The diversity-multiplexing tradeoff of the MIMO Z interference channel

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***Abstract*—The fundamental diversity-multiplexing tradeoff (DMT) of the quasi-static fading, MIMO Z interference channel (ZIC), with M_1 and M_2 antennas at the transmitters and N_1 and N_2 antennas at the corresponding receivers, respectively, is derived. Channel state information at the transmitters (CSIT) and a short-term average power constraint is assumed. The achievability of the DMT is proved by showing that a simple Gaussian superposition coding scheme can achieve a rate region which is within a constant (independent of signal-to-noise ratio (SNR)) number of bits from an upper bound to the capacity region of the ZIC. We also characterize an achievable DMT of the ZIC with No-CSIT and show that in a small region of multiplexing gains (MG), the full CSIT DMT of the ZIC can be achieved with no CSIT at all. The size of this MG region depends on the system parameters such as the number of antennas at the four nodes (referred to hereafter as "antenna configuration"), SNRs and interference-to-noise ratio (INR) of the direct and cross links. Interestingly, for some antenna configurations this MG region covers the entire MG region of the ZIC. Thus, under these circumstances, the optimal DMT of the MIMO ZIC with F-CSIT is same as that of a corresponding ZIC with No-CSIT and availability of CSIT can not further improve the DMT. Finally, we identify a class of ZICs with $M_1 = M_2 = M \leq \frac{N_1}{2}$, $N_1 \leq N_2$ and SNR $\leq$ INR where the achievable DMT with No-CSIT coincides with the optimal DMT with F-CSIT.**

I. Introduction

ZICs emerge as the natural information theoretic model for several practical wireless communication scenario such as femto-cells [1]. Also, the ZIC is a special case of the 2-user IC. Thus optimal (in some metric) coding and decoding schemes on a ZIC might reveal useful insights for the 2-user IC also. For instance, the optimal DMT of the ZIC is an upper bound for the 2-user MIMO IC (with and without CSIT). These facts make the analysis of the ZIC an important step towards a better understanding of the general multiuser wireless system. Motivated by the aforementioned facts in this paper we analyze the DMT of the MIMO ZIC. However, unlike the DMT framework in a point-to-point (PTP) channel, [2] where there is a single communication link which can be characterized by a single SNR, in a multiuser setting such as the one at hand, it is only natural to allow the SNRs and INRs of different links to vary with different exponentials with respect to (w.r.t.) a nominal SNR, denoted as ρ. This technique was first used in [3] to analyze the DoF region which the authors referred to as the Generalized DoF (GDoF) region, of the 2-user SISO IC. Later, this technique was extended to the DMT scenario of the SISO IC in [4] and [5]. Following similar approach, we allow the differerent INR and SNRs at the receivers to vary exponentially with respect to (w.r.t.) ρ with different scaling factors. We refer to the corresponding DMT as the generalized DTM (GDMT) to distinguish it from the case when SNR=INR in all the links.

In this paper, we first derive the DMT of the MIMO ZIC with CSIT and arbitrary number of antennas at each node. The computation of the DMT of the MIMO ZIC involves the asymptotic joint eigenvalue distribution of two specially correlated random Wishart matrices which was recently derived by the authors in [6] in a different context. Using this distribution result, the fundamental DMT of the MIMO ZIC channel with CSIT is established as the solution of a convex optimization problem. While it is argued that in general the optimization problem can be solved using numerical methods, closed-form solutions are computed for several special cases. We also characterize an achievable DMT of the MIMO ZIC with No-CSIT. Comparing this result with the DMT with CSIT, we found that for some specific system parameters, these two DMTs are identical. Encouraged by this result, we find a class of MIMO ZICs, with $M_1 = M_2 = M \leq \frac{N_1}{2}$, $N_1 \leq N_2$ and SNR $\leq$ INR, where the achievable GDMT with No-CSIT coincides with the GDMT with CSIT.

An early work in this direction is [7], where the authors derive an achievable DMT on a SISO ZIC with No-CSIT. The optimal DMT of the SISO ZIC with F-CSIT can be obtained from [8], where the optimal DMT (F-CSIT) of the 2-user SISO IC was derived. In this work, we focus on the MIMO case. In [9], an upper bound to the DMT of a 2-user MIMO IC was derived for the case in which all nodes have same number of antennas and the direct and cross links have the same SNRs and INRs, respectively. This result, if specialized for the MIMO ZIC, provides only an upper bound. Our result will prove that for the special case considered in [9], this upper bound is actually tight on a ZIC with F-CSIT. Further, the result of this paper on MIMO ZIC is much more general, in the sense that we consider arbitrary number of antennas at each node and arbitrary scaling parameters for the different SNRs and the INR of the system.

Notations: We denote the conjugate transpose of the matrix A as $A^\dagger$ and its determinant as $|A|$. $\mathbb{C}$ and $\mathbb{R}$ represent the field of complex and real numbers, respectively. The set of real numbers $\{x \in \mathbb{R} : a \leq x \leq b\}$ will be denoted by $[a, b]$. Furthermore, $(x \wedge y)$, $(x \vee y)$ and $(x)^+$ represent the minimum of x and y, the maximum of x and y, and the maximum of x and 0, respectively. All the logarithms in this

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paper are with base 2. We denote the distribution of a complex circularly symmetric Gaussian random vector with zero mean and covariance matrix Q as $\mathcal{CN}(0, Q)$. If $\mathcal{R}$ represents a set of points in $\mathbb{R}^2$ then $\mathcal{R} \pm (c_1, c_2) = \{(R_1 \pm c_1, R_2 \pm c_2) : (R_1, R_2) \in \mathcal{R}\}$, and finally, any two functions $f(\rho)$ and $g(\rho)$ of ρ, where ρ is the signal to noise ratio (SNR) defined later, are said to be exponentially equal and denoted as $f(\rho) \doteq g(\rho)$ if, $\lim_{\rho\to\infty} \frac{\log(f(\rho))}{\log(\rho)} = \lim_{\rho\to\infty} \frac{\log(g(\rho))}{\log(\rho)}$. The same is true for $\dot{\geq}$ and $\dot{\leq}$.

II. Channel Model and Preliminaries

We consider a MIMO ZIC as shown in Figure 1, where user 1 (Tx_1) and user 2 (Tx_2) have M_1 and M_2 antennas and receiver 1 (Rx_1) and 2 (Rx_2) have N_1 and N_2 antennas, respectively. This channel will be referred hereafter as a (M_1, N_1, M_2, N_2) ZIC. $H_{ij} \in \mathbb{C}^{N_j \times M_i}$ is the channel matrix between Tx_i and Rx_j. H_{11}, H_{21} and H_{22} are mutually independent and contain mutually independent and identically distributed (i.i.d.) $\mathcal{CN}(0, 1)$ entries. Following [5], we also incorporate a real-valued attenuation factor for the signal transmitted from Tx_i to the receiver Rx_j (denoted as η_{ij}). At time t, Tx_i chooses a vector $X_{it} \in \mathbb{C}^{M_i \times 1}$ and sends $\sqrt{P_i} X_{it}$ over the channel, where for the input signals we assume the following short term average power constraint:

$$\text{tr}(Q_{it}) \leq M_i, \forall\ i = 1, 2, \ \text{where}\ Q_{it} = \mathbb{E}\left(X_{it} X_{it}^{\dagger}\right). \quad (1)$$

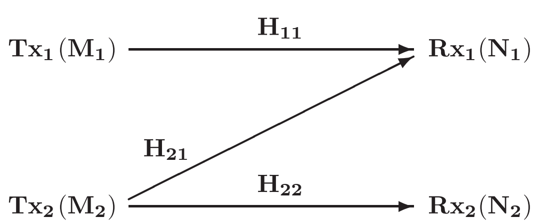

Fig. 1: Channel model for the Z IC.

With these aforementioned assumptions, the received signals at time t can be written as

$$Y_{1t} = \eta_{11}\sqrt{P_1} H_{11} X_{1t} + \eta_{21}\sqrt{P_2} H_{21} X_{2t} + Z_{1t},$$
$$Y_{2t} = \eta_{22}\sqrt{P_2} H_{22} X_{2t} + Z_{2t},$$

where $Z_{it} \in \mathbb{C}^{N_i \times 1}$ are i.i.d as $\mathcal{CN}(\mathbf{0}, I_{N_i})$ across i and t. The above equations can be equivalently written in the following form.

$$Y_{1t} = \sqrt{\text{SNR}_{11}} H_{11} \hat{X}_{1t} + \sqrt{\text{INR}_{21}} H_{21} \hat{X}_{2t} + Z_{1t}; \quad (2)$$
$$Y_{2t} = \sqrt{\text{SNR}_{22}} H_{22} \hat{X}_{2t} + Z_{2t}, \quad (3)$$

where the normalized inputs $\hat{X}_i$s satisfy equation (1) with equality and SNR_{ii} and INR_{ji} is the signal-to-noise ratio and interference-to-noise ratio at receiver i. In the analysis that follows, we will assume the following scaling parameters (with respect to a nominal SNR, ρ) for the different SNRs and INR.

$$\alpha_{11} = \frac{\log(\text{SNR}_{11})}{\log(\rho)},\ \alpha_{22} = \frac{\log(\text{SNR}_{22})}{\log(\rho)}, \quad (4)$$
$$\alpha_{21} = \frac{\log(\text{INR}_{21})}{\log(\rho)}. \quad (5)$$

For ease of notations, in the sequel, we use the following notations: $\text{SNR}_{ii} = \rho_{ii}$, $\text{INR}_{21} = \rho_{21}$, $H = \{H_{11}, H_{21}, H_{22}\}$, $\bar{\rho} = [\rho_{11}, \rho_{21}, \rho_{22}]$ and $\bar{\alpha} = [\alpha_{11}, \alpha_{21}, \alpha_{22}]$.

To define the DMT notations we follow [2]. We assume that user i uses a coding scheme $\mathscr{C}_i$ and is operating at a rate $R_i = r_1 \log(\rho)$ bits per channel use. Let us denote $\mathscr{C} = \{\mathscr{C}_1, \mathscr{C}_2\}$. The diversity order of the ZIC with coding scheme $\mathscr{C}$ and rates (R_1, R_2) is defined as follows

$$d_{ZC}(r_1, r_2, \mathscr{C}) = \lim_{\rho\to\infty} \frac{\log\left(P_e(\bar{\rho})\right)}{\log(\rho)}, \quad (6)$$

where $P_{e_i}(\bar{\rho})$ represents the probability of error (averaged over channel statistics) at receiver i and $P_e(\bar{\rho}) = (P_{e_1}(\bar{\rho}) \vee P_{e_2}(\bar{\rho}))$. Finally, the optimal DMT of the ZIC, denoted as $d^*_{ZC}(r_1, r_2)$, is defined as follows

$$d^*_{ZC}(r_1, r_2) = \max_{\mathscr{C} \in \widetilde{\mathscr{C}}} d_{ZC}(r_1, r_2, \mathscr{C}), \quad (7)$$

where $\widetilde{\mathscr{C}}$ represents the collection of all coding schemes that uses CSIT and short term power constraint (equation (1)). Note the diversity order $d^*_{ZC}(r_1, r_2)$ is a function of the relative scaling parameters of the different links $\bar{\alpha}$. However, for brevity of notation, we will not mention them explicitly.

A. Approximate capacity region

In this subsection, we will first find a set of upper bounds to the rate region of the MIMO ZIC. Next, we will propose a simple superposition coding scheme, which can achieve a rate region within constant (independent of SNR and channel coefficients) number of bits to the set of upper bounds. This approximate capacity result will then be used to derive the fundamental DMT in the next section. Note that most proofs are omitted in this paper due to space constraints but will be reported in a full-length journal version of this paper [10].

Lemma 1: The capacity region of the 2-user MIMO ZIC (of Figure 1) with F-CSIT, for a given realization of channel matrices H, denoted by $\mathcal{C}(H, \bar{\rho})$, is outer bounded by the rate region

$$\mathcal{R}^c(H, \bar{\rho}) + (N_1 \log(M_1 \vee M_2), N_2 \log(M_1 \vee M_2)),$$

where $\mathcal{R}^c(H, \bar{\rho})$ represents the set of rate pairs (R_1, R_2) such that $R_1, R_2 \geq 0$ and satisfies the following constraints:

$$R_i \leq \log\left|\left(I_{N_i} + \rho_{ii} H_{ii} H_{ii}^{\dagger}\right)\right| \triangleq I_{bi},\ i \in \{1, 2\};$$
$$R_1 + R_2 \leq \log\left|\left(I_{N_1} + \rho_{21} H_{21} H_{21}^{\dagger} + \rho_{11} H_{11} H_{11}^{\dagger}\right)\right|$$
$$\log\left|\left(I_{M_2} + \rho_{22} H_{22}\left(I_{M_2} + \rho_{21} H_{21}^{\dagger} H_{21}\right)^{-1} H_{22}^{\dagger}\right)\right| \triangleq I_{b3}.$$

In what follows, we find an achievable rate region. Consider a coding scheme where the first transmitter uses a random

Gaussian code book and the second user uses a superposition code as follows

$$X_2 = U_2 + W_2, \tag{8}$$

where U_2 (hereafter mentioned as the private part of the message) and W_2 (common part of the message) are mutually independent complex Gaussian random vectors with covariance matrices as follows:

$$\mathbb{E}(X_1X_1^\dagger) = I_{M_1}, \quad \mathbb{E}(W_2W_2^\dagger) = \frac{I_{M_2}}{2} \text{ and}$$
$$\mathbb{E}(U_2U_2^\dagger) = \frac{1}{2}\left(I_{M_2} + \rho^{\alpha_{21}}H_{21}^\dagger H_{21}\right)^{-1}. \tag{9}$$

Remark 1: Note that this covariance split satisfies the power constraint in equation (1).

Lemma 2: For a given channel realization H, the above described coding scheme can achieve the following rate region

$$\mathcal{R}^c(H, \bar{\rho}) - (2N_1, 2N_2),$$

where $\mathcal{R}^c(H, \bar{\rho})$ is as given in Lemma 1.

Using Lemma 1 and 2, respectively and similar method as in the proof of Theorem 2 in [2] it can be proved that (for more details refer to [10])

$$d_{IC}^*(r_1, r_2) = \min_{i\in\mathcal{I}} d_{O_i}(r_i), \tag{10}$$
$$\text{where } \rho^{-d_{O_i}(r_i)} \doteq \Pr\left(I_{bi} \le r_i\right), \tag{11}$$

for $i \in \mathcal{I} = \{1, \cdots, 7\}$ and $r_3 = r_4 = r_5 = (r_1 + r_2)$, $r_6 = (2r_1 + r_2)$ and $r_7 = (r_1 + 2r_2)$.

III. Explicit DMT of the ZIC

In this section we will evaluate the $d_{O_i}(r_i)$'s given in equation (10) which would yield the explicit DMT expressions for the ZIC. Using the first and second bound of Lemma 1 in equations (11) it can be proved that

$$d_{O_i}(r_i) = \alpha_{ii} d_{M_i,N_i}\left(\frac{r_i}{\alpha_{ii}}\right),$$
$$\forall\, r_i \in [0,\ (M_i \wedge N_i)\alpha_{ii}] \text{ and } i \in \{1, 2\},$$

where $d_{m,n}(r)$ is the optimal diversity order of a point-to-point (PTP) MIMO channel with m transmit and n receive antennas. To evaluate $d_{O_3}(r_3)$, we write the bound I_{b3} of Lemma 1 in the following way

$$I_{b3} = \log\left|\left(I_{M_1} + \rho_{11}H_{11}^\dagger\left(I_{N_1} + \rho_{21}H_{21}H_{21}^\dagger\right)^{-1}H_{11}\right)\right|$$
$$+ \log\left|\left(I_{M_2} + \rho_{22}H_{22}\left(I_{M_2} + \rho_{21}H_{21}^\dagger H_{21}\right)^{-1}H_{22}^\dagger\right)\right|$$
$$+ \log\left|\left(I_{N_1} + \rho_{21}H_{21}H_{21}^\dagger\right)\right|,$$
$$\overset{(a)}{=} \left\{\sum_{i=1}^{p}(1 + \rho^{\alpha_{21}}\lambda_i) + \sum_{j=1}^{q_1}(1 + \rho^{\alpha_{11}}x_j) + \sum_{k=1}^{q_2}(1 + \rho^{\alpha_{22}}y_k)\right\},$$

where in step (a), we denoted the ordered non-zero (w.p.1) eigenvalues of $W_1 = H_{11}^\dagger\left(I_{N_1} + \rho_{21}H_{21}H_{21}^\dagger\right)^{-1}H_{11}$, $W_2 = H_{22}\left(I_{M_2} + \rho_{21}H_{21}^\dagger H_{21}\right)^{-1}H_{22}^\dagger$ and $W_3 = H_{21}H_{21}^\dagger$ as $x_1 \ge \cdots \ge x_{q_1} > 0$, $y_1 \ge \cdots \ge y_{q_2} > 0$ and $\lambda_1 \ge \cdots \ge \lambda_p > 0$, respectively, and $p = (M_2 \wedge N_1)$, $q_1 = (M_1 \wedge N_1)$ and $q_2 = (M_2 \wedge N_2)$. Now putting $\lambda_i = \rho^{-\alpha_i}$, for $1 \le i \le p$, $x_j = \rho^{-\beta_j}$, for $1 \le j \le q_1$ and $y_k = \rho^{-\gamma_k}$, $1 \le k \le q_2$ in the above equation and putting it in equation (11) we get

$$\rho^{-d_{O_3}(r_1+r_2)} = \Pr\left(\left\{\sum_{i=1}^{p}(\alpha_{21} - \alpha_i)^+ + \sum_{j=1}^{q_1}(\alpha_{11} - \beta_j)^+ + \sum_{k=1}^{q_2}(\alpha_{22} - \gamma_k)^+\right\} < (r_1 + r_2)\right).$$

To evaluate this expression we need to derive the joint distribution of $\vec{\gamma}, \vec{\beta}$ and $\vec{\alpha}$ where $\vec{\gamma} = \{\gamma_1, \cdots, \gamma_{q_2}\}$ and similarly $\vec{\alpha}$ and $\vec{\beta}$. However, note that W_1, W_2 and W_3 are not independent and hence neither are $\vec{\gamma}, \vec{\beta}$ and $\vec{\alpha}$. Using Theorems 1 and 2 of [6] this distribution can be computed (see [10] for more details). Now using this joint distribution, equation (11) and a similar argument as in [2], we have

$$d_{O_3}(r_1 + r_2) = \min_{\vec{\alpha}, \vec{\gamma}, \vec{\beta}} \mathbb{O}_f\left(\vec{\alpha}, \vec{\gamma}, \vec{\beta}, \bar{\alpha}\right)$$

subject to the following constraints:

$$\sum_{i=1}^{p}(\alpha_{21} - \alpha_i) + \sum_{j=1}^{q_1}(\alpha_{11} - \beta_j) + \sum_{k=1}^{q_2}(\alpha_{22} - \gamma_k) < (r_1 + r_2);$$
$$0 \le \alpha_1 \le \cdots \le \alpha_p \le \alpha_{21}; 0 \le \beta_1 \le \cdots \le \beta_{q_1} \le \alpha_{11};$$
$$0 \le \gamma_1 \le \cdots \le \gamma_{q_2} \le \alpha_{22}; (\alpha_i + \beta_j) \ge \alpha_{21},\ \forall (i + j) \ge (N_1 + 1);$$
$$(\alpha_i + \gamma_k) \ge \alpha_{21},\ \forall (i + k) \ge (M_2 + 1),$$

where $\mathbb{O}_f\left(\vec{\alpha}, \vec{\gamma}, \vec{\beta}, \bar{\alpha}\right)$ is given in equation (12). Following similar arguments as in [6], it can be easily proved that for arbitrary M_1, M_2, N_1 and N_2 this is a convex optimization problem and hence can be solved using numerical methods. However, in what follows, we will provide closed-form solutions for two specific antenna configurations. In the first example, we consider the case where $M_1 = M_2 = N_1 = N_2$.

Lemma 3: Consider the MIMO ZIC as shown in Figure 1, with $M_1 = M_2 = N_1 = N_2 = n$ and SNRs and INRs of different links are as described in Section II with $\alpha_{21} = \alpha$ and $\alpha_{22} = 1 = \alpha_{11}$. The DMT with F-CSIT (and short term average power constraint, (1)) of this channel at multiplexing gain pair (r_1, r_2) is given by

$$d_{ZC_1}^*(r_1, r_2) = \min\left\{d_{n,n}(r_1), d_{n,n}(r_2), d_z(r_1 + r_2)\right\} \tag{13}$$

where if $\alpha \le 1$,

$$d_z(r) = \begin{cases} \alpha d_{n,3n}(\frac{r}{\alpha}) + 2n^2(1-\alpha), & \text{for } 0 \le r \le n\alpha; \\ 2(1-\alpha)d_{n,n}(\frac{(r-n\alpha)}{2(1-\alpha)}), & \text{for } n\alpha \le r \le n(2-\alpha). \end{cases}$$

and if $1 \le \alpha$,

$$d_z(r) = \begin{cases} d_{n,3n}(r) + n^2(\alpha - 1), & 0 \le r \le n; \\ (\alpha - 1)d_{n,n}\left(\frac{r-n}{\alpha-1}\right), & n \le r \le n\alpha. \end{cases} \tag{14}$$

Remark 2: Note that for $\alpha = 1$, the optimal DMT becomes $d_{ZC_1}^*(r_1, r_2) = \min\left\{d_{n,n}(r_1), d_{n,n}(r_2), d_{n,3n}(r_1 + r_2)\right\}$ which is exactly the upper bound derived in [9].

Remark 3: Note from Figure 2 that, for low multiplexing gains the optimal DMT is dominated by the terms

$$\mathbb{O}_f\left(\vec{\alpha},\vec{\gamma},\vec{\beta},\bar{\alpha}\right)=\sum_{i=1}^{p}(M_2+N_1+M_1+N_2+1-2i)\alpha_i+\sum_{j=1}^{q_1}(M_1+N_1+1-2j)\beta_j+\sum_{k=1}^{q_2}(M_2+N_2+1-2k)\gamma_k$$
$$-(M_1+N_2)(M_2\wedge N_1)\alpha_{21}+\sum_{k=1}^{q_2}\sum_{i=1}^{(M_2-k)\wedge N_2}(\alpha_{21}-\alpha_i-\gamma_k)^{+}+\sum_{j=1}^{q_1}\sum_{i=1}^{(N_1-j)\wedge M_1}(\alpha_{21}-\alpha_i-\beta_j)^{+};\quad (12)$$

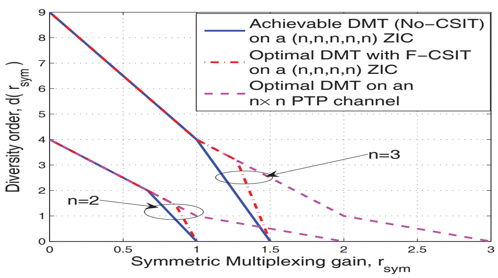

Fig. 2: Comparison of optimal DMT on ZIC with $\bar{\alpha}=[1,1,1]$ to PTP performance.

$d_{n,n}(r_i),\ i\in\{1,2\}$, i.e., single user point-to-point (PTP) DMT is optimal. This should be compared to a MIMO MAC channel, where there is a low multiplexing gain region where the single user performance can be obtained with No-CSIT. This suggests that, if the first receiver of the ZIC is made to operate as a MAC receiver then in an appropriately defined LMG region the optimal DMT of the IC can be attained with no CSIT at all.

Consider a decoder (at the interfered receiver) which does joint maximum-likelihood (ML) decoding of both the messages. However, the event where only the second user's message is decoded incorrectly is not considered as an error event. Hereafter, we will refer to this decoder as the *Individual ML (IML) decoder*. We also assume that both the transmitters use random Gaussian codes. The DMT of such a system is given by the following.

Lemma 4: On a MIMO ZIC as in Lemma 3, the IML decoder can achieve the following DMT

$$d^{IML}_{(n,n,n)}(r_1,r_2)=\min\{d_{n,n}(r_1),d_{n,n}(r_2),d_z(r_1+r_2)\}$$

where if $\alpha\le 1,\ d_z(r)=$

$$\begin{cases}\alpha d_{n,2n}(\frac{r}{\alpha})+n^2(1-\alpha),\ 0\le r\le k_0\alpha;\\ \alpha d_{n,2n}(\frac{(r-(k-k_0)(1-\alpha))}{\alpha})+(1-\alpha)(n-k+k_0)^2,\ r\in\mathcal{S}_1;\\ \alpha d_{n,2n}(n-k+1)+(1-\alpha)d_{n,n}(\frac{(r-(k+1)\alpha)}{(1-\alpha)}),\quad r\in\mathcal{S}_2;\\ (1-\alpha)d_{n,n}(\frac{(r-(k+1)\alpha)}{(1-\alpha)}),k_0\alpha+(n-k_0)\le r\le n,\end{cases}$$

for $\mathcal{S}_1=[k\alpha,\ (k+1)\alpha]$ and $\mathcal{S}_2=[(k+1)\alpha,\ k\alpha+1]$, $k_0=\lfloor\frac{n}{2}\rfloor$ and $k_0\le k\le n$ and if $1\le\alpha$,

$$d_z(r)=\begin{cases}d_{n,2n}(r)+n^2(\alpha-1),\ 0\le r\le n;\\ (\alpha-1)d_{n,n}(\frac{r-n}{\alpha-1}),\ n\le r\le n\alpha.\end{cases}\quad (15)$$

Figure 2 illustrates that, indeed the IML decoder can achieve the optimal DMT (with F-CSIT) of the MIMO ZIC on a region of low multiplexing gains (LMG), whereas comparing equations (14) to (15) we see that this is true at high MG values also for $\alpha\ge 1$. Next, we formally define this LMG region.

Definition 1: Consider a MIMO ZIC of Figure 1 with a diversity order of $d^*_{ZC}(r_1,r_2)$ at MG tuple (r_1,r_2). The MG tuple (r_1,r_2) is said to lie in the LMG region if $d^{MAC}_{(M_1,M_2,N_1)}(r_1,r_2,\bar{\alpha})=d^*_{ZC}(r_1,r_2)$, where $d^{MAC}_{(M_1,M_2,N_1)}(r_1,r_2,\bar{\alpha})$ represents the optimal DMT of a 2-user MAC with the common receiver having N_1 antennas and the two users having M_1 and M_2 antennas.

Example 1: From this definition, for $n=1$ and $\alpha_{ij}=1,\ i,j\in\{1,2\}$, we have $\mathcal{MG}_L=\{(r_1,r_2):((1-r_1)\wedge(1-r_2))\le 2(1-r_1-r_2)\}=\{(r_1,r_2):(r_1\vee r_2)+2(r_1\wedge r_2)\le 1\}$. Note that this region is same as that of Definition 3 of [11], where this region was defined for a SISO IC. However, our analysis in this paper is much more general since we consider MIMO case and SNR$\neq$INR.

Motivated by the femto-cell (FC) model, in [1] the optimal DMT of the SISO ZIC was derived, we now consider the corresponding MIMO case.

Lemma 5: Consider the ZIC, as shown in Figure 1 with, $M_1=M_2=N_1=N_2=n$ and $\alpha_{22}=\alpha$ and $\alpha_{11}=\alpha_{21}=1$. The optimal DMT of this channel, with F-CSIT (and short term average power constraint, (1)), at a MG tuple (r_1,r_2), is given by

$$d^*_{ZC_2}(r_1,r_2)=\min\{d_{n,n}(r_1),d_{n,n}(r_2),d_s(r_1+r_2)\}$$

where if $\alpha\le 1$,

$$d_s(r)=\begin{cases}d_{n,3n}(r)+n(n-k)(\alpha-1)+n(r-k-\alpha)^{+},\\ \quad\text{for } k\le r\le(k+1)\text{ and }0\le k\le(n-1).\end{cases}$$

and if $1\le\alpha$,

$$d_s(r)=\begin{cases}d_{n,3n}(r)+n^2(\alpha-1),\ \text{for } 0\le r\le n;\\ (\alpha-1)d_{n,n}(\frac{(r-n)}{(\alpha-1)}),\ \text{for } n\le r\le n\alpha.\end{cases}$$

Remark 4: Note that the fundamental DMT of the ZIC with single antenna nodes and $\alpha_{22}=\alpha,\alpha_{11}=\alpha_{21}=1$ was derived in [1]. This clearly is a special case of Lemma 5 and can be obtained by putting $n=1$.

Since in general, the BS can host more antennas than the user sets, in what follows, we consider a case where $M_1=M_2=M\le N_1\le N_2$. Further, to maintain simplicity, here we will consider only the case of $\alpha_{11}=\alpha_{22}=1\le\alpha_{21}$ (the general case will be reported in [10]).

Lemma 6: Consider the ZIC, as shown in Figure 1 with, $M_1=M_2=M\le N_1\le N_2$, $\alpha_{11}=\alpha_{22}=1$ and $\alpha_{21}=\alpha\ge 1$. The optimal DMT of this channel with F-CSIT (and short term average power constraint (1)), at multiplexing gain pair (r_1,r_2), is given by

$$d^*_{ZC_3}(r_1,r_2)=\min\{d_{M,N_1}(r_1),d_{M,N_2}(r_2),d_{ZC_s}(r_1+r_2)\}$$

where for $k \in \{0, 1 \cdots (M-1)\}$

$$d_{ZC_s}(r_s) = \begin{cases} \alpha d_{M,(M+N_1+N_2)}(\frac{r_s}{\alpha}) + (M+N_2)((r_s - k\alpha - 1)^+ \\ +(M-k)(1-\alpha)) + M(N_1 - M), \ \forall r_s \in [k\alpha, (k+1)\alpha]; \\ d_{M,(N_1-M)}(r_s - M\alpha), \ \forall r_s \in [M\alpha, M(\alpha-1)+N_1]. \end{cases}$$

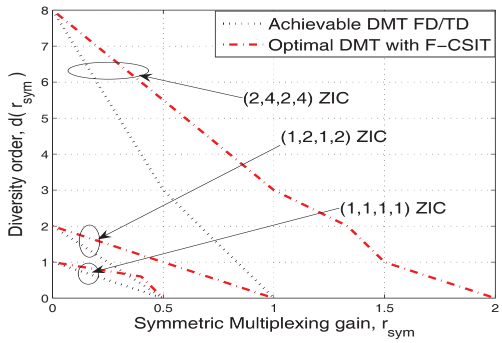

Fig. 3: Optimal DMT of different ZICs with $\bar{\alpha} = [1, 1, 1]$.

In Figure 3, explicit DMT curves of the MIMO ZIC for a few antenna configurations are plotted and compared against the performance of orthogonal schemes such as frequency division (FD) or time division (TD) multiple-access. It can be noticed from the figure that the gain of interference management technique over the orthogonal access schemes can be significant, particularly in MIMO ZICs. These orthogonal schemes (FD/TD) are the simplest coding schemes with No-CSIT. In the following, we will show that much better performance (than the FD/TD schemes) can be achieved by other No-CSIT schemes.

Lemma 7: Consider the MIMO ZIC as in Lemma 6. The DMT achieved by the IML decoder on this channel with No-CSIT, at multiplexing gain pair (r_1, r_2), is given by

$$d^{IML}_{(M,M,N_1)}(r_1, r_2) = \min\{d_{M,N_1}(r_1), d_{M,N_2}(r_2), d_s(r_1 + r_2)\}$$

where for $k \in \{0, 1 \cdots (M-1)\}$

$$d_s(r_s) = \begin{cases} \alpha d_{M,(M+N_1)}(\frac{r_s}{\alpha}) + M((r_s - k\alpha - 1)^+ \\ +(M-k)(1-\alpha)) + M(N_1 - M), \ \forall r_s \in [k\alpha, (k+1)\alpha]; \\ d_{M,(N_1-M)}(r_s - M\alpha), \ \forall r_s \in [M\alpha, M(\alpha-1)+N_1]. \end{cases}$$

The explicit DMT of the (M, N_1, M, N_2) IC under the symmetric MG requirement i.e., $r_1 = r_2 = r_{sym}$, is plotted in Figure 4. Comparing the performance improvement of the IML decoder on the $(2, 4, 2, 4)$ ZIC with respect to that on the $(2, 2, 2, 4)$ ZIC, we realize that a larger number of antennas at the interfered receiver can completely compensate for the lack of CSIT. Another important fact brought out by this figure is that on some ZICs the optimal DMT with F-CSIT is identical to that of the corresponding ZIC with No-CSIT. In what follows, we characterize such a class of ZICs.

Corollary 1: The optimal DMT of a MIMO ZIC as considered in Lemma 6, with F-CSIT and symmetric MG requirement $r_1 = r_2 = r_{sym}$, is identical to that of a corresponding ZIC with No-CSIT. The explicit expression for the DMT can be obtained by putting $r_1 = r_2$ in Lemma 7.

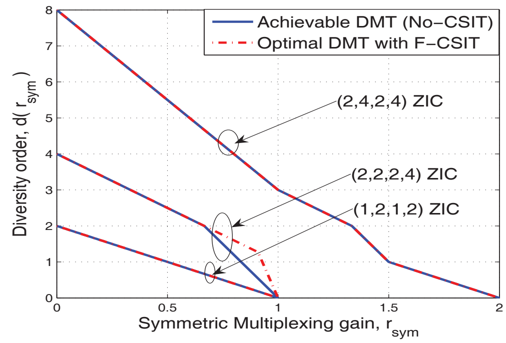

Fig. 4: Optimal DMT of different ZICs with $\bar{\alpha} = [1, 1, 1]$.

IV. Conclusion

The DMT of the MIMO ZIC with CSIT is characterized. Besides serving as an upper bound to the DMT of the 2-user MIMO IC, it reveals several insights about this channel. It is found that the gain of interference management schemes over that of orthogonal schemes, such as TDMA/FDMA, is much more on a MIMO system than that achievable on a SISO channel. It is also illustrated that a larger number of antennas at the interfered receiver can completely compensate for the lack of CSIT on a ZIC. A class of ZICs is found for which it was proved that availability of the CSIT does not improve the DMT performance of the system.

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