange messages by sharing a single relay. While the capacity of this simple channel is still unknown in general, it has been shown that physical layer network coding [2] and analog network coding [3] increase the achievable sum-rates of the two-way relay channels because they allow users to exploit their transmit signal as side-information. Recently, the two-way relay channel has been generalized in a number of ways: multi-pair two-way relay channels [4] and multi-user multi-way relay channels [5]-[7].

In spite of extensive studies on multi-way relay channels, relatively little work has addressed on the understanding of their capacity, especially when the nodes are fully-connected in the network due to difficulty in managing interference. Note that when the direct links are considered in the multi-way relay channel, it can be viewed equivalently as an interference network with a relay. In general, if the networks are fully-connected, i.e., a non-layered structure, then a node receives signals arriving along different paths, which causes more inter-user interference than that of the layered network. In particular, for the fully-connected interference network with relays, it has been shown that the relay *cannot* improve the DoF of such a network regardless of how many antennas the relay has [8]. In this paper, we provide counter examples of the claim

that the relay cannot increase the DoF of the fully-connected interference network. In other words, the relay is *useful* in improving the DoF of the multi-user interference networks when multi-way information exchange is considered between users.

In this paper, we consider a fully-connected interference network with a relay where K users with a single antenna exchange unicast messages with each other via a relay with N multiple antennas. We refer to this network as a fully-connected Y channel as a generalized network model of [7]. In particular, we assume that all nodes have half-duplex constraint due to hardware limitations, implying that transmission and reception occurs in different orthogonal time slots.

The main contribution of this paper is to characterize the optimal DoF for a fully-connected Y channel. Specifically, it is demonstrated that the sum-DoF of $\frac{K(K-1)}{2K-2} = \frac{K}{2}$ is the optimal if the relay has $N \geq K - 1$ antennas. Both achievability and the converse are provided. Further, as an extension, by considering three distributed relays each of which has a single antenna, it turns out that the three distributed relays are sufficient to achieve the optimal sum-DoF for the three-user fully-connected Y channel. One major implication of our results is that the available DoF of fully-connected interference network that supports multi-directional information exchange can be improved substantially by allowing a relay with multiple antennas or allowing multiple distributed relays even if all nodes operate in *half-duplex*. The DoF gain comes from two mechanisms. One is the side-information inherently given by multi-way communication, i.e., *caching gain*. The second is due to the fact that the relay can make sure each user does not see an undesired interference signal by using the proposed space-time interference neutralization. We refer to it as *interference shaping gain*. To acquire two different gains, the multiple antenna relay or multiple relays with a single antenna controls the information flow of the multi-way communication so that each user exploits side-information efficiently, which leads to increase the DoF by the use of a relay.

II. SYSTEM MODEL

Let us consider an interference network comprised of K users with a single antenna each and a relay with N antennas. All the users and the relay are completely-connected as illustrated in Fig. 1. User k , $k \in \mathcal{U} \triangleq \{1, 2, \dots, K\}$, wants to

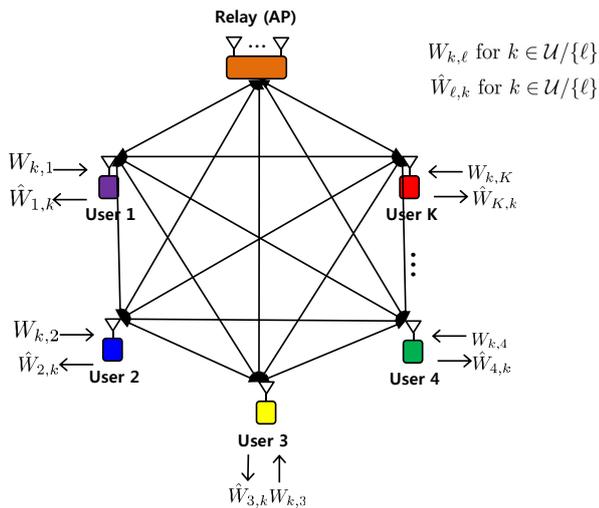


Fig. 1. K -user fully-connected Y channel. User ℓ wants to send $K - 1$ messages $W_{k,\ell}$ and receive $\hat{W}_{\ell,k}$ for $k \in \mathcal{U}/\{\ell\}$ in this network.

send $K - 1$ unicast messages $W_{\ell,k}$ for $\ell \in \mathcal{U}/\{k\} \triangleq \mathcal{U}_k^c$ to user ℓ and intends to decode $K - 1$ messages $W_{k,\ell}$ for $\ell \in \mathcal{U}_k^c$ sent by all other users. In this channel, it is assumed that the relay and all nodes operate in half-duplex mode, implying that transmission and reception span orthogonal time slots.

Let $x_\ell[n] = f(W_{k,\ell})$ for $k \in \mathcal{U}_\ell^c$ be the transmitted signal by user ℓ at time slot n where $f(\cdot)$ represents an encoding function. Also, let \mathcal{S}_n and \mathcal{D}_n denote the set of source and destination nodes at time slot n . Due to the fully-connected property and the half-duplex constraint, when the users belonging to the source set \mathcal{S}_n send their signals at the n -th time slot simultaneously, user $k \in \mathcal{D}_n$ and the relay receives the signals

$$y_k[n] = \sum_{\ell \in \mathcal{S}_n} h_{k,\ell}[n]x_\ell[n] + z_k[n], \quad k \in \mathcal{D}_n, \quad (1)$$

$$\mathbf{y}_R[n] = \sum_{\ell \in \mathcal{S}_n} \mathbf{h}_{R,\ell}[n]x_\ell[n] + \mathbf{z}_R[n], \quad (2)$$

where $y_k[n]$ and $\mathbf{y}_R[n] \in \mathbb{C}^{N \times 1}$ represent the received signal at user k and the relay; $z_k[n]$ and $\mathbf{z}_R[n]$ denote the additive noise signal at user k and at the relay at time slot n whose elements are Gaussian random variable with zero mean and unit variance, i.e., $\mathcal{CN}(0, 1)$; and $h_{k,\ell}[n]$ and $\mathbf{h}_{R,\ell}[n] = [h_{R,\ell}^1[n], \dots, h_{R,\ell}^N[n]]$ represent the channel coefficients from user ℓ to user k and the channel vector from user ℓ to the relay, respectively.

When the relay and user $\ell \in \mathcal{S}_n$ cooperatively transmit at the n -th time slot, at the same time, user $k \in \mathcal{D}_n$ receives the signal as

$$y_k[n] = \sum_{\ell \in \mathcal{S}_n} h_{k,\ell}[n]x_\ell[n] + \mathbf{h}_{k,R}^*[n]\mathbf{x}_R[n] + z_k[n], \quad k \in \mathcal{D}_n, \quad (3)$$

where $\mathbf{h}_{j,R}^*[n] = [h_{j,R}^1[n], \dots, h_{j,R}^N[n]]^*$ denotes the (downlink) channel vector from the relay to user k and $\mathbf{x}_R[n]$ represents

the transmit signal vector at the relay when the n -th channel is used.

The transmit power at each user and the relay is assumed to be P , i.e., $\mathbb{E}[|x_j[n]|^2] \leq P$ and $\mathbb{E}[\|\mathbf{x}_R[n]\|_2^2] \leq P$. Further, it is assumed that all the entries of all channel values in $h_{\ell,k}[n]$, $\mathbf{h}_{R,\ell}[n]$, and $\mathbf{h}_{k,R}^*[n]$ are drawn from a continuous distribution and the absolute value of all the channel coefficients is bounded between a nonzero minimum value and a finite maximum value. The channel state information (CSI) is assumed to be perfectly known at the users in receiving mode and the relay has global channel knowledge for all links.

User k sends an independent message $W_{\ell,k}$ for one intended user ℓ with rate $R_{\ell,k}(P) = \frac{\log_2 |W_{\ell,k}|}{n}$ for $\ell, k \in \mathcal{U}$ and $\ell \neq k$, a rate tuple $\mathcal{R} = (R_{1,2}, R_{1,3}, \dots, R_{K,K-1}) \in \mathbb{R}^{K(K-1)}$ is achievable if every receiver can decode the desired message with an error probability that is arbitrarily small with sufficient channel uses n . Then, the sum-DoF characterizing the approximate sum-rates in the high SNR regime is defined as

$$d_{\text{sum}} = \sum_{k=1, k \neq \ell}^K \sum_{\ell=1}^K d_{k,\ell} = \lim_{P \rightarrow \infty} \frac{\sum_{k=1, k \neq \ell}^K \sum_{\ell=1}^K R_{k,\ell}(P)}{\log(P)}. \quad (4)$$

III. DOF OF K -USER FULLY-CONNECTED Y CHANNEL

In this section, we derive the DoF of K -user fully-connected channel. The following theorem is the main result of this paper.

Theorem 1: For the fully-connected Y channel where K users have a single antenna and a relay has $N \geq K - 1$ antennas, the maximum sum-DoF equals $\frac{K}{2}$.

A. Converse

We provide the converse of Theorem 1 by using the cut-set theorem [9]. Using the fact that user cooperation does not deteriorate the DoF of the channel, let us first consider a special case where all users except user k fully cooperate. This cooperation allows us to view the fully-connected Y channel as a two-way relay channel equivalently where the user group has $K - 1$ antennas but user 1 has a single antenna as illustrated in Fig 2. Further, let us set the messages to be null between cooperating users, i.e., $W_{i,j} = \phi$ for $i, j \in \mathcal{U}_1^c$, by the using the fact that the null-messages cannot degrade the performance of the non-null messages. In this two-way relay channel, the user group wants to send the message $W_{1,k}$ for $k \in \mathcal{U}_1^c$ and user 1 wants to send the messages $W_{k,1}$ for $k \in \mathcal{U}_1^c$. The converse follows from the following lemma which serves an outer bound of the equivalent two-way relay channel.

Lemma 1: Let $d_{k,\ell}$ be the DoF for message $W_{k,\ell}$ for $k, \ell \in \mathcal{U}$. Then, the following inequality holds:

$$\sum_{\ell=1, \ell \neq k}^K d_{k,\ell} + \sum_{k=1, k \neq \ell}^K d_{k,\ell} \leq 1, \quad \text{for } k, \ell \in \mathcal{U}. \quad (5)$$

Proof: We would like to defer the rigorous proof into our full paper [10] due to space limitation.

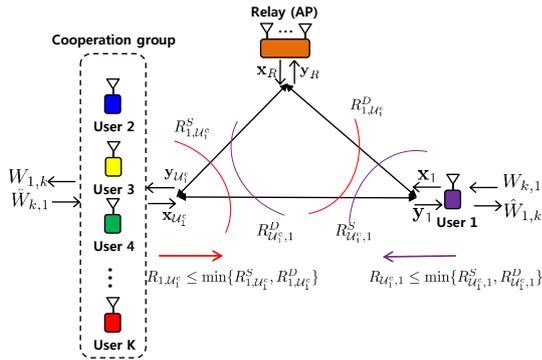


Fig. 2. A user cooperation scenario when all users except for user 1 cooperate with each other by sharing the messages and antennas.

To attain the converse result of Theorem 1, we add K inequalities from Lemma 1, which gives us

$$2 \left(\sum_{\ell \neq k}^K \sum_{k=1}^K d_{\ell,k} \right) \leq K \Rightarrow \sum_{\ell \neq k}^K \sum_{k=1}^K d_{\ell,k} \leq \frac{K}{2}, \quad (6)$$

which completes the proof. \square

B. Achievability

Due to page limitation, we provide achievability of Theorem 1 when $K = 3$ and $N = 2$. For the general proof, please see the journal version of this paper [10].

1) *Phase One (Round-Robin Multiple-Access Channel (MAC))*: This phase comprises of three time slots. In each channel use, two users send one message to one intended user so that the intended user has one equation that contains two desired data symbols. Specifically, at time slot 1, user 2 and 3 send information symbols $s_{1,2}$ and $s_{1,3}$ for user 1. While user 1 and the relay listen the signals, user 2 and 3 do not receive any signals in this time slot due to half-duplex constraint, i.e., $\mathcal{S}_1 = \{2, 3\}$ and $\mathcal{D}_1 = \{1, R\}$. When noise is ignored, user 1 and the relay have

$$D_1[1] = h_{1,2}[1]s_{1,2} + h_{1,3}[1]s_{1,3}, \quad (7)$$

$$\mathbf{D}_R[1] = \mathbf{h}_{R,2}[1]s_{1,2} + \mathbf{h}_{R,3}[1]s_{1,3}. \quad (8)$$

Since the relay has two antennas, it resolves the transmitted data symbols $s_{1,2}$ and $s_{1,3}$ by using a zero-forcing (ZF) decoder.

In the second time slot, user 1 and user 3 transmit data symbols $s_{2,1}$ and $s_{2,3}$ to user 2, i.e., $\mathcal{S}_2 = \{1, 3\}$ and $\mathcal{D}_2 = \{2, R\}$. The received equations at user 1 and the relay are

$$D_2[2] = h_{2,1}[2]s_{2,1} + h_{2,3}[2]s_{2,3}, \quad (9)$$

$$\mathbf{D}_R[2] = \mathbf{h}_{R,1}[2]s_{2,1} + \mathbf{h}_{R,3}[2]s_{2,3}. \quad (10)$$

By taking an advantage of multiple antennas, the relay decodes $s_{2,1}$ and $s_{2,3}$.

Finally, at time slot 3, user 1 and user 2 deliver data symbols $s_{3,1}$ and $s_{3,2}$ to user 3, i.e., $\mathcal{S}_3 = \{1, 2\}$ and $\mathcal{D}_3 = \{3, R\}$.

Hence, the received equation at user 3 and the relay are given by

$$D_3[3] = h_{3,1}[3]s_{3,1} + h_{3,2}[3]s_{3,2}, \quad (11)$$

$$\mathbf{D}_R[3] = \mathbf{h}_{R,1}[3]s_{3,1} + \mathbf{h}_{R,2}[3]s_{3,2}. \quad (12)$$

Similarly, the relay obtains $s_{3,1}$ and $s_{3,2}$ by using a ZF decoder. As a result, during the phase one, each user obtains one equation consisted of two desired symbols and the relay acquires all six independent data symbols in the network.

2) *Phase Two (Relay Broadcast)*: The second phase spans one time slot. In this time slot, the relay sends a superposition of six data symbols obtained during the phase one, i.e., $\mathcal{S}_4 = \{R\}$ and $\mathcal{D}_4 = \{1, 2, 3\}$. The transmitted signal at the relay is given by

$$\mathbf{x}_R[4] = \sum_{i=1}^3 \sum_{j=1, j \neq i}^3 \mathbf{v}_{i,j}[4]s_{i,j}, \quad (13)$$

where $\mathbf{v}_{i,j}[4] \in \mathbb{C}^{2 \times 1}$ denotes the beamforming vector used for carrying symbol $s_{i,j}$ at time slot 4. The main design principle of $\mathbf{v}_{i,j}[4]$ is to control the interference propagation on the network so that each user receives an equation that consists of desired data symbols or self interference data symbols which can be eliminated by using side-information at each user. For instance, user 1 wants to receive an additional equation consisted of $s_{1,2}$ and $s_{1,3}$ and can cancel the self-interference signals caused by $s_{2,1}$ and $s_{3,1}$ by exploiting side-information. Thus, the relay picks the beamforming vectors $\mathbf{v}_{2,3}[4]$ and $\mathbf{v}_{3,2}[4]$ carrying $s_{2,3}$ and $s_{3,2}$ so that user 1 does not receive them. To accomplish this, $\mathbf{v}_{2,3}[4]$ and $\mathbf{v}_{3,2}[4]$ are selected as

$$\mathbf{v}_{2,3}[4] \in \text{null}(\mathbf{h}_{1,R}^*[4]) \text{ and } \mathbf{v}_{3,2}[4] \in \text{null}(\mathbf{h}_{1,R}^*[4]). \quad (14)$$

Applying the same principle, the other relay beamforming vectors are designed as

$$\mathbf{v}_{1,3}[4] \in \text{null}(\mathbf{h}_{2,R}^*[4]), \quad \mathbf{v}_{3,1}[4] \in \text{null}(\mathbf{h}_{2,R}^*[4]), \quad (15)$$

$$\mathbf{v}_{1,2}[4] \in \text{null}(\mathbf{h}_{3,R}^*[4]), \text{ and } \mathbf{v}_{2,1}[4] \in \text{null}(\mathbf{h}_{3,R}^*[4]). \quad (16)$$

To give some intuition on the proposed precoding solution, we rewrite the transmit signal at the relay as

$$\mathbf{x}_R[4] = \mathbf{v}_1^c[4](s_{2,3} + s_{3,2}) + \mathbf{v}_2^c[4](s_{1,3} + s_{3,1}) + \mathbf{v}_3^c[4](s_{1,2} + s_{2,1}), \quad (17)$$

where $\mathbf{v}_i^c[4] \in \text{null}(\mathbf{h}_{i,R}^*[4])$. Thus, we can interpret the transmitted signal at the relay at the second phase as a class of *superposition coding*.

3) *Decoding*: We explain a decoding method used by user 1. Recall that user 1 received an equation consisting of two desired symbols $s_{1,2}$ and $s_{1,3}$ at time slot 1 in the form of $D_1[1] = h_{1,2}[1]s_{1,2} + h_{1,3}[1]s_{1,3}$. In time slot 4, user 1 obtained an equation containing both two desired and two self-interference data symbols given by

$$\begin{aligned} y_1[4] &= \mathbf{h}_{1,R}^*[4]\mathbf{x}_R[4], \\ &= \mathbf{h}_{1,R}^*[4] \{ \mathbf{v}_2^c[4](s_{1,3} + s_{3,1}) + \mathbf{v}_3^c[4](s_{1,2} + s_{2,1}) \}. \end{aligned} \quad (18)$$

Assuming that user 1 preserves the transmitted information symbols $s_{2,1}$ and $s_{3,1}$ by caching memory as side-information and it has the effective channel $\mathbf{h}_{1,R}^*[4]\mathbf{v}_i^c[4]$ for $i \in \{1, 2, 3\}$, user 1 can generate an interference equation $M_1[4] = \mathbf{h}_{1,R}^*[4]\mathbf{v}_2^c[4]s_{3,1} + \mathbf{h}_{1,R}^*[4]\mathbf{v}_3^c[4]s_{2,1}$. Thus, user 1 extracts one equation that contains two desired symbols by eliminating the self-interference as

$$y_1[4] - M_1[4] = \mathbf{h}_{1,R}^*[4]\mathbf{v}_2^c[4]s_{1,2} + \mathbf{h}_{1,R}^*[4]\mathbf{v}_3^c[4]s_{1,3}. \quad (19)$$

From self-interference cancellation, we acquired a new equation for the two desired symbols. Therefore, the two desired symbols are obtained by solving the following matrix equation,

$$\begin{bmatrix} y_1[1] \\ y_1[4] - M_1[4] \end{bmatrix} = \underbrace{\begin{bmatrix} h_{1,2}[1] & h_{1,3}[1] \\ \mathbf{h}_{1,R}^*[4]\mathbf{v}_2^c[4] & \mathbf{h}_{1,R}^*[4]\mathbf{v}_3^c[4] \end{bmatrix}}_{\tilde{\mathbf{H}}_1} \begin{bmatrix} s_{1,2} \\ s_{1,3} \end{bmatrix}. \quad (20)$$

Note that beamforming vectors were selected independently of the direct channel between users, i.e. $h_{1,2}[1]$ and $h_{1,3}[1]$. Therefore, the rank of the effective channel becomes 2 with probability one, which allows user 1 to decode two desired symbols $s_{1,2}$ and $s_{1,3}$. For user 2 and user 3, the same method applies. Consequently, it is possible to exchange a total of six data symbols within 4 channel uses by using the relay employing multiple antennas in the 3-user fully-connected Y channel, which completes the proof. \square

Remark 1 (The required CSIT knowledge): To achieve the optimal DoF, while CSIT at users is not needed, the users require to know the effective channel value $\mathbf{h}_{j,R}^*[4]\mathbf{v}_j^c[4]$ for performing self-interference cancellation. This channel knowledge can be easily obtained from demodulation reference signals when a multi-carrier system is applied by the channel coherence property in the frequency domain. Alternatively, CSIT at the relay plays an important role to attain the DoF gains.

Remark 2 (Comparison with a four-user fully-connected X network): Let us consider a four-user, fully-connected, and half-duplex X network where each user wants to exchange three unicast messages with the other users. Since no relays is involved in this network, the optimal DoF of such a channel equals $\frac{4}{3}$ as shown in [8], which can be achieved by interference alignment. Meanwhile, our result shows that a total of $\frac{K}{2} = 2$ DoF are achievable by the use of a relay employing $N = 3$ antennas, which is a 50% DoF improvement.

IV. DISTRIBUTED SINGLE-ANTENNA RELAYS

In this section, we consider a fully-connected 3-user Y channel with three distributed relays each has a single antenna as depicted in Fig. 3. For a such channel, we will establish the following theorem.

Theorem 2: For the 3-user fully-connected Y channel with three distributed single-antenna relays, the optimal $\frac{3}{2}$ of DoF are achievable.

Proof: Since the converse argument is direct from Theorem 1, the achievability is shown by the proposed *space-time interference neutralization*. The first phase comprises three time slots, i.e., $T_1 = \{1, 2, 3\}$. For $k \in T_1$ time slot, all users

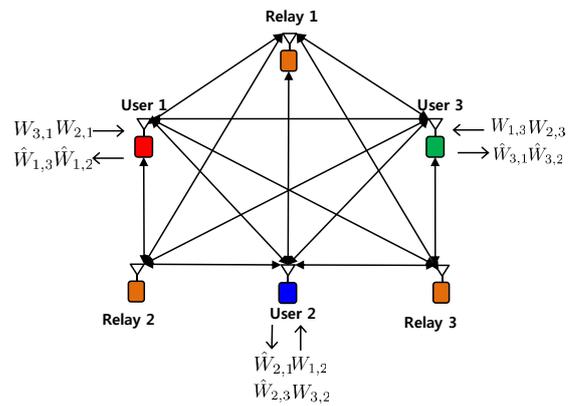


Fig. 3. 3-user fully-connected Y channel with three distributed relays employing a single antenna.

except for user k transmit signals intended for user k . Thus, the received signals at user k and relay $n \in \{1, 2, 3\}$ are given by

$$y_k[k] = \sum_{\ell=1, \ell \neq k}^3 h_{k,\ell}[k]s_{k,\ell} + z_k[k], \quad (21)$$

$$y_R^n[k] = \sum_{\ell=1, \ell \neq k}^3 h_{R,\ell}^n[k]s_{k,\ell} + z_R^n[k]. \quad (22)$$

During phase one, each user k has one desired equation and each relay has three equations. In the second phase, one time slot is used for the relay transmission, $T_2 = \{4\}$. During time slot 4, the three relays cooperatively send signals to the users based on what they obtained during the first phase. The transmitted signal at the n -th relay is given by

$$x_R^n[4] = \sum_{k=1}^3 v^n[k]y_R^n[k], \quad (23)$$

where $v^n[k]$ denotes the precoding coefficient used at the n -th relay for the k -th time slot observation $y_R^n[k]$. When the three relays send at time slot 4, the received signal at user j is given by

$$\begin{aligned} y_j[4] &= \sum_{n=1}^3 h_{j,R}^n[4]x_R^n[4] = \sum_{n=1}^3 h_{j,R}^n[4] \sum_{k=1}^3 v^n[k]y_R^n[k], \\ &= \sum_{n=1}^3 h_{j,R}^n[4] \sum_{k=1}^3 v^n[k] \left(\sum_{\ell=1, \ell \neq k}^3 h_{R,\ell}^n[k]s_{k,\ell} \right), \end{aligned} \quad (24)$$

$$= \mathbf{h}_{j,R}^*[4]\mathbf{V}^R \begin{bmatrix} \mathbf{H}_{R,\mathcal{U}_1^c}[1] & \mathbf{H}_{R,\mathcal{U}_2^c}[2] & \mathbf{H}_{R,\mathcal{U}_3^c}[3] \end{bmatrix} \begin{bmatrix} \mathbf{s}_{1,\mathcal{U}_1^c} \\ \mathbf{s}_{2,\mathcal{U}_2^c} \\ \mathbf{s}_{3,\mathcal{U}_3^c} \end{bmatrix}, \quad (25)$$

where $\mathbf{h}_{j,R}^*[4] = [h_{j,R}^1[4], h_{j,R}^2[4], h_{j,R}^3[4]] \in \mathbb{C}^{1 \times 3}$ denotes the channel vector from the three relays to user $j \in \{1, 2, 3\}$ at time slot 4; $\mathbf{V}^R \in \mathbb{C}^{3 \times 3}$ denotes a space-time relay network coding matrix at time slot 4 whose (n, k) element is defined as $\mathbf{V}^R(n, k) = [v^n[k]]$; $\mathbf{H}_{R,\mathcal{U}_k^c}[k] \in \mathbb{C}^{3 \times 2}$ denotes the effective

channel matrix from user group \mathcal{U}_k^c to the three relays at time slot $k \in T_1$; $\mathbf{s}_{j,\mathcal{U}_j^c}$ represents the desired symbol vector at user j , which comes from user group \mathcal{U}_j^c . Note that user j receives a linear combination of six symbols from the relays. Let us decompose these six symbols into three terms: two desired symbols $\mathbf{s}_{j,\mathcal{U}_j^c}$, two self-interference symbols $\mathbf{s}_{\mathcal{U}_j^c,j}$, and two inter-user interference symbols $\mathbf{s}_{I_j^c} = [s_{k,\ell}]$ where $k \neq j$ or $\ell \neq j$. Then, let define a permutation matrix $\mathbf{P}_j \in \mathbb{Z}^{6 \times 6}$ that changes the order of transmitted symbols such that

$$\begin{bmatrix} \mathbf{s}_{1,\mathcal{U}_1^c} \\ \mathbf{s}_{2,\mathcal{U}_2^c} \\ \mathbf{s}_{K,\mathcal{U}_3^c} \end{bmatrix} = \mathbf{P}_j \begin{bmatrix} \mathbf{s}_{j,\mathcal{U}_j^c} \\ \mathbf{s}_{\mathcal{U}_j^c,j} \\ \mathbf{s}_{I_j^c} \end{bmatrix}. \quad (26)$$

Using the permutation matrix, we can rewrite the equation in (25) as

$$\begin{aligned} y_j[4] &= \mathbf{h}_{j,R}^*[4] \mathbf{V}^R [\mathbf{H}_{R,\mathcal{U}_1^c}[1] \ \mathbf{H}_{R,\mathcal{U}_2^c}[2] \ \mathbf{H}_{R,\mathcal{U}_3^c}[3]] \mathbf{P}_j \begin{bmatrix} \mathbf{s}_{j,\mathcal{U}_j^c} \\ \mathbf{s}_{\mathcal{U}_j^c,j} \\ \mathbf{s}_{I_j^c} \end{bmatrix}, \quad (27) \\ &= \mathbf{h}_{j,R}^*[4] \mathbf{V}^R \mathbf{A}_j \mathbf{s}_{j,\mathcal{U}_j^c} + \mathbf{h}_{j,R}^*[4] \mathbf{V}^R \mathbf{B}_j \mathbf{s}_{\mathcal{U}_j^c,j} + \mathbf{h}_{j,R}^*[4] \mathbf{V}^R \mathbf{C}_j \mathbf{s}_{I_j^c}, \quad (28) \end{aligned}$$

where the effective channel matrices $\mathbf{A}_j \in \mathbb{C}^{3 \times 2}$, $\mathbf{B}_j \in \mathbb{C}^{3 \times 2}$, and $\mathbf{C}_j \in \mathbb{C}^{3 \times 2}$ are defined as

$$[\mathbf{A}_j \ \mathbf{B}_j \ \mathbf{C}_j] = [\mathbf{H}_{R,\mathcal{U}_1^c}[1] \ \mathbf{H}_{R,\mathcal{U}_2^c}[2] \ \mathbf{H}_{R,\mathcal{U}_3^c}[3]] \mathbf{P}_j. \quad (29)$$

To eliminate interference signal $\mathbf{s}_{I_j^c}$ for user j , the relays cooperatively design *space-time relay network coding matrix* \mathbf{V}^R so that the following interference neutralization condition is satisfied,

$$\mathbf{h}_{j,R}^*[4] \mathbf{V}^R \mathbf{C}_j = \mathbf{0}_{1 \times 2}, \quad \text{for } j \in \mathcal{U}. \quad (30)$$

To solve the matrix equations in (30) for all K users, we convert them into vector forms by exploiting Kronecker product operation property, $\text{vec}(\mathbf{A}\mathbf{X}\mathbf{B}) = (\mathbf{B}^T \otimes \mathbf{A})\text{vec}(\mathbf{X})$. The combined vector form of interference neutralization in (30) is given by

$$\underbrace{\begin{bmatrix} \mathbf{C}_1^T \otimes \mathbf{h}_{1,R}^*[4] \\ \mathbf{C}_2^T \otimes \mathbf{h}_{2,R}^*[4] \\ \mathbf{C}_3^T \otimes \mathbf{h}_{3,R}^*[4] \end{bmatrix}}_{\mathbf{F} \in \mathbb{C}^{8 \times 9}} \underbrace{\vec{\mathbf{v}}^R}_{9 \times 1} = \mathbf{0}_{8 \times 1}. \quad (31)$$

where $\vec{\mathbf{v}}^R = \text{vec}(\mathbf{V}^R)$ is the vector representation of relay beamforming matrix \mathbf{V}^R by stacking the column vectors of it. Because the elements of the channels are drawn from a continuous random variable and the size of the unified system matrix \mathbf{F} is 8×9 , \mathbf{F} has a null subspace almost surely. Therefore, the relay beamforming vector eliminating all interference signals on the network are obtained as

$$\vec{\mathbf{v}}^R \in \text{null}(\mathbf{F}). \quad (32)$$

By reshaping the vector solution $\vec{\mathbf{v}}^R$ into a matrix, we obtain the network-wise space-time relay precoding matrix \mathbf{V}^R .

Last, let us consider the decoding procedure at user 1. Recall that user 1 received an equation consisting of two desired symbols $\mathbf{s}_{1,\mathcal{U}_1^c} = [s_{1,2} \ s_{1,3}]^T$ in time slot 1. In addition, in time

slot 4, user 1 received a signal from the relay in the form of $y_1[4] = \mathbf{h}_{1,R}^*[4] \mathbf{V}^R \mathbf{A}_1 \mathbf{s}_{1,\mathcal{U}_1^c} + \mathbf{h}_{1,R}^*[4] \mathbf{V}^R \mathbf{B}_1 \mathbf{s}_{\mathcal{U}_1^c,1}$. Since user 1 has knowledge of $\mathbf{s}_{\mathcal{U}_1^c,1}$, it cancels known interference symbols from $y_1[4]$. Thus, the input-output relationship is given by

$$\begin{bmatrix} y_1[1] \\ y_1[4] - \mathbf{h}_{1,R}^*[4] \mathbf{V}^R \mathbf{B}_1 \mathbf{s}_{\mathcal{U}_1^c,1} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{h}_{1,\mathcal{U}_1^c}[1] \\ \mathbf{h}_{1,R}^*[4] \mathbf{V}^R \mathbf{A}_1 \end{bmatrix}}_{\tilde{\mathbf{H}}_1} \mathbf{s}_{1,\mathcal{U}_1^c}, \quad (33)$$

where $\mathbf{h}_{1,\mathcal{U}_1^c}[1] = [h_{1,2}[1], h_{1,3}[1]]$. Since the effective channel matrix $\tilde{\mathbf{H}}_1$ has full rank two, user 1 decodes two desired symbols. By the symmetry, user 2 and 3 obtain their desired information symbols as well. As a result, it is possible to achieve $\frac{3}{2}$ DoF. This completes the proof. \square

V. CONCLUSION

In this paper, we studied a fully-connected interference network with relay nodes. By considering the multi-way information flows, we characterized the sum-DoF for the network by yielding converse based on the cut-set theorem and achievability using the proposed multi-phase transmission scheme. From the derived DoF result, we verified the intuition that the relay is useful in increasing the DoF of multi-user interference network when the multi-way information flows are considered, even if the relay and users operate in half-duplex. This DoF increase is due to two gains: the caching gain that inherently given by multi-way communications and the interference shaping gain by the space-time relay transmission that cancels the interference signals.

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