

Modeling and Control for a 6-DOF Platform Manipulator

Marwa Jouini, Mohamed Sassi, Neji Amara and Anis Sellami

Research unit C3S, ESSTT, University of Tunis, 5 av. Taha Hussein BP 56 – 1008 Tunis, Tunisia
marwajouini7@gmail.com ; mohamed.sassi@esstt.rnu.tn ; amaraneji@gmail.com ; Anis.Sellami@esstt.rnu.tn

Abstract—This paper deals with the modeling and the control of a parallel robot with six degree of freedom (dof). The mathematical model of the 6-DOF parallel manipulator includes dynamics model which is on the Lagrange method. The model is built in generalized coordinate system. The kinematics model is based on the closed-form solutions. The latter has six electric actuators at six legs. The model-based controller is presented with feedback of platform positions. Two control laws of the actuators positions of the robot are proposed: PID control and Sliding Mode Control (SMC). Simulation results are given to show the comparison performance in term of robustness.

Keywords: Parallel manipulator, Model kinematics, Model dynamics, Platform, Sliding mode control, PID control.

I. INTRODUCTION

GENERALLY, the parallel link manipulators provide better accuracy, higher rigidity, higher load-to-weight ratio, and more uniform load distribution than the serial manipulators. Parallel robot adaptation in various fields has given rise to different geometries of robots, with 3, 4, 5 or 6 degrees of freedom. In the context of this paper, we will cover a particular interest in parallel robots hexapods [1] (composed of 6 identical kinematic chains connecting a base platform), in particular, robots with electric actuators that drive the hexapod robot. The Stewart platform manipulator is a 6-DOF mechanism with two bodies connected together by six extensible legs [2], [1]. The equations, forward kinematics and dynamics of parallel manipulators are very complicated and difficult to solve. In recent years, many research works have been conducted on the dynamics and kinematics of the Gough-Stewart platform manipulator. Geng and al. [3] developed Lagrange equations of motion, regarding the geometry and inertia distribution of the manipulator. The Lagrange formulation is well structured and can be expressed in closed form, but a large amount of symbolic computation is needed to find partial derivatives of the Lagrangian in this method. The forward kinematics and inverse kinematics models are described with closed-form solution and Newton-Raphson method. The proper coordination of the actuators length enables the top plate to follow the desired trajectory with high accuracy.

Recent researches have been focused on the control of parallel

manipulator. Chifu Yang [4] developed in 2008 a PID controller with gravity compensation with the feedback of cylinder length of platform. Lee and Kim [5] presented a model based on sliding mode control for the Stewart platform. In 2006, Iqbal and Bhatti [6] developed a control design for tracking and regulation of a robot platform without any knowledge of the system's mass properties in presence of nonlinearities.

The novelties in this paper is modeling of the robot, and make a comparative study between PID control and sliding mode control of the actuators positions of robot platform.

This paper is structured as follow. We first define the studied system. Afterward, kinematics and dynamics are explained in section II. Then, section III, deals with the PID control and the sliding mode control of the six electric actuators of the manipulator. Simulation results are discussed in section IV and finally some comments conclude the work in section V.

II. SYSTEM MODELING

The 6 - DOF electric driven parallel manipulator composed by a fixed base (down platform), a moveable platform (upper platform) and six legs stretch, as shown in figure 1.

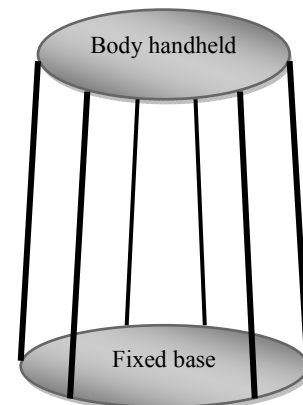


Fig.1: 6-DOF parallel manipulator.

A. Kinematic Model

Kinematics is the science of motion that treats the subject without regard to the forces that cause it [7].

The kinematics of 6-DOF Stewart-Gough platform mechanism include inverse kinematics and forward kinematics.

The length of each actuator of Stewart platform for a given orientation can be determined using inverse kinematics and can be written as:

$$\frac{d\rho_i}{dt} = J^{-1} \frac{dq}{dt} \quad (1)$$

Where $\frac{d\rho_i}{dt} \in R^{6 \times 1}$ is the vector of joint velocity of the platform, $q = [x \ y \ z \ \Phi \ \theta \ \Psi]^T$ is the generalized coordinates vector of the platform, x, y, z , are the platform centre of mass Cartesian coordinates, Φ, θ, Ψ , are the platform Euler angles and \dot{q} is the $R^{6 \times 1}$ vector of the operating speed, $i=1..6$ is the iterative number, J^{-1} is the $R^{6 \times 6}$ transition matrix for speeds of operational space to the joint space (inverse Jacobian).

Inverse kinematics of parallel manipulator is different from serial manipulator. The length of leg of platform can be solved by closed-form solution, and it can be described as:

$$\rho_i = \|\overline{A_i B_i}\| = \|\overline{A_i O} + \overline{OC} + P \cdot \overline{CB_i}\| \quad (2)$$

Where ρ_i is the length of leg of platform, P is matrix of transformation from body coordinates to global coordinates, A_i and B_i are respectively the connection points of the jacks in the base and the mobile platform, O is the origin of the absolute coordinate system and C is the center linked to the moving frame movable platform.

B. Dynamic Model

The dynamic equation of the platform considers inertia and Coriolis. Lebret in [8] developed the dynamic equation using Lagrange method as:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau \quad (3)$$

Where $M \in R^{6 \times 6}$ is an inertial matrix, $G \in R^{6 \times 1}$ is the vector of the gravity terms, $C \in R^{6 \times 6}$ is the Coriolis /centripetal matrix, and $\tau \in R^{6 \times 1}$ is the applied torque vector. Some relevant properties are given as below:

Property1:

M is a symmetric and positive definite matrix for all $q \in R$.

Property 2:

q and \dot{q} are bounded.

C. Actuators model

The robot controller is based on the parallel electric actuators. The state space of the robot actuator platform as follows:

$$\begin{cases} K_t I(t) = J \dot{\Omega}(t) \\ U(t) = K_e \Omega(t) + (RI(t) + L\dot{I}(t)) \end{cases} \quad (4)$$

The actuators are represented by DC motors, where U the control is input voltage, I is the current, L is the inductance of winding, R is the resistance of the armature, J is the inertia of motor, K_e, K_t are the coefficients of speed and torque, and $\Omega(t) = \dot{\theta}(t)$ is the rotational speed of the motor shaft.

III. CONTROL DESIGN

The purpose of the control structure is to calculate the length of six legs for each position of the actuator of the robot. The overall model of the system is shown in figure 2:

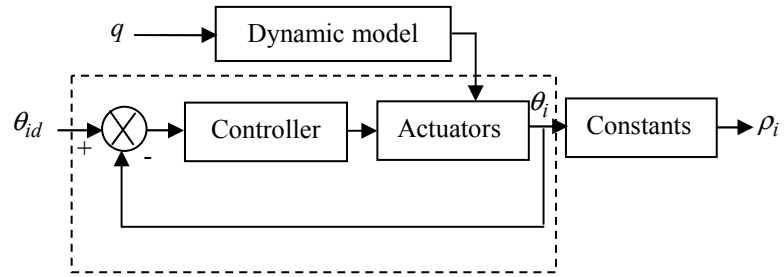


Fig.2: Global model.

A. PID control

In order to suitably settle the PID controller, first we need to calculate the transfer function of the closed loop servo loop, the block diagram used is as follows:

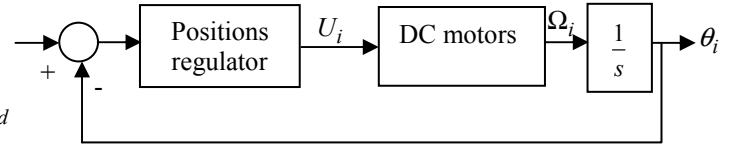


Fig.3: Block diagram of the servo loop.

The transfer function of the PID is written in the form:

$$H_{PID}(s) = P + \frac{K_i}{s} + K_d s = \frac{K_i + Ps + K_d s^2}{s} \quad (5)$$

From the transfer function of the controller and the process, the transfer function of a closed loop in canonical form is the following:

$$H(s) = \frac{\frac{K_i K}{T} + \frac{PK}{T}s + \frac{K_d K}{T}s^2}{\frac{KK_i}{T} + \frac{KP}{T}s + \frac{(KK_d + 1)}{T}s^2 + s^3} \quad (6)$$

Where $k = \frac{1}{k_e}$ and $T = \frac{RJ}{k_e k_t}$.

The denominator of the equation (6) can be written in the form:

$$D(s) = (s + w_n)(s^2 + 2\xi w_n s + w_n^2) \quad (7)$$

Where ξ and w_n are positive constants.

PID controller parameters are attained as follows:

$$\begin{cases} P = \frac{T w_n^2 (1 + 2\xi)}{K} \\ K_i = \frac{T w_n^3}{K} \\ K_d = \frac{T w_n (1 + 2\xi) - 1}{K} \end{cases} \quad (8)$$

B. Sliding mode control

The main objective of the design approach of sliding mode control is to force the error of the robot actuator positions and its derivative to zero. The switching surface design comprises the construction of the sliding function. Positions of errors and derivatives are the coordinates in the selected robot platform.

The robot actuator position errors are introduced by:

$$e = \theta_{id} - \theta_i \quad (9)$$

Where $\theta_i \in \mathbb{R}^{6 \times 1}$ is the position vector and $\theta_{id} \in \mathbb{R}^{6 \times 1}$ is the desired positions vector.

Linear sliding surfaces $S \in \mathbb{R}^{6 \times 1}$ are introduced in terms of errors and its derivative positions are given below:

$$S = \lambda e + \dot{e} \quad (10)$$

Where $\lambda \in \mathbb{R}^{6 \times 6}$ is a diagonal strictly positive matrix. Equation (10) is characterized by deviations from the desired state. The sliding surfaces are achieved and maintained when $S = 0$.

The simple input control to reach $S = 0$ through a control equivalent, is given by:

$$\begin{cases} U_{eq} = -1 & \text{si } S < 0 \\ U_{eq} = +1 & \text{si } S > 0 \end{cases} \quad (11)$$

Where U_{eq} is the equivalent control signal system, to compensate the system dynamics.

The complete control circuit block diagram of these actuators of manipulator is shown in figure 4. Here U is adjusted to positive and negative voltages for -1 and +1 values.

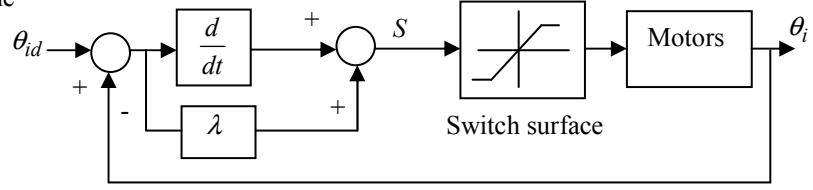


Fig.4: Block diagram of sliding mode.

IV. SIMULATION RESULTS

Simulation has been performed in-order to examine the effectiveness of the proposed controller design. The robot platform has six legs, six motors containing six regulators. The simulation parameters for the 6-DOF electric Stewart platform are given with the following nominal parameters: mass of platform $m=1\text{kg}$, $R=1.05\Omega$, $L=0.5\text{mH}$, $I=17.2\text{A}$

$$J=1.6\text{kg.m}^2, k_e=1\text{Vrad}^{-1}\text{s}, U=9\text{V}, k_t=1.77\text{N.m.A}^{-1}$$

$$\text{and } q = (0.9\text{m}, 0.9\text{m}, 0.5\text{m}, 0.3\text{rad}, 0.2\text{rad}, 0.3\text{rad})^t.$$

The proposed control procedure is simulated using the Matlab/Simulink environment. Then to test the robustness of the proposed controls, we study the influence of parameter variations on the performance of the position settings. The parameters are varied simultaneously at the motor start. The value of the inertia of the motor is thus multiplied by 0.5, the resistance of the armature is multiplied by 2 and the inductance of winding is multiplied by 3.

A. Simulation with the PID controller

The simulation result of the PID control is shown in figure 5. The figure shows the evolution of the real position with respect to the desired position in the absence of parameters variation.

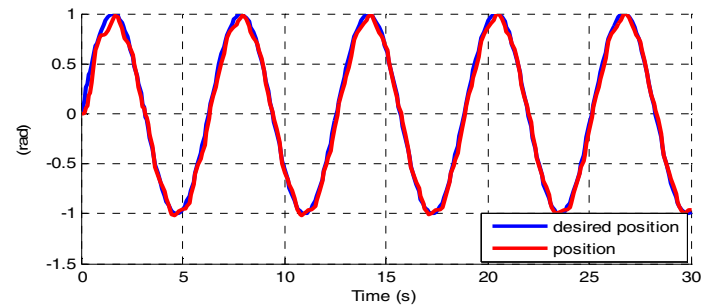


Fig.5: Position of one actuator.

B. Simulation with the sliding mode controller

The simulation result of the sliding mode control is shown in figure 6. The figure shows the evolution of the real position with respect to the desired position in the absence of parameters variation.

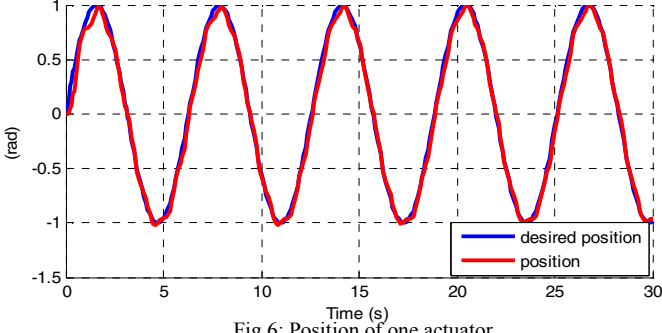


Fig.6: Position of one actuator.

C. Robustness test

The comparison between the PID control and the sliding mode control is introduced in the following figure in presence of parameters variation:

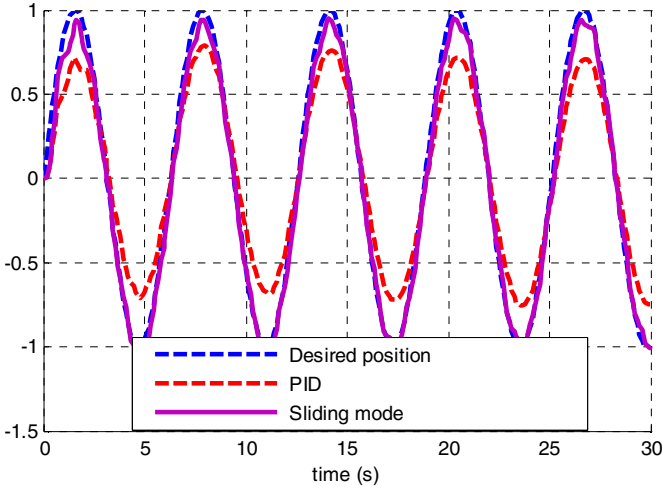


Fig.7: Comparison between the PID and sliding mode.

Following these simulations, the PID controller and the sliding mode controller are compared. These two methods gives us good results. In fact, the six actuator positions of the robot platform towards the six desired values.

The main difference between both methods is in the robustness. In fact, the variation of the parameters of the actuator (DC motor) shows that the PID controller is not robust to parameter uncertainties unlike the sliding mode controller, which guarantees good results. Then we can conclude that the sliding mode controller is better and more robust than the PID controller.

D. Simulation of the global model

After the test of robustness we note that the sliding mode control is robust. The simulation of the global model of the manipulator platform is presented in the following figures, which confirms the stability of the six lengths of the robot legs ρ_i .

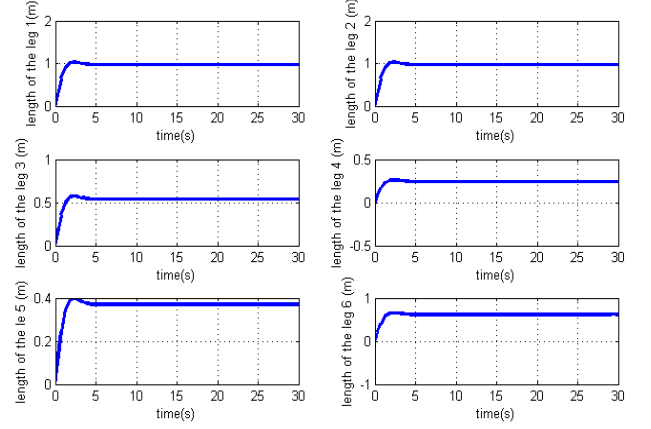


Fig.8: Lengths of the six legs for the sliding mode control.

From these figures we note that sliding mode control ensures the stability of each length of the platform after a certain time.

V. CONCLUSION

The robot platform system studied in this paper is a manipulator with a closed kinematic chain. The latter is more complicated for control and modeling. We perform a performance study between PID control and sliding mode control of robot platform actuators. Simulation results show the stability of the system. In meantime, the sliding mode control gives high robustness performance in comparison with the PID control.

APPENDIX

Here we explore each component of the Stewart platform dynamic equation. The inertial matrix M can be written as:

$$M = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{11}I_1 & P_{12}I_2 & P_{13}I_3 \\ 0 & 0 & 0 & P_{21}I_1 & P_{22}I_2 & P_{23}I_3 \\ 0 & 0 & 0 & P_{31}I_1 & P_{32}I_2 & P_{33}I_3 \end{bmatrix}$$

Where

$$P \text{ is the transition matrix, } P = R(y_r, \theta).R(x_r, \Phi).R(z_r, \Psi)$$

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} = \begin{bmatrix} \cos \theta \cos \psi - \sin \Phi \sin \theta \sin \psi & & \\ & -\cos \Phi \sin \psi & \\ \sin \theta \cos \psi + \sin \Phi \cos \theta \sin \psi & & \end{bmatrix}$$

$$\begin{bmatrix} \cos \theta \sin \psi + \sin \Phi \sin \theta \cos \psi & -\cos \Phi \sin \theta \\ \cos \Phi \cos \psi & \sin \Phi \\ \sin \theta \sin \psi - \sin \Phi \cos \theta \cos \psi & \cos \Phi \cos \theta \end{bmatrix}$$

I a diagonal matrix of inertia, $I = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}$

The Coriolis and Centrifugal matrix C can be written as:

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{11}I_1 & Q_{12}I_2 & Q_{13}I_3 \\ 0 & 0 & 0 & Q_{21}I_1 & Q_{22}I_2 & Q_{23}I_3 \\ 0 & 0 & 0 & Q_{31}I_1 & Q_{32}I_2 & Q_{33}I_3 \end{bmatrix}$$

Where

$$[\dot{P}] = [Q] = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{bmatrix}$$

Moreover, the Jacobean inverse matrix can be written as:

$$J^{-1} = \begin{bmatrix} \bar{U}_1^T & (\bar{U}_1 \wedge \bar{B}_1 \bar{C})^T \\ \bar{U}_2^T & (\bar{U}_2 \wedge \bar{B}_2 \bar{C})^T \\ \bar{U}_3^T & (\bar{U}_3 \wedge \bar{B}_3 \bar{C})^T \\ \bar{U}_4^T & (\bar{U}_4 \wedge \bar{B}_4 \bar{C})^T \\ \bar{U}_5^T & (\bar{U}_5 \wedge \bar{B}_5 \bar{C})^T \\ \bar{U}_6^T & (\bar{U}_6 \wedge \bar{B}_6 \bar{C})^T \end{bmatrix}$$

where

$$\bar{U}_i = \frac{\overline{A_i B_i}}{\rho_i}$$

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