

System Quality Factors for LPI Communications

Lawrence L. Gutman
Wright Research and Development Center
Wright Patterson AFB, OH

Glenn E. Prescott
Air Force Institute of Technology
Wright-Patterson AFB, OH

Abstract

Quality factors are developed for a Low Probability of Intercept (LPI) communications system in order to provide a quantitative technique which allows the system engineer to evaluate LPI effectiveness in the presence of jammers and intercept receivers. LPI quality factors are derived from the system link equations which describe the signal power gains and losses as a function of system link parameters. In this paper, quality factors are described for the essential components of the LPI system which provide some advantage to the cooperative transmitter and receiver over the jammer and intercept receiver. These include quality factors for the antennas, type of modulation, atmospheric propagation conditions, and interference rejection capability.

Introduction

A typical LPI scenario is illustrated in Figure 1, representing any situation in which a cooperative transmitter and receiver are targeted by both jammers - which disrupt the communications receiver; and intercept receivers - which attempt to detect and exploit the transmitted signal. Since all players are likely to be present in any realistic situation, both the communications receiver and the intercept receiver must be able to function in the presence of jamming.

The purpose of any LPI communication system is to successfully conduct communications between a cooperative transmitter and receiver in such a manner as to minimize the probability of interception by an unauthorized receiver. The communications system is assumed to have a variety of techniques available for reducing the probability of interception. For example, the transmitter may employ steerable high gain antennas and emit a signal with low power density and large time bandwidth product. The communications receiver may have null steering antennas, adaptive interference suppression filters, and employ coherent processing. On the other hand, the intercept receiver has similar available technologies - steerable antennas with low side lobes for eliminating inadvertent (or intentional) jamming, and adaptive filters for excising narrowband interference, for example.

The principle players in the LPI scenario each have critical characteristics which can be easily evaluated and compared. For example, the communications transmitter is characterized by its transmission power, type of modulation and antenna characteristics. The communications receiver characteristics are defined based on some minimum received bit energy per noise power density ratio, E_b/N_o at the receiver input required to provide some acceptable bit error performance, P_e . Antenna gain characteristics, receiver bandwidth and noise figure are also important parameters to be considered.

The adversaries to the transmitter and receiver are the interceptor and jammer, respectively. The intercept receiver is typically a non-coherent energy detector whose performance is established by its probability of detection, P_d and probability of false alarm, P_{fa} . Its performance will also be influenced by intercept antenna characteristics, intercept receiver bandwidth and noise figure. The jammer targets the receiver, therefore the essential parameter in this case is the jammer power spectral density - that is, the amount of power that can be distributed over the operating frequency range of the receiver. Therefore, both power and antenna gain play a major role in determining jammer effectiveness.

We can analyze the relationships among these players and reveal potential trade-offs that may exist by performing a simple link analysis. The link analysis reveals strengths and vulnerabilities, and provides the system designer the insight to determine how to most effectively concentrate system resources.

The Communications Link

We begin by determining the signal power available at the communications receiver. We naturally assume that there is some performance requirement imposed upon the communications receiver, expressed in terms of bit error probability. This requirement will dictate some minimum required signal energy necessary to provide the specified performance level. The signal power available at the detector input of the communications receiver, S_c can be expressed in decibels as:

$$S_c = \frac{P_t G_{tc} G_{ct}}{(4\pi R_c / \lambda)^2 L_c} \quad (1)$$

where in the numerator,

- P_t - transmitter power
- G_{tc} - gain of transmitter antenna in direction of comm receiver
- G_{ct} - gain of comm receiver in direction of transmitter

The denominator terms represent losses, where $(4\pi R_c / \lambda)^2$ is free space propagation loss and L_c accounts for atmospheric losses due to rain, water vapor and oxygen absorption. Expressing L_c in terms of propagation path length and path attenuation,

$$L_c = \log^{-1} \left(\frac{1}{10} \xi_c R_c \right) \quad (2)$$

This loss is assumed to be linear as a function of range so that ξ_c is a generalized average loss factor expressed in units of dB/Km, for example. Such loss factors are not truly linear since the transmission medium is not homogeneous and will vary with season, temperature, time of day and altitude. However, the cumulative "average" effect is represented here by ξ_c .

To establish the pre-detection signal-to-noise ratio at the receiver input, the noise power density, N_{oc} at the communications receiver can be expressed as

$$N_{oc} = kT_{ac} + T_o(F_c - 1) + \sum_{n=1}^N \sum_{m=1}^M g_{cn} g_{cm} \frac{J_{nmc}}{B_c} \quad (3)$$

where

- k - Boltzman's constant
- B_c - Bandwidth of comm receiver
- T_{ac} - Temperature of comm antenna
- T_o - Thermal noise (290° K)
- F_c - Noise figure of comm receiver

The first term represents noise at the antenna output, while the second term indicates the receiver sensitivity as expressed by the noise figure. The double summation component accounts for the power spectral density in each discrete frequency component transmitted by each jammer through the parameter J_{nmc} . The subscript n represents the jammer from the n th direction of arrival. The communication receiver is assumed to be jammed by N jammers from N different directions (angles-of-arrival). The subscript m represents the m th frequency component from the n th jammer. Therefore, m is a frequency domain parameter which represents a spectral interfering tone much narrower in bandwidth than the desired signal.

The effects of the N jammers can be countered by null steering antennas (represented here by g_{cn}), and the M interfering tones can be eliminated by adaptive interference suppression techniques (represented here by g_{cm}). Both of these gain factors have the effect of reducing the level of the jammer by some factor. For example, with a 20 dB

null in the direction of the n th jammer, $g_{cn} = .01$. In a dense jamming environment N_{oc} will be dominated by the double summation component.

A similar development can be made for the link between the transmitter and the intercept receiver.

The Interceptor Link

The carrier power available to the intercept receiver is:

$$S_i = \frac{P_t G_{ti} G_{it}}{(4\pi R_i / \lambda)^2 L_i} \quad (4)$$

where R_i is the transmitter to interceptor path length and L_i accounts for the losses (other than free space loss) along that path. Also,

$$L_i = \log^{-1} \left(\frac{1}{10} \xi R_i \right) \quad (5)$$

G_{it} and G_{ti} account for the gain of the transmitter antenna in the interceptor direction, and the gain of the interceptor receiver in the transmitter direction, respectively.

The noise at the intercept receiver can be expressed as,

$$N_{oi} = kT_{ai} + T_o(F_i - 1) + \sum_{n=1}^N \sum_{m=1}^M g_{in} g_{im} \frac{J_{nmi}}{B_i} \quad (6)$$

The antenna noise and receiver sensitivity are represented here as are the interfering effects of jammers. To improve the effectiveness of the interceptor in a dense jamming environment, it is likely that null steering antennas and adaptive interference suppression filters will be employed. Therefore, the intercept receiver can provide both spatial and spectral discrimination to gain an advantage over the LPI system.

LPI System Quality Factors

The relative merit of the LPI system with respect to the interceptor can be observed by comparing the received signal-to-noise ratios at the communications receiver and the intercept receiver [1, 2]. S_c/N_{oc} is the signal-to-noise density ratio required to achieve some minimum error performance at the communications receiver, while S_i/N_{oi} is the signal-to-noise density ratio available to the intercept receiver. Taking the ratio of these,

$$\frac{S_c/N_{oc}}{S_i/N_{oi}} = \frac{P_t G_{ct} G_{tc} (4\pi R_i / \lambda)^2 L_i N_i}{P_t G_{ti} G_{it} (4\pi R_c / \lambda)^2 L_c N_c} \quad (7)$$

Cancelling common terms and rearranging

$$\left(\frac{R_c}{R_i} \right)^2 = \frac{G_{tc} G_{ct}}{G_{ti} G_{it}} \cdot \frac{L_i}{L_c} \cdot \frac{N_i}{N_c} \cdot \frac{S_i/N_{oi}}{S_c/N_{oc}} \quad (8)$$

Where we can define the overall LPI quality factor in terms of R_c and R_i

$$Q_{LPI} = 20 \log \left(\frac{R_c}{R_i} \right) \quad (9)$$

which indicates that any improvement in LPI effectiveness will either allow the communications system to operate over a longer range, or will require the intercept receiver to move closer to the transmitter in order to achieve some specified level of performance.

We have expressed the link equations, (1 - 4) in terms of groups of parameters which can be identified as quality factors relating to the antenna, path loss, modulation, and interference as follows:

$$Q_{ANT} = 10 \log \left(\frac{G_{tc} G_{ct}}{G_{ti} G_{it}} \right) \quad (10)$$

$$Q_{ATM} = \xi_i R_i - \xi_c R_c \quad (11)$$

$$Q_{MOD} = 10 \log \left(\frac{S_i / N_{oi}}{S_c / N_{oc}} \right) \quad (12)$$

$$Q_{ADA} = 10 \log \left\{ \frac{k T_{ai} + T_o (F_i - 1) + \sum_{n=1}^N \sum_{m=1}^M g_{in} g_{im} J_{nmi} / B_i}{k T_{ac} + T_o (F_c - 1) + \sum_{n=1}^N \sum_{m=1}^M g_{cn} g_{cm} J_{nmc} / B_c} \right\} \quad (13)$$

The antenna quality factor, Q_{ANT} , accounts for the advantage provided by steerable narrow beam antennas at the transmitter, receiver, and intercept receiver. Therefore, G_{tc} , G_{ct} and G_{it} are designed to be large. On the other hand, the transmitter needs the transmitter sidelobe gain, G_{ti} to be as small as possible. Therefore, high gain, low sidelobe antennas at the communications transmitter and receiver keep the antenna quality factor large.

The LPI system has no control over G_{it} , which is the intercept antenna gain. If G_{it} is large, the antenna quality factor will suffer. However, as the intercept receiver antenna gain increases, the beamwidth gets narrower and eventually the probability of intercept diminishes significantly. The intercept is now required to scan so slowly, in order to integrate enough energy in each of the directions of arrival, that a short transmitted pulse is likely to be missed. So there is obviously a tradeoff in the amount of antenna gain the intercept can effectively use with the time required to dwell on each look angle and the number of look angles to search.

The atmospheric quality factor, Q_{ATM} is a function only of atmospheric effects and range. In most cases $\xi_i \approx \xi_c$, since the intercept and the communications receiver are usually located close enough to one another that they operate under common atmospheric conditions.

The modulation quality factor, Q_{MOD} contains the detectability parameters which describe the quality of communications for some acceptable bit error rate P_e , and the quality of interception for some acceptable probability of detection, P_d and probability of false alarm, P_{fa} . Only the parameters of the signal and the method used to detect the signal are important. Any factors causing the intercept receiver to require a larger signal to noise ratio to achieve a specified performance level increases the modulation qual-

ity factor. The modulation quality factor for a radiometer (non-coherent energy detector) has been shown to be [3]:

$$Q_{MOD} = \frac{N_{oc}}{E_b B_d} \sqrt{\frac{X_o B_i}{T_i}} \rho_T \rho_B \quad (14)$$

where

$$\begin{aligned} X_o &<< 16 T_i B_i \\ B_i &- \text{intercept receiver bandwidth} \\ T_i &- \text{interceptor integration time} \\ \rho_T &- \text{reciprocal of signal duty cycle} \\ \rho_B &- \text{maximum of } [B_i, B_c] \div B_i \end{aligned}$$

and,

$$X_o = 2 [\sigma f c^{-1}(2 P_{fa}) - \sigma f c^{-1}(2 P_d)]^2 \quad (15)$$

where X_o is the post detection signal-to-noise ratio, and

$$\frac{E_b}{N_{oc}} = [\sigma f c^{-1}(2 P_e)]^2 \quad (16)$$

for binary PSK modulation, Substituting (15) and (16) into (14) we obtain the following expression for Q_{MOD} :

$$Q_{MOD} = \frac{\sigma f c^{-1}(2 P_{fa}) - \sigma f c^{-1}(2 P_d)}{[\sigma f c^{-1}(2 P_e)]^2} \sqrt{\frac{2 B_i}{T_i}} \rho_T \rho_B \quad (17)$$

and $\sigma f c^{-1}$ is the inverse of the complementary error function as defined in Reference [3].

The adaptive technologies quality factor, Q_{ADA} compares the ability of both the communications and intercept receivers to adaptively filter interference. When the adaptive processes are successful in eliminating interference, the adaptive technologies quality factor is dominated by the respective receiver noise figures and antenna noise temperatures. Any factors causing the intercept receiver have more noise or interference, such as large noise figure or inability to null jammers, will cause Q_{ADA} to be large. On the other hand, if the intercept can successfully null a jammer, the gain factor g_{in} becomes less than unity, reducing the received jammer power and reducing the quality factor.

Likewise, the communications receiver's ability to null out and adaptively filter interference provides an added advantage over the intercept receiver in the presence of jamming, thus increasing the quality factor.

When all quality factor terms are taken together, the LPI quality factor, Q_{LPI} can be expressed as,

$$Q_{LPI} = 20 \log \left(\frac{R_c}{R_i} \right) = Q_{MOD} + Q_{ANT} + Q_{ADA} + Q_{ATM} \quad (18)$$

where we can define the first three terms as system dependent quality factors unrelated to atmospheric conditions:

$$Q_S = Q_{MOD} + Q_{ANT} + Q_{ADA} \quad (19)$$

where Q_S is the clear atmosphere system quality factor. When the atmosphere quality factor, Q_{ATM} is added, the change in Q_{LPI} due to atmospheric effects can be observed.

Combining (11), (18) and (19) with the assumption that $\xi_i = \xi_c = \xi$, we have

$$Q_s = 20 \log \left(\frac{R_c}{R_i} \right) + \xi R_c \left(1 - \frac{R_i}{R_c} \right) \quad (20)$$

as shown in Figure 2.

Conclusions

The object of defining quality factors for the LPI communications system is to give a physical interpretation to a grouping of terms obtained from the system link equations. These quality factors are useful for revealing features which allow a cooperative transmitter and receiver to conduct communications while interception is attempted by unauthorized users.

References

1. J. D. Edell, "Wideband, Non-coherent, Frequency-Hopped Waveforms and Their Hybrids in Low-Probability of Intercept Communications," Naval Research Laboratory Report 8025, November 8, 1976.
2. Paul Crepeau, "LPI and AJ Modulation Quality Factors," Naval Research Laboratory Report 3436, January 1977.
3. H. F. Engler, Jr., and D. H. Howard, "A Compendium of Analytic Models for Coherent and Non-coherent Receivers," Air Force Wright Aeronautical Laboratories Technical Report, AFWAL-TR-85-1118, September 1985.

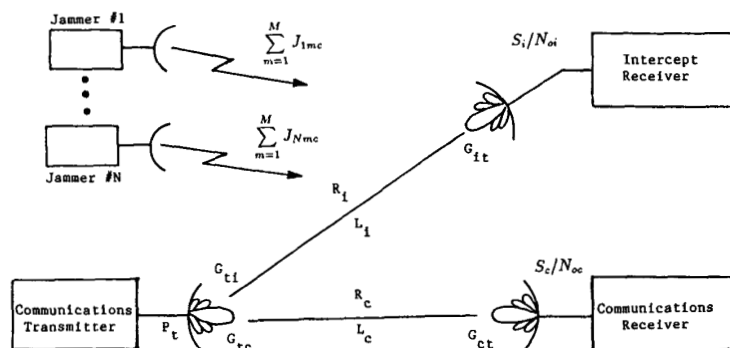


Figure 1 - LPI Scenario

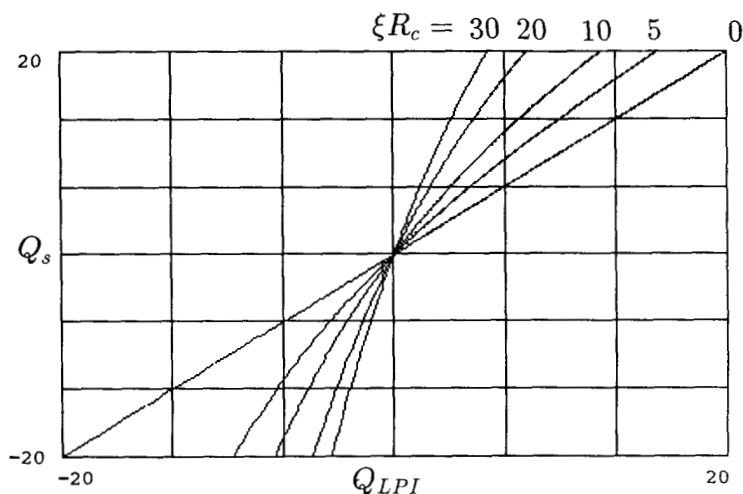


Figure 2 - LPI Quality Factor and System Quality Factor as a Function of Path Loss